Math 250, Linear Algebra: Homework 3 (Due Sept. 27 in class)
Fall 2024
"Algebra is nothing more than geometry in words; geometry
is nothing more than algebra in pictures." – Sophie Germain

Name:

Read the directions *very carefully* and then solve the problem, providing thorough details and steps. If you are not sure about what a particular symbol or word means, please ask me!

1. 1.4.11.
2. 1.4.13.
3. 1.4.21.
4. 1.5.5.
5. 1.5.19 (1.5.15 in 5E).
6. 1.5.27, 1.5.29, 1.5.31, 1.5.33, 1.5.35 (just 1.5.23 in 5E: I don't know why they broke it up into five problems in the newer edition).
7. 1.7.1.
8. 1.7.7.
9. 1.7.9.

Rayyan Syed

Given A and $\mathbf{b}$ in Exercises 11 and 12, write the augmented matrix for the linear system that corresponds to the matrix equation $A\mathbf{x} = \mathbf{b}$. Then solve the system and write the solution as a vector.

11. $A = \begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & 5 \\ -2 & -4 & -3 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} -2 \\ 2 \\ 9 \end{bmatrix}$

$$\begin{bmatrix} 1 & 2 & 4 & -2 \\ 0 & 1 & 5 & 2 \\ -2 & -4 & -3 & 9 \end{bmatrix} \xrightarrow{R_3+2R_1} \begin{bmatrix} 1 & 2 & 4 & -2 \\ 0 & 1 & 5 & 2 \\ 0 & 0 & 5 & 5 \end{bmatrix} \xrightarrow{R_1-2R_2} \begin{pmatrix} 1 & 0 & -4 & -4 \\ 0 & 1 & 5 & 2 \\ 0 & 0 & 5 & 5 \end{pmatrix} \xrightarrow{R_3/5} \begin{bmatrix} 1 & 0 & -4 & -4 \\ 0 & 1 & 5 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$\xrightarrow{R_2-5R_3} \begin{bmatrix} 1 & 0 & -4 & -4 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 1 \end{bmatrix} \xrightarrow{R_1+4R_3} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 1 \end{bmatrix} \quad \begin{matrix} x_1 = 0 \\ x_2 = -3 \\ x_3 = 1 \end{matrix} \quad \boxed{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ -3 \\ 1 \end{bmatrix}}$$

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13. Let $\mathbf{u} = \begin{bmatrix} 0 \\ 4 \\ 4 \end{bmatrix}$ and $A = \begin{bmatrix} 3 & -5 \\ -2 & 6 \\ 1 & 1 \end{bmatrix}$. Is $\mathbf{u}$ in the plane in $\mathbb{R}^3$

$$\begin{pmatrix} 3 & -5 & 0 \\ -2 & 6 & 4 \\ 1 & 1 & 4 \end{pmatrix} \xrightarrow{R_2+\frac{2}{3}R_1} \begin{pmatrix} 3 & -5 & 0 \\ 0 & \frac{8}{3} & 4 \\ 1 & 1 & 4 \end{pmatrix} \xrightarrow{R_3-\frac{1}{3}R_1} \begin{pmatrix} 3 & -5 & 0 \\ 0 & \frac{8}{3} & 4 \\ 0 & \frac{8}{3} & 4 \end{pmatrix} \xrightarrow{R_3-R_2} \begin{pmatrix} 3 & -5 & 0 \\ 0 & \frac{8}{3} & 4 \\ 0 & 0 & 0 \end{pmatrix} \to \frac{8}{3}x_2 = 4$$

$\frac{8x_2}{8} = \frac{12}{8} = \frac{3}{2} = x_2$ $\quad 3x_1 - 5\left(\frac{3}{2}\right) = 0 \quad x_1 = \frac{5}{2}$

Yes, u is in the plane spanned by A, $\mathbb{R}^3$

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21. Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 0 \\ -1 \\ 0 \\ 1 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}$.

Does $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ span $\mathbb{R}^4$? Why or why not?

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 1 & -1 \end{bmatrix}$$

According to theorem 4, $\{v_1, v_2, v_3\}$ doesn't span $\mathbb{R}^4$ because there isn't pivot position every row

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In Exercises 5 and 6, follow the method of Examples 1 and 2 to write the solution set of the given homogeneous system in parametric vector form.

5.
$$\begin{aligned} x_1 + 3x_2 + x_3 &= 0 \\ -4x_1 - 9x_2 + 2x_3 &= 0 \\ -3x_2 - 6x_3 &= 0 \end{aligned}$$

$$\begin{pmatrix} 1 & 3 & 1 & 0 \\ -4 & -9 & 2 & 0 \\ 0 & -3 & -6 & 0 \end{pmatrix} \xrightarrow{R_2+4R_1} \begin{bmatrix} 1 & 3 & 1 & 0 \\ 0 & 3 & 6 & 0 \\ 0 & -3 & -6 & 0 \end{bmatrix} \xrightarrow{R_3+R_2} \begin{bmatrix} 1 & 3 & 1 & 0 \\ 0 & 3 & 6 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$x_1 + 3(-2x_3) + x_3 = 0 \quad x_1 = 5x_3$

