Linear Algebra Midterm 2 Practice

2.1 T&F

Exercises 15–24 concern arbitrary matrices A, B, and C for which the indicated sums and products are defined. Mark each statement True or False **(T/F)**. Justify each answer.

15. $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ with a_1, a_2; $B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$ with b_1, b_2

$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

15. **(T/F)** If A and B are 2×2 with columns $\mathbf{a}_1, \mathbf{a}_2$, and $\mathbf{b}_1, \mathbf{b}_2$, respectively, then $AB = [\mathbf{a}_1\mathbf{b}_1 \quad \mathbf{a}_2\mathbf{b}_2]$.

16. **(T/F)** If A and B are 3×3 and $B = [\mathbf{b}_1 \quad \mathbf{b}_2 \quad \mathbf{b}_3]$, then $AB = [A\mathbf{b}_1 + A\mathbf{b}_2 + A\mathbf{b}_3]$.

17. **(T/F)** Each column of AB is a linear combination of the columns of B using weights from the corresponding column of A.

18. **(T/F)** The second row of AB is the second row of A multiplied on the right by B.

19. **(T/F)** $AB + AC = A(B + C)$

20. **(T/F)** $A^T + B^T = (A + B)^T$

21. **(T/F)** $(AB)C = (AC)B$ Order Matters

22. **(T/F)** $(AB)^T = A^T B^T$ $B^T \cdot A^T$

23. **(T/F)** The transpose of a product of matrices equals the product of their transposes in the same order.

24. **(T/F)** The transpose of a sum of matrices equals the sum of their transposes.

2.2 TF

In Exercises 11–20, mark each statement True or False **(T/F)**. Justify each answer.

11. **(T/F)** In order for a matrix B to be the inverse of A, both equations $AB = I$ and $BA = I$ must be true.

12. **(T/F)** A product of invertible $n \times n$ matrices is invertible, and the inverse of the product is the product of their inverses in the same order. $(AB)^{-1} = B^{-1}A^{-1}$

11? ASIC

13. **(T/F)** If A and B are $n \times n$ and invertible, then $A^{-1}B^{-1}$ is the inverse of AB. $B^{-1}A^{-1}$ is the inverse of AB

14. **(T/F)** If A is invertible, then the inverse of A^{-1} is A itself. $(A^{-1})^{-1} = A$

15. **(T/F)** If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $ab - cd \neq 0$, then A is invertible. ?

16. **(T/F)** If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $ad = bc$, then A is not invertible. Det A would = 0, not Inv.

17. **(T/F)** If A is an invertible $n \times n$ matrix, then the equation $A\mathbf{x} = \mathbf{b}$ is consistent for *each* $\mathbf{b}$ in $\mathbb{R}^n$.

18. **(T/F)** If A can be row reduced to the identity matrix, then A must be invertible. $[A : I_n]$

19. **(T/F)** Each elementary matrix is invertible.

20. **(T/F)** If A is invertible, then the elementary row operations that reduce A to the identity I_n also reduce A^{-1} to I_n.

21. Let A be an invertible $n \times n$ matrix, and let B be an $n \times p$ matrix. Show that the equation $AX = B$ has a unique solution $A^{-1}B$.

2.3

In Exercises 11–20, the matrices are all $n \times n$. Each part of the exercises is an *implication* of the form "If 'statement 1', then 'statement 2'." Mark an implication as True if the truth of "statement 2" *always* follows whenever "statement 1" happens to be true. An implication is False if there is an instance in which "statement 2" is false but "statement 1" is true. Justify each answer.

