5.1.5 $\begin{bmatrix}3&7&9\\-4&-5&1\\2&4&4\end{bmatrix}\begin{bmatrix}4\\-3\\1\end{bmatrix} = \begin{bmatrix}0\\-30\\24\end{bmatrix}$ No this is not an eigenvector

5.1.13 Did $(A-\lambda I)x=0$ for all eigenvalues $\overset{\lambda=1}{\left\{\begin{bmatrix}0\\1\\0\end{bmatrix}\right\}}$ $\overset{\lambda=2}{\left\{\begin{bmatrix}-1\\2\\2\end{bmatrix}\right\}}$ $\overset{\lambda=3}{\left\{\begin{bmatrix}-1\\1\\1\end{bmatrix}\right\}}$

5.1.33 Let $A=\begin{bmatrix}2&0\\0&2\end{bmatrix}$ $\lambda=1$ $A-\lambda I=\begin{bmatrix}1&0\\0&1\end{bmatrix}$ $(A-\lambda I)^{-1}=\begin{bmatrix}1&0\\0&1\end{bmatrix}$ $\lambda=1$ multiplicity 2 for both A and A^{-1} and $\lambda^{-1}=1$

5.2.5 $\begin{bmatrix}2-\lambda&1\\-1&4-\lambda\end{bmatrix}$ $(2-\lambda)(4-\lambda)+1 \to \lambda^2-6\lambda+9 \to \lambda=3$ mult(2)

5.2.9 6 $\begin{bmatrix}1-\lambda&-1\\2&-1-\lambda\end{bmatrix} \to ((1-\lambda)(-1-\lambda))+2 = \lambda^2-1$

5.2.19

detA is a product of the n evals of A because the evals of are the roots of the characteristic polynomial equation. And the det is equal to the product of evals

5.3.1 $A^4 = \overset{P}{\begin{bmatrix}5&7\\2&3\end{bmatrix}}\overset{D^k}{\begin{bmatrix}16&0\\0&1\end{bmatrix}} = \overset{PD^k}{\begin{bmatrix}80&7\\32&3\end{bmatrix}}\cdot\overset{P^{-1}}{\begin{bmatrix}3&-7\\-2&5\end{bmatrix}} = \begin{bmatrix}226&-525\\90&-209\end{bmatrix} = A^4$

5.3.5 $\lambda=5,1,1$ $\lambda=5\to\left\{\begin{bmatrix}1\\1\\1\end{bmatrix}\right\}$ $\lambda=1\to\left\{\begin{bmatrix}1\\0\\-1\end{bmatrix}\right\}$ $\lambda=1\to\left\{\begin{bmatrix}2\\-1\\0\end{bmatrix}\right\}$

Basis of each eigenspace are the coorespoding column vectors in P

5.3.11 $\lambda=1,2,3$ $\begin{bmatrix}-1&4&-2\\-3&4&0\\-3&1&3\end{bmatrix}$ $\underset{\lambda=1}{\begin{bmatrix}-2&4&-2&0\\-3&3&0&0\\-3&1&2&0\end{bmatrix}} \rightsquigarrow \begin{bmatrix}1&0&-1&0\\0&1&-1&0\\0&0&0&0\end{bmatrix} \to x_3\begin{bmatrix}1\\1\\1\end{bmatrix}$

$\underset{\lambda=2}{\begin{bmatrix}-3&4&-2&0\\-3&2&0&0\\-3&1&1&0\end{bmatrix}} \rightsquigarrow \begin{bmatrix}1&0&-\frac{2}{3}&0\\0&1&-1&0\\0&0&0&0\end{bmatrix}$ $x_1=\frac{2}{3}x_3$, $x_2=x_3$, x_3 free $= x_3\begin{bmatrix}\frac{2}{3}\\1\\1\end{bmatrix}$

$\lambda = 3$ $\quad \begin{bmatrix} -4 & 4 & -2 & 0 \\ -3 & 1 & 0 & 0 \\ -3 & 1 & 0 & 0 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 1 & 0 & -\frac{1}{4} & 0 \\ 0 & 1 & -\frac{3}{4} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $\quad x_1 = \frac{1}{4}x_3$, $x_2 = \frac{3}{4}x_3$, x_3 free $\quad x_3\begin{bmatrix} \frac{1}{4} \\ \frac{3}{4} \\ 1 \end{bmatrix}$

$P = \begin{bmatrix} 1 & \frac{2}{3} & \frac{1}{4} \\ 1 & 1 & \frac{3}{4} \\ 1 & 1 & 1 \end{bmatrix}$ $\quad D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ $\quad AP = PD$

$AP = \begin{bmatrix} -1 & 4 & -2 \\ -3 & 4 & 0 \\ -3 & 1 & 3 \end{bmatrix} \begin{bmatrix} 1 & \frac{2}{3} & \frac{1}{4} \\ 1 & 1 & \frac{3}{4} \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 4/3 & 3/4 \\ 1 & 2 & \frac{9}{4} \\ 1 & 2 & 3 \end{bmatrix}$

$PD = \begin{bmatrix} 1 & 4/3 & 3/4 \\ 1 & 2 & \frac{9}{4} \\ 1 & 2 & 3 \end{bmatrix}$

5.3.15

Did Same Process as above and got $P = \begin{bmatrix} -5 & -4 & -2 \\ 1 & 0 & -1 \\ 1 & 1 & 1 \end{bmatrix}$ $D = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

5.3.21-27

21. T, matrix is only diagable if it can be written as $A = PDP^{-1}$

23. F, A must have n LI eigenvectors

25. T, Above

27. T, P consists of the LI eigenvectors

1. Given a linear transformation $T : V \to V$ of a vector space to itself, an **eigenvector** $\mathbf{v} \in V$ of T with **eigenvalue** $\lambda \in \mathbf{R}$ is one which satisfies $T(\mathbf{v}) = \lambda\mathbf{v}$. This generalizes the idea of an eigenvector-eigenvalue pair for a matrix, since any matrix defines a linear transformation. (a) Let $\mathbf{P}$ denote the set of all polynomials with $\mathbf{R}$ coefficients (e.g., $p(x) = x^3 + \pi x - \sqrt{2}$ is such an object), and let $T : \mathbf{P} \to \mathbf{P}$ be $T(p) = p'$, where $p'(x) = dp/dx$. T is a linear transformation.[1] Does it have any eigenvectors? *Why or why not?*
(b) Let $D(\mathbf{R})$ denote the set of all differentiable functions on $\mathbf{R}$. $\mathbf{P}$ is a subset of $D(\mathbf{R})$, but–as you know–$D(\mathbf{R})$ contains much more than just polynomials (e.g., it contains $f(x) = \cos x$). If we consider $T : D(\mathbf{R}) \to D(\mathbf{R})$ in the obvious way that $T(f) = f'$, does it have eigenvectors? If so, what are the eigenvalues associated to the eigenvectors you found?