$3x_2 + 6x_3 = 0 \quad \frac{3x_2}{3} = \frac{-6x_3}{3} \quad x_2 = -2x_3$

x_3 is free say its 1 $\quad \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x\begin{bmatrix} 5 \\ -2 \\ 1 \end{bmatrix}$

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19. Follow the method of Example 3 to describe the solutions of the following system in parametric vector form. Also, give a geometric description of the solution set and compare it to that in Exercise 5.

$$\begin{aligned} x_1 + 3x_2 + x_3 &= 1 \\ -4x_1 - 9x_2 + 2x_3 &= -1 \\ -3x_2 - 6x_3 &= -3 \end{aligned}$$

$$\begin{bmatrix} 1 & 3 & 1 & 1 \\ -4 & -9 & 2 & -1 \\ 0 & -3 & -6 & -3 \end{bmatrix} \xrightarrow{R_2+4R_1} \begin{bmatrix} 1 & 3 & 1 & 1 \\ 0 & 3 & 6 & 3 \\ 0 & -3 & -6 & -3 \end{bmatrix} \xrightarrow{R_3+R_2} \begin{bmatrix} 1 & 3 & 1 & 1 \\ 0 & 3 & 6 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{R_2/3} \begin{bmatrix} 1 & 3 & 1 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$x_1 = 5t - 2$

$x_2 = 1 - 2t$

$x_3 = t$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix} + t\begin{pmatrix} 5 \\ -2 \\ 1 \end{pmatrix}$$

In Exercises 27–36, mark each statement True or False (T/F). Justify each answer.

27. (T/F) A homogeneous equation is always consistent. T, A(x)=0, Homogeneous will always have a trivial solution.

29. (T/F) The equation $A\mathbf{x} = \mathbf{0}$ gives an explicit description of its solution set. F, gives implicit description

31. (T/F) The homogeneous equation $A\mathbf{x} = \mathbf{0}$ has the trivial solution if and only if the equation has at least one free variable. F, Ax=0 is Always in homogeneous system, where as, free variable always means non trivial

33. (T/F) The equation $\mathbf{x} = \mathbf{p} + t\mathbf{v}$ describes a line through $\mathbf{v}$ parallel to $\mathbf{p}$. F, opposite

35. (T/F) The solution set of $A\mathbf{x} = \mathbf{b}$ is the set of all vectors of the form $\mathbf{w} = \mathbf{p} + \mathbf{v}_h$, where $\mathbf{v}_h$ is any solution of the equation $A\mathbf{x} = \mathbf{0}$. T, IdK why

In Exercises 1–4, determine if the vectors are linearly independent. Justify each answer.

1. $\begin{bmatrix} 5 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 7 \\ 2 \\ -6 \end{bmatrix}, \begin{bmatrix} -2 \\ -1 \\ 6 \end{bmatrix}$

$$\begin{bmatrix} 5 & 7 & -2 \\ 1 & 2 & -1 \\ 0 & -6 & 6 \end{bmatrix} \xrightarrow{R_2 - \frac{1}{5}R_1} \begin{bmatrix} 5 & 7 & -2 \\ 0 & \frac{3}{5} & -\frac{3}{5} \\ 0 & -6 & 6 \end{bmatrix} \xrightarrow{R_1/5} \begin{bmatrix} 1 & \frac{7}{5} & -\frac{2}{5} \\ 0 & \frac{3}{5} & -\frac{3}{5} \\ 0 & -6 & 6 \end{bmatrix} \xrightarrow{\frac{5}{3}R_2} \begin{bmatrix} 1 & \frac{7}{5} & -\frac{2}{5} \\ 0 & 1 & -1 \\ 0 & -6 & 6 \end{bmatrix} \xrightarrow{R_1 - \frac{7}{5}R_2} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & -6 & 6 \end{bmatrix}$$

$$\xrightarrow{R_3 + 6R_2} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

Linearly Dependent because there's a free variable

In Exercises 5–8, determine if the columns of the matrix form a linearly independent set. Justify each answer.

7. $\begin{bmatrix} 1 & 4 & -3 & 0 \\ -2 & -7 & 5 & 1 \\ -4 & -5 & 7 & 5 \end{bmatrix}$

Has a rank of 3, which is same number of rows, so LD

In Exercises 9 and 10, (a) for what values of h is $\mathbf{v}_3$ in Span $\{\mathbf{v}_1, \mathbf{v}_2\}$, and (b) for what values of h is $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly *dependent*? Justify each answer.

9. $\mathbf{v}_1 = \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} -3 \\ 10 \\ -6 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} 2 \\ -7 \\ h \end{bmatrix}$

$$\begin{bmatrix} 1 & -3 & 2 \\ -3 & 10 & -7 \\ 2 & -6 & h \end{bmatrix} \xrightarrow{R_2 + 3R_1} \begin{bmatrix} 1 & -3 & 2 \\ 0 & 1 & -1 \\ 2 & -6 & h \end{bmatrix} \xrightarrow{R_3 - 2R_1} \begin{bmatrix} 1 & -3 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & h-4 \end{bmatrix}$$

when $h=4$
v_3 is in Span $\{v_1, v_2\}$

$p < n$
So Linearly Dependent?