COMP2011 -- **Networks**, Databases and Graphics

1. Channel Capacity

Dr. Shi Zhou
Department of Computer Science
University College London

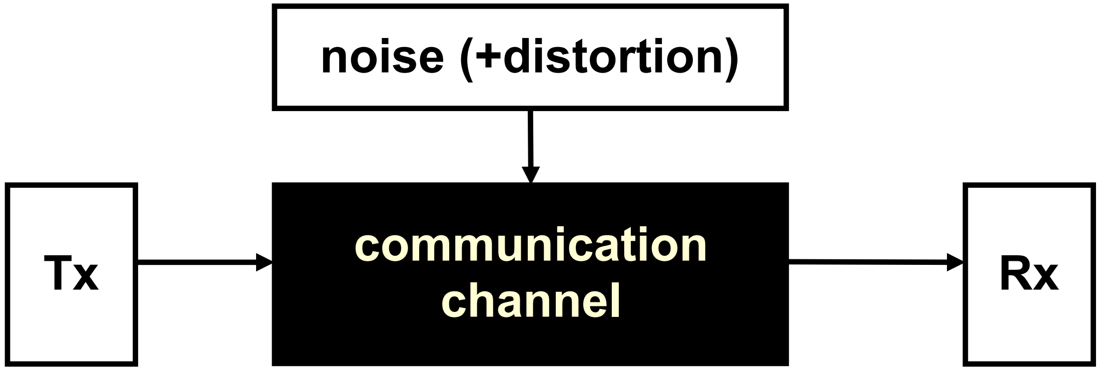
Questions

- What is the difference between analogue and digital signals?
  - Why use digital?
- How do we describe and characterise signals?
- Characteristics of a communication channel
  - Constraints: noise and bandwidth
  - Why can we get 100Mb/s on the office network, but only 30Kb/s over phone line using dial up?
  - Why broadband achieve much higher rates over the same phone line?

Part 1

Channel, signal and noise

Communication channel

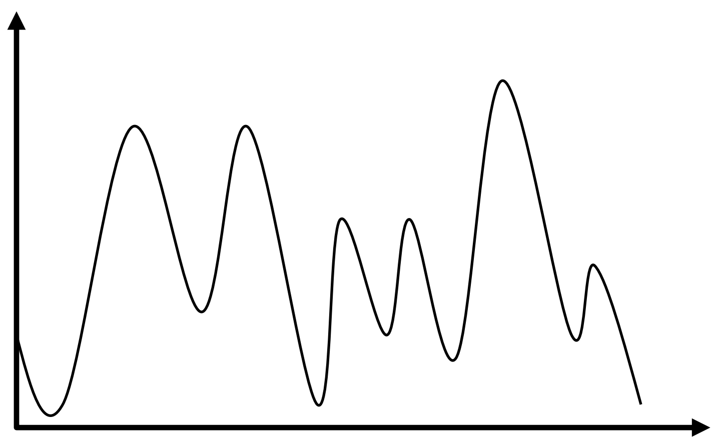


- Transmitter (Tx), receiver (Rx)
- Channel: a physical medium over which a signal is transmitted
  - copper wire, optical fibre, radio
- Signal: a presentation of information
  - analogue or digital
- Noise: anything that interferes, always present

Analogue and digital signals

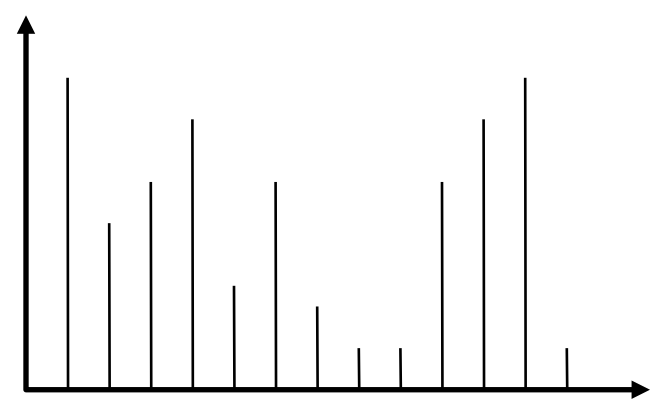
Analogue

- Signal level is *analogue* of information value
  - e.g. microphone – electrical wave mirrors sound wave (varying air pressure)
- Continuous, smoothly varying
- Mature (old!) technology



