9/20/24

Thm: $A\vec{x} = b$ be consistent & let $\vec{p}$ be a sltn. Then the solution set of $A\vec{x} = \vec{b}$ has the form $\vec{p} + \vec{v}_h$ is any solution of $A\vec{x} = \vec{0}$

1.7 Linear independence

ex: $x_1\begin{pmatrix}1\\2\\3\end{pmatrix} + x_2\begin{pmatrix}4\\5\\6\end{pmatrix} + x_3\begin{pmatrix}2\\1\\0\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix}$ $x_1 = x_2 = x_3 = 0$ is a sltn

Are there others

Def: A set of vectors in $\mathbb{R}^n$ is linearly independent (LI) if the only solution of $x_1\vec{v}_1 + \dots x_p\vec{v}_p = \vec{0}$ is $x_1 = x_2 = \dots 0$.

the set $\{\vec{v}_1 \dots \vec{v}_p\}$ is linearly dependent (LD) if coefficients $c_1, \dots, c_p$ not all zero s.t. $c_1\vec{v}_1 + \dots + c_p\vec{v}_p = \vec{0}$

ex: Let $v_1 = \begin{pmatrix}1\\2\\3\end{pmatrix}$, $v_2 = \begin{pmatrix}4\\5\\6\end{pmatrix}$, & $v_3 = \begin{pmatrix}2\\1\\0\end{pmatrix}$. LI?

$$\begin{pmatrix}1 & 4 & 2 & 0\\2 & 5 & 1 & 0\\3 & 6 & 0 & 0\end{pmatrix} \to \begin{pmatrix}1 & 4 & 2 & 0\\0 & -3 & -3 & 0\\0 & 0 & 0 & 0\end{pmatrix}$$

∞ solutions

NOT LI

How are they LD?

$$\begin{pmatrix}1 & 0 & -2 & 0\\0 & 1 & 1 & 0\\0 & 0 & 0 & 0\end{pmatrix} \Rightarrow \vec{x} = \begin{pmatrix}2\\-1\\1\end{pmatrix}$$

$x_3 = 1$ choose

$2\vec{v}_1 - \vec{v}_2 + \vec{v}_3 = \vec{0}$

$\vec{v_2} = 2\vec{v_1} + \vec{v_3}$

Fact: The columns of a matrix A are <u>LI</u>
$\iff$ $A\vec{x} = \vec{0}$ has only the trivial solution $\vec{x} = \vec{0}$

note: A set w/ one vector, it's LI unless $\vec{v} = 0$

A set w/ 2 vectors is LI as long as the first vector is not a scalar multiple of the second one

ex: $\left\{ \begin{pmatrix} 3 \\ 1 \end{pmatrix}, \begin{pmatrix} 6 \\ 2 \end{pmatrix} \right\} = LD$

ex $\left\{ \begin{pmatrix} 3 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \end{pmatrix} \right\} = LI$

thm: A set $S = \{\vec{v_1} \dots \vec{v_p}\}$ for $p \geq 2$ is LD $\iff$ at least one of the $\vec{v_i}$ for $1 \leq i \leq p$ is a linear combination of the other vecs

ex: $\vec{u} = \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix}$ $\vec{v} = \begin{pmatrix} 1 \\ 6 \\ 0 \end{pmatrix}$ what is span$\{\vec{u}, \vec{v}\}$?

Not scalars of each other

In xy plane

$\vec{u}$ & $\vec{v}$ both lie in the plane $x_3 = 0$ in $\mathbb{R}^3$

span$\{\vec{u}, \vec{v}\} = \{(x_1, x_2, x_3) : x_3 = 0\}$ = "xy plane"

Thm: $\{\vec{v}_1, \dots, \vec{v}_p\}$ in $\mathbb{R}^n$ is LD if $p > n$

Proof: let $A = (\vec{v}_1, \vec{v}_2 \dots \vec{v}_p)$. This is $(n \cdot p)$. If $p > n$ there must be a free variable in $(A \,\vdots\, 0)$ $\square$

ex: $\left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 6 \end{pmatrix}, \begin{pmatrix} 8 \\ -17 \end{pmatrix}, \begin{pmatrix} 9 \\ 23 \end{pmatrix}, \begin{pmatrix} 6 \\ 10^{23} \end{pmatrix} \right\}$

Span $\{ \dots\dots \} = \mathbb{R}^2$

Thm: If $S = \{\vec{v}_1 \dots \vec{v}_p\}$ contains the 0 vector, then S is LD

✓Prove yourself ✓