Lecture 5.
Algebraic IR Models

Generalized Vector Model

Latent Semantic Indexing

Generalized Vector Model

Generalized Vector Model

- Classic models enforce independence of index terms
- For instance, in the Vector model
  - A set of term vectors $\{\vec{k}_1, \vec{k}_2, \ldots, \vec{k}_t\}$ are linearly independent
  - Frequently, this is interpreted as $\forall_{i,j} \Rightarrow \vec{k}_i \bullet \vec{k}_j = 0$
- In the generalized vector space model, two index term vectors might be non-orthogonal

	K_1	K_2	K_3	$q \bullet d_j$
d_1	2	0	1	5
d_2	1	0	0	1
d_3	0	1	3	11
d_4	2	0	0	2
d_5	1	2	4	17
d_6	1	2	0	5
d_7	0	5	0	10
q	1	2	3	

Key Idea

- As before, let $w_{i,j}$ be the weight associated with $[k_i, d_j]$ and V = {k_1, k_2, ..., k_t} be the set of all terms
- If the $w_{i,j}$ weights are binary, all patterns of occurrence of terms within docs can be represented by minterms:

$$
\begin{array}{rcl}
 & & (k_1, k_2, k_3, \ldots, k_t) \\
m_1 & = & (0,\ 0,\ 0,\ \ \ldots,\ 0) \\
m_2 & = & (1,\ 0,\ 0,\ \ \ldots,\ 0) \\
m_3 & = & (0,\ 1,\ 0,\ \ \ldots,\ 0) \\
m_4 & = & (1,\ 1,\ 0,\ \ \ldots,\ 0) \\
 & \vdots & \\
m_{2^t} & = & (1,\ 1,\ 1,\ \ \ldots,\ 1)
\end{array}
$$

For instance, m_2 indicates documents in which solely the term k_1 occurs

Key Idea

- For any document d_j, there is a minterm m_r that includes exactly the terms that occur in the document
- Let us define the following set of minterm vectors $\vec{m}_r$,

$$\begin{array}{rcl} & & 1, 2, \ldots, 2^t \\ \vec{m}_1 & = & (1, 0, \ldots, 0) \\ \vec{m}_2 & = & (0, 1, \ldots, 0) \\ & \vdots & \\ \vec{m}_{2^t} & = & (0, 0, \ldots, 1) \end{array}$$

Notice that we can associate each unit vector $\vec{m}_r$ with a minterm m_r, and that $\vec{m}_i \bullet \vec{m}_j = 0$ for all $i \neq j$

Key Idea

- Pairwise orthogonality among the $\vec{m}_r$ vectors does not imply independence among the index terms
- On the contrary, index terms are now correlated by the $\vec{m}_r$ vectors
  - For instance, the vector $\vec{m}_4$ is associated with the minterm $m_4 = (1, 1, \ldots, 0)$
  - This minterm induces a dependency between terms k_1 and k_2
  - Thus, if such document exists in a collection, we say that the minterm m_4 is active
- The model adopts the idea that co-occurrence of terms induces dependencies among these terms

Forming the Term Vectors

- Let $on(i, m_r)$ return the weight $\{0, 1\}$ of the index term k_i in the minterm m_r
- The vector associated with the term k_i is computed as:

$$\vec{k}_i = \frac{\sum_{\forall r} on(i, m_r)\ c_{i,r}\ \vec{m}_r}{\sqrt{\sum_{\forall r} on(i, m_r)\ c_{i,r}^2}}$$

$$c_{i,r} = \sum_{d_j \mid c(d_j)=m_r} w_{i,j}$$

- Notice that for a collection of size N, only N minterms affect the ranking (and not 2^t)

