DSD Exercise

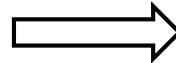# Gauss-Seidel Iteration Machine

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Background

❖ Solving large systems of linear equations is required in many engineering simulations and scientific computing applications

❖ Several iterative methods are used to accelerate the computing due to their simplicity, such as Jacobi Iteration, **Gauss-Seidel Iteration**

$$\begin{bmatrix} a_{11} & \cdots & a_{1N} \\ \vdots & \ddots & \vdots \\ a_{N1} & \cdots & a_{NN} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_N \end{bmatrix} = \begin{bmatrix} b_1 \\ \vdots \\ b_N \end{bmatrix} \implies$$

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1N}x_N = b_1$$
$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2N}x_N = b_2$$
$$\vdots$$
$$a_{N1}x_1 + a_{N2}x_2 + \cdots + a_{NN}x_N = b_N$$

Gauss-Seidel Iteration Machine (GSIM)

❖ **Iterative method** to solve a linear system of equations

❖ **Ax = b** ⟹

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} \quad (1)$$

x_3^0: initial value of x_3

x_2^1: first iteration result of x_2

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1N}x_N = b_1$$
$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2N}x_N = b_2$$
$$\vdots$$
$$a_{N1}x_1 + a_{N2}x_2 + \cdots + a_{NN}x_N = b_N$$

$$x_1^1 = \frac{1}{a_{11}}(b_1 - a_{12}x_2^0 - \cdots - a_{1N}x_N^0)$$
$$x_2^1 = \frac{1}{a_{22}}(b_2 - a_{21}x_1^1 - a_{23}x_3^0 - \cdots - a_{2N}x_N^0)$$
$$\vdots$$
$$x_N^1 = \frac{1}{a_{NN}}(b_N - a_{N1}x_1^1 - a_{N2}x_2^1 - \cdots - a_{NN-1}x_{N-1}^1)$$

$$x_i^{k+1} = \frac{1}{a_{ii}}\left[b_i - \sum_{j=1}^{i-1} a_{ij}x_j^{k+1} - \sum_{j=i+1}^{N} a_{ij}x_j^k \right]$$

- Expand eq (1)
- Change the order
- Final equation

Project Description

Initialization M iterations (define by yourself!)   Output result **x**

$$x^0 = \begin{bmatrix} x_1^0 \\ x_2^0 \\ \vdots \\ x_N^0 \end{bmatrix}$$

$$x^k = \begin{bmatrix} x_1^k \\ x_2^k \\ \vdots \\ x_N^k \end{bmatrix}$$

Gauss-Seidel Iteration

$$x^{k+1} = \begin{bmatrix} x_1^{k+1} \\ x_2^{k+1} \\ \vdots \\ x_N^{k+1} \end{bmatrix}$$
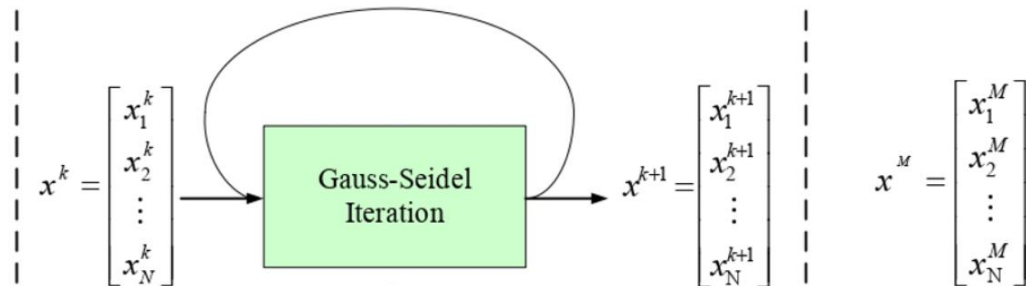
$$x^M = \begin{bmatrix} x_1^M \\ x_2^M \\ \vdots \\ x_N^M \end{bmatrix}$$

❖ Given a fixed 16 × 16 matrix **A**

$$A = \begin{bmatrix}
20 & -13 & 6 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-13 & 20 & -13 & 6 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
6 & -13 & 20 & -13 & 6 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 6 & -13 & 20 & -13 & 6 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 6 & -13 & 20 & -13 & 6 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 6 & -13 & 20 & -13 & 6 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 6 & -13 & 20 & -13 & 6 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 6 & -13 & 20 & -13 & 6 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 6 & -13 & 20 & -13 & 6 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 6 & -13 & 20 & -13 & 6 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 6 & -13 & 20 & -13 & 6 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 6 & -13 & 20 & -13 & 6 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 6 & -13 & 20 & -13 & 6 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 6 & -13 & 20 & -13 & 6 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 6 & -13 & 20 & -13 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 6 & -13 & 20
\end{bmatrix}$$

