11/6 Finish 5.3, 5.5-ish, 6.1?

Thm: If $A \in Mat(n,n)$ has n distinct evals, then A is diagable

Q: what if the eigenvalues are not distinct ex I_n

Thm: Let $A \in Mat(n,n)$ with distinct eigenvalues $\lambda_1 \dots \lambda_p$ all of which have multiplicity $mult(\lambda_i) \geq 1$ and $p \leq n$

(i) For all $1 \leq k \leq p$, $\dim E(A, \lambda_k) \leq mult(\lambda_k)$

(ii) A is diagable iff $\sum_{k=1}^{p} \dim E(A, \lambda_k) = n \iff p_A(\lambda) := \det(A - \lambda I)$ factors into linear factors ⟨⟨ $(\lambda - \lambda_1)^{n_1} (\lambda - \lambda_2)^{n_2} \dots (\lambda - \lambda)^{n_p}$

$\iff \dim E = mult(\lambda_k)$ for all $1 \leq k \leq p$

(iii) If a is diagable & $\mathcal{B}$ is a basis for $E(A, \lambda_k)$ for each k, then $\{\mathcal{B}_1, \dots \mathcal{B}_k\}$ is a basis for $\mathbb{R}^n$

Triangular

ex: Diagonalize $A = \begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 1 & 4 & -3 & 0 \\ -1 & -2 & 0 & -3 \end{bmatrix}$ Eigenvalues of 5 & -3 both multiplicity 2

Solve (DIY): $E(A,5) := Nul(A - 5I) = span\left\{\begin{pmatrix} -8 \\ 4 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -16 \\ 4 \\ 0 \\ 1 \end{pmatrix}\right\}$

and $E(A,-3) := Nul(A + 3I) = span\left\{\begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}\right\}$

thus: $A = PDP^{-1}$, where $D = \begin{pmatrix} 5 & & & 0 \\ & 5 & & \\ & & -3 & \\ 0 & & & -3 \end{pmatrix}$ & $P = \begin{pmatrix} -8 & -16 & 0 & 0 \\ 4 & 4 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}$

5.5-ish **Complex Eigenvalues**

Def: $\mathbb{C} := \{ a+bi : a, b \in \mathbb{R} \}$ & $i := \sqrt{-1}$

☆ **Thm:** Fundamental theorem of Algebra: (1) Every quadratic polynomial over $\mathbb{C}$ has 2 roots

(2) Every degree n polynomial in $\mathbb{C}$ factors completely: $p(z) = (z-\alpha_1)(z-\alpha_2)\dots(z-\alpha_n)$, roots of p

ex: $p(z) = z^2+1 = (z-i)(z+i) = (z^2+iz-iz-i^2) = z^2+1$

ex: $R_\theta = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \rightsquigarrow \det(R_\theta - \lambda I) = \det\begin{pmatrix} \cos\theta-\lambda & -\sin\theta \\ \sin\theta & \cos\theta-\lambda \end{pmatrix} = — — \exists$

$(\cos\theta-\lambda)^2 + \sin^2\theta = \cos^2\theta - 2\lambda\cos\theta + \sin^2\theta + \lambda^2 \dots = \lambda^2 - 2\lambda\cos\theta + 1$ $\quad \lambda = \dfrac{2\cos\theta \pm \sqrt{4\cos^2\theta - 4}}{2}$

$\cos\theta \pm \sqrt{-\sin^2\theta} = \cos\theta \pm i\sin\theta$ or $e^{i\theta}$ $\quad$ θ can be any $n\cdot\pi$ ex $\pi, 2\pi$

$\rightsquigarrow$ The $\mathbb{R}$-evals are ± 1, but there are no evecs in $\mathbb{R}^2$

$\hookrightarrow$ (@ $\theta = 0, \pi$)

Case, $\theta = \pi/2$: $R_{\frac{\pi}{2}} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \Longleftrightarrow \lambda = \pm i$

Solve: $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\vec{v} = i\vec{v} \Longleftrightarrow \left(\begin{array}{cc|c} -i & -1 & 0 \\ 1 & -i & 0 \end{array}\right) \xrightarrow{iR_1} \left(\begin{array}{cc|c} 1 & -i & 0 \\ 1 & -i & 0 \end{array}\right) \xrightarrow[R_2-R_1]{\text{then}} \left(\begin{array}{cc|c} 1 & -i & 0 \\ 0 & 0 & 0 \end{array}\right)$

Solution set: $\vec{v} = v_2\begin{pmatrix} i \\ 1 \end{pmatrix} = \operatorname{span}\left\{\begin{pmatrix} i \\ 1 \end{pmatrix}\right\} = E(R_{\frac{\pi}{2}}, i)$

DIY: $E(R_{\frac{\pi}{2}}, -i) = \operatorname{span}\left\{\begin{pmatrix} -i \\ 1 \end{pmatrix}\right\}$

Hello