15.
$$\begin{aligned} x_1 \qquad + 3x_3 \qquad &= 2 \\ x_2 \qquad - 3x_4 &= 3 \\ -2x_2 + 3x_3 + 2x_4 &= 1 \\ 3x_1 \qquad\qquad + 7x_4 &= -5 \end{aligned}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 3 & 0 & 2 \\ 0 & 1 & 0 & -3 & 3 \\ 0 & -2 & 3 & 2 & 1 \\ 3 & 0 & 0 & 7 & -5 \end{array}\right] \xrightarrow{R_4 - 3R_1}$$

3 - 3
0-0
0-9
7-0
-5-6

$$\left[\begin{array}{cccc|c} 1 & 0 & 3 & 0 & 2 \\ 0 & 1 & 0 & -3 & 3 \\ 0 & -2 & 3 & 2 & 1 \\ 0 & 0 & -9 & 7 & -11 \end{array}\right] \xrightarrow{R_3 + 2R_2}$$

0+0 , -2+2(1), 3+0 , 2+2(-3), 1+2(3)

$$\left[\begin{array}{cccc|c} 1 & 0 & 3 & 0 & 2 \\ 0 & 1 & 0 & -3 & 3 \\ 0 & 0 & 3 & -4 & 7 \\ 0 & 0 & -9 & 7 & -11 \end{array}\right] \to \frac{R_3}{R_1}$$

$\frac{0}{1}$ $\frac{0}{0}$ $\frac{3}{3}$ $\frac{-4}{0}$ $\frac{7}{2}$

$$\left[\begin{array}{cccc|c} 1 & 0 & 3 & 0 & 2 \\ 0 & 1 & 0 & -3 & 3 \\ 0 & 0 & 1 & 0 & 3.5 \\ 0 & 0 & -9 & 7 & -11 \end{array}\right] R_1 + \tfrac{1}{3}R_4$$

$3 + \frac{-9}{3}$ 3+-3 0

1+0 , 0+0 , 3+-3 , $0+\frac{7}{3}$, $2+\frac{-11}{3}$

$\frac{6}{3}$ $-\frac{11}{3}$ $\frac{-5}{3}$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & \frac{7}{3} & \frac{-5}{3} \\ 0 & 1 & 0 & -3 & 3 \\ 0 & 0 & 1 & 0 & 3.5 \\ 0 & 0 & -9 & 7 & -11 \end{array}\right] \xrightarrow{R_4 + 9R_3}$$

0+0 , 0+0, -9+9, 7+0, -11+31.5
205

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & \frac{7}{3} & \frac{-5}{3} \\ 0 & 1 & 0 & -3 & 3 \\ 0 & 0 & 1 & 0 & 3.5 \\ 0 & 0 & 0 & 7 & 20.5 \end{array}\right] \frac{R_4}{7} \to$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & \frac{7}{3} & \frac{-5}{3} \\ 0 & 1 & 0 & -3 & 3 \\ 0 & 0 & 1 & 0 & 3.5 \\ 0 & 0 & 0 & 1 & \frac{41}{14} \end{array}\right] R_2 + 3R_4$$

$20 \cdot \frac{1}{2}$
41/2 · 7

$\frac{41}{14}$

$\frac{41}{2} \cdot \frac{1}{7}$

-3+3(1)
-3+3 = 0

$3 + 3\left(\frac{41}{4}\right)$
$\frac{12}{4} + \frac{123}{4}$

$\frac{135}{4}$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & \frac{7}{3} & \frac{-5}{3} \\ 0 & 1 & 0 & 0 & \frac{135}{4} \\ 0 & 0 & 1 & 0 & 3.5 \\ 0 & 0 & 0 & 1 & \frac{41}{14} \end{array}\right]$$

$R_1 - \frac{7}{3}R_4$

$\frac{7}{3} - \frac{7}{3}$

$\frac{-5}{3}$ -

$\left(\frac{7}{3}\right)\left(\frac{41}{14}\right)$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & \frac{287}{42} \\ 0 & 1 & 0 & 0 & \frac{135}{4} \\ 0 & 0 & 1 & 0 & 3.5 \\ 0 & 0 & 0 & 1 & \frac{41}{14} \end{array}\right]$$

$\frac{41}{287}$ $\frac{287}{42}$

287, 14, 3, 42

$$\boxed{x_1 = \frac{287}{42} \quad x_2 = \frac{135}{4} \quad x_3 = \frac{7}{2} \quad x_4 = \frac{41}{4}}$$

In Exercises 19–22, determine the value(s) of h such that the matrix is the augmented matrix of a consistent linear system.

19. $\begin{bmatrix} 1 & h & : & 4 \\ 3 & 6 & : & 8 \end{bmatrix}$

[redacted]

For what value of "h" is the System Consistent?

$R_1 - \frac{1}{3}R_2$

$$\begin{bmatrix} 0 & h-2 & : & \frac{4}{3} \\ 3 & 6 & : & 8 \end{bmatrix}$$

$\frac{12}{3} \cdot \frac{8}{3}$

$0x_1 + (h-2)x_2 = \frac{4}{3}$

$\frac{4}{3} \cdot \frac{1}{(h-2)}$

$\frac{(h-2)x_2}{h-2} = \frac{\frac{4}{3}}{h-2}$

$\frac{4}{3h-6}$.

$x_2 = \frac{4}{3h-6}$

$3h-6=0$

$\frac{3h}{3} = \frac{6}{3}$

$h=2$

$h \neq 2$

$\boxed{\text{System is consistent } \forall h \text{ except } h=2}$

21. $\begin{bmatrix} 1 & 3 & -2 \\ -4 & h & 8 \end{bmatrix}$ $R_2 + 4R_1$

$8+$

$x_1 + 3x_2 = -2$

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & h+12 & 0 \end{bmatrix}$$

$(h+12)x_2 = 0$

$h \neq -12$

But if

$h=12$, $0=0$, so infinite, therefore All h

In Exercises 1 and 2, determine which matrices are in reduced echelon form and which others are only in echelon form.

1. a. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$ b. $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Non zero Digits above Pivot Point

RREF

REF

Staircase & zeros Above Pivot Points Reduced to ones

Row reduce the matrices in Exercises 3 and 4 to reduced echelon form. Circle the pivot positions in the final matrix and in the original matrix, and list the pivot columns.

3. $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 4 & 5 & 6 & 7 \\ 6 & 7 & 8 & 9 \end{bmatrix}$ [redacted]

$R_2 - 4R_1 \downarrow$

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -3 & -6 & -9 \\ 6 & 7 & 8 & 9 \end{bmatrix}$$

$R_3 - 6R_1$

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -3 & -6 & -9 \\ 0 & -5 & -10 & -15 \end{bmatrix}$$

$R_2 / -3$

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & -5 & -10 & -15 \end{bmatrix}$$

$R_3 + 5R_2$

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_1 - 2R_2$

$$\begin{bmatrix} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$