By the previous equation:

$$x_1^1 = \tfrac{1}{20}[b_1 + 13(x_2^0 + 0) - 6(x_3^0 + 0) + (x_4^0 + 0)]$$
$$x_2^1 = \tfrac{1}{20}[b_2 + 13(x_3^0 + x_1^1) - 6(x_4^0 + 0) + (x_5^0 + 0)]$$
$$x_3^1 = \tfrac{1}{20}[b_3 + 13(x_4^0 + x_2^1) - 6(x_5^0 + x_1^1) + (x_6^0 + 0)]$$
$$x_4^1 = \tfrac{1}{20}[b_4 + 13(x_5^0 + x_3^1) - 6(x_6^0 + x_2^1) + (x_7^0 + x_1^1)]$$
$$\vdots$$
$$x_{16}^1 = \tfrac{1}{20}[b_{16} + 13(0 + x_{15}^1) - 6(0 + x_{14}^1) + (0 + x_{13}^1)]$$

At most 7 non-zero terms one time   Only divided by 20

Score Criteria

❖ 評分一：Error rate E^2

　❖ $E^2 = \sum_{i=1}^{16}\sum_{j=1}^{16}\left(a_{ij}x_j^k - b_i\right)^2$

　❖ Achieve Level A

A 級：		E^2	< 0.000001
B 級：	$0.000001 \leqq$	E^2	< 0.000005
C 級：	$0.000005 \leqq$	E^2	< 0.000010
D 級：	$0.000010 \leqq$	E^2	< 0.000050
E 級：	$0.000050 \leqq$	E^2	< 0.000100
F 級：	$0.000100 \leqq$	E^2	< 0.001000
G 級：	$0.001000 \leqq$	E^2	< 0.005000
H 級：	$0.005000 \leqq$	E^2	< 0.010000
I 級：	$0.010000 \leqq$	E^2	< 0.100000
J 級：	$0.100000 \leqq$	E^2	< 0.300000
K 級：	$0.300000 \leqq$	E^2	

❖ 評分二：AT score

　❖ $AT = area \times total\ timing$

　❖ Area: synthesis cell area

　❖ Timing: total execution time (tb1+tb2+…+tb5)

```
----------------------------------------
Your Score Level: A

Congratulations! GSIM's Function Successfully!

------------------------PASS----------------------

Simulation complete via $finish(1) at time 3734500 PS + 0
./testfixture5.v:213        #(`CYCLE/2); $finish;
ncsim> exit
```

|	
Combinational area:	3875.164193
Buf/Inv area:	434.534396
Noncombinational area:	1147.442383
Macro/Black Box area:	0.000000
Net Interconnect area:	48580.242432
Total cell area:	5022.606576
Total area:	53602.849008

Design Guidelines

❖ Architecture level
- ❖ Data path scheduling
- ❖ Parallel processing (unfolding)
- ❖ Pipelining

❖ Computation unit level
- ❖ Constant multiplier, constant divider
- ❖ Decimal analysis

Architecture Level Optimization (1/3)