Dependency between Index Terms

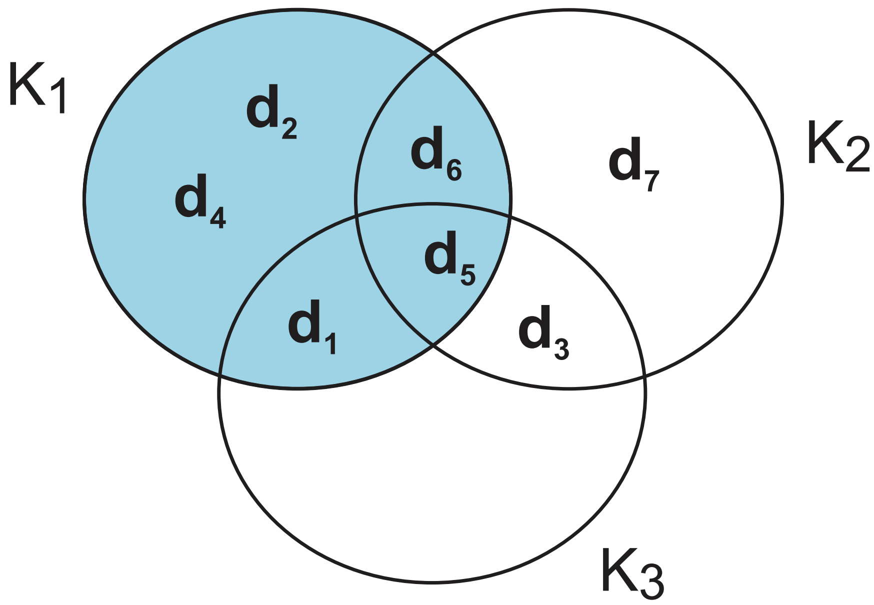
- A degree of correlation between the terms k_i and k_j can now be computed as:

$$\vec{k}_i \bullet \vec{k}_j = \sum_{\forall r} on(i, m_r) \times c_{i,r} \times on(j, m_r) \times c_{j,r}$$

- This degree of correlation sums up the dependencies between k_i and k_j induced by the docs in the collection

The Generalized Vector Model

- An Example



	K_1	K_2	K_3
d_1	2	0	1
d_2	1	0	0
d_3	0	1	3
d_4	2	0	0
d_5	1	2	4
d_6	1	2	0
d_7	0	5	0
q	1	2	3

Computation of $c_{i,r}$

	K_1	K_2	K_3
d_1	2	0	1
d_2	1	0	0
d_3	0	1	3
d_4	2	0	0
d_5	1	2	4
d_6	0	2	2
d_7	0	5	0
q	1	2	3

	K_1	K_2	K_3
$d_1 = m_6$	1	0	1
$d_2 = m_2$	1	0	0
$d_3 = m_7$	0	1	1
$d_4 = m_2$	1	0	0
$d_5 = m_8$	1	1	1
$d_6 = m_7$	0	1	1
$d_7 = m_3$	0	1	0
$q = m_8$	1	1	1

	$c_{1,r}$	$c_{2,r}$	$c_{3,r}$
m_1	0	0	0
m_2	3	0	0
m_3	0	5	0
m_4	0	0	0
m_5	0	0	0
m_6	2	0	1
m_7	0	3	5
m_8	1	2	4

Assume wij=fij in this example.

m1	0	0	0
m2	1	0	0
m3	0	1	0
m4	1	1	0
m5	0	0	1
m6	1	0	1
m7	0	1	1
m8	1	1	1

$$\vec{k}_i = \frac{\sum_{\forall r} on(i, m_r)\ c_{i,r}\ \vec{m}_r}{\sqrt{\sum_{\forall r} on(i, m_r)\ c_{i,r}^2}}$$

$$c_{i,r} = \sum_{d_j \mid c(d_j)=m_r} w_{i,j}$$

Computation of $\overrightarrow{k_i}$

- $\overrightarrow{k_1} = \frac{(3\vec{m}_2+2\vec{m}_6+\vec{m}_8)}{\sqrt{3^2+2^2+1^2}}$
- $\overrightarrow{k_2} = \frac{(5\vec{m}_3+3\vec{m}_7+2\vec{m}_8)}{\sqrt{5+3+2}}$
- $\overrightarrow{k_3} = \frac{(1\vec{m}_6+5\vec{m}_7+4\vec{m}_8)}{\sqrt{1+5+4}}$

	$c_{1,r}$	$c_{2,r}$	$c_{3,r}$
m_1	0	0	0
m_2	3	0	0
m_3	0	5	0
m_4	0	0	0
m_5	0	0	0
m_6	2	0	1
m_7	0	3	5
m_8	1	2	4

