HW2

Self-driving Car HW2 學號:309512074
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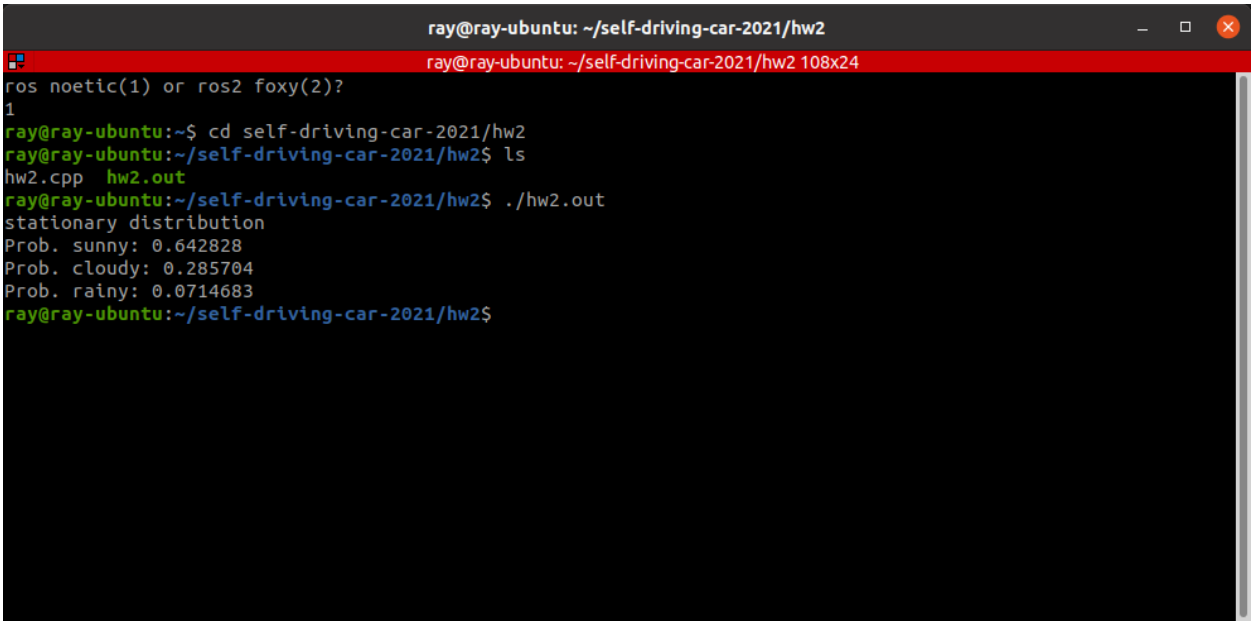
Exercise 2 (b)(c)

hw2.cpp

```cpp
#include <iostream>
using namespace std;

enum weather{sunny=0, cloudy, rainy};

weather nextweather(weather);
void distribution(void);

int main(){
    srand(time(NULL));
    distribution();
    return 0;
}

weather nextweather(weather today){
    double transition[3][3]={
        {0.8, 0.2, 0},
        {0.4, 0.4, 0.2},
        {0.2, 0.6, 0.2}
    };
    double num = (double)(rand()%100) / 100;
    if(num < transition[today][0]) return sunny;
    else if(num < transition[today][0]+transition[today][1]) return cloudy;
    else return rainy;

}

void distribution(void){
    //Run N times to find the weather of a random day
    int sunny_count=0, cloudy_count=0, rainy_count=0;
    int N = 10000, length=10000;
    for(int i=0; i< N; i++){
        weather w = (weather)(rand()%3);
        for(int j =0; j<length; j++){
            if((weather)w == sunny) sunny_count++;
            else if((weather)w == cloudy) cloudy_count++;
            else rainy_count++;
            w = nextweather(w);
        }
    }
    cout << "stationary distribution" << endl;
    cout << "Prob. " << "sunny: "<< ((double)sunny_count / (N* length)) << endl;
    cout << "Prob. " << "cloudy: "<< ((double)cloudy_count / (N* length)) << endl;
    cout << "Prob. " << "rainy: "<< ((double)rainy_count / (N* length)) << endl;
}
```

results



Exercise 3

3(a)

