9/20 Hw Rayyan Syed

**11.** $\begin{bmatrix} 3 & -4 & 2 & 0 \\ -9 & 12 & -6 & 0 \\ -6 & 8 & -4 & 0 \end{bmatrix} \frac{R_1}{3} \begin{pmatrix} 1 & -\frac{4}{3} & \frac{2}{3} & 0 \\ -9 & 12 & -6 & 0 \\ -6 & 8 & -4 & 0 \end{pmatrix} \xrightarrow[R_3+6R_1]{R_2+9R_1} \begin{pmatrix} 1 & -\frac{4}{3} & \frac{2}{3} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

$x_1 - \frac{4}{3}x_2 + \frac{2}{3}x_3 = 0$

$x_1 = \frac{4}{3}x_2 - \frac{2}{3}x_3$

$x_2$ & $x_3$ Free variables

Exercises 19 and 20 use the notation of Example 1 for matrices in echelon form. Suppose each matrix represents the augmented matrix for a system of linear equations. In each case, determine if the system is consistent. If the system is consistent, determine if the solution is unique.

**19.** a. $\begin{bmatrix} \blacksquare & * & * & * \\ 0 & \blacksquare & * & * \\ 0 & 0 & \blacksquare & 0 \end{bmatrix}$

Consistent & unique
Can tell by looking at it

In Exercises 23 and 24, choose $h$ and $k$ such that the system has (a) no solution, (b) a unique solution, and (c) many solutions. Give separate answers for each part.

**23.** $x_1 + hx_2 = 2$
$4x_1 + 8x_2 = k$

$\begin{pmatrix} 1 & h & 2 \\ 4 & 8 & k \end{pmatrix} \xrightarrow{R_2 - 4R_1} \begin{pmatrix} 1 & h & 2 \\ 0 & 8-4h & k-8 \end{pmatrix}$

$8-4h=0$ $\quad k-8=0$

$\frac{4h}{4}=\frac{8}{4}$ $h=2$ $\quad k\neq 8$

No Solution
$h=2$
$k\neq 8$

Unique Solution
As long as $R_2$ is not all 0
so as long as $h\neq 2$

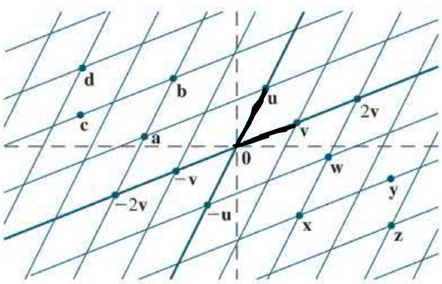
Many solutions
$0=0$
$h=2$ $\quad k=8$

In Exercises 5 and 6, write a system of equations that is equivalent to the given vector equation.

**5.** $x_1 \begin{bmatrix} 6 \\ -1 \\ 5 \end{bmatrix} + x_2 \begin{bmatrix} -3 \\ 4 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ -7 \\ -5 \end{bmatrix}$

$6x_1 - 3x_2 = 1$
$-x_1 + 4x_2 = -7$
$5x_1 = -5$

Use the accompanying figure to write each vector listed in Exercises 7 and 8 as a linear combination of **u** and **v**. Is every vector in $\mathbb{R}^2$ a linear combination of **u** and **v**?




**7.** Vectors **a**, **b**, **c**, and **d**

$\vec{a} = -2v+u$ $\quad \vec{b} = -2v+2u$ $\quad \vec{c} = -3.5v+2u$ $\quad \vec{d} = -4v+3u$

Every vector in $\mathbb{R}^2$ in the second quadrant is a linear combination of u & v

It seems this remains true for the vectors in Quadrant 4

In Exercises 11 and 12, determine if **b** is a linear combination of $\mathbf{a}_1$, $\mathbf{a}_2$, and $\mathbf{a}_3$.

**11.** $\mathbf{a}_1 = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}, \mathbf{a}_2 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \mathbf{a}_3 = \begin{bmatrix} 5 \\ -6 \\ 8 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 2 \\ -1 \\ 6 \end{bmatrix}$ $\left(\begin{array}{cccc} 1 & 0 & 5 & 2 \\ -2 & 1 & -6 & -1 \\ 0 & 2 & 8 & 6 \end{array}\right) \xrightarrow{R_2+2R_1} \begin{pmatrix} 1 & 0 & 5 & 2 \\ 0 & 1 & 4 & 3 \\ 0 & 2 & 8 & 6 \end{pmatrix} \xrightarrow{R_3-2R_2} \begin{pmatrix} 1 & 0 & 5 & 2 \\ 0 & 1 & 4 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

$x_1 = 2 - 5x_3$ Therefor b is a linear combination of $a_1, a_2, a_3$

$x_2 = 3 - 4x_3$

$x_3$ is free

In Exercises 13 and 14, determine if **b** is a linear combination of the vectors formed from the columns of the matrix $A$.