Computation of Document Vectors

- $\overrightarrow{d_1} = 2\overrightarrow{k_1} + \overrightarrow{k_3}$
- $\overrightarrow{d_2} = \overrightarrow{k_1}$
- $\overrightarrow{d_3} = \overrightarrow{k_2} + 3\overrightarrow{k_3}$
- $\overrightarrow{d_4} = 2\overrightarrow{k_1}$
- $\overrightarrow{d_5} = \overrightarrow{k_1} + 2\overrightarrow{k_2} + 4\overrightarrow{k_3}$
- $\overrightarrow{d_6} = 2\overrightarrow{k_2} + 2\overrightarrow{k_3}$
- $\overrightarrow{d_7} = 5\overrightarrow{k_2}$
- $\overrightarrow{q} = \overrightarrow{k_1} + 2\overrightarrow{k_2} + 3\overrightarrow{k_3}$

	K_1	K_2	K_3
d_1	2	0	1
d_2	1	0	0
d_3	0	1	3
d_4	2	0	0
d_5	1	2	4
d_6	0	2	2
d_7	0	5	0
q	1	2	3

Conclusions

- Model considers correlations among index terms
- Not clear in which situations it is superior to the standard Vector model
- Computation costs are higher
- Model does introduce interesting new ideas

Latent Semantic Indexing

Latent Semantic Indexing

- Classic IR might lead to poor retrieval due to:
  - unrelated documents might be included in the answer set Index Match 並不一定相關
  - relevant documents that do not contain at least one index term are not retrieved Index不match不一定不相關
  - **Reasoning**: retrieval based on index terms is vague and noisy
- The user information need is more related to concepts and ideas than to index terms
- A document that shares concepts with another document known to be relevant might be of interest

Latent Semantic Indexing

- The idea here is to map documents and queries into a dimensional space composed of concepts
- Let
  - t: total number of index terms
  - N: number of documents
  - $\mathbf{M} = [m_{ij}]$: term-document matrix $t \times N$
- To each element of $\mathbf{M}$ is assigned a weight $w_{i,j}$ associated with the term-document pair $[k_i, d_j]$
  - The weight $w_{i,j}$ can be based on a *tf-idf* weighting scheme

Latent Semantic Indexing

- The matrix $\mathbf{M} = [m_{ij}]$ can be decomposed into three components using singular value decomposition

$$\mathbf{M} = \mathbf{K} \cdot \mathbf{S} \cdot \mathbf{D}^T$$

- were
  - $\mathbf{K}$ is the matrix of eigenvectors derived from $\mathbf{C} \overset{\text{term matrix}}{=} \mathbf{M} \cdot \mathbf{M}^T$
  - $\mathbf{D}^T$ is the matrix of eigenvectors derived from $\mathbf{M}^T \cdot \mathbf{M}$ $\quad \text{document matrix}$
  - $\mathbf{S}$ is an $r \times r$ diagonal matrix of singular values where $r = \min(t, N)$ is the rank of $\mathbf{M}$

Computing an Example

- Let $\mathbf{M}^T = [m_{ij}]$ be given by

	K_1	K_2	K_3	$q \bullet d_j$
d_1	2	0	1	5
d_2	1	0	0	1
d_3	0	1	3	11
d_4	2	0	0	2
d_5	1	2	4	17
d_6	1	2	0	5
d_7	0	5	0	10
q	1	2	3	

- Compute the matrices $\mathbf{K}$, $\mathbf{S}$, and $\mathbf{D}^t$

Latent Semantic Indexing

- In the matrix $\mathbf{S}$, consider that only the s largest singular values are selected
- Keep the corresponding columns in $\mathbf{K}$ and $\mathbf{D}^T$
- The resultant matrix is called $\mathbf{M}_s$ and is given by

$$\mathbf{M}_s = \mathbf{K}_s \cdot \mathbf{S}_s \cdot \mathbf{D}_s^T$$

- where s, $s < r$, is the dimensionality of a reduced concept space
- The parameter s should be
  - large enough to allow fitting the characteristics of the data
  - small enough to filter out the non-relevant representational details

