Math 250, Linear Algebra: Homework 5
Fall 2024
"The purpose of computing is insight, not numbers." – Richard Hamming

Name:

Read the directions *very carefully* and then solve the problem, providing thorough details and steps. If you are not sure about what a particular symbol or word means, please ask me!

1. 2.1.1.
2. 2.1.9.
3. 2.1.25 (2.1.17 in 5E).
4. 2.2.9 (2.2.7 in 5E).
5. 2.2.11, 13, 15, 17, 19 (2.2.9 in 5E).
6. 2.2.23 (2.2.13 in 5E).
7. 2.2.31 (2.2.21 in 5E).
8. 2.3.11, 13, 15, 17, 19 (2.3.11 in 5E).
9. 3.1.5.
10. 3.1.13.
11. 3.1.20 (3.1.21 in 5E, somehow).

2.1.1 $-2\begin{bmatrix} 2 & 0 & -1 \\ 4 & -3 & 2 \end{bmatrix} = \begin{bmatrix} -4 & 0 & 2 \\ -8 & 6 & -4 \end{bmatrix} = -2A$ $\quad B-2A = \begin{bmatrix} 7 & -5 & 1 \\ 1 & -4 & -3 \end{bmatrix} - \begin{bmatrix} -4 & 0 & 2 \\ -8 & 6 & -4 \end{bmatrix} = \begin{bmatrix} 11 & -5 & -1 \\ 9 & -10 & 1 \end{bmatrix}$

AC not possible because A has 3 columns while C has 2 rows. Number of columns in A must match the number of rows in C

$CD = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 3 & 5 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 13 \\ -7 & -6 \end{bmatrix}$

2.1.9 $AB = \begin{bmatrix} 23 & -10+5K \\ -9 & 15+K \end{bmatrix}$ $\quad BA = \begin{bmatrix} 23 & 15 \\ 6-3K & 15+K \end{bmatrix}$

$-10+5K = 15 \quad K=5$
$-9 = 6-3K \quad -15 = -3K \quad K=5$
$15+K = 15+K \quad K=5$

$\boxed{AB = BA \text{ when } K=5}$

$\begin{bmatrix} 1 & -2 \\ -2 & 5 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ 6 \end{bmatrix}$

$1x - 2y = -1 \qquad x = 2y-1$
$-2x+5y = 6 \qquad -2(2y-1)+5y = 6$
$-4y+2+5y = 6$
$y = 4$
$x = 7$

$\begin{bmatrix} 1 & -2 \\ -2 & 5 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ -9 \end{bmatrix}$

$x - 2y = 2 \qquad x = 2+2y$
$-2x+5y = -9 \qquad -2(2+2y)+5y = -9 \qquad x = -8$
$-4-4y+5y = -9$
$-4+y = -9 \qquad y = -5$

$\begin{bmatrix} 7 \\ 4 \end{bmatrix}$ = first column of b $\qquad \begin{bmatrix} -8 \\ -5 \end{bmatrix}$ = 2nd column of b

2.1.25

2.2.9 $A = \begin{bmatrix} 1 & 2 \\ 5 & 12 \end{bmatrix}$ $\begin{matrix} d & -b \\ -c & a \end{matrix}$ $\quad \begin{bmatrix} 12 & -2 \\ -5 & 1 \end{bmatrix}\frac{1}{2}$ $\quad \det = 2$ $\quad A^{-1} = \begin{bmatrix} 6 & -1 \\ -\frac{5}{2} & \frac{1}{2} \end{bmatrix}$ $\begin{bmatrix} 3 \\ 5 \end{bmatrix}$

a. $A^{-1} \times b_1 = \begin{bmatrix} -9 \\ 4 \end{bmatrix}$ $\quad A^{-1} \times b_2 = \begin{bmatrix} 11 \\ -5 \end{bmatrix}$ $\quad A^{-1} \times b_3 = \begin{bmatrix} 6 \\ -2 \end{bmatrix}$ $\quad A^{-1} \times b_4 = \begin{bmatrix} 13 \\ -5 \end{bmatrix}$