11. **(T/F)** If the equation $A\mathbf{x} = \mathbf{0}$ has only the trivial solution, then A is row equivalent to the $n \times n$ identity matrix. T

12. **(T/F)** If there is an $n \times n$ matrix D such that $AD = I$, then there is also an $n \times n$ matrix C such that $CA = I$. T

13. **(T/F)** If the columns of A span $\mathbb{R}^n$, then the columns are linearly independent. T

14. **(T/F)** If the columns of A are linearly independent, then the columns of A span $\mathbb{R}^n$. T

15. **(T/F)** If A is an $n \times n$ matrix, then the equation $A\mathbf{x} = \mathbf{b}$ has at least one solution for each $\mathbf{b}$ in $\mathbb{R}^n$. T

16. **(T/F)** If the equation $A\mathbf{x} = \mathbf{b}$ has at least one solution for each $\mathbf{b}$ in $\mathbb{R}^n$, then the solution is unique for each $\mathbf{b}$. T

17. **(T/F)** If the equation $A\mathbf{x} = \mathbf{0}$ has a nontrivial solution, then A has fewer than n pivot positions. T

18. **(T/F)** If the linear transformation $\mathbf{x} \mapsto A\mathbf{x}$ maps $\mathbb{R}^n$ into $\mathbb{R}^n$, then A has n pivot positions. F

19. **(T/F)** If A^T is not invertible, then A is not invertible. $(A^T)^{-1} = (A^{-1})^T$

20. **(T/F)** If there is a $\mathbf{b}$ in $\mathbb{R}^n$ such that the equation $A\mathbf{x} = \mathbf{b}$ is inconsistent, then the transformation $\mathbf{x} \mapsto A\mathbf{x}$ is not one-to-one.

21. An $m \times n$ **upper triangular matrix** is one whose entries *below* the main diagonal are 0's (as in Exercise 8). When is a square upper triangular matrix invertible? Justify your answer.

When the products of its main diagonal is not 0!, therefore when $\det \neq 0$

3.1

In Exercises 39 through 42, A is an $n \times n$ matrix. Mark each statement True or False (**T/F**). Justify each answer.

39. **(T/F)** An $n \times n$ determinant is defined by determinants of $(n-1) \times (n-1)$ submatrices.

40. **(T/F)** The (i, j)-cofactor of a matrix A is the matrix A_{ij} obtained by deleting from A its ith row and jth column.

41. **(T/F)** The cofactor expansion of det A down a column is equal to the cofactor expansion along a row.

42. **(T/F)** The determinant of a triangular matrix is the sum of the entries on the main diagonal.

3.2

In Exercises 27–34, A and B are $n \times n$ matrices. Mark each statement True or False (**T/F**). Justify each answer.

- 5

27. **(T/F)** A row replacement operation does not affect the determinant of a matrix.

28. **(T/F)** If det A is zero, then two rows or two columns are the same, or a row or a column is zero. LD

29. **(T/F)** If the columns of A are linearly dependent, then det $A = 0$.

30. **(T/F)** The determinant of A is the product of the diagonal entries in A. if triangular

31. **(T/F)** If three row interchanges are made in succession, then the new determinant equals the old determinant. − + −

32. **(T/F)** The determinant of A is the product of the pivots in any echelon form U of A, multiplied by $(-1)^r$, where r is the number of row interchanges made during row reduction from A to U.

33. **(T/F)** $\det(A + B) = \det A + \det B$.

34. **(T/F)** $\det A^{-1} = (-1) \det A$.

4.1

In Exercises 23–32, mark each statement True or False **(T/F)**. Justify each answer.

23. **(T/F)** If **f** is a function in the vector space V of all real-valued functions on $\mathbb{R}$ and if $\mathbf{f}(t) = 0$ for some t, then **f** is the zero vector in V.

only know for that value of t

24. **(T/F)** A vector is any element of a vector space.

25. **(T/F)** An arrow in three-dimensional space can be considered to be a vector.

26. **(T/F)** If **u** is a vector in a vector space V, then $(-1)\,\mathbf{u}$ is the same as the negative of **u**.

27. **(T/F)** A subset H of a vector space V is a subspace of V if the zero vector is in H.

needs to be closed under vector addition & scalar multiplication

28. **(T/F)** A vector space is also a subspace.