Latent Ranking

- The relationship between any two documents in s can be obtained from the $\mathrm{M}_s^T \cdot \mathrm{M}_s$ matrix given by

$$
\begin{aligned}
\mathrm{M}_s^T \cdot \mathrm{M}_s &= (\mathbf{K}_s \cdot \mathbf{S}_s \cdot \mathbf{D}_s^T)^T \cdot \mathbf{K}_s \cdot \mathbf{S}_s \cdot \mathbf{D}_s^T \\
&= \mathbf{D}_s \cdot \mathbf{S}_s \cdot \mathbf{K}_s^T \cdot \mathbf{K}_s \cdot \mathbf{S}_s \cdot \mathbf{D}_s^T \\
&= \mathbf{D}_s \cdot \mathbf{S}_s \cdot \mathbf{S}_s \cdot \mathbf{D}_s^T \\
&= (\mathbf{D}_s \cdot \mathbf{S}_s) \cdot (\mathbf{D}_s \cdot \mathbf{S}_s)^T
\end{aligned}
$$

- In the above matrix, the (i, j) element quantifies the relationship between documents d_i and d_j

Latent Ranking

- The user query can be modelled as a pseudo-document in the original $\mathbf{M}$ matrix
- Assume the query is modelled as the document numbered 0 in the $\mathbf{M}$ matrix
- The matrix $\mathbf{M}_s^T \cdot \mathbf{M}_s$ quantifies the relationship between any two documents in the reduced concept space
- The first row of this matrix provides the rank of all the documents with regard to the user query

Information Retrieval using a Singular Value Decomposition Model

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The Singular Value Decomposition (SVD) Model

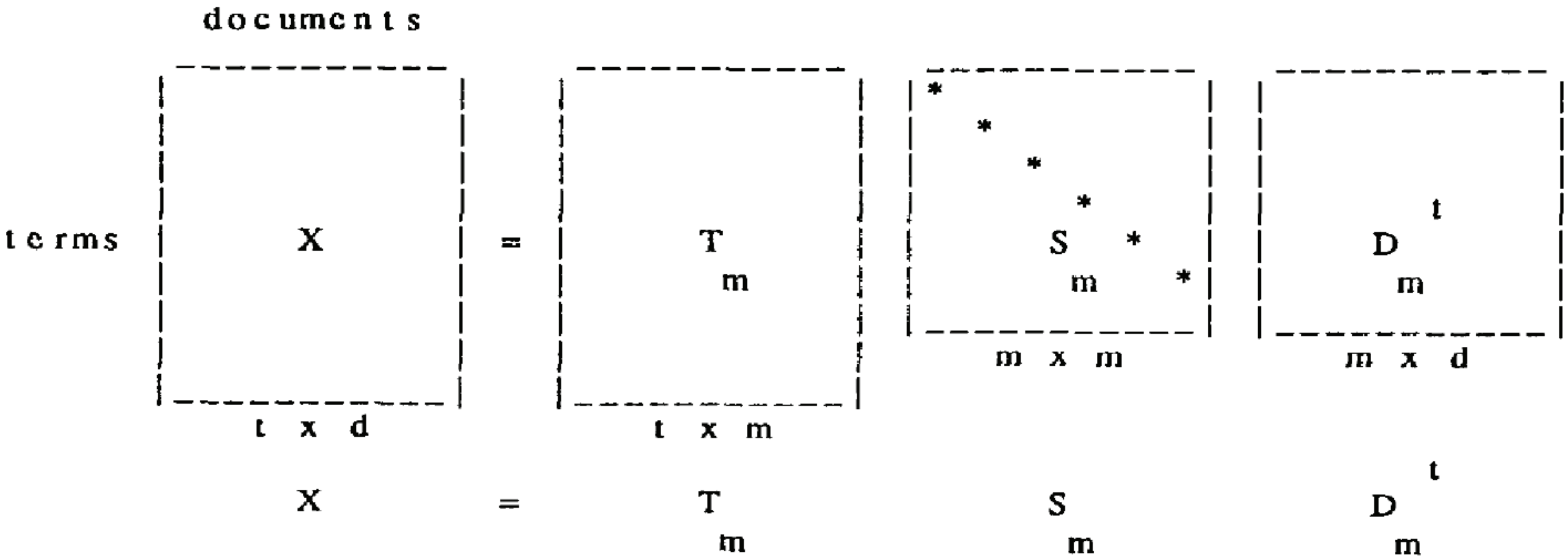


Figure 3. Singular value decomposition of the term x document matrix, **X**. Where :