$\begin{bmatrix} 1 & 2 & -9 & 11 & 6 & 13 \\ 5 & 12 & 4 & -5 & -2 & -5 \end{bmatrix} \xrightarrow{R_2 - 5R_1} \begin{bmatrix} 1 & 2 & -9 & 11 & 6 & 13 \\ 0 & 2 & 4 & -5 & -2 & -5 \end{bmatrix} \xrightarrow{R_2/2} \begin{bmatrix} 1 & 2 & -9 & 11 & 6 & 13 \\ 0 & 1 & 4 & -5 & -2 & -5 \end{bmatrix} \xrightarrow{R_1 - 2R_2} \begin{bmatrix} 1 & 0 & -9 & 11 & 6 & 13 \\ 0 & 1 & 4 & -5 & -2 & -5 \end{bmatrix}$

2.2.11, 13, 15, 17, 19

11. T, Invertible if there is an $n \times n$ matrix C such that $CA = I$ & $AC = I$
13. F, Inverse of AB + $(AB)^{-1}$ is $B^{-1}A^{-1}$
15. F, would have to be $ad - bc \neq 0$ for A to be invertible
17. T, $x = A^{-1}b$
19. T, Row operations are reversible, so elementary matrices are invertible

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2.2.23 $AB = AC$ $\qquad A$ is invertible $\qquad A^{-1}A = I$

$A^{-1}(AB) = A^{-1}(AC) \quad A^{-1}A(B) = A^{-1}A(C) \rightarrow \cancel{I}B = \cancel{I}C \quad B = C$

Not generally true when A is not invertible

2.2.31 Linear independence is when the vectors cannot be written as scalars of eachother

For example, a 2x2 matrix has the determinant $ad - bc \neq 0$ if it $= 0$, then it is linearly dependent. if $\neq 0$, you can find the inverse of the matrix. I believe the determinant determines if the Matrix is invertible, the determinant is only able to do that if vectors are linearly Independent

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2.3.11, 13, 15, 17, 19

11. T, according to statements b & d in the invertible matrix theorem, All statements are equivalent
13. T, according to statements h & e in the IMT, TFA
15. T, statement g in IMT. IMT only applies to $n \times n$ (Square Matrices)
17. T, only when $Ax = 0$ has the trivial solution, can A have n pivot positions.

3.1.5. $\begin{vmatrix} 2 & 3 & -3 \\ 4 & 0 & 3 \\ 6 & 1 & 5 \end{vmatrix}$ $A_{11} = \begin{bmatrix} 0 & 3 \\ 1 & 5 \end{bmatrix} \to A_{12} = \begin{bmatrix} 4 & 3 \\ 6 & 5 \end{bmatrix} \to A_{13} = \begin{bmatrix} 4 & 0 \\ 6 & 1 \end{bmatrix}$

$0-3=-3=\det(A_{11})$ $20-18=2=\det(A_{12})$ $4-0=4=\det(A_{13})$

$2(-3)-3(2)+-3(4)=\boxed{-24}$ Matrix is invertible

$-6\ -6\ -12$

3.1.13.

13.	4	0	−7	3	−5
	0	0	2	0	0
	7	3	−6	4	−8
	5	0	5	2	−3
	0	0	9	−1	2

$A_{23} = -2 \cdot \begin{bmatrix} 4 & 0 & 3 & -5 \\ 7 & 3 & 4 & -8 \\ 5 & 0 & 2 & -3 \\ 0 & 0 & -1 & 2 \end{bmatrix}$ $A_{22} = 3 \cdot \begin{bmatrix} 4 & 3 & -5 \\ 5 & 2 & -3 \\ 0 & -1 & 2 \end{bmatrix}$ $A_{32} = \begin{bmatrix} 4 & -5 \\ 5 & -3 \end{bmatrix}$ $-12-(-5\cdot5)$ $-12\cdot-25$ $=13$

$A_{33} = \begin{bmatrix} 4 & 3 \\ 5 & 2 \end{bmatrix}$ $8-15=-7\cdot 2$

$13-14$
$=$
-1

$-2\cdot3\cdot-1=\boxed{6}$

3.1.20 $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $\underset{K\cdot R_2}{\begin{bmatrix} a & b \\ kc & kd \end{bmatrix}}$ Scales determinant by a factor of K

ad-bc

$a(Kd)-b(KC)$

$K(ad)-K(bc)$