5.1

In Exercises 21–30, A is an $n \times n$ matrix. Mark each statement True or False **(T/F)**. Justify each answer.

21. **(T/F)** If $A\mathbf{x} = \lambda\mathbf{x}$ for some vector $\mathbf{x}$, then λ is an eigenvalue of A. True, $Ax = \lambda x$ must be satisfied

22. **(T/F)** If $A\mathbf{x} = \lambda\mathbf{x}$ for some scalar λ, then $\mathbf{x}$ is an eigenvector of A. True, x scaled by λ gives A

23. **(T/F)** A matrix A is invertible if and only if 0 is an eigenvalue of A. F, non-invertible matrices have eval 0

24. **(T/F)** A number c is an eigenvalue of A if and only if the equation $(A - cI)\mathbf{x} = 0$ has a nontrivial solution. $A - CI$ must be singular meaning $(A-CI)$ det $= 0$

25. **(T/F)** Finding an eigenvector of A may be difficult, but checking whether a given vector is in fact an eigenvector is easy. Neither hard but checking if evec is faster

26. **(T/F)** To find the eigenvalues of A, reduce A to echelon form. $(A - \lambda I)x = 0$

27. **(T/F)** If $\mathbf{v}_1$ and $\mathbf{v}_2$ are linearly independent eigenvectors, then they correspond to distinct eigenvalues. Can have same evals and multiplicity exists

28. **(T/F)** The eigenvalues of a matrix are on its main diagonal. only if triangular or already diagonal

29. **(T/F)** If $\mathbf{v}$ is an eigenvector with eigenvalue 2, then $2\mathbf{v}$ is an eigenvector with eigenvalue 4. has the same eval, 2

30. **(T/F)** An eigenspace of A is a null space of a certain matrix.

5.2

In Exercises 21–30, A and B are $n \times n$ matrices. Mark each statement True or False **(T/F)**. Justify each answer.

21. **(T/F)** If 0 is an eigenvalue of A, then A is invertible. Non-invertible

22. **(T/F)** The zero vector is in the eigenspace of A associated with an eigenvalue λ.

23. **(T/F)** The matrix A and its transpose, A^T, have different sets of eigenvalues. Same

24. **(T/F)** The matrices A and $B^{-1}AB$ have the same sets of eigenvalues for every invertible matrix B. Similar matrices have same evals

25. **(T/F)** If 2 is an eigenvalue of A, then $A - 2I$ is not invertible. Satisfies $Ax = \lambda x$ or $(A - \lambda I)x = 0$ so $\det(A - \lambda I)x = 0$ and if it does, not invertible

26. **(T/F)** If two matrices have the same set of eigenvalues, then they are similar. Also need same characteristic polynomial, and satisfy $A = PBP^{-1}$

27. **(T/F)** If $\lambda + 5$ is a factor of the characteristic polynomial of A, then 5 is an eigenvalue of A. $\lambda + 5 = 0$ $\lambda = -5$

28. **(T/F)** The multiplicity of a root r of the characteristic equation of A is called the algebraic multiplicity of r as an eigenvalue of A.

29. **(T/F)** The eigenvalue of the $n \times n$ identity matrix is 1 with algebraic multiplicity n.

30. **(T/F)** The matrix A can have more than n eigenvalues. Not possible

In Exercises 21–28, A, P, and D are $n \times n$ matrices. Mark each statement True or False (**T/F**). Justify each answer. (Study Theorems 5 and 6 and the examples in this section carefully before you try these exercises.)

5.3

21. (T/F) A is diagonalizable if $A = PDP^{-1}$ for some matrix D and some invertible matrix P. T, P must be invertible

22. (T/F) If $\mathbb{R}^n$ has a basis of eigenvectors of A, then A is diagonalizable.

n

23. (T/F) A is diagonalizable if and only if A has n eigenvalues, counting multiplicities. F, must also have n LI e-vecs

24. (T/F) If A is diagonalizable, then A is invertible. different concepts

25. (T/F) A is diagonalizable if A has n eigenvectors. Not necessarily

26. (T/F) If A is diagonalizable, then A has n distinct eigenvalues. Only if they are linearly independent eigenvectors

27. (T/F) If $AP = PD$, with D diagonal, then the nonzero columns of P must be eigenvectors of A. T

28. (T/F) If A is invertible, then A is diagonalizable. Complex numbers

6.1

In Exercises 19–28, all vectors are in $\mathbb{R}^n$. Mark each statement True or False (**T/F**). Justify each answer.

