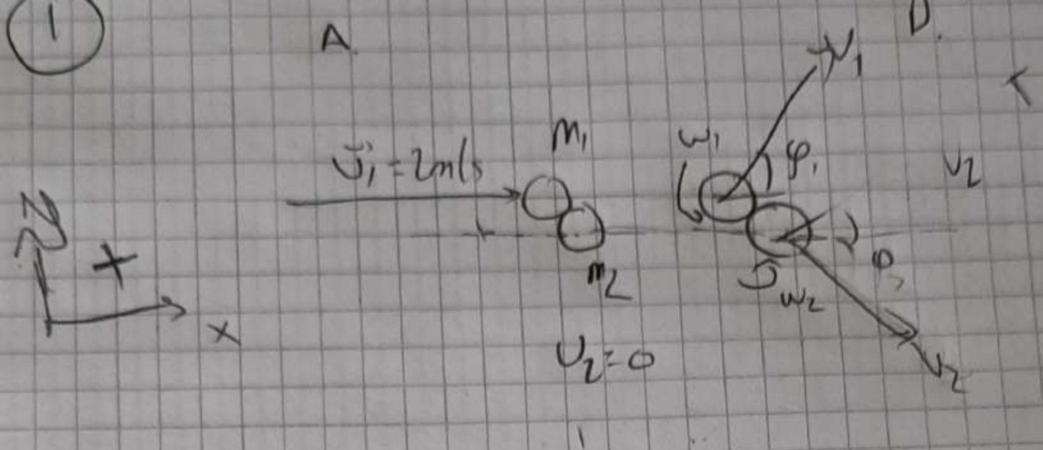
Valentina García - Ray Díaz

(1)



(A) se llama

$$v_{1x} = v_1 \cos \varphi_1$$
$$v_{1y} = v_1 \operatorname{sen} \varphi_1$$

$$v_{2x} = v_2 \cos \varphi_2$$
$$v_{2y} = v_2 \operatorname{sen} \varphi_2$$

$$\boxed{\vec{P}_I = \vec{P}_F}$$ Conv. M. Lineal

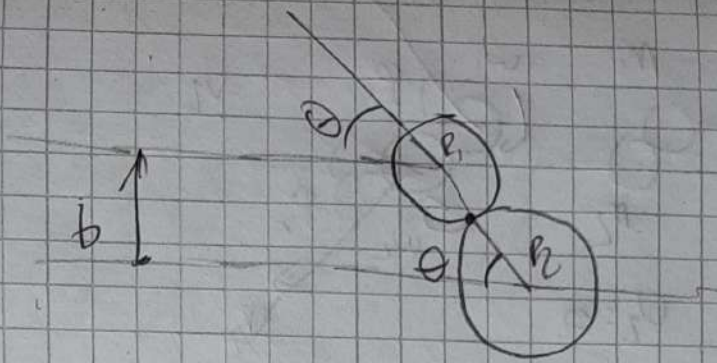
x: $m_1 v_1 = m_1 v_{1x} + m_2 v_{2x}$ (1)

y: $0 = m_1 v_{1y} + m_2 v_{2y}$ (2)

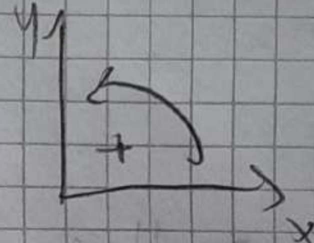
(B). Conv. Mon. angular.

