Math 250, Linear Algebra: Homework 4 (Due **Oct. 8th** in class)
Fall 2024
"In the high country of the mind one has to become adjusted
to the thinner air of uncertainty." – Robert M. Pirsig

Name:

Read the directions *very carefully* and then solve the problem, providing thorough details and steps. If you are not sure about what a particular symbol or word means, please ask me!

1. 1.7.11.
2. 1.7.21, 23, 25, and 27 (1.7.21 in 5E). Remember to **justify** your response.
3. 1.8.5.
4. 1.8.9.
5. 1.8.17.
6. 1.8.21, 23, 25, 27, and 29 (1.8.21 in 5E). Remember to **justify** your response.
7. 1.9.1 (but you should practice all of 1-10 for your own benefit).
8. 1.9.19.
9. 1.9.21.
10. 1.9.23, 25, 27, 29, and 31 (1.9.23 in 5E). Remember to **justify** your response.

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1.7.11

$$\begin{bmatrix} 1 & 3 & -1 \\ -1 & -5 & 5 \\ 4 & 7 & h \end{bmatrix} \xrightarrow{R_2+R_1} \begin{bmatrix} 1 & 3 & -1 \\ 0 & -2 & 4 \\ 4 & 7 & h \end{bmatrix} \xrightarrow{R_3-4R_1} \begin{bmatrix} 1 & 3 & -1 \\ 0 & -2 & 4 \\ 0 & -5 & h+4 \end{bmatrix} \xrightarrow{R_2/-2} \begin{bmatrix} 1 & 3 & -1 \\ 0 & 1 & -2 \\ 0 & -5 & h+4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & -1 \\ 0 & 1 & -2 \\ 0 & -5 & h+4 \end{bmatrix} \xrightarrow{R_3+5R_2} \begin{bmatrix} 1 & 3 & -1 \\ 0 & 1 & -2 \\ 0 & 0 & h-6 \end{bmatrix}$$

$h=6$
LD because a free variable is created & there are only 2 pivot columns

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1.7.21, 23, 25, 27

21. T, Columns of Matrix A are LI if & only if equation $Ax=0$ has trivial solution

23. F, not each vector, only 1 has to be a linear combo of another

25. T, Linearly dependent if number of columns > number of rows, $p>n$

27. T, Linear dependence; 1 vector can be written as a LC of another, so if $\{x, y, z\}$ is linearly dependent, then z spans $\{x, y\}$, which is the same thing as z being a LC of $\{x, y\}$

1.8.5

$A: \begin{bmatrix} 1 & -5 & -7 \\ -3 & 7 & 5 \end{bmatrix}$ $b = \begin{bmatrix} -2 \\ -2 \end{bmatrix}$ $\begin{bmatrix} 1 & -5 & -7 & -2 \\ -3 & 7 & 5 & -2 \end{bmatrix} \xrightarrow{R_2+3R_1} \begin{bmatrix} 1 & -5 & -7 & -2 \\ 0 & -8 & -16 & -8 \end{bmatrix} \xrightarrow{\frac{R_2}{-8}} \begin{bmatrix} 1 & -5 & -7 & -2 \\ 0 & 1 & 2 & 1 \end{bmatrix} \xrightarrow{R_1+5R_2} \begin{bmatrix} 1 & 0 & 3 & 3 \\ 0 & 1 & 2 & 1 \end{bmatrix}$

$x_1+3x_3=3$
$x_2+2x_3=1$
$x_2 = 1-2x_3$
$x_1 = 3-3x_3$
x_3 is free

vector is $\begin{bmatrix} 3-3x_3 \\ 1-2x_3 \\ x_3 \end{bmatrix}$, not unique because x_3 is free

