$-R_1$

$$\begin{pmatrix} -1 & 6 & 6 \\ 3 & -8 & 3 \\ 1 & -2 & 6 \\ 1 & -4 & -3 \end{pmatrix} \xrightarrow{R_2 - 3R_3} \begin{bmatrix} 1 & -6 & -6 \\ 0 & -2 & -15 \\ 1 & -2 & 6 \\ 1 & -4 & -3 \end{bmatrix} \xrightarrow{R_3 - R_4} \begin{pmatrix} 1 & -6 & -6 \\ 0 & 2 & -15 \\ 0 & 2 & 9 \\ 1 & -4 & -3 \end{pmatrix} \xrightarrow{R_4 - R_1} \begin{pmatrix} 1 & -6 & -6 \\ 0 & 2 & -15 \\ 0 & 2 & 9 \\ 0 & 2 & 3 \end{pmatrix} \xrightarrow{R_2/2} \begin{pmatrix} 1 & -6 & -6 \\ 0 & 1 & -7/2 \\ 0 & 2 & 9 \\ 0 & 2 & 3 \end{pmatrix}$$

$✓$

$R_3 - R_4$

$$\begin{pmatrix} 1 & -6 & -6 \\ 0 & 1 & -7/2 \\ 0 & 0 & 6 \\ 0 & 2 & 3 \end{pmatrix} \xrightarrow{R_3/6} \begin{pmatrix} 1 & -6 & -6 \\ 0 & 1 & -7/2 \\ 0 & 0 & 1 \\ 0 & 2 & 3 \end{pmatrix}$$

$-2 - -4$ $\quad -2+4$ $\quad 6 - -3$ $\quad -4 - -6$ $\quad -3 - -6$ $\quad -3+6$

$$\begin{pmatrix} -1 & 6 & 6 \\ 3 & -8 & 3 \\ 1 & -2 & 6 \\ 1 & -4 & -3 \end{pmatrix}$$

$-6 -24 -2 -4$

$-32 -4$

$$\frac{-36}{12} = -3 \begin{bmatrix} -1 \\ 3 \\ 1 \\ 1 \end{bmatrix} \quad \begin{pmatrix} 3 \\ -9 \\ -3 \\ -3 \end{pmatrix} \quad \begin{pmatrix} 6 \\ -8 \\ -2 \\ -4 \end{pmatrix} - \begin{bmatrix} 3 \\ -9 \\ -3 \\ -3 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 1 \\ -1 \end{bmatrix} = v_2$$

Pivot in each

$\cancel{-6} +9 \cancel{+6} -3$

$1\,9\,1\,1$

$$\frac{6}{12} \qquad \frac{x_3 \cdot v_1}{v_1 \cdot v_1} \cdot v_1 = \frac{1}{2} \begin{bmatrix} -1 \\ 3 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1/2 \\ 3/2 \\ 1/2 \\ 1/2 \end{bmatrix} + \frac{x_3 \cdot v_2}{v_2 \cdot v_2} \cdot v_2 = \frac{30}{12} = \frac{5}{2} \begin{bmatrix} 3 \\ 1 \\ 1 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} -1/2 \\ 3/2 \\ 1/2 \\ 1/2 \end{bmatrix} + \begin{pmatrix} 15/2 \\ 5/2 \\ 5/2 \\ -5/2 \end{pmatrix}$$

$$\begin{pmatrix} 7 \\ 4 \\ 3 \\ -2 \end{pmatrix}$$

$$\begin{matrix} 6 \\ 3 \\ 6 \\ -3 \end{matrix} - \begin{matrix} 7 \\ 4 \\ 3 \\ -2 \end{matrix} \quad \begin{bmatrix} -1 \\ -1 \\ 3 \\ -1 \end{bmatrix} = v_3$$

$$\left\{ \begin{bmatrix} -1 \\ 3 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \\ 3 \\ -1 \end{bmatrix} \right\}$$

1. Find an orthonormal basis for V

$\begin{matrix}2\\1\\2\\1\end{matrix} - \cancel{\frac{4}{2} = 2\begin{bmatrix}1\\0\\1\\0\end{bmatrix}}$ $\begin{bmatrix}2\\0\\2\\0\end{bmatrix}$

$\begin{matrix}2\\1\\2\\1\end{matrix} - \begin{matrix}2\\0\\2\\0\end{matrix}$ $\begin{bmatrix}0\\1\\0\\1\end{bmatrix}$

$y \cdot y = 2$

$\begin{bmatrix}1 & 0\\0 & 1\\1 & 0\\0 & 1\end{bmatrix}\begin{bmatrix}\frac{1}{\sqrt{2}}\\0\\1/\sqrt{2}\\0\end{bmatrix}\begin{bmatrix}0\\1/\sqrt{2}\\0\\1/\sqrt{2}\end{bmatrix}$

$$\begin{bmatrix}5-\lambda & 0 & 0\\0 & 1-\lambda & 2\\0 & 2 & 4-\lambda\end{bmatrix}$$