❖ Reading data

  ❖ Arbitrary reading: Using several MUXs to load data
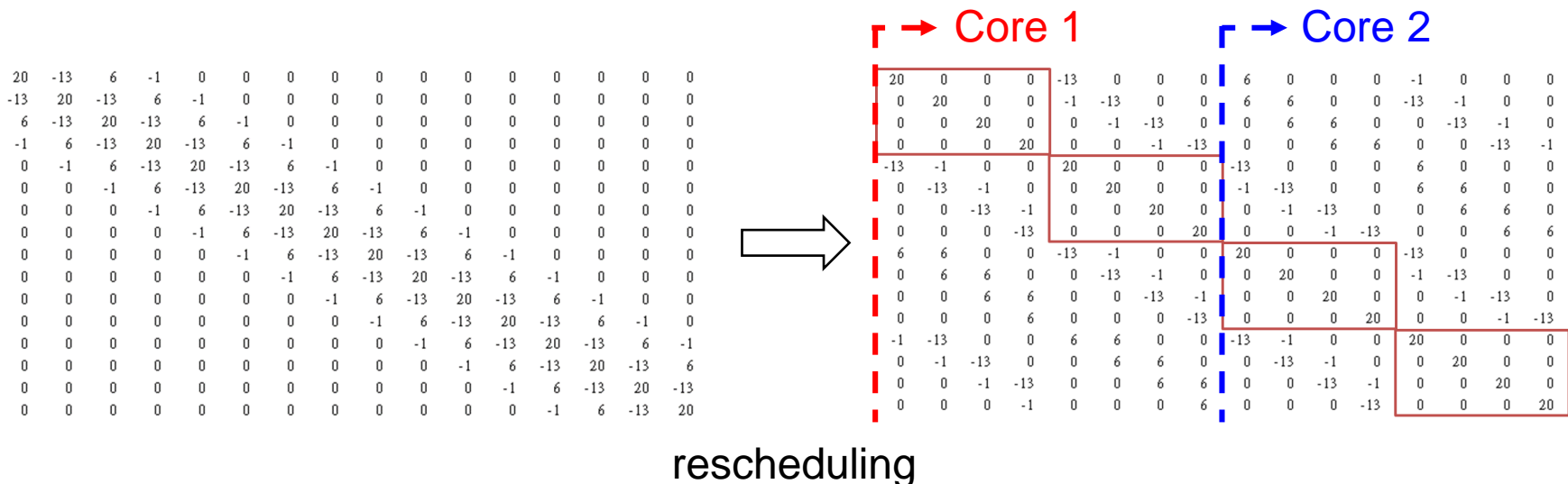
  ❖ Structural reading: Similar computation dataflow
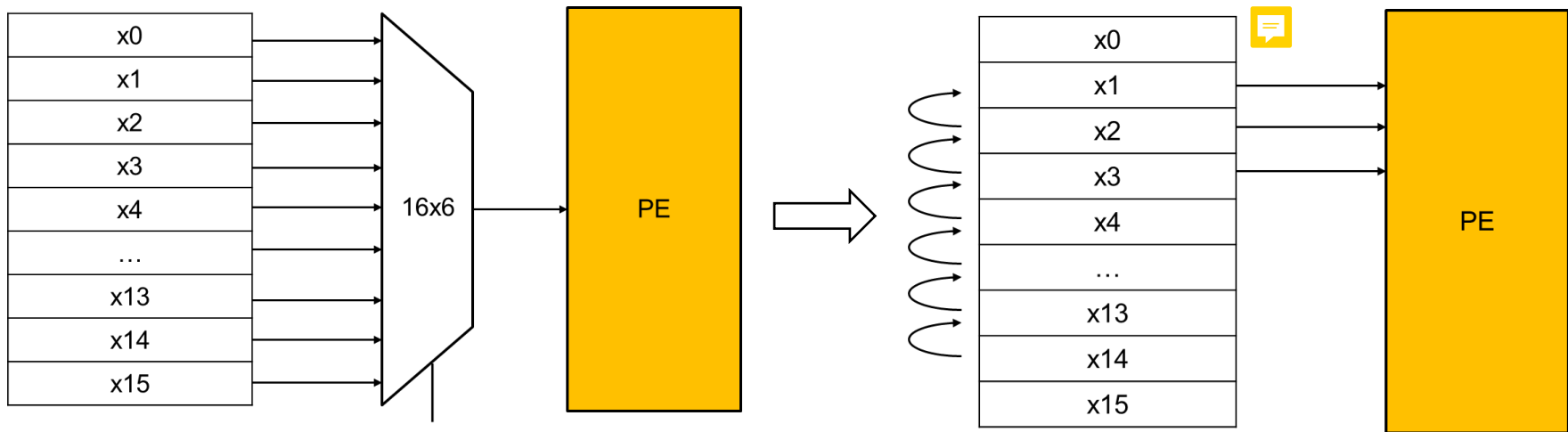
Scheduling by removing 6 MUXs

Architecture Level Optimization (2/3)

❖ Parallel processing (unfolding)