3.1

t	z_t
2	2
3	2
4	3
5	1

$$P(x_2=1|z_2) = \frac{1}{P(z_2)} \cdot P(z_2=2|x_2=1)\,P(x_2=1)$$

$$= \eta \cdot 0.4 \cdot 0.8 = \eta \cdot 0.32 = 0.70$$

$$P(x_2=2|z_2) = \eta \cdot P(z_2=2|x_2=2)\,P(x_2=2)$$

$$= \eta \cdot 0.7 \cdot 0.2 = \eta \cdot 0.14 = 0.3$$

$$P(x_2=3|z_2) = 0$$

$$0.46\eta = 1 \quad \eta = 2.17$$

$$P(x_3=1|z_{2:3}) = \eta \cdot P(z_3=2|x_3=1, z_2=2) \cdot P(x_3=1|z_2=2)$$

$$= \eta \cdot P(z_3=2|x_3=1) \cdot \sum_{i=1}^{3} P(x_3=1|x_2=i, z_2) \times P(x_2=i|z_2)$$

$$= \eta \cdot P(z_3=2|x_3=1) \cdot \sum_{i=1}^{3} P(x_3=1|x_2=i) \times P(x_2=i|z_2)$$

$$= \eta \times 0.4 \times (0.8 \cdot 0.7 + 0.4 \cdot 0.3 + 0) = 0.271\eta = 0.6$$

$$P(x_3=2|z_{2:3}) = \eta\, P(z_3=2|x_3=2) \times \sum_{i=1}^{3} P(x_3=2|x_2=i) \times P(x_2=i|z_2)$$

$$= \eta \cdot 0.7 \times 0.2609 = 0.183\eta = 0.4$$

$$P(x_3=3|z_{2:3}) = \eta\, P(z_3=2|x_3=3) \times P(x_3=3|z_2) = 0$$

$$\eta = \frac{1}{0.271 + 0.183} = \frac{1}{0.4539}$$

$$P(X_4=1|z_{2:4}) = \eta P(z_4=3|X_4=1,z_{2:3}) \cdot P(X_4=1|z_{2:3}) = 0$$

$$P(X_4=2|z_{2:4}) = \eta P(z_4=3|X_4=2,z_{2:3}) \cdot P(X_4=2|z_{2:3}) = 0$$

$$P(X_4=3|z_{2:4}) = 1$$

$$P(X_5=1|z_{2:5}) = \eta P(z_5=1|X_5=1,z_{2:4}) \times P(X_5=1|z_{2:4})$$

$$= \eta P(z_5=1|X_5=1) \times \sum_{i=1}^{3} P(X_5=1|X_4=i,z_{2:4}) \times P(X_4=i|z_{2:4})$$

$$= \eta P(z_5=1|X_5=1) \times P(X_5=1|X_4=3) \times 1$$

$$= \eta \cdot 0.6 \cdot 0.2 = 0.12\eta = 0.4$$

$$P(X_5=2|z_{2:5}) = \eta P(z_5=1|X_5=2,z_{2:4}) \times P(X_5=2|z_{2:4})$$

$$= \eta P(z_5=1|X_5=2) \cdot \sum_{i=1}^{3} P(X_5=1|X_4=i,z_{2:4}) \times P(X_4=i|z_{2:4})$$

$$= \eta P(z_5=1|X_5=2) \cdot P(X_5=1|X_4=3) \cdot 1$$

$$= 0.18\eta = 0.6 \quad \eta = \frac{1}{0.3}$$

(a) A: 0.4

$$P(X_5=3|z_{2:5}) = 0$$

3(b)

3.

(b)

t	z_t
2	1
3	1
4	3

$$P(X_2=1|z_2) = \eta P(z_2=1|X_2=1) \times P(X_2=1)$$

$$= 0.48\eta = 0.89$$

$$P(X_2=2|z_2) = \eta P(z_2=1|X_2=2) \cdot P(X_2=2$$

$$= 0.06\eta = 0.11$$

$$P(X_2=3|z_2) = 0$$

$$\eta = \frac{1}{0.48+0.06} = \frac{1}{0.54}$$

$$P(X_3=1|z_{2:3}) = \eta P(z_3=1|X_3=1,z_2) \times P(X_3=1|z_2)$$

$$= \eta P(z_3=1|X_3=1) \cdot \sum_{i=1}^{3} P(X_3=1|X_2=i) \times P(X_2=i|z_2)$$

$$= \eta \cdot 0.3 \cdot (0.8 \times 0.89 + 0.4 \times 0.11) = 0.4536\eta = 0.87$$

$$P(X_3=2|z_{2:3}) = \eta P(z_3=1|X_3=2,z_2) \times P(X_3=2|z_2)$$

$$= \eta P(z_3=1|X_3=2) \times \sum_{i=1}^{3} P(X_3=2|X_2=i) \cdot P(X_2=i|z_2)$$

$$= \eta \times 0.3 \times 0.222 = 0.0666\eta = 0.13$$

$$P(X_3=3|z_{2:3}) = 0$$

$$\eta = \frac{1}{0.4536+0.0666} = \frac{1}{0.5202}$$

$$P(X_4=1 \mid z_{2:4}) = \eta P(z_4=3 \mid X_4=1, z_{2:3}) \times P(X_4=1 \mid z_{2:3})$$