29. **(T/F)** A subspace is also a vector space.

30. **(T/F)** $\mathbb{R}^2$ is a subspace of $\mathbb{R}^3$. Generally no

31. **(T/F)** The polynomials of degree two or less are a subspace of the polynomials of degree three or less.

32. **(T/F)** A subset H of a vector space V is a subspace of V if the following conditions are satisfied: (i) the zero vector of V is in H, (ii) $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{u}+\mathbf{v}$ are in H, and (iii) c is a scalar and $c\mathbf{u}$ is in H.

Satisfies All axioms

In Exercises 25–38, A denotes an $m \times n$ matrix. Mark each statement True or False **(T/F)**. Justify each answer.

4.2

25. **(T/F)** The null space of A is the solution set of the equation $A\mathbf{x} = \mathbf{0}$. T, Null space is all x that map to 0

26. **(T/F)** A null space is a vector space. T, bc null space is a subspace, therefore vector

27. **(T/F)** The null space of an $m \times n$ matrix is in $\mathbb{R}^m$. F, is in $\mathbb{R}^n$, Col space is in $\mathbb{R}^m$

28. **(T/F)** The column space of an $m \times n$ matrix is in $\mathbb{R}^m$. Above

29. **(T/F)** The column space of A is the range of the mapping $\mathbf{x} \mapsto A\mathbf{x}$.

30. **(T/F)** Col A is the set of all solutions of $A\mathbf{x} = \mathbf{b}$.

31. **(T/F)** If the equation $A\mathbf{x} = \mathbf{b}$ is consistent, then Col $A = \mathbb{R}^m$.

32. **(T/F)** Nul A is the kernel of the mapping $\mathbf{x} \mapsto A\mathbf{x}$. Same thing

33. **(T/F)** The kernel of a linear transformation is a vector space. Nul space is same as kernel of T

34. **(T/F)** The range of a linear transformation is a vector space.

35. **(T/F)** Col A is the set of all vectors that can be written as $A\mathbf{x}$ for some $\mathbf{x}$. Definition of Col space

36. **(T/F)** The set of all solutions of a homogeneous linear differential equation is the kernel of a linear transformation.

37. **(T/F)** The row space of A is the same as the column space of A^T.

38. **(T/F)** The null space of A is the same as the row space of A^T.

In Exercises 21–32, mark each statement True or False (T/F). Justify each answer.

4.3

21. **(T/F)** A single vector by itself is linearly dependent.

22. **(T/F)** A linearly independent set in a subspace H is a basis for H. only if it spans as well

23. **(T/F)** If $H = \text{Span}\{\mathbf{b}_1, \ldots, \mathbf{b}_p\}$, then $\{\mathbf{b}_1, \ldots, \mathbf{b}_p\}$ is a basis for H. only if LI too

24. **(T/F)** If a finite set S of nonzero vectors spans a vector space V, then some subset of S is a basis for V. Spanning set theorem

25. **(T/F)** The columns of an invertible $n \times n$ matrix form a basis for $\mathbb{R}^n$.

26. **(T/F)** A basis is a linearly independent set that is as large as possible. LI set is maximum

27. **(T/F)** A basis is a spanning set that is as large as possible. spanning set small

28. **(T/F)** The standard method for producing a spanning set for Nul A, described in Section 4.2, sometimes fails to produce a basis for Nul A.

29. **(T/F)** In some cases, the linear dependence relations among the columns of a matrix can be affected by certain elementary row operations on the matrix.

30. **(T/F)** If B is an echelon form of a matrix A, then the pivot columns of B form a basis for Col A.

31. **(T/F)** Row operations preserve the linear dependence relations among the rows of A.

32. **(T/F)** If A and B are row equivalent, then their row spaces are the same.

4.4

In Exercises 15–20, mark each statement True or False (**T/F**). Justify each answer. Unless stated otherwise, $\mathcal{B}$ is a basis for a vector space V.