$5-\lambda\begin{vmatrix}1-\lambda & 2\\2 & 4-\lambda\end{vmatrix}$

$(1-\lambda)(4-\lambda) - 4$

$\cancel{4-\lambda-4\lambda+\lambda^2-4}$

$5-\lambda\,(\lambda^2-5\lambda)$

$\lambda(\lambda-5)(5-\lambda)$ $A = \begin{bmatrix}5 & 0 & 0\\0 & 1 & 2\\0 & 2 & 4\end{bmatrix}$

$\lambda=0, \lambda=5\ \lambda=5$

$\lambda = 0, 5$ twice

$\lambda=0\ \left\{\begin{bmatrix}0\\-2\\1\end{bmatrix}\right\}$

$x_3\begin{bmatrix}0\\-2\\1\end{bmatrix}$

$x_1=0$

$x_2=-2x_3$

x_3 free

$A-\lambda 0$ $\begin{bmatrix}0 & 0 & 0 & 0\\0 & 1 & 2 & 0\\0 & 2 & 4 & 0\end{bmatrix}$

0 0 0
0

$$A - 5I = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -4 & 2 & 0 \\ 0 & 2 & -1 & 0 \end{bmatrix} \xrightarrow[R_3/2]{R_2/2} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -2 & 1 & 0 \\ 0 & 1 & -\frac{1}{2} & 0 \end{bmatrix}$$

x_1 free
$2x_2 = x_3$
$x_2 = \frac{1}{2}x_3$

$$x_1\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1/2 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix}$$

Yes A is diagonalizable bc eigenbasis had dim=3

$P(\lambda) =$ 2 mult 2, and 1

E-vals of A = 2, 2, 1

$Ax = \lambda x \to (A - \lambda I)x = 0 \qquad Ax - 3\cancel{I}x = \lambda v - 3v$ or $v(\lambda - 3)$

Midterm 2

$$A = \begin{bmatrix} 1 & 2 & -5 & 0 & -1 \\ 2 & 5 & -8 & 4 & 3 \\ -3 & -9 & 9 & -7 & -2 \\ 3 & 10 & -7 & 11 & 7 \end{bmatrix} \xrightarrow{R_3+R_4} \begin{bmatrix} 1 & 2 & -5 & 0 & -1 \\ 2 & 5 & -8 & 4 & 3 \\ 0 & -1 & 2 & 4 & 9 \\ 3 & 10 & -7 & 11 & 7 \end{bmatrix} \xrightarrow[R_3-3R_1]{R_2-2R_1} \begin{bmatrix} 1 & 2 & -5 & 0 & -1 \\ 0 & 1 & 2 & 4 & 5 \\ 0 & -1 & 2 & 4 & 9 \\ 0 & 4 & 8 & 11 & 10 \end{bmatrix} \xrightarrow{R_1-2R_2} \begin{bmatrix} 1 & 0 & -9 & -8 & -11 \\ 0 & 1 & 2 & 4 & 5 \\ 0 & -1 & 2 & 4 & 9 \\ 0 & 4 & 8 & 11 & 10 \end{bmatrix}$$

$$2\begin{bmatrix} 1 & 0 & -9 & -8 & -11 \\ 0 & 1 & 2 & 4 & 5 \\ 0 & -1 & 2 & 4 & 9 \\ 0 & 4 & 8 & 11 & 10 \end{bmatrix} \xrightarrow[R_4-4R_2]{R_3+R_2} \begin{bmatrix} 1 & 0 & -9 & -8 & -11 \\ 0 & 1 & 2 & 4 & 5 \\ 0 & 0 & 4 & 8 & 14 \\ 0 & 0 & 0 & -5 & -10 \end{bmatrix} \xrightarrow{R_3/2} \begin{bmatrix} 1 & 0 & -9 & -8 & -11 \\ 0 & 1 & 2 & 4 & 5 \\ 0 & 0 & 2 & 4 & 7 \\ 0 & 0 & 0 & -5 & -10 \end{bmatrix} \xrightarrow{R_2-R_3} \begin{bmatrix} 1 & 0 & -9 & -8 & -11 \\ 0 & 1 & 0 & 0 & -2 \\ 0 & 0 & 2 & 4 & 7 \\ 0 & 0 & 0 & -5 & -10 \end{bmatrix} \xrightarrow[R_4/-5]{R_3/2} \begin{bmatrix} 1 & 0 & -9 & -8 & -11 \\ 0 & 1 & 0 & 0 & -2 \\ 0 & 0 & 1 & 2 & 3/2 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix} \xrightarrow{R_1+4R_3} \begin{bmatrix} 1 & 0 & -5 & 0 & -5 \\ 0 & 1 & 0 & 0 & -2 \\ 0 & 0 & 1 & 2 & 3/2 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & -9 & 0 & 5 \\ 0 & 1 & 2 & 0 & -3 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{matrix} 1 & 2 & 0 \\ 2 & 5 & 4 \\ -3 & -9 & -7 \\ 3 & 10 & 11 \end{matrix} = \text{Span}\left\{ \begin{bmatrix} 1 \\ 2 \\ -3 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \\ -9 \\ 10 \end{bmatrix}, \begin{bmatrix} 0 \\ 4 \\ -7 \\ 11 \end{bmatrix} \right\} = \text{Col} A$$

$$\begin{bmatrix} 1 & 0 & -9 & 0 & 5 & 0 \\ 0 & 1 & 2 & 0 & -3 & 0 \\ 0 & 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$x_1 = 9x_3 - 5x_5$
$x_2 = -2x_3 + 3x_5$
$x_4 = -2x_5$
x_5 free x_3 free

$x_5 = 1$

$$x_5\begin{bmatrix} -5 \\ 3 \\ 0 \\ -2 \\ 1 \end{bmatrix} + x_3\begin{bmatrix} 9 \\ -2 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 3 & -1 & 2 & -5 \\ 0 & 5 & -3 & -6 \\ -6 & 7 & -7 & 4 \\ 5 & -8 & 0 & 9 \end{bmatrix} \xrightarrow{R_3+2R_1} \begin{bmatrix} 3 & -1 & 2 & -5 \\ 0 & 5 & -3 & -6 \\ 0 & 5 & -3 & -6 \\ 5 & 8 & 0 & 9 \end{bmatrix}$$

- Linear dependence
det = 0
also
Means non-invertible

$$\begin{vmatrix} 1 & -5 & 1 \\ -1 & 7 & 1 \\ 3 & 8 & 0 \end{vmatrix} \quad \cancel{0\begin{bmatrix} 1 & -5 \\ -1 & 7 \end{bmatrix}} - 1\begin{bmatrix} 1 & -5 \\ 3 & 8 \end{bmatrix} + 1\begin{bmatrix} -1 & 7 \\ 3 & 8 \end{bmatrix}$$

$-8 - -21$

$8 - (-5 \cdot 3)$

$8 + 15$ $\quad -8 - 21$ $\quad -8 + 21$

$-23 + 13$ $\quad -29$ $\quad 13$

$\det \neq 0$

$8 -$

so col vectors r Linearly Independent and form a basis for $\mathbb{R}^3$

$A = \begin{pmatrix} 4 & -1 & 6 \\ 2 & 1 & 6 \\ 2 & -1 & 8 \end{pmatrix}$ find e-basis of $\lambda = 2$

$$\begin{matrix} 2 & -1 & 6 \\ 2 & -1 & 6 \\ 2 & -1 & 6 \end{matrix} \quad \begin{bmatrix} 2 & -1 & 6 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\frac{2x_1}{2} = \frac{x_2 - 6x_3}{2}$

$x_1 = \frac{1}{2}x_2 - 3x_3$

x_2 & x_3 free

$x_2\begin{bmatrix} 1/2 \\ 1 \\ 0 \end{bmatrix} + x_3\begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}$

$\lambda = 2$: $\left\{\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}\right\}$

$$\left(\begin{matrix} 1 & -3 & 1 \\ 3 & -5 & 1 \\ 2 & 2 & 1 \end{matrix}\right.\left|\begin{matrix} 0 \\ 0 \\ 0 \end{matrix}\right. \xrightarrow{R_2-3R_1} \begin{pmatrix} 1 & -3 & 1 & 0 \\ 0 & 4 & -2 & 0 \\ 2 & 2 & 1 & 0 \end{pmatrix}$$

$$\xrightarrow{R_3-2R_1} \begin{pmatrix} 1 & -3 & 1 & 0 \\ 0 & 4 & -2 & 0 \\ 0 & 8 & -1 & 0 \end{pmatrix} \xrightarrow[R_3/2]{R_2/2} \begin{pmatrix} 1 & -3 & 1 & 0 \\ 0 & 2 & -1 & 0 \\ 0 & 4 & -\frac{1}{2} & 0 \end{pmatrix} \text{ LI}$$

$-5 - 3(-3)$

$-5 + 9$ $\quad 2 - 2(-3)$ $\quad 1 - 2$

$2 + 6$

$$1\begin{bmatrix} -5 & 1 \\ 2 & 1 \end{bmatrix} - 3\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix} + 1\begin{bmatrix} 3 & -5 \\ 2 & 2 \end{bmatrix}$$

-7 $\quad 1$ $\quad 6 + 10 = 16$

$-7-3+16$

$-10+16=6$ LI bc det $\neq 0$, not eigenvalue

$Ax=\lambda x \quad A^3x = \lambda^3 x$

$\begin{bmatrix} 1 & 2 & 6 \\ 3 & 6 & 0 \end{bmatrix}$ Yes LD

$\begin{bmatrix} -3 & 1 \\ -3 & 8 \end{bmatrix}\begin{bmatrix} 1 \\ 4 \end{bmatrix} = \begin{matrix} -3+4 \\ -3+32 \end{matrix} \quad \begin{bmatrix} 1 \\ 29 \end{bmatrix}$ NO