$$\vec{L}_I = \vec{L}_F$$

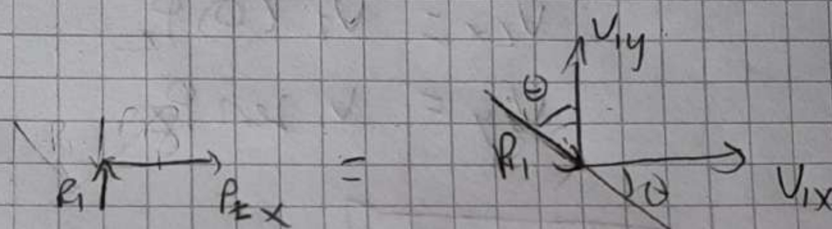
en el momento de la colisión



se mantiene el parámetro b constante



x: $L_{1I} = L_{1F}$



$$-m_1 v_1 R_1 = m_1 R_1 (V_{1x} \operatorname{sen}(90+\theta))$$

$$+ m_1 R_1 V_{1y} \operatorname{sen}\theta + I_1 \omega_1$$

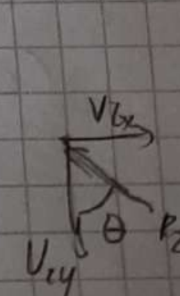
$$-m_1 v_1 R_1 = m_1 R_1 V_{1x} \cos\theta + m_1 R_1 V_{1y} \operatorname{sen}\theta$$

$$I_1 \omega_1 \qquad (3)$$

$$L_{2I} = L_{2F}$$

$$0 = I_2 \omega_2 - m_2 R_2 V_{2x} \operatorname{sen}(90-\theta)$$

$$+ m_2 V_{2y} R_2 \operatorname{sen}\theta$$

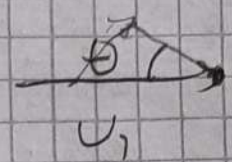


$0 = I_2 \omega_2 + M_2 R_2 V_{2x} \cos\theta + M_2 V_2 R \operatorname{sen}\theta$

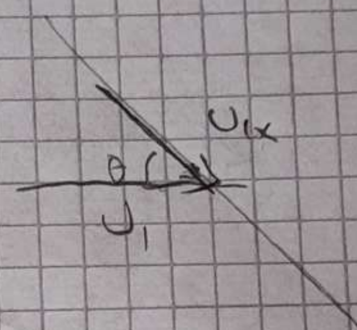
(4).

c). $e = \dfrac{\text{Impulso Final}}{\text{Impulso inicial}}$

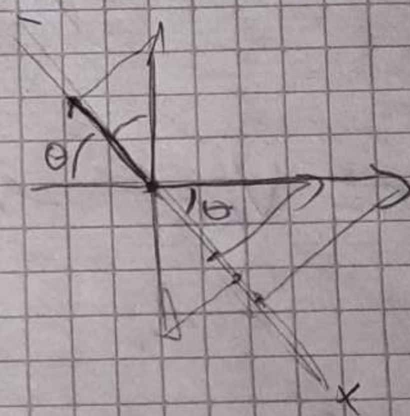
Impulso inicial →



$$\boxed{I = M_1 V_1 \cos\theta}$$



Impulso final



$V_{1y} \cos(90-\theta) = V_{1y} \operatorname{sen}\theta$

$V_{1x} \cos\theta = V_{1x} \cos\theta$

$V_{2x} \cos\theta = V_{2x} \cos\theta$

$V_{2y} \cos(90-\theta) = V_{2y} \operatorname{sen}\theta$

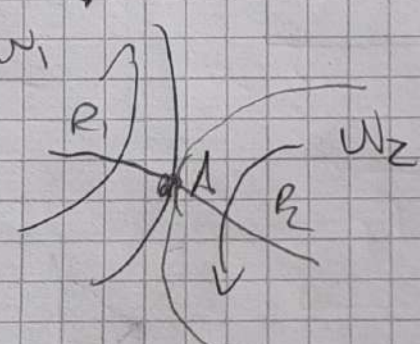
Por estar en la misma línea de acción del choque

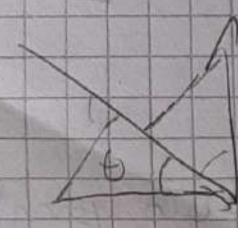
Imp ricid:

$$V_{2y} \sin\theta + V_{2x} \cos\theta - V_{1x} \cos\theta - V_{1y} \sin\theta$$

$$e = -\left[\frac{(V_{1y} \sin\theta + V_{1x} \cos\theta) - (V_{2y} \sin\theta + V_{2x} \cos\theta)}{U_1 \cos\theta}\right]$$

