Sabemos que:

$$\vec{a}^L = f(z) = f\left(\vec{b} + \overrightarrow{W} \cdot \vec{X}\right)$$

$$C = \frac{1}{2}(a^L - y_i)^2$$

Además:

$$\delta^L = \frac{\partial C}{\partial b^L} = (Utilizando\ regla\ de\ la\ cadena) = \frac{\partial C}{\partial a^L}\frac{\partial a^L}{\partial z^L}\frac{\partial z^L}{\partial b^L}$$

$$\delta^L = \frac{\partial C}{\partial a^L}\frac{\partial a^L}{\partial z^L} \cdot 1 = \frac{\partial C}{\partial a^L}\frac{\partial a^L}{\partial z^L} = \frac{\partial C}{\partial z^L}$$

Ahora demostremos la segunda parte:

$$\delta^{l-1} = \frac{\partial C}{\partial z^{l-1}} = \frac{\partial C}{\partial z^l}\frac{\partial z^l}{\partial z^{l-1}} = \delta^l \frac{\partial z^l}{\partial z^{l-1}}$$

Pero sabemos que

$$z^l = b^l + W^l \cdot X^l = b^l + W^l a^{l-1}\ (X\ es\ la\ función\ de\ activación\ de\ la\ capa\ anterior)$$

Derivamos:

$$\frac{\partial z^l}{\partial z^{l-1}} = \frac{\partial}{\partial z^{l-1}}(b^l + W^l a^{l-1}) = W^l \frac{\partial a^{l-1}}{\partial z^{l-1}}$$

$$\delta^{l-1} = \delta^l W^l \frac{\partial a^{l-1}}{\partial z^{l-1}}$$