  ❖ Using multi-core to compute

  ❖ Necessity: No data dependancy

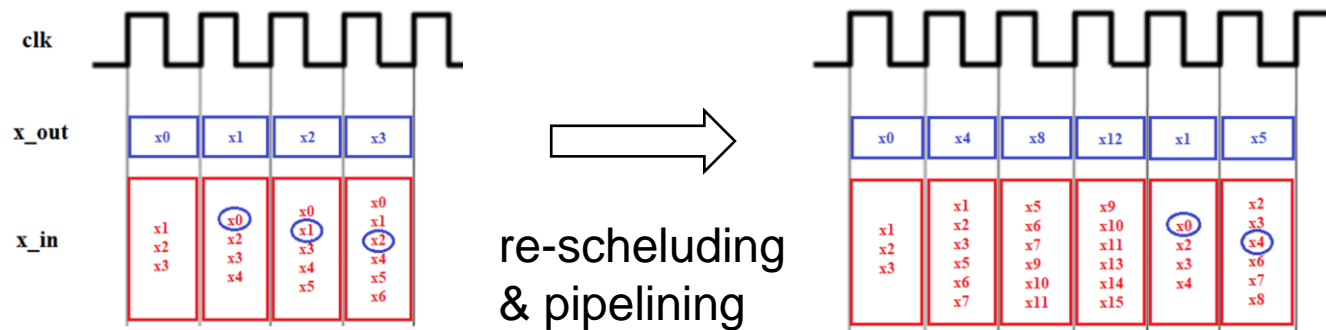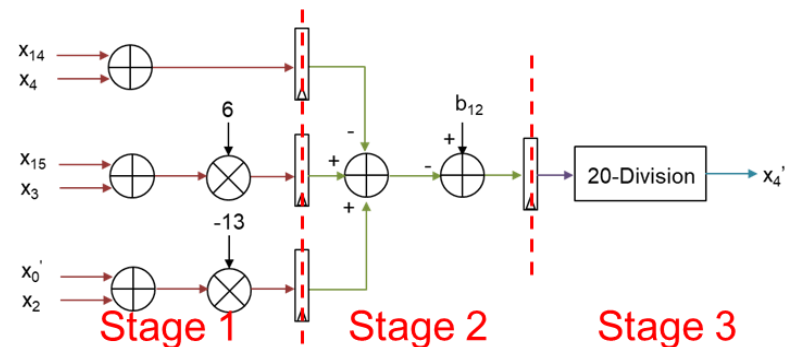❖ Reordering computation

  ❖ Processing elements: 1, 5, 9, 13, …
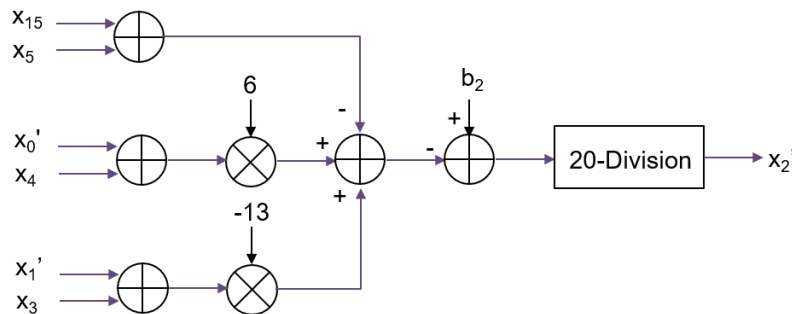

rescheduling

Architecture Level Optimization (3/3)

❖ Pipelining

    ❖ Divide computation into several cycles
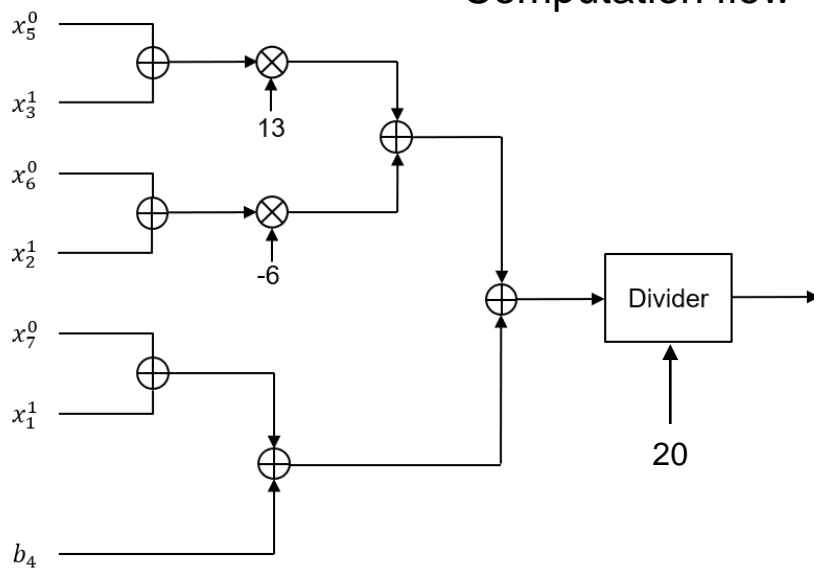

re-scheluding
& pipelining

e.g. 3-stages pipeling

Computation Unit Level (1/2)