d) Si





$V_{AR_1} = V_{AR_2}$ →

$r = \vec{r}$

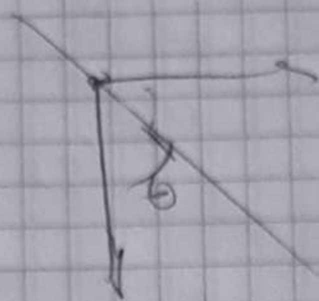
$V = R\dot{\theta} + \dot{R}$

$$\boxed{V = R\omega + V}$$

$R_1 w_1 + [V_{1x}, V_{1y}] = V_A$

$R_1 w_1 + V_{1x} \cos(90-\theta) + V_{1y} \cos\theta$

$R_1 w_1 + V_{1x} \sin\theta + V_{1y} \cos\theta = V_A$



$$V_2 = R_2 w_2 + V_{2T} = V_{el\ radial} + V_{tang}$$

$$= R_2 W_2 + [V_{2x}, V_{2y}]$$

$$= R_2 W_2 + V_{2x} \cos(90-\theta) + V_{2y} \cos\theta$$

$$= R_2 W_2 + V_{2x} \sin\theta + V_{2y} \cos\theta.$$

$$V_{A1} = V_{A2}$$

$$R_1 \omega_1 + V_{1x} \sin\theta + V_{1y} \cos\theta = R_2 \omega_1 + V_{2x} \sin\theta + V_{2y} \cos$$

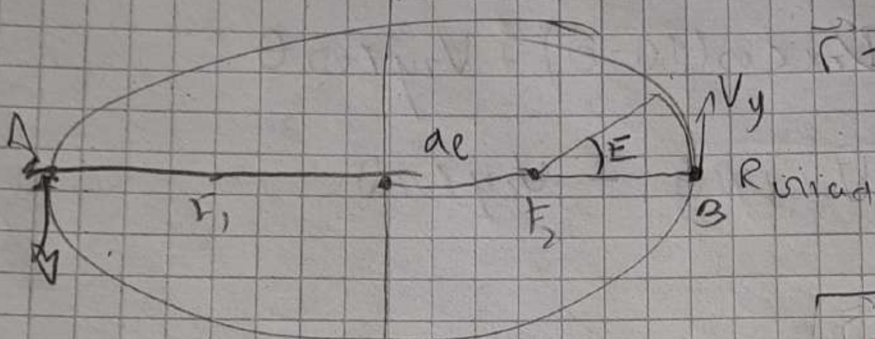
2

$$R = [a(1+e), 0, 0]$$

R_x = posición en x inicial

R_y = Pos. en y inicial

Carroll - Astrophysics



$\vec{r} =$

$$r = \frac{a(1-e^2)}{1+e\cos\theta}$$

como el punto inicial corresponde al afelio

$$R_{Afelio} = \frac{a(1+e)(1-e)}{1+e\cos(180)} = \frac{a(1+e)\cancel{(1-e)}}{\cancel{1-e}}$$

$$\boxed{R_0 = a(1+e)}$$

Para velocidad $L = \mu R V$

$$r_a = \frac{L^2/\mu^2}{GM(1 + e\cos\theta)} \quad , \theta = 180°$$

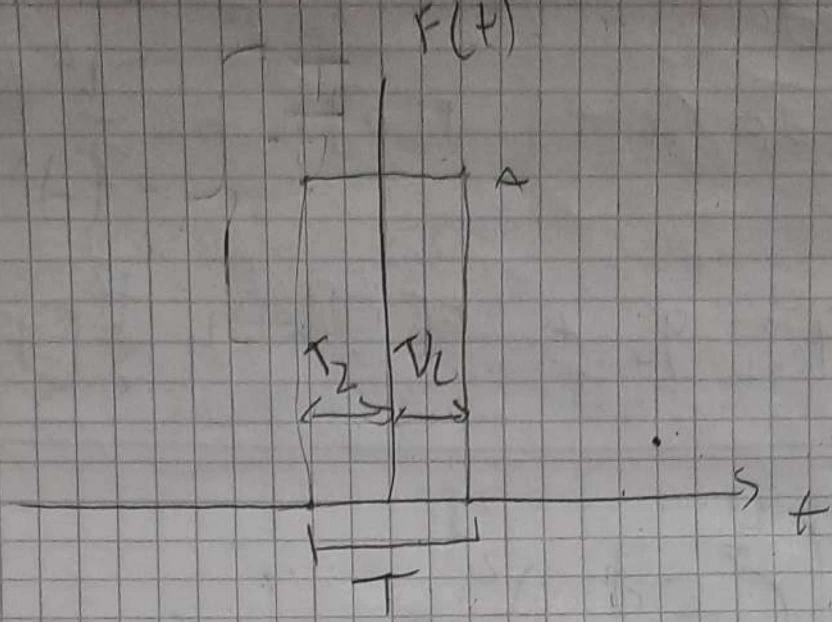
$$r_a = \frac{(\cancel{\mu^2} R_a^2 V_a^2 / \cancel{\mu^2})}{GM(1 - e)}$$