19. (T/F) $\mathbf{v} \cdot \mathbf{v} = \|\mathbf{v}\|^2$. $\|v\| = \sqrt{v_1^2+v_2^2}$ and $\left(\sqrt{v_1^2+v_2^2}\right)^2 = v_1 \cdot v_2$

20. (T/F) $\mathbf{u} \cdot \mathbf{v} - \mathbf{v} \cdot \mathbf{u} = 0$. Dot prod is commutative

21. (T/F) If the distance from $\mathbf{u}$ to $\mathbf{v}$ equals the distance from $\mathbf{u}$ to $-\mathbf{v}$, then $\mathbf{u}$ and $\mathbf{v}$ are orthogonal. A them

22. (T/F) If $\|\mathbf{u}\|^2 + \|\mathbf{v}\|^2 = \|\mathbf{u} + \mathbf{v}\|^2$, then $\mathbf{u}$ and $\mathbf{v}$ are orthogonal. orthogonal iff is true

23. (T/F) If vectors $\mathbf{v}_1, \dots, \mathbf{v}_p$ span a subspace W and if $\mathbf{x}$ is orthogonal to each $\mathbf{v}_j$ for $j = 1, \dots, p$, then $\mathbf{x}$ is in $W^\perp$.

24. (T/F) If $\mathbf{x}$ is orthogonal to every vector in a subspace W then $\mathbf{x}$ is in $W^\perp$.

25. (T/F) For any scalar c, $\|c\mathbf{v}\| = c\|\mathbf{v}\|$. $|C| \cdot \|v\|$

26. (T/F) For any scalar c, $\mathbf{u} \cdot (c\mathbf{v}) = c(\mathbf{u} \cdot \mathbf{v})$.

27. (T/F) For a square matrix A, vectors in Col A are orthogonal to vectors in Nul A.

28. (T/F) For an $m \times n$ matrix A, vectors in the null space of A are orthogonal to vectors in the row space of A.

$\text{Nul}A = \text{Row } A^\perp$

6.2

In Exercises 23–32, all vectors are in $\mathbb{R}^n$. Mark each statement True or False (T/F). Justify each answer.

23. (T/F) Not every linearly independent set in $\mathbb{R}^n$ is an orthogonal set. However every orthogonal set is LI

24. (T/F) Not every orthogonal set in $\mathbb{R}^n$ is linearly independent.

25. (T/F) If $\mathbf{y}$ is a linear combination of nonzero vectors from an orthogonal set, then the weights in the linear combination can be computed without row operations on a matrix. $c = \frac{y \cdot u}{u \cdot u}$ but you can do row operations too

26. (T/F) If a set $S = \{\mathbf{u}_1, \dots, \mathbf{u}_p\}$ has the property that $\mathbf{u}_i \cdot \mathbf{u}_j = 0$ whenever $i \neq j$, then S is an orthonormal set. - orthogonal set

27. (T/F) If the vectors in an orthogonal set of nonzero vectors are normalized, then some of the new vectors may not be orthogonal.

28. (T/F) If the columns of an $m \times n$ matrix A are orthonormal, then the linear mapping $\mathbf{x} \mapsto A\mathbf{x}$ preserves lengths. $\|Ux\| = \|x\|$ & $(Ux)\cdot(Uy) = x \cdot y$

29. (T/F) A matrix with orthonormal columns is an orthogonal matrix. Orthonormality always means orthogonal too but must be square in this case

30. (T/F) The orthogonal projection of $\mathbf{y}$ onto $\mathbf{v}$ is the same as the orthogonal projection of $\mathbf{y}$ onto $c\mathbf{v}$ whenever $c \neq 0$.

31. (T/F) If L is a line through $\mathbf{0}$ and if $\hat{\mathbf{y}}$ is the orthogonal projection of $\mathbf{y}$ onto L, then $\|\hat{\mathbf{y}}\|$ gives the distance from $\mathbf{y}$ to L. $\|y - \hat{y}\|$

32. (T/F) An orthogonal matrix is invertible. Always

6.3

In Exercises 21–30, all vectors and subspaces are in $\mathbb{R}^n$. Mark each statement True or False (T/F). Justify each answer.

