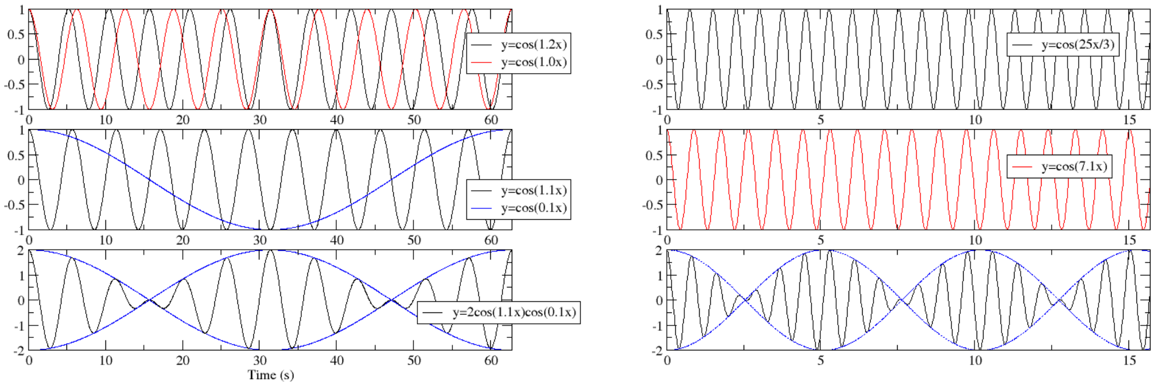

Figure 3: (a) The result of two driving forces on one SHO with different frequencies but the same amplitude. Top: two forces shown together. Middle: the sum and difference oscillations. Bottom: the resultant motion with the envelope superimposed in dashed lines. (b) Two driving forces on one SHO (the frequencies do not have integer relationship). Top: first force. Middle: second force; note that the peaks and troughs of the two forces do not *quite* coincide. Bottom: the resultant motion with the envelope superimposed in dashed lines.

Figure 3(a) shows an illustration of exactly this behaviour for the frequencies $\omega_1 = 1.2s^{-1}$ and $\omega_2 = 1.0s^{-1}$ which gives a resulting motion at frequency $\omega = 1.1s^{-1}$ modulated by an envelope with frequency $\Delta\omega = 0.1s^{-1}$.

The resulting motion will show a true periodicity if the ratio of the frequencies can be written as a ratio of integers (i.e. $\omega_1/\omega_2 = n_1/n_2$). The motion from two frequencies which are almost non-periodic is shown in Fig. 3(b) (I used 25/3 and 7.1 here but we could have made it properly non-periodic if we'd tried harder).

General Case If the amplitudes and frequencies all differ, then the phasor diagram must be dynamic as illustrated in Fig. 2(c), and the tip of the resultant will trace out a shape in time called a *cycloid*. For instance, the path followed by the tip of the resultant vector in the complex plane for different amplitudes and frequencies differing by a factor of two is plotted in Fig. 4.

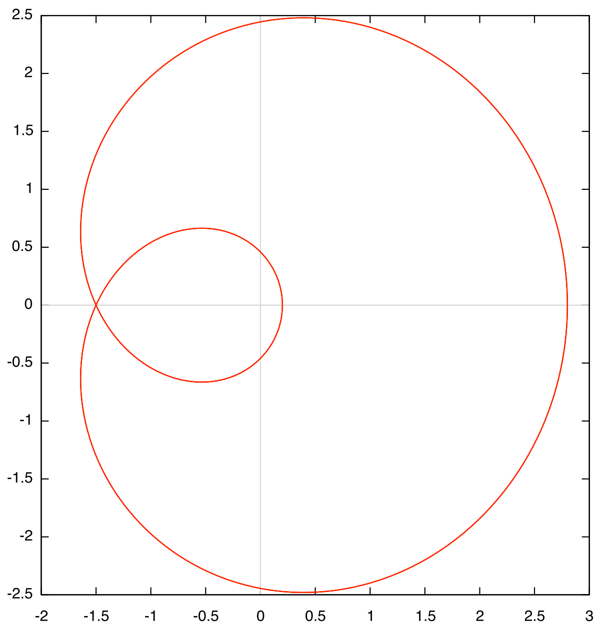

Figure 4: The location of the tip of the resultant vector for the two driving forces $1.5\cos 1.2t$ and $1.3\cos 0.6t$, which draws out a cycloid.