6.1.1 $\sqrt{\left(\frac{7}{4}\right)^2+\left(\frac{1}{2}\right)^2+(1)^2} = \sqrt{\frac{69}{16}} = \frac{\sqrt{69}}{4} \to \frac{1}{v} = \frac{4}{\sqrt{69}} \times \begin{bmatrix} 7/4 \\ 1/2 \\ 1 \end{bmatrix} = \begin{bmatrix} 7/\sqrt{69} \\ 2/\sqrt{69} \\ 4/\sqrt{69} \end{bmatrix}$

6.1.13 $\sqrt{(10--1)^2+(-3--5)^2} = \sqrt{121+4} = \sqrt{125} = 5\sqrt{5}$

6.1.15 $a = \begin{bmatrix} 8 \\ -5 \end{bmatrix}$ $b = \begin{bmatrix} -2 \\ -3 \end{bmatrix}$ $a \cdot b = -1$, not orthogonal

Orthogonal

6.1.17 $u \cdot v = -12+2+10+0 = 0$

6.1.37 If $W := \text{span}\{v_1 \dots v_p\}$ then $W = c_1v_1 + c_2v_2 + \dots + c_pv_p \to x \cdot W = x(c_1v_1 + c_2v_2 + \dots + c_pv_p)$

Same as $\dots + c_p(x \cdot v_p) + \dots$, $x \cdot v_j = 0$ so $x \cdot w = c_1(0) + c_2(0) + \dots + c_p(0) = 0$

6.2.9 $u_1 \cdot u_2 = 0$ $u_1 \cdot u_3 = 0$ $u_2 \cdot u_3 = 0$ Form a basis / orthogonal set

$\begin{bmatrix} 1 & -1 & 2 & 8 \\ 0 & 4 & 1 & -4 \\ 1 & 1 & -2 & -3 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 1 & 0 & 0 & 2.5 \\ 0 & 1 & 0 & -1.5 \\ 0 & 0 & 1 & 2 \end{bmatrix}$ $2.5u_1 - 1.5u_2 + 2u_3 = X$ or $\begin{bmatrix} 8 \\ -4 \\ 3 \end{bmatrix}$

$y - \text{proj}_u y =$

6.2.13 $\frac{u \cdot y}{u \cdot u} u = \frac{-13}{65} \cdot \begin{bmatrix} 4 \\ -7 \end{bmatrix} \to \begin{bmatrix} -52/65 \\ 91/65 \end{bmatrix}$, $\begin{bmatrix} 2 \\ 3 \end{bmatrix} - \begin{bmatrix} -52/65 \\ 91/65 \end{bmatrix} = \begin{bmatrix} 2.8 \\ 1.6 \end{bmatrix}$ $y = \begin{bmatrix} -.8 \\ 1.4 \end{bmatrix} + \begin{bmatrix} 2.8 \\ 1.6 \end{bmatrix}$

6.2.17 $v_1 \cdot v_2 = 0$ Orthogonal ✓ $\sqrt{\left(\frac{1}{3}\right)^2+\left(\frac{1}{3}\right)^2+\left(\frac{1}{3}\right)^2} = \frac{1}{\sqrt{3}}$ $\sqrt{\left(\frac{1}{2}\right)^2+(0)^2+\left(\frac{1}{2}\right)^2} = \frac{1}{\sqrt{2}}$

$\frac{\frac{1}{3}}{\frac{1}{\sqrt{3}}} = \frac{1}{\sqrt{3}}$ $\begin{bmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{bmatrix}$ $\frac{\frac{1}{2}}{\frac{1}{\sqrt{2}}} = \begin{bmatrix} -1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{bmatrix} \to \begin{bmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{bmatrix}, \begin{bmatrix} -1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{bmatrix}$

6.2.23 T, however every orthogonal set is Linear independent

.25 T, can usually tell by looking at it.

.27 F, normalizing is just scaling it to 1, orthogonality is not Affected

.29 T, orthonormality implies orthogonality

.31 F, $\|\hat{y}\|$ only represents the length of the Projection

6.3.1 $\frac{u_1 \cdot x}{u \cdot u} = \frac{-16}{18} = \frac{-8}{9}$ $\frac{u_2 \cdot x}{u_2 u_2} = \frac{-8}{36} = \frac{-2}{9}$ $\frac{u_3 \cdot x}{u_3 \cdot u_3} = \frac{12}{18} = \frac{2}{3}$ $\frac{u_4 \cdot x}{u_4 \cdot u_4} = \frac{28}{36} = \frac{7}{9}$

Vector 1 = $-\frac{8}{9}(u_1) + -\frac{2}{9}(u_2) + \frac{2}{3}(u_3)$, vector 2 = $\frac{7}{9}(u_4)$

6.3.5 $u_1 \cdot u_2 = 0$ $\hat{y} = \frac{y \cdot u_1}{u_1 \cdot u_1} \cdot u_1 + \frac{y \cdot u_2}{u_2 \cdot u_2} \cdot u_2$ $\begin{matrix} 3/2 \\ -1/2 \\ 1 \end{matrix} + \begin{matrix} -15/6 \\ 15/6 \\ 30/6 \end{matrix} = \begin{bmatrix} -1 \\ 2 \\ 6 \end{bmatrix}$

6.3.9 $u_1 \cdot u_2 = 0$ $u_2 \cdot u_3 = 0$ $u_1 \cdot u = 0$ $\hat{y} = 2u_1 + \frac{2}{3}u_2 - \frac{2}{3}u_3$

$\frac{y \cdot u_1}{u_1^2} u_1 = \begin{bmatrix} 2 \\ 2 \\ 0 \\ 2 \end{bmatrix}$ $\frac{y \cdot u_2}{u_2^2} \cdot u_2 = \begin{bmatrix} -2/3 \\ 2 \\ 2/3 \\ -4/3 \end{bmatrix}$ $\frac{y \cdot u_3}{u_3^2} \cdot u_3 = \begin{pmatrix} 2/3 \\ 0 \\ -2/3 \\ -2/3 \end{pmatrix}$, summing all $\hat{y} = \begin{pmatrix} 10/3 \\ 4 \\ 0 \\ 0 \end{pmatrix}$ $z = (\frac{2}{3}, -1, 3, -1)$ $y = \hat{y} + z$

6.3.21 T, If W is a subspace of $\mathbb{R}^n$ where $\hat{y} \in W$, then z is in $W^\perp$

23. F because the Projection only depends on the subspace, not the orthogonal basis in it

25. F, the best approximation of y is just $\text{Proj}_W y$

27. T, when dim W > 1 each term itself is an orthogonal Projection of y

29. T, $\text{Proj}_{Col(U)} y = UU^T y$, if cols of U are ON, $U^T U = I_p$

6.4.3

6.4.7 $\longrightarrow$

6.4.9