1 Division   2 Multiplication

$$Ex: x_4^1 = \frac{1}{20}[b_4 + 13 \times (x_5^0 + x_3^1) + (-6) \times (x_6^0 + x_2^1) + (x_7^0 + x_1^1)]$$

Computation flow

Division and multiplication is complicated.
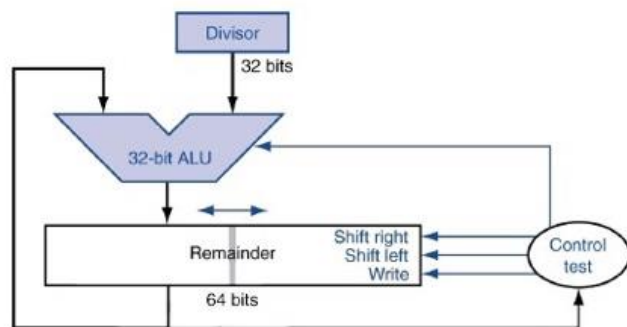It need large area and long computation time.

Computation Unit Level (2/2)

❖ Multiplier
  ❖ Power-of-two method (e.g. $6 = 110_2$, $13 = 1101_2$)
❖ Divider
  ❖ Conventional (for arbitrary input): requires 32 cycles
  ❖ Constant divider: requires 3 cycles
    ➢ Canonic Signed Digit (CSD) code
    ➢ For each odd integer d, there exists an odd integer m such that $d \times m = 2^n - 1$

$$\frac{1}{d} = \frac{m}{2^n - 1} = \frac{m}{2^n(1 - 2^{-n})}$$

$$= \frac{m}{2^n}(1 + 2^{-n})(1 + 2^{-2n})(1 + 2^{-4n})\dots$$

Need: 1 Adder, 1 shifter
Time: 32 cycles

Need: 1 Adder, 1 shifter
Time: 3 cycles

Simulation & Synthesis

❖ Check "doc.pdf"

❖ 3 Major Things

  ❖ RTL coding & Simulation

  ❖ Synthesizable coding & Logic Synthesis

  ❖ Gate-level simulation & Debugging/refinement

❖ Files needed for simulation

  ❖ RTL code: GSIM.v

  ❖ Gate-level code: GSIM _syn.v,

  ❖ Timing info (SDF file): GSIM _syn.sdf,

  ❖ Design library (DDC file): GSIM _syn.ddc

Notice

❖ Do not fit the given test pattern, there will be hidden cases!

❖ Latches are not allowed in gate level code after synthesis, use Flip-flop instead.

❖ Negative Slack and Timing Violations are not allowed after synthesis.

❖ The tsmc13_neg.v file is not allowed to be downloaded! Or you may offend the copyright protected by NTU & TSRI!

Grading Policy

- ❖ RTL (40%): Function correctness, Rank A
  - ❖ 10% for hidden tb
- ❖ Synthesis (40%): Pass baseline AT score: 1.0×10^{10}
  - ❖ 10% for hidden tb
- ❖ Ranking (15%): AT ranking. Get 0% if not passing the hidden tb
- ❖ Report (5%)

- ❖ For each team, you need to submit 4 files + 1 report
  - ❖ RTL code: *GSIM.v*
  - ❖ Synthesis: *GSIM _syn.v,*
    GSIM _syn.sdf,
    GSIM _syn.ddc
  - ❖ Report: *report.pdf*

Report

1. **Simulated cycle time (ns)**

    ❖ Gate-level simulation clock cycle

    (i.e. The cycle you passed testbench after synthesis)

2. **Area (um^2)**

    ❖ report_area

3. **AT score**

    ❖ $AT = area \times timing$

    ❖ Area: total cell area

    ❖ Timing: total execution time (tb1 + tb2 + … + tb5)

4. **Screenshot**

    ❖ Inferred memory devices in process

    　　(※No latch should be inferred!)

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Submission

❖ *DSD_Exercise_學號/*

           GSIM.v

           GSIM _syn.v

           GSIM _syn.sdf

           GSIM _syn.ddc

           report.pdf

❖ Compress all the files into one **ZIP** file

    ❖ File name: DSD_ Exercise _學號.zip

    ❖ EX: DSD_ Exercise _b09901001.zip

❖ Upload the file to NTUCOOL

❖ Deadline: 2024/05/01 23:59 ※Late submission is not allowed