21. (T/F) If $\mathbf{z}$ is orthogonal to $\mathbf{u}_1$ and to $\mathbf{u}_2$ and if $W = \text{Span}\{\mathbf{u}_1, \mathbf{u}_2\}$, then $\mathbf{z}$ must be in $W^\perp$.

22. (T/F) For each $\mathbf{y}$ and each subspace W, the vector $\mathbf{y} - \text{proj}_W \mathbf{y}$ is orthogonal to W.

23. (T/F) The orthogonal projection $\hat{\mathbf{y}}$ of $\mathbf{y}$ onto a subspace W can sometimes depend on the orthogonal basis for W used to compute $\hat{\mathbf{y}}$.

24. (T/F) If $\mathbf{y}$ is in a subspace W, then the orthogonal projection of $\mathbf{y}$ onto W is $\mathbf{y}$ itself.

~~25.~~ (T/F) The best approximation to $\mathbf{y}$ by elements of a subspace W is given by the vector $\mathbf{y} - \text{proj}_W \mathbf{y}$.

26. (T/F) If W is a subspace of $\mathbb{R}^n$ and if $\mathbf{v}$ is in both W and $W^\perp$, then $\mathbf{v}$ must be the zero vector.

27. (T/F) In the Orthogonal Decomposition Theorem, each term in formula (2) for $\hat{\mathbf{y}}$ is itself an orthogonal projection of $\mathbf{y}$ onto a subspace of W.

28. (T/F) If $\mathbf{y} = \mathbf{z}_1 + \mathbf{z}_2$, where $\mathbf{z}_1$ is in a subspace W and $\mathbf{z}_2$ is in $W^\perp$, then $\mathbf{z}_1$ must be the orthogonal projection of $\mathbf{y}$ onto W.

29. (T/F) If the columns of an $n \times p$ matrix U are orthonormal, then $UU^T\mathbf{y}$ is the orthogonal projection of $\mathbf{y}$ onto the column

30. (T/F) If an $n \times p$ matrix U has orthonormal columns, then $UU^T\mathbf{x} = \mathbf{x}$ for all $\mathbf{x}$ in $\mathbb{R}^n$.

Idk

$$\frac{\frac{1}{2}}{\frac{3}{4}} \qquad \frac{1}{2} \cdot \frac{4}{3} \qquad \frac{4}{6} = \frac{2}{3}$$

In Exercises 17–22, all vectors and subspaces are in $\mathbb{R}^n$. Mark each statement True or False **(T/F)**. Justify each answer.

17. **(T/F)** If $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is an orthogonal basis for W, then multiplying $\mathbf{v}_3$ by a scalar c gives a new orthogonal basis $\{\mathbf{v}_1, \mathbf{v}_2, c\mathbf{v}_3\}$. 6.4

18. **(T/F)** If $W = \text{Span}\{\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3\}$ with $\{\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3\}$ linearly independent, and if $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is an orthogonal set in W, then $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is a basis for W.

19. **(T/F)** The Gram–Schmidt process produces from a linearly independent set $\{\mathbf{x}_1, \dots, \mathbf{x}_p\}$ an orthogonal set $\{\mathbf{v}_1, \dots, \mathbf{v}_p\}$ with the property that for each k, the vectors $\mathbf{v}_1, \dots, \mathbf{v}_k$ span the same subspace as that spanned by $\mathbf{x}_1, \dots, \mathbf{x}_k$.

20. **(T/F)** If $\mathbf{x}$ is not in a subspace W, then $\mathbf{x} - \text{proj}_W \mathbf{x}$ is not zero.

21. **(T/F)** If $A = QR$, where Q has orthonormal columns, then $R = Q^T A$.

22. **(T/F)** In a QR factorization, say $A = QR$ (when A has linearly independent columns), the columns of Q form an orthonormal basis for the column space of A.

x_3

$$\text{Proj}_{W_2} x_3 = \frac{x_3 \cdot v_1}{v_1 \cdot v_1} \cdot v_1 + \frac{x_3 \cdot v_2}{v_2 \cdot v_2} \cdot v_2$$