$\mathbf{T}_m$ has orthogonal, unit-length columns ($\mathbf{T}_m{}^t\mathbf{T}_m = \mathbf{I}$)
$\mathbf{D}_m$ has orthogonal, unit-length columns ($\mathbf{D}_m{}^t\mathbf{D}_m = \mathbf{I}$)
$\mathbf{S}_m$ is the diagonal matrix of singular values
t is the number of rows of **X**
d is the number of columns of **X**

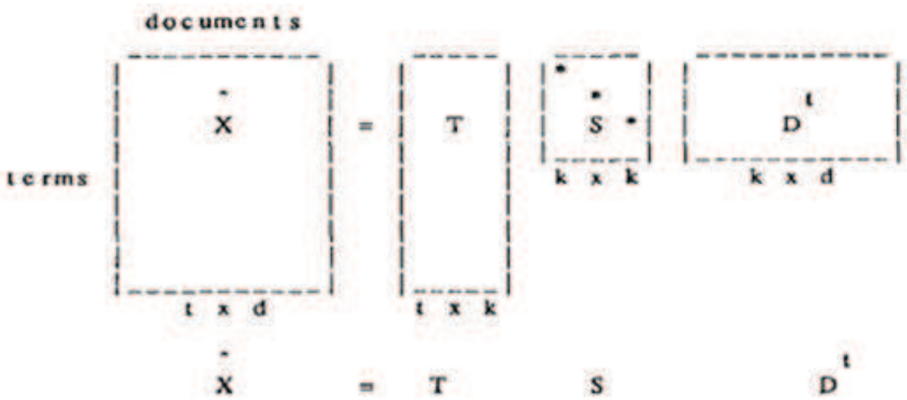
T_m is the matrix of eigenvectors of the square symmetric matrix $Y = XX^t$, and D_m is the matrix of cigenvectors of $Y = X^tX$.

Reduced Model

In general, if $\mathbf{X}=\mathbf{T}_m\mathbf{S}_m\mathbf{D}_m{}^t$ is of full rank, then the matrices $\mathbf{T}_m$, $\mathbf{D}_m$, and $\mathbf{S}_m$ must be also. However, if only the k largest singular values of $\mathbf{S}_m$ are kept along with their corresponding columns in the $\mathbf{T}_m$ and $\mathbf{D}_m$ matrices, and the rest deleted (yielding matrices $\mathbf{S}_k$, $\mathbf{T}_k$ and $\mathbf{D}_k$), the resulting matrix, $\hat{\mathbf{X}}$, is the unique matrix of rank k which is closest in the least squares sense to $\mathbf{X}$:

$$(2) \qquad \hat{\mathbf{X}} = \mathbf{T}_k\mathbf{S}_k\mathbf{D}_k{}^t \approx \mathbf{X}$$

contain only the k largest independent linear components of X
capture the major associational structure of the data and throw out much of the noise
use the value of k which maximizes the retrieval



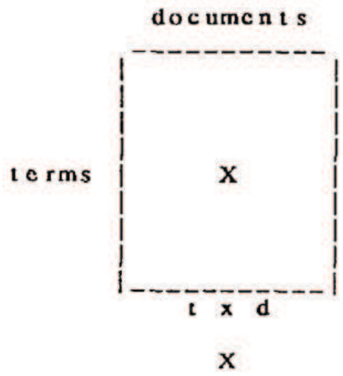
Interpretation of the row vectors of the SVD matrices, T and D:

coordinates in a k-dimensional space,
terms and documents are points in a vector space

The diagonal matrix, S, serves to stretch or shrink the orthogonal axes of this space

Term Matching Paradigm

- The simiIarity of two documents is obtained by using an cosine measure of the corresponding two column vectors of the raw data matrix X.
- A query is represented as a sort of pseudo-document, i.e., a column vector of term frequencies, $X_{\bullet q}$, which is similarly compared against columns of X, and the best matches found.
- The appropriate “cleaned up” version of the query column-vector, $X_{\bullet q}$, is given by $\sim X_{\bullet q} = TT^{t}X_{\bullet q}$.