15. (**T/F**) If $\mathbf{x}$ is in V and if $\mathcal{B}$ contains n vectors, then the $\mathcal{B}$-coordinate vector of $\mathbf{x}$ is in $\mathbb{R}^n$.

16. (**T/F**) If $\mathcal{B}$ is the standard basis for $\mathbb{R}^n$, then the $\mathcal{B}$-coordinate vector of an $\mathbf{x}$ in $\mathbb{R}^n$ is $\mathbf{x}$ itself.

17. (**T/F**) If $P_{\mathcal{B}}$ is the change-of-coordinates matrix, then $[\mathbf{x}]_{\mathcal{B}} = P_{\mathcal{B}}\,\mathbf{x}$, for $\mathbf{x}$ in V.

18. (**T/F**) The correspondence $[\,\mathbf{x}\,]_{\mathcal{B}} \mapsto \mathbf{x}$ is called the coordinate mapping.

19. (**T/F**) The vector spaces $\mathbb{P}_3$ and $\mathbb{R}^3$ are isomorphic. false, isomorphic to $\mathbb{R}^4$

20. (**T/F**) In some cases, a plane in $\mathbb{R}^3$ can be isomorphic to $\mathbb{R}^2$.

4.5

In Exercises 17–26, V is a vector space and A is an $m \times n$ matrix. Mark each statement True or False **(T/F)**. Justify each answer.

17. **(T/F)** The number of pivot columns of a matrix equals the dimension of its column space.

dim Col A = num of pivot columns

18. **(T/F)** The number of variables in the equation $A\mathbf{x} = \mathbf{0}$ equals the nullity A.

Nullity A = num free variables

19. **(T/F)** A plane in $\mathbb{R}^3$ is a two-dimensional subspace of $\mathbb{R}^3$.

20. **(T/F)** The dimension of the vector space $\mathbb{P}_4$ is 4.

its $n+1$, in this case, 5

~~**21.** **(T/F)**~~ The dimension of the vector space of signals, $\mathbb{S}$, is 10.

22. **(T/F)** The dimensions of the row space and the column space of A are the same, even if A is not square.

dim Col A = dim Row A

23. **(T/F)** If B is any echelon form of A, then the pivot columns of B form a basis for the column space of A.

Tell you where pivot positions are but may not correspond to A

24. **(T/F)** The nullity of A is the number of columns of A that are not pivot columns.

num of free variables so yes

25. **(T/F)** If a set $\{\mathbf{v}_1, \dots, \mathbf{v}_p\}$ spans a finite-dimensional vector space V and if T is a set of more than p vectors in V, then T is linearly dependent.

Some theorem or something

26. **(T/F)** A vector space is infinite-dimensional if it is spanned by an infinite set.

?

43. **(T/F)** If there exists a set $\{\mathbf{v}_1, \dots, \mathbf{v}_p\}$ that spans V, then $\dim V \leq p$.

44. **(T/F)** If there exists a linearly dependent set $\{\mathbf{v}_1, \dots, \mathbf{v}_p\}$ in V, then $\dim V \leq p$.

45. **(T/F)** If there exists a linearly independent set $\{\mathbf{v}_1, \dots, \mathbf{v}_p\}$ in V, then $\dim V \geq p$.

46. **(T/F)** If $\dim V = p$, then there exists a spanning set of $p + 1$ vectors in V.

47. **(T/F)** If every set of p elements in V fails to span V, then $\dim V > p$.

48. **(T/F)** If $p \geq 2$ and $\dim V = p$, then every set of $p - 1$ nonzero vectors is linearly independent.

49. Justify the following equality: $\dim \text{Row } A + \text{nullity } A = n$, the number of columns of A

50. Justify the following equality: $\dim \text{Row } A + \text{nullity } A^T = m$, the number of rows of A

Exercises 51 and 52 concern finite-dimensional vector spaces V and W and a linear transformation $T : V \to W$.

4.6