$$V_a^2 = \frac{GM(1-e)}{r_a} = \frac{GM}{a}\frac{(1-e)}{(1+e)}$$

$$\boxed{V_a = \sqrt{\frac{GM(1-e)}{a(1+e)}}}$$

//

4



$$F(t) = \begin{cases} 0 & \text{dlc} \\ A & -\frac{T}{2} \leq t \leq T/2 \end{cases}$$

$$F(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t) \right]$$

$$a_0 = \frac{1}{T} \int_{-T/2}^{T/2} F(t)\,dt = \frac{1}{T} A t \Big|_{-T/2}^{T/2}$$

$$= \frac{1}{T} \cdot A \left[\frac{T}{2} - \left(-T/2 \right) \right] =$$

$$\boxed{a_0 = \frac{1}{T} AT} \rightarrow \boxed{a_0 = A}$$

Dada $F(t) = \frac{A}{2} + \sum_{n=1}^{\infty} (a_n \cos(\omega_0 n t)) + \sum_{n=0}^{\infty} (b_n \operatorname{sen}(\omega_0 n t))$

$F(t) \to$ es continua si

$$\lim_{t \to T/2^+} F(t) = \lim_{t \to T/2^-} F(t)$$

limite derecha = limite por izq. → eso garantiza continuidad

$F(t)$ es derivable entre $[-T/2, T/2]$, si es continua

como en nuestro caso $F(t)$ es continua $[-T/2, T/2]$

lo que explica que es diferenciable

$\boxed{F'(t) = 0}$ → expandiendo en series Fourier

$$F(t) = \frac{A}{2} + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + b_n \,\text{sen}(n\omega_0 t)$$

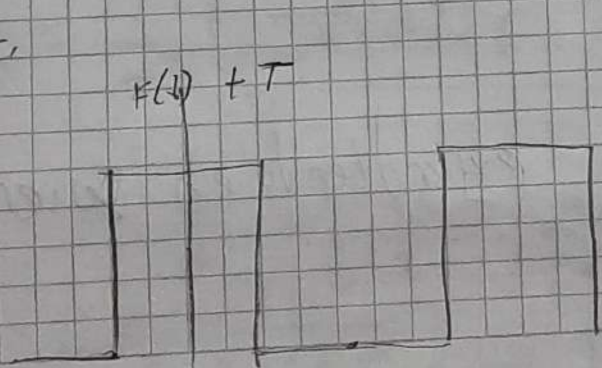
$$F'(t) = \frac{d}{dt}\left[\sum a_n \cos(n\omega_0 t) + b_n \,\text{sen}(\omega_n t)\right]$$

$$= \sum \left[a_n \left(\frac{d(\cos(n\omega t))}{dt}\right) + b_n \left(\frac{d\,\text{sen}(n\omega t)}{dt}\right)\right]$$

$$= \sum_{n=1}^{\infty} \left[a_n \left(-\text{sen}(n\omega_0 t)\right)(n\omega_0) + b_n \left(\cos(\omega_n t)\, n\omega_0\right)\right]$$

$$F'(t) = \sum n\omega_0 \left(b_n \cos(\omega_n t) - a_n \,\text{sen}(n\omega t)\right)$$

