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Start vector equations

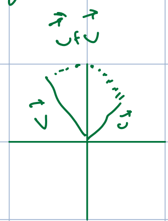
Def: A vector in $\mathbb{R}^2$ is a pair of notes $\vec{v} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$ denoting the line segment from $(0,0)$ to (v_1, v_2)

Fact: You can add & scale vectors; ie: you can compute "linear combinations"

ex: let $\vec{u} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ & $\vec{v} = \begin{pmatrix} 2 \\ -5 \end{pmatrix}$: compute: $4\vec{u} - 3\vec{v}$

$4\vec{u} = \begin{pmatrix} 4 \\ 8 \end{pmatrix}$ $\quad 3\vec{v} = \begin{pmatrix} 6 \\ -15 \end{pmatrix}$ $\quad 4\vec{u} - 3\vec{v} = \begin{pmatrix} 4 \\ 8 \end{pmatrix} + \begin{pmatrix} 6 \\ -15 \end{pmatrix} = \begin{pmatrix} 10 \\ -7 \end{pmatrix}$

Parallelogram rule: If $\vec{u}, \vec{v} \in \mathbb{R}^2$, the $\vec{u} + \vec{v}$ is the 4th vertex of the parallelogram whose other vertices are $\vec{0}, \vec{u}, \vec{v}$



We have vectors like $\vec{u} = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} \in \mathbb{R}^3$, $\vec{u} = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ \vdots \\ u_n \end{pmatrix} \in \mathbb{R}^n$

Proposition: for vectors $\vec{u}, \vec{v}, \vec{w} \in \mathbb{R}^n$ & scalars $c, d \in \mathbb{R}$: (i) $\vec{u} + \vec{v} = \vec{v} + \vec{u}$ (commutative)

(ii). $(\vec{u} + \vec{v}) + \vec{w} = \vec{u} + (\vec{v} + \vec{w})$ (Associative)

(iii). If add 0 vector, get same vector $0 + \vec{u} = \vec{u}$

(iiii) $\vec{u} + (-\vec{u}) = 0$,

(iiiii): $c(\vec{v} + \vec{u}) = c\vec{u} + c\vec{v}$

(vi): $(c+d)\vec{u} = c\vec{u} + d\vec{u}$

(vii) $= (cd)\vec{u}$ " = same

Note: unambiguously write $\vec{u} + (-\vec{v})$ as $\vec{u} - \vec{v}$

Def: Let $\vec{v}_1, \vec{v}_2, \dots, \vec{v_p} \in \mathbb{R}^n$ & let $c_1, c_2, \dots, c_p \in \mathbb{R}$. The vector $\vec{y} = c_1 \vec{v}_1 + c_2 \vec{v}_2 + \dots$ is a linear combination of $\vec{v}_1, \dots, \vec{v_p}$ with weights / coefficients $c_1, \dots, c_p$.

ex. Let $\vec{a}_1 = \begin{pmatrix} 1 \\ -2 \\ 5 \end{pmatrix}$ $\vec{a}_2 = \begin{pmatrix} 2 \\ 5 \\ 6 \end{pmatrix}$ $\vec{b} = \begin{pmatrix} 7 \\ 4 \\ -3 \end{pmatrix}$

Q.: Are these numbers x_1 & x_2 such that $x_1 \vec{a}_1 + x_2 \vec{a}_2 = \vec{b}$?

$$x_1 \begin{pmatrix} 1 \\ -2 \\ -5 \end{pmatrix} + x_2 \begin{pmatrix} 2 \\ 5 \\ 6 \end{pmatrix} = \begin{matrix} 7 \\ 4 \\ -3 \end{matrix} \rightarrow \begin{matrix} x_1 + 2x_2 = 7 \\ -2x_1 + 5x_2 = 4 \\ -5x_1 + 6x_2 = -3 \end{matrix}$$

$$\left[\begin{array}{cc:c} 1 & 2 & 7 \\ -2 & 5 & 4 \\ -5 & 6 & -3 \end{array}\right] \xrightarrow[R_3+5R_1]{R_2+2R_1} \begin{pmatrix} 1 & 2 & 7 \\ 0 & 9 & 18 \\ 0 & 16 & 32 \end{pmatrix} \xrightarrow[\frac{1}{16}R_3]{\frac{1}{9}R_2} \begin{pmatrix} 1 & 2 & 7 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{pmatrix} \rightarrow R_3 - R_2$$

$$\begin{pmatrix} 1 & 2 & 7 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix} \xrightarrow{R_1 - 2R_2} \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix} \Leftrightarrow \left\{ \begin{matrix} x_1 = 3 \\ x_2 = 2 \end{matrix} \right\}$$

Answer: Yes

Fact: A vector equation $x_1 \vec{a}_1 + x_2 \vec{a}_2 + \dots x_n \vec{a}_n = b$ has same sltn set as the linear system whose augmented matrix is

$$\left(\vec{a}_1 \ \vec{a}_2 \ \dots \ \vec{a}_n \ \vdots \ \vec{b} \right)$$

Def: For $\vec{v}_1, \dots, \vec{v_p} \in \mathbb{R}^n$, the set of all linear combinations of them is called their span: $\{ \vec{v}_1 \dots \vec{v_p} \}$

ex: $\begin{pmatrix} 7 \\ 4 \\ -3 \end{pmatrix} \in \text{span} \left\{ \begin{pmatrix} 1 \\ -2 \\ -5 \end{pmatrix} \begin{pmatrix} 2 \\ 5 \\ 6 \end{pmatrix} \right\}$.

ex: $\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \right\}$: "xy plane" in $\mathbb{R}^3$.

Q: Given $\vec{v}$ $\mathbb{R}^2$, what is span $\{\vec{v}\}$? All c, entire line $\vec{v}$ is inside of

1.4 Matrix equation $A\vec{x} = \vec{b}$.

Def: For A an $m \cdot n$ matrix (m rows, n columns) with columns $\vec{a}_1, \dots, \vec{a}_n$ & $\vec{x} \in \mathbb{R}^n$, the product $A\vec{x}$ is:

$$A\vec{x} := (\vec{a}_1 \dots \vec{a}_n)\begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = x_1\vec{a}_1 + x_2\vec{a}_2 + \dots + x_n\vec{a}_n$$

(is in $\mathbb{R}^m$)

ex: $\begin{pmatrix} 1 & -2 & 2 \\ 0 & 5 & 3 \end{pmatrix}\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$ ✓ num of col = num of rows

$$2\begin{pmatrix} 1 \\ 0 \end{pmatrix} + 0\begin{pmatrix} -2 \\ 5 \end{pmatrix} + 1\begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$$

Idea: $m \cdot n$ matrix A is an operation which eats an n-vector & deposits an m-vector.