Rayyan Syed Homework 8

4.4.1 $\begin{bmatrix}1&0&1\\0&1&2\\1&1&1\end{bmatrix}\begin{bmatrix}c_1\\c_2\\c_3\end{bmatrix}=\begin{bmatrix}1\\4\\7\end{bmatrix} \overset{RREF}{\rightsquigarrow} \begin{bmatrix}1&0&0&2\\0&1&0&6\\0&0&1&-1\end{bmatrix} \to \boxed{\begin{bmatrix}2\\6\\-1\end{bmatrix}}$

4.4.15-19

15. True, If $\mathcal{B}$ contains n vectors, then the coordinate system will make V act like $\mathbb{R}^n$
17. False, correct would be $[x]_B = P_B[x]$
19. False, ex: $P_3 = 1 + t + t^2 + t^3$ so $P_3 \in \mathbb{R}^4$

4. 5. 5 $\begin{bmatrix}1&-4&-2\\2&5&-4\\-1&0&2\\-3&7&6\end{bmatrix} \overset{RREF}{\rightsquigarrow} \begin{bmatrix}1&0&-2\\0&1&0\\0&0&0\\0&0&0\end{bmatrix}$ so basis = $\left\{\begin{pmatrix}1\\2\\-1\\-3\end{pmatrix}, \begin{pmatrix}-4\\5\\0\\7\end{pmatrix}\right\} \in \mathbb{R}^2$

4.5.7 Reducing the augmented matrix shows linear dependence amongst all colums, so no basis

4.5.9 $\begin{bmatrix}1&3&9&-7\\0&1&4&-3\\2&1&-2&1\end{bmatrix} \overset{RREF}{\rightsquigarrow} \begin{bmatrix}1&0&-3&2\\0&1&4&-3\\0&0&0&0\end{bmatrix}$ 2 pivot columns so dimension of subspace spanned is 2

4.5.11 After Augmenting Matrix with 0 and Reducing

Nul A = 2, Then Col A & Row A = 3 because dimension of Col A = Row A

Nul A = 2 Row A = 2
Col A = 2

4.5.13 $\begin{pmatrix}1&0&9&5&0\\0&0&1&-4&0\end{pmatrix}$ $R_1 - 9R_2$ $\begin{bmatrix}1&0&0&41&0\\0&0&1&-4&0\end{bmatrix}$ using same matrix, just removing last column vector for Col A

4.5.27 $\begin{bmatrix}1\\0\\0\\0\end{bmatrix}\begin{bmatrix}0\\2\\0\\0\end{bmatrix}\begin{bmatrix}-2\\0\\4\\0\end{bmatrix}\begin{bmatrix}0\\-12\\0\\8\end{bmatrix}$ these are linearly independent, therefore a basis of $\mathbb{P}_3$

4.5.29 $\begin{bmatrix}1\\0\\0\\0\end{bmatrix}\begin{bmatrix}0\\2\\0\\0\end{bmatrix}\begin{bmatrix}-2\\0\\4\\0\end{bmatrix}\begin{bmatrix}0\\-12\\0\\8\end{bmatrix}\begin{bmatrix}7\\-12\\-8\\12\end{bmatrix}$ Pretend this is augmented $\overset{RREF}{\rightsquigarrow}$ $\begin{bmatrix}1&0&0&0&3\\0&1&0&0&3\\0&0&1&0&-2\\0&0&0&1&\frac{3}{2}\end{bmatrix}$ $[P]_B = \left(3, 3, -2, \frac{3}{2}\right)$

4.5.43-47?

43. T, there cannot be more than p vectors and $\dim V$ can be less than or equal to that amount

45. T, the linearly independent set could have the same dimension of the basis or the basis could have more so $\dim V \geq p$

47. T, must be greater than p because $\dim V$ is atleast the number of LI vectors

49. ex: $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ span $\mathbb{R}^3$ so Col space $= \mathbb{R}^3$ & $\dim \text{Col} A \Leftrightarrow \dim \text{Row} A$ $\left[\begin{array}{ccc:c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$ Nul space is $\{0\}$ so Nullity is also 0

$3 + 0 = 3$ which is num of Col in A

Therefore $\dim \text{Row } A + \text{Nullity } A = n$

4.6.1 a. $\begin{bmatrix} 6 & 9 \\ -2 & -4 \end{bmatrix}$ b. $[x]_C = \begin{bmatrix} 6 & 9 \\ -2 & -4 \end{bmatrix} \begin{bmatrix} -3 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \end{bmatrix}$

4.6.5 a. $\begin{bmatrix} 4 & -1 & 0 \\ -1 & 1 & 1 \\ 0 & 1 & -2 \end{bmatrix}$ b. $[x]_B = \begin{bmatrix} 4 & -1 & 0 \\ -1 & 1 & 1 \\ 0 & 1 & -2 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} 8 \\ 2 \\ 2 \end{bmatrix}$