Ahora



$$F(t_2) = F(t+T) = \begin{cases} A & \frac{3T}{2} \leq t_2 \leq \frac{5T}{2} \\ 0 & \text{dlc} \end{cases}$$

$$F(t_1) \begin{cases} A & -T/2 \leq t_1 \leq T/2 \\ 0. & \text{dlc} \end{cases}$$

$$F(t_1) = \frac{a_0}{2} + \sum a_n \cos(n\omega_0 t_1) + b_n \sin(n\omega_0 t_1)$$

$$F(t_2) = \frac{a_0}{2} + \sum a_n \cos(n\omega_0 t_2) + b_n \sin(\omega_n t_2)$$

$$\int_0^{t_1} F(t_1)\, dt_1 = \int_0^{t_1} \left[\frac{a_0}{2} + \left[a_n \cos(\omega_n t_1) + b_n \sin(\omega_n t_1)\right]\right] dt_1$$

$$\int_0^{t_2} F(t_2)\, dt_2 = \int_0^{t_2} \left[\frac{a_0}{2} + (a_n \cos(\omega_n t_2) + b_n \sin(\omega_n t_2)\right] dt_2$$

$$\int_{t_1}^{t_2} F(t)\, dt = \int_{t_1}^{t_2} \left[a_0/2 + (a_n \cos(\omega_n t) + b_n \sin(\omega_n t)\right] dt$$

$$\int_{T_1}^{T_2} F(t)\,dt = \frac{a_0}{2}(t_2 - t_1)$$

$$+ \sum_{n=1}^{\infty} \frac{a_n}{n\omega_0}\left(\int_{t_1}^{t_2} \cos(\omega_n t)\,dt\right)$$

$$+ \sum \frac{b_n}{n\omega_0}\left(\int_{t_1}^{t_2} \sin(\omega_n t)\,dt\right)$$

$$= \frac{a_0(t_2 - t_1)}{2} + \sum_{n=1}^{\infty} \frac{a_n}{n\omega_0}\left(\sin(\omega_n t_2) - \sin(\omega_n t_1)\right)$$

$$+ \sum_{n=1}^{\infty} \frac{b_n}{n\omega_0}\left(-\cos(\omega_n t_2) + \cos(\omega_n t_1)\right)$$

$\rightarrow F(t) = t^2 \qquad -\pi < t < \pi$

$$a_0 = \frac{1}{T}\int_{-\pi}^{\pi} t^2\,dt = \frac{1}{T}\left[\frac{t^3}{3}\right]_{-\pi}^{\pi}$$

$$= \frac{1}{T}\left[\frac{\pi^3}{3} - \left(\frac{-\pi}{3}\right)^2\right]$$

$$\boxed{a_0 = \frac{2\pi^3}{3T}}$$

$$a_n = \frac{2}{T}\int_{-\pi}^{\pi} t^2 \cos(n\omega t)\,dt$$

$$a_n = \frac{2}{T}\,\frac{4n\omega_0\pi\cos(n\omega_0\pi) + 2(-2+(n\omega)^2\pi^2)\sin(n\omega\pi)}{(n\omega)^3}$$

$$\boxed{a_n = \frac{2\omega_0}{2\pi}\,\frac{4\pi\cos(n\omega_0\pi)}{n^2\omega^2}}$$

$n \to$ par $+$

$n \to$ impar $-$

$$a_n = \frac{4}{\omega_0}\cdot\frac{1}{n^2}\cdot\cos(n\omega_0\pi)$$

$$a_n = \frac{4}{w_0} \cdot \frac{1}{n^2} \cdot (-1)^n$$

$$b_n = \frac{2}{T} \int_{-\pi}^{\pi} F(t) \sin(n w_0 t)\, dt$$

$$= \frac{2}{T} \left[-\frac{1}{n w_0} \cancel{\left[t^2 \cos(n w t) \right]_{-\pi}^{\pi}}^{\to 0} + \frac{2}{n w_0} \int_{-\pi}^{\pi} t \cos(n w_0 t)\, dt \right]$$

$$= \frac{2}{n \pi} \int_{-\pi}^{\pi} t \left(\cos(n w_0 t) \right) dt$$

$$= \frac{2}{\pi n} \left[\cancel{\frac{1}{n w_0} (0 - 0)}^{\to 0} - \frac{1}{n w_0} \int_{-\pi}^{\pi} \sin(n w_0 t)\, dt \right]$$

$$= \frac{-2}{n^2 \pi w_0} \cancel{\int_{-\pi}^{\pi} \sin(w n t)\, dt}^{\to 0}$$

$$b_n = 0$$