$$= 0$$

$$P(X_4=2 \mid z_{2:4}) = \eta P(z_4=3 \mid X_4=2, z_{2:3}) \times P(X_4=2 \mid z_{2:3})$$

$$= 0$$

$$P(X_4=3 \mid z_{2:4}) = 1$$

i	$P(X_2=i \mid z_2)$
1	0.89
2	0.11
3	0

i	$P(X_3=i \mid z_{2:3})$
1	0.87
2	0.13
3	0

i	$P(X_4=i \mid z_{2:4})$
1	0
2	0
3	1

3(c)

3. (c)

$$P(X_2=1 \mid z_{2:4}) = \eta P(z_2=1 \mid X_2=1, z_{3:4}) \times P(X_2=1 \mid z_{3:4})$$

$$= \eta P(z_2=1 \mid X_2=1) \times P(z_3=1 \mid X_2=1, z_4) \times P(X_2=1 \mid z_4)$$

$$= \eta P(z_2=1 \mid X_2=1) \times \left(\sum_{i=1}^{3} P(z_3=1 \mid X_3=i, X_2=1, z_4) \times P(X_3=i \mid X_2=1, z_4) \right) \times P(X_2=1 \mid z_4)$$

$$= \eta P(z_2=1 \mid X_2=1) \times \left(\sum_{i=1}^{3} P(z_3=1 \mid X_3=i) \times P(X_3=i \mid X_2=1) \right) \times P(z_4=3 \mid X_2=1) \times P(X_2=1)$$

$$= \eta P(z_2=1 \mid X_2=1) \times \underbrace{\left(\sum_{i=1}^{3} P(z_3=1 \mid X_3=i) \times P(X_3=i \mid X_2=1) \right)}_{①} \times \underbrace{\left(\sum_{j=1}^{3} P(z_4=3 \mid X_4=j, X_2=1) \times P(X_4=j \mid X_2=1) \right)}_{②} \times P(X_2=1)$$

$$① = 0.6 \times 0.8 + 0.3 \times 0.2 = 0.54$$

$$② = \sum_{j=1}^{3} P(z_4=3|X_4=j) \times \left(\sum_{k=1}^{3} P(X_4=j|X_3=k) \times P(X_3=k|X_2=1) \right)$$

$$= \sum_{j=1}^{3} \sum_{k=1}^{3} P(z_4=3|X_4=j) \times P(X_4=j|X_3=k) \times P(X_3=k|X_2=1)$$

$$= \sum_{k=1}^{3} P(X_4=3|X_3=k) \times P(X_3=k|X_2=1)$$

$$= 0.2 \times 0.2 + 0.2 \times 0 + 0 = 0.04$$

$$CA = \begin{bmatrix} 0.54 & 0.36 & 0.3 \\ 0.46 & 0.44 & 0.5 \\ 0 & 0.2 & 0.2 \end{bmatrix}$$

$$CAA = \begin{bmatrix} 0.504 & 0.42 & 0.384 \\ 0.456 & 0.46 & 0.456 \\ 0.04 & 0.12 & 0.16 \end{bmatrix}$$

$$P(X_2=1|z_{2:4}) = n \times 0.6 \times 0.54 \times 0.04 \times 0.8 = 0.0104n$$

$$P(X_2=2|z_{2:4}) = nP(z_2=1|X_2=2) \times CA_{1,2} \times CAA_{3,2} \times P(X_2=2) = 0.0026n$$

$$P(X_2=3|z_{2:4}) = 0 \qquad n = \frac{1}{0.013}$$

$$P(X_3=1|z_{2:4}) = n \sum_{i=1}^{3} P(X_4=i|X_3=1, z_{2:3}) \times P(z_4=3|X_4=i, z_{2:3}) \times P(X_3=1|z_{2:3})$$

$$= 0$$

$$P(X_3=2|z_{2:4}) = n P(X_4=3|X_3=2) \times P(z_4=3|X_4=3) \times P(X_3=2|z_{2:3})$$

$$= n \times 0.2 \times 1 \times 0.2 = 0.042n$$

$$P(X_3=3|z_{2:4}) = 0$$

i	$P(X_2=i, z_{2:4})$
1	0.8
2	0.2
3	0

i	$P(X_3=i\|z_{2:4})$
1	0
2	1
3	0

1	$P(X_4=i, z_{2:4})$
1	0
2	0
3	1

hence the most likely sequence is sunny, cloudy, rainy

$P = 0.8 \times 1 \times 1 = 0.8$