**13.** $A = \begin{bmatrix} 1 & -4 & 2 \\ 0 & 3 & 5 \\ -2 & 8 & -4 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 3 \\ -7 \\ -3 \end{bmatrix}$ $\begin{pmatrix} 1 & -4 & 2 & 3 \\ 0 & 3 & 5 & -7 \\ -2 & 8 & -4 & -3 \end{pmatrix} \xrightarrow{R_3+2R_1} \begin{pmatrix} 1 & -4 & 2 & 3 \\ 0 & 3 & 5 & -7 \\ 0 & 0 & 0 & 3 \end{pmatrix}$ B is not a linear combination of A $0 \neq 3$

**17.** Let $\mathbf{a}_1 = \begin{bmatrix} 1 \\ 4 \\ -2 \end{bmatrix}, \mathbf{a}_2 = \begin{bmatrix} -2 \\ -3 \\ 7 \end{bmatrix}$, and $\mathbf{b} = \begin{bmatrix} 4 \\ 1 \\ h \end{bmatrix}$. For what value(s) of $h$ is **b** in the plane spanned by $\mathbf{a}_1$ and $\mathbf{a}_2$?

$\begin{pmatrix} 1 & -2 & 4 \\ 4 & -3 & 1 \\ -2 & 7 & h \end{pmatrix} \xrightarrow{R_2-4R_1} \begin{pmatrix} 1 & -2 & 4 \\ 0 & 5 & -15 \\ -2 & 7 & h \end{pmatrix} \xrightarrow{R_3+2R_1} \begin{pmatrix} 1 & -2 & 4 \\ 0 & 5 & -15 \\ 0 & 3 & h+8 \end{pmatrix} \xrightarrow{R_2/5}$

$\begin{pmatrix} 1 & -2 & 4 \\ 0 & 1 & -3 \\ 0 & 3 & h+8 \end{pmatrix} \xrightarrow{R_1+2R_2} \begin{pmatrix} 1 & 0 & -2 \\ 0 & 1 & -3 \\ 0 & -3 & h+8 \end{pmatrix}$ $x_1 = -2$, $x_2 = -3$, $3x_2 = h+8$

$3(-3) = h+8$

$\boxed{h = -17}$

Compute the products in Exercises 1–4 using (a) the definition, as in Example 1, and (b) the row–vector rule for computing $A\mathbf{x}$. If a product is undefined, explain why.

**1.** $\begin{bmatrix} -4 & 2 \\ 1 & 6 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ 7 \end{bmatrix}$ undefined bc number of rows and columns are not the same

**3.** $\begin{bmatrix} 6 & 5 \\ -4 & -3 \\ 7 & 6 \end{bmatrix} \begin{bmatrix} 1 \\ -3 \end{bmatrix}$ $1\begin{pmatrix} 6 \\ -4 \\ 7 \end{pmatrix} + -3\begin{pmatrix} 5 \\ -3 \\ 6 \end{pmatrix}$ $\begin{matrix} 6-15 \\ -4-3(-3) \\ 7-18 \end{matrix}$ $\boxed{\begin{bmatrix} -9 \\ 5 \\ -11 \end{bmatrix}}$

2C ✓ 2R

In Exercises 5–8, use the definition of $A\mathbf{x}$ to write the matrix equation as a vector equation, or vice versa.

**5.** $\begin{bmatrix} 5 & 1 & -8 & 4 \\ -2 & -7 & 3 & -5 \end{bmatrix} \begin{bmatrix} 5 \\ -1 \\ 3 \\ -2 \end{bmatrix} = \begin{bmatrix} -8 \\ 16 \end{bmatrix}$ $5\begin{pmatrix} 5 \\ -2 \end{pmatrix} - 1\begin{pmatrix} 1 \\ -7 \end{pmatrix} + 3\begin{pmatrix} -8 \\ 3 \end{pmatrix} - 2\begin{pmatrix} 4 \\ 5 \end{pmatrix} = \begin{pmatrix} -8 \\ 16 \end{pmatrix}$

**7.** $x_1 \begin{bmatrix} 4 \\ -1 \\ 7 \\ -4 \end{bmatrix} + x_2 \begin{bmatrix} -5 \\ 3 \\ -5 \\ 1 \end{bmatrix} + x_3 \begin{bmatrix} 7 \\ -8 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 6 \\ -8 \\ 0 \\ -7 \end{bmatrix}$

$\begin{matrix} x_1 \\ x_2 \\ x_3 \end{matrix}$ $\begin{pmatrix} 4 & -5 & 7 \\ -1 & 3 & -8 \\ 7 & -5 & 0 \\ -4 & 1 & 2 \end{pmatrix} = \begin{pmatrix} 6 \\ 8 \\ 0 \\ -7 \end{pmatrix}$