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1.8.9

$A = \begin{bmatrix} 1 & -4 & 7 & -5 \\ 0 & 1 & -4 & 3 \\ 2 & 6 & 6 & 4 \end{bmatrix} \xrightarrow{R_3-2R_1} \begin{bmatrix} 1 & -4 & 7 & -5 & 0 \\ 0 & 1 & -4 & 3 & 0 \\ 0 & 2 & -8 & 14 & 0 \end{bmatrix} \xrightarrow{R_1+4R_2} \begin{bmatrix} 1 & 0 & -9 & 7 & 0 \\ 0 & 1 & -4 & 3 & 0 \\ 0 & 2 & -8 & 14 & 0 \end{bmatrix} \xrightarrow{R_3-2R_2} \begin{bmatrix} 1 & 0 & -9 & 7 & 0 \\ 0 & 1 & -4 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

$x_1-9x_3+7x_4=0$ $x_1 = 9x_3-7x_4$
$x_2-4x_3+3x_4=0$ $x_2 = 4x_3-3x_4$
x_4 free x_3 free

$$\boxed{x = x_3\begin{bmatrix} 9 \\ 4 \\ 1 \\ 0 \end{bmatrix} + x_4\begin{bmatrix} -7 \\ -3 \\ 0 \\ 1 \end{bmatrix}}$$

1.8.17

$3u$: $3\begin{pmatrix} 2 \\ 1 \end{pmatrix} \rightarrow \begin{bmatrix} 6 \\ 3 \end{bmatrix}$

$2v$: $2\begin{pmatrix} -1 \\ 3 \end{pmatrix} \rightarrow \begin{bmatrix} -2 \\ 6 \end{bmatrix}$

$3u+2v$: $\begin{pmatrix} 6 \\ 3 \end{pmatrix} + \begin{pmatrix} -2 \\ 6 \end{pmatrix}$ $\begin{pmatrix} 4 \\ 9 \end{pmatrix}$

1.8.21, 23, 25, 27, 29

21. T, "Linear transformations preserve the operations of vector addition & scalar multiplication

23. F, Domain is $\mathbb{R}^5$, co domain is $\mathbb{R}^3$, $\mathbb{R}^n$ is Domain, $\mathbb{R}^m$ is codomain $n=5$, $m=3$ $\overset{3\ 5}{(m,n)}$ Mat

25. F, not necessarily, could be between $\mathbb{R}^m \rightarrow \mathbb{R}^n$ I think

27. F, every matrix transformation is a linear transformation, but not every LT is a MT

29. T, multiplication property

1.9.1

$$\begin{bmatrix} 2 & -5 \\ 1 & 2 \\ 2 & 0 \\ 1 & 0 \end{bmatrix}$$

1.9.18 $\begin{bmatrix} 1 & -5 & 4 \\ 0 & 1 & -6 \end{bmatrix}$

1.9.21

$$\begin{bmatrix} 1 & 1 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 8 \end{bmatrix} \qquad \begin{bmatrix} 1 & 1 & 3 \\ 4 & 5 & 8 \end{bmatrix} \xrightarrow{R_2 - 4R_1} \begin{bmatrix} 1 & 1 & 3 \\ 0 & 1 & -4 \end{bmatrix}$$

$$x_1\begin{bmatrix} 1 \\ 4 \end{bmatrix} + x_2\begin{bmatrix} 1 \\ 5 \end{bmatrix} = \begin{bmatrix} 3 \\ 8 \end{bmatrix}$$

$x_2 = -4$

$x_1 + x_2 = 3$

$x_1 - 4 = 3$

$x_1 = 7$

$$\boxed{x = \begin{bmatrix} 7 \\ -4 \end{bmatrix}}$$

1.9.23, 25, 27, 29, 31

23. T, transformations are based on the identity matrixes, which will always be $n \times n$

25. T, vector addition and scalar multiplication are preserved, $T(u+v)$ is same as $Tu + Tv$ & $cT(u)$ and $cT(v)$ is $T(cu)$ and $T(cv)$ or something like that

27. F, Linear transformations are Linear, why would their result not be linear transformations

29. F, For example $\mathbb{R}^5$ to $\mathbb{R}^7$, Can't get 7 dimensions from 5, must already exist in $\mathbb{R}^n$

31. F, it can be one to one because you can get different results

5 TF (10pts) 4

10 Reduce 10

10 find value to make LD 7

10 10 ~~10~~

5 3

31

5 2

36/50