Latent Structure Paradigm

- similarities for all pairs of documents

$$\hat{X}^t\hat{X} = (TSD^t)^t TSD^t = DST^t TSD^t = DSSD^t = (DS)(DS)^t.$$

comparison of document i and document j :
the inner product of rows i and j of the matrix DS

- similarities for all pairs of terms

$$\hat{X}\hat{X}^t = TSD^t(TSD^t)^t = TSD^t DST^t = TSST^t = (TS)(TS)^t.$$

comparison of term i and term j:
inner product of rows i and j of the matrix TS

- association between term i and document j

$$\hat{X} = TSD^t = (TS^{1/2})(DS^{1/2})^t,$$

the association between term i and document j:
the inner product of row i of the matrix, $TS^{1/2}$, and
row j of the matrix, $DS^{1/2}$

Geometric Interpretation

- If the axes of the space are resealed by the associated diagonal values of S, the inner-product between term points or document points can be used to make the algebraic comparisons of interest.
- The axes must be resealed by the associated diagonal values of $S^{1/2}$ for comparisons between a term and a document.

The Procedure

- A collection of documents has its content terms tabulated to give a frequency matrix, which is taken as X.
- A *k*-dimensional SVD decomposition of X is computed yielding matrices T, S, and D.
- The rows of T and D are taken as index vectors for corresponding terms and documents, respectively.
- The diagonal elements of S (or $S^{1/2}$, as needed), are taken as component-wise weights in ensuing similarity calculations.
- A query, treated as vector of term frequencies (albeit very sparse), is converted to a pseudo-document, $D_{q\bullet}$, in the factor space following equation.

$$\mathbf{D}_{q\bullet} = \mathbf{X}_{\bullet q}^{\mathbf{t}} \mathbf{TS}^{-1}$$

row column

- This query factor-vector is then compared to the factor-vectors of all the documents, and the documents ordered according to the results.

$X=TSD^t \rightarrow X^t=DS^tT^t$

$S^t=S$ and $T^tT=I$

$X^t=DS^tT^t \rightarrow D=X^tTS^{-1}$

$D_{q\bullet}=X_{\bullet q}{}^tTS^{-1}$

c1: *Human* machine *interface* for Lab ABC *computer* applications
c2: A *survey* of *user* opinion of *computer system response time*
c3: The *EPS user interface* management *system*
c4: *System* and *human system* engineering testing of *EPS*
c5: Relation of *user*-perceived *response time* to error measurement

m1: The generation of random, binary, unordered *trees*
m2: The intersection *graph* of paths in *trees*
m3: *Graph minors* IV: Widths of *trees* and well-quasi-ordering
m4: *Graph minors*: A *survey*

Terms	**Documents**								
	c1	c2	c3	c4	c5	m1	m2	m3	m4
human	1	0	0	1	0	0	0	0	0
interface	1	0	1	0	0	0	0	0	0
computer	1	1	0	0	0	0	0	0	0
user	0	1	1	0	1	0	0	0	0
system	0	1	1	2	0	0	0	0	0
response	0	1	0	0	1	0	0	0	0
time	0	1	0	0	1	0	0	0	0
EPS	0	0	1	1	0	0	0	0	0
survey	0	1	0	0	0	0	0	0	1
trees	0	0	0	0	0	1	1	1	0
graph	0	0	0	0	0	0	1	1	1
minors	0	0	0	0	0	0	0	1	1