Digital

- Information represented by a form of encoding
- Discrete symbols or signal levels
  - e.g. text, numbers, Morse code sampled speech
- Anything can be converted to a digital representation
- Computer use binary codes
- New(er) technology



Noise

- Noise - anything that interferes, **always** present in a communication channel
- Electro-magnetic (EM) interference
  - Signals from nearby wires (cross-talk); induced by magnetic fields in the environment, e.g. electric motors; from the atmosphere, e.g. radio signal and broadcast transmission; and cosmic rays (radio signals from space)
  - Optical transmission is immune from EM interference
- Thermal noise
  - Due to agitation of electrons, impossible to remove
- Physical nature of channel medium
  - Attenuation – signal gets weaker with distance
  - Distortion – frequency dependent attenuation

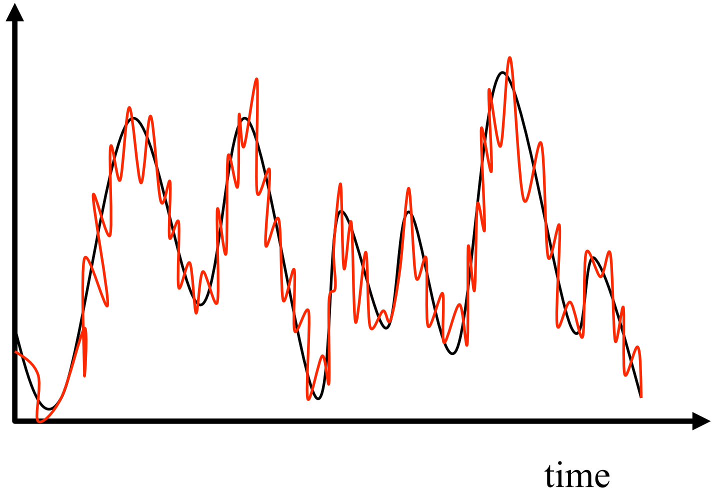
Signal to noise ratio - SNR

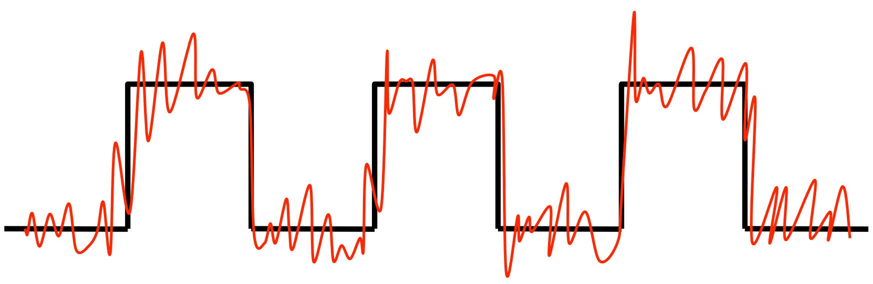
- SNR = S/N
  - Ratio of signal power (S) to noise power (N)
  - Relative strength of noise to the strength of transmitted signal
  - A meaningful assessment of noise
  - Assessing performance of a channel with respect to noise
- Usually quoted in dB (deciBels)
  - $SNR_{dB} = 10\log_{10}(S/N)$
  - Typical SNR is $1000 \equiv 30$dB
  - $10\log_{10}(2\times S/N) \approx 3 + 10\log_{10}(S/N)$

Signