1. 3.2.5.
2. 3.2.7.
3. 3.2.11.
4. 3.2.15.
5. 3.2.17.
6. 3.2.19.
7. 3.2.25.
8. 3.2.27, 29, 31, 33 (3.2.27 in 5E).
9. 3.2.39 (3.2.33 in 5E).
10. 4.1.1.
11. 4.1.5.
12. 4.1.7.

1. 3.2.5. $\begin{vmatrix}1&5&-4\\-1&-4&5\\-2&-8&7\end{vmatrix} \xrightarrow[R_3+2R_1]{R_2+R_1} \begin{vmatrix}1&5&-4\\0&-1&1\\0&-2&-1\end{vmatrix} \xrightarrow{R_3-2R_2} \begin{vmatrix}1&5&-4\\0&-1&1\\0&0&-3\end{vmatrix}$ $1\cdot-1\cdot-3 = \boxed{-3}$ Also means invertible

$\boxed{\text{RAYYAN SYED}}$

2. 3.2.7. $\begin{vmatrix}1&3&0&2\\-2&-5&7&4\\3&5&2&1\\1&-1&2&-3\end{vmatrix} \xrightarrow[R_3-3R_1]{R_2+2R_1} \xrightarrow{R_4-R_1} \begin{vmatrix}1&3&0&2\\0&1&7&8\\0&-4&2&-5\\0&-4&2&-5\end{vmatrix}$ $\boxed{\text{2 identical rows so det} = 0}$ Means not invertible

3. 3.2.11. $\begin{vmatrix}3&4&-3&-1\\3&0&1&-3\\-6&0&-4&3\\6&8&-4&-1\end{vmatrix} \xrightarrow[R_2-R_1]{R_3+R_4} \begin{vmatrix}3&4&-3&-1\\0&-4&4&-2\\0&8&-8&2\\6&8&-4&-1\end{vmatrix} \xrightarrow{R_4-2R_1} \begin{vmatrix}3&4&-3&-1\\0&-4&4&-2\\0&8&-8&2\\0&0&2&1\end{vmatrix}$ $A_{11} = -3\begin{vmatrix}-4&4&-2\\8&-8&2\\0&2&1\end{vmatrix} \xrightarrow{R_2+2R_1} \begin{vmatrix}-4&4&-2\\0&0&-2\\0&2&1\end{vmatrix}$ R_2 swap R_3 $\begin{vmatrix}-4&4&-2\\0&2&1\\0&0&-2\end{vmatrix}$ $\boxed{-3(-4\cdot2\cdot-2) = -48}$

4. 3.2.15. $\begin{vmatrix}a&b&c\\d&e&f\\g&h&i\end{vmatrix} = 7.$ $\begin{vmatrix}a&b&c\\d&e&f\\3g&3h&3i\end{vmatrix}$ If one row is multiplied by k to produce B, then det B = k det A, $\boxed{\text{so } 7\times3 = 21}$

5. 3.2.17. $\begin{vmatrix}a+d&b+e&c+f\\d&e&f\\g&h&i\end{vmatrix}$ If a multiple of one row of A is added to another row to produce a matrix B, then $\boxed{\det B = \det A = 7}$

6. 3.2.19. 19. $\begin{vmatrix}a&b&c\\2d+a&2e+b&2f+c\\g&h&i\end{vmatrix}$ $2R_2+R_1$ so det = 14 because of the 2 rules listed in 3.2.15 & 3.2.17

7. 3.2.25. 25. $\begin{bmatrix}7\\-4\\-6\end{bmatrix}, \begin{bmatrix}-8\\5\\7\end{bmatrix}, \begin{bmatrix}7\\0\\-5\end{bmatrix}$ $7\begin{vmatrix}5&0\\7&-5\end{vmatrix} - -8\begin{vmatrix}-4&0\\-6&-5\end{vmatrix} + 7\begin{vmatrix}-4&5\\-6&7\end{vmatrix}$

-25 $\quad 20 \quad$ $-28-(-6(5))$ $= 2$

$7(-25) - (-8(20)) + 7(2)$

$-175 + 160 + 14$

$-15 + 14 = -1$

$\boxed{-1 \neq 0 \text{ so LI}}$

8. 3.2.27, 29, 31, 33

27. If the row replacement operation is simply taking one row & adding to another, then T

29. T, if columns of A are LI, then det ≠ 0

31. F, one interchange yields the negative determinant. so 3 would also be the negative determinant

33. F, you would have to add matrices A & B together, completely changing the determinant, so adding the det A + det B is not the same. However det(AB) = det A · det B

9. 3.2.39 Let $A = \begin{bmatrix}6&1\\3&2\end{bmatrix}$, $B = \begin{bmatrix}4&3\\1&2\end{bmatrix}$ $AB = \begin{bmatrix}25&20\\14&13\end{bmatrix}$ $(25\cdot13) - (14\cdot20) = 45$

$\det A = 12-3 = 9$ $\quad \det B = 8-3 = 5$ $\quad \det A \cdot \det B = \det AB = 45 = \det BA$

$9\cdot5 = 45$

10. 4.1.1.

a. The sum of u & v, denoted by $u+v$ is in V.

1. Let V be the first quadrant in the xy-plane; that is, let

$$V = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} : x \geq 0, y \geq 0 \right\}$$

a. If **u** and **v** are in V, is $\mathbf{u}+\mathbf{v}$ in V? Why?

b. Find a specific vector **u** in V and a specific scalar c such that $c\mathbf{u}$ is *not* in V. (This is enough to show that V is *not* a vector space.)

If they are individually & $\not\leq 0$ in V, their sums are too

Let $u = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $c = -1$

$cu = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$ $x \not\geq 0$, $y \not\geq 0$, so not in V

11. 4.1.5.

In Exercises 5–8, determine if the given set is a subspace of $\mathbb{P}_n$ for an appropriate value of n. Justify your answers.

5. All polynomials of the form $\mathbf{p}(t) = at^2$, where a is in $\mathbb{R}$.

Yes Subspace of $\mathbb{P}_2$

If $a=0$, $p(t)=0$ ✓

$p_1(t) = a_1t^2$ $p_2(t) = a_2t^2 \rightarrow (a_1+a_2)t^2$ ✓

$c \cdot p(t) = c(at^2) = (ca)t^2$ ✓

12. 4.1.7. Let H be the set of all vectors of the form $\begin{bmatrix} s \\ 3s \\ 2s \end{bmatrix}$. Find a vector **v** in $\mathbb{R}^3$ such that $H = \text{Span}\{\mathbf{v}\}$. Why does this show that H is a subspace of $\mathbb{R}^3$?

$\text{Span}(v) = \left(s \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} : s \in \mathbb{R} \right)$

This shows that H is a subspace because contains 0 vector when $s=0$, closed addition & closed scalar multiplication