4.1.11

$U=\begin{bmatrix}5b+2c\\b\\c\end{bmatrix}$ $V=\begin{bmatrix}5b+2c\\b\\c\end{bmatrix}$ $b=1$ $c=0$ $\downarrow$ $v=\begin{bmatrix}5\\1\\0\end{bmatrix}$ $b=0$, $c=1$ $\downarrow$ $v=\begin{bmatrix}2\\0\\1\end{bmatrix}$ This shows that W is a subspace of $\mathbb{R}^3$ because it contains the 0 vector & is closed under addition & scalar multiplication

4.1.23-31

23. F because it is only true for some t. Also depends on function.

25. T, For example $\begin{bmatrix}1\\0\\0\end{bmatrix}$ represents an arrow in 3 dimensional space.

27. F, H is a subset of of V assuming it is atleast $\mathbb{R}^{v-1}$. Then a subspace if contains $\vec{0}$ & closed under addition & scalar multiplication

29. T, satisfies same conditions to be a subspace

31. T, Same conditions

4.2.3 $\begin{bmatrix}1&3&5&0&0\\0&1&4&-2&0\end{bmatrix} \xrightarrow{R_1-3R_2} \begin{bmatrix}1&0&-7&6&0\\0&1&4&-2&0\end{bmatrix}$ $x_1=7x_3-6x_4$, $x_2=-4x_3+2x_4$, x_3, x_4 $\quad x_3\begin{bmatrix}7\\-4\\1\\0\end{bmatrix}+x_4\begin{bmatrix}-6\\2\\0\\1\end{bmatrix}$ $NulA=\left\{\begin{pmatrix}7\\-4\\1\\0\end{pmatrix},\begin{pmatrix}-6\\2\\0\\1\end{pmatrix}\right\}$

4.2.5 $\begin{bmatrix}1&-2&0&4&0&0\\0&0&1&-9&0&0\\0&0&0&0&1&0\end{bmatrix}$ $x_1-2x_2+4x_4$, x_3-9x_4, $x_5=0$ $\quad x_1=2x_2-4x_4$, x_2, $x_3=-9x_4$, x_4, $x_5=0$ $\quad x_2\begin{bmatrix}2\\1\\0\\0\\0\end{bmatrix}+x_4\begin{bmatrix}-4\\0\\-9\\1\\0\end{bmatrix}$ $NulA=\left\{\begin{bmatrix}2\\1\\0\\0\\0\end{bmatrix},\begin{bmatrix}-4\\0\\-9\\1\\0\end{bmatrix}\right\}$

4.2.7 Not subspace $0+0+0\neq 2$

4.2.23 $A=\begin{bmatrix}-6&12\\-3&6\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}$ $x=0$ $y=\frac{1}{6}$ Yes in ColA $\quad \begin{bmatrix}-6&12\\-3&6\end{bmatrix}\begin{bmatrix}2\\1\end{bmatrix}$ $2\begin{pmatrix}-6\\-3\end{pmatrix}+1\begin{pmatrix}12\\6\end{pmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$ W is in NulA $\quad p(0), p(1)=0$ so $p(t)=t^2-t$

Range of T is $\mathbb{R}^2$

4.2.43 $(p+q)0 = p(0)+q(0) = T(p)+T(q)$ $\quad cp(0) = c\begin{pmatrix}p(0)\\p(1)\end{pmatrix} = cT(p)$

$(p+q)1 = p(1)+q(1)$ $\quad cp(1)$

4.3.1 Det $\neq 0$ therefore LI, there for a basis for $\mathbb{R}^3$

4.3.3 $\begin{bmatrix}1&3&-3\\0&2&-5\\-2&-4&1\end{bmatrix} \xrightarrow{R_3+2R_1} \begin{bmatrix}1&3&-3\\0&2&-5\\0&2&-5\end{bmatrix}$ R_2 & R_3 are the same therefore LD & do not span $\mathbb{R}^3$

4.3.11 $z=-x-2y$ $\begin{bmatrix}x\\y\\-x-2y\end{bmatrix}\to x\begin{bmatrix}1\\0\\-1\end{bmatrix}+y\begin{bmatrix}0\\1\\-2\end{bmatrix}$ so $\left\{\begin{bmatrix}1\\0\\-1\end{bmatrix},\begin{bmatrix}0\\1\\-2\end{bmatrix}\right\}$

4.3.15 Reduced Matrix $\rightsquigarrow$ $\begin{bmatrix}1&0&-3&0&4\\0&1&4&0&-5\\0&0&0&1&2\\0&0&0&0&0\end{bmatrix}\to\left\{\begin{bmatrix}1\\0\\0\\0\end{bmatrix},\begin{bmatrix}0\\1\\0\\0\end{bmatrix},\begin{bmatrix}0\\0\\1\\0\end{bmatrix}\right\}\to\{v_1, v_2, v_4\}$

4.3.41 $T(c_1v_1 + c_2v_2 + \dots c_pv_p) = T(0) \rightarrow c_1T(v_1) + c_2T(v_2) + \dots + c_pT(v_p) = 0$

Shows $\{T(v_1) \dots T(v_p)\}$ is LD

4.4.1 $5\begin{pmatrix} 3 \\ -5 \end{pmatrix} + 3\begin{pmatrix} -4 \\ 6 \end{pmatrix}$ $\quad [x]_B = \begin{bmatrix} 3 \\ -7 \end{bmatrix}$

4.4.7 $\begin{bmatrix} 1 & -3 & 2 \\ -1 & 4 & -2 \\ -3 & 9 & 4 \end{bmatrix}\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} 8 \\ -9 \\ 6 \end{bmatrix}$ $\quad \begin{bmatrix} 1 & -3 & 2 & 8 \\ -1 & 4 & -2 & -9 \\ -3 & 9 & 4 & 6 \end{bmatrix} \xrightarrow{\text{Reduce}} \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 3 \end{bmatrix}$

So $\begin{bmatrix} -1 \\ -1 \\ 3 \end{bmatrix}$

$$\sum_{n=1}^{\infty}$$