T_m =

0.22	-0.11	0.29	-0.41	-0.11	-0.34	0.52	-0.06	-0.41
0.20	-0.07	0.14	-0.55	0.28	0.50	-0.07	-0.01	-0.11
0.24	0.04	-0.16	-0.59	-0.11	-0.25	-0.30	0.06	0.49
0.40	0.06	-0.34	0.10	0.33	0.38	0.00	0.00	0.01
0.64	-0.17	0.36	0.33	-0.16	-0.21	-0.17	0.03	0.27
0.27	0.11	-0.43	0.07	0.08	-0.17	0.28	-0.02	-0.05
0.27	0.11	-0.43	0.07	0.08	-0.17	0.28	-0.02	-0.05
0.30	-0.14	0.33	0.19	0.11	0.27	0.03	-0.02	-0.17
0.21	0.27	-0.18	-0.03	-0.54	0.08	-0.47	-0.04	-0.58
0.01	0.49	0.23	0.03	0.59	-0.39	-0.29	0.25	-0.23
0.04	0.62	0.22	0.00	-0.07	0.11	0.16	-0.68	0.23
0.03	0.45	0.14	-0.01	-0.30	0.28	0.34	0.68	0.18

S_m =

3.34								
	2.54							
		2.35						
			1.64					
				1.50				
					1.31			
						0.85		
							0.56	
								0.36

D_m =

0.20	-0.06	0.11	-0.95	0.05	-0.08	0.18	-0.01	-0.06
0.61	0.17	-0.50	-0.03	-0.21	-0.26	-0.43	0.05	0.24
0.46	-0.13	0.21	0.04	0.38	0.72	-0.24	0.01	0.02
0.54	-0.23	0.57	0.27	-0.21	-0.37	0.26	-0.02	-0.08
0.28	0.11	-0.51	0.15	0.33	0.03	0.67	-0.06	-0.26
0.00	0.19	0.10	0.02	0.39	-0.30	-0.34	0.45	-0.62
0.01	0.44	0.19	0.02	0.35	-0.21	-0.15	-0.76	0.02
0.02	0.62	0.25	0.01	0.15	0.00	0.25	0.45	0.52
0.08	0.53	0.08	-0.03	-0.60	0.36	0.04	-0.07	-0.45

$\hat{X} =$

T

0.22	-0.11
0.20	-0.07
0.24	0.04
0.40	0.06
0.64	-0.17
0.27	0.11
0.27	0.11
0.30	-0.14
0.21	0.27
0.01	0.49
0.04	0.62
0.03	0.45

S

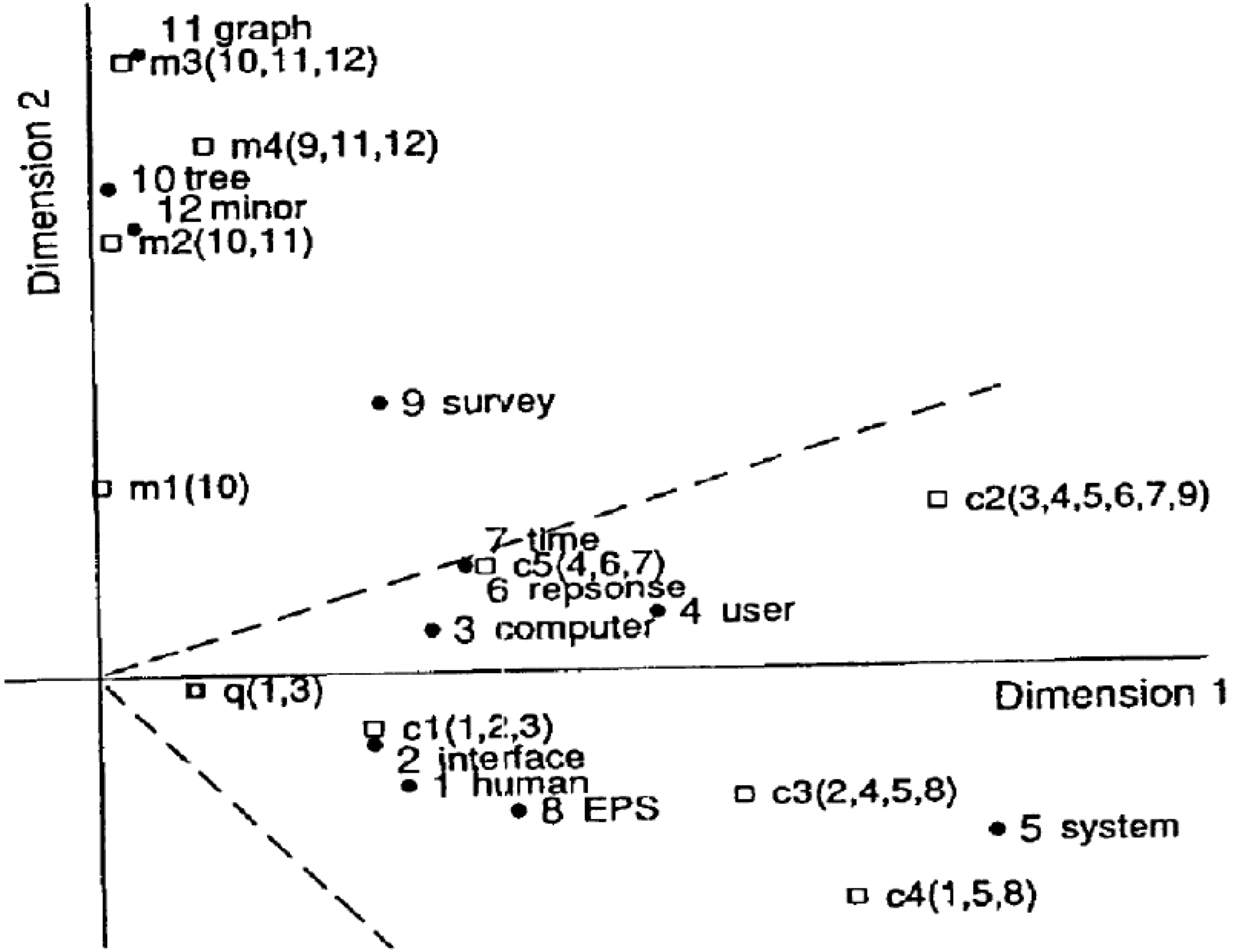
3.34	
	2.54

D′

0.20	0.61	0.46	0.54	0.28	0.00	0.02	0.02	0.08
-0.06	0.17	-0.13	-0.23	0.11	0.19	0.44	0.62	0.53

$\hat{X} =$

	c1	c2	c3	c4	c5	m1	m2	m3	m4
human	0.16	0.40	0.38	0.47	0.18	-0.05	-0.12	-0.16	-0.09
interface	0.14	0.37	0.33	0.40	0.16	-0.03	-0.07	-0.10	-0.04
computer	0.15	0.51	0.36	0.41	0.24	0.02	0.06	0.09	0.12
user	0.26	0.84	0.61	0.70	0.39	0.03	0.08	0.12	0.19
system	0.45	1.23	1.05	1.27	0.56	-0.07	-0.15	-0.21	-0.05
response	0.16	0.58	0.38	0.42	0.28	0.06	0.13	0.19	0.22
time	0.16	0.58	0.38	0.42	0.28	0.06	0.13	0.19	0.22
EPS	0.22	0.55	0.51	0.63	0.24	-0.07	-0.14	-0.20	-0.11
survey	0.10	0.53	0.23	0.21	0.27	0.14	0.31	0.44	0.42
trees	-0.06	0.23	-0.14	-0.27	0.14	0.24	0.55	0.77	0.66
graph	-0.06	0.34	-0.15	-0.30	0.20	0.31	0.69	0.98	0.85
minors	-0.04	0.25	-0.10	-0.21	0.15	0.22	0.50	0.71	0.62



Dimension 2
Dimension 1
11 graph
m3(10,11,12)
m4(9,11,12)
10 tree
12 minor
m2(10,11)
9 survey
m1(10)
c2(3,4,5,6,7,9)
7 time
c5(4,6,7)
6 repsonse
4 user
3 computer
q(1,3)
c1(1,2,3)
2 interface
1 human
8 EPS
c3(2,4,5,8)
5 system
c4(1,5,8)

Conclusions

- Latent semantic indexing provides an interesting conceptualization of the IR problem
- Thus, it has its value as a new theoretical framework
- From a practical point of view, the latent semantic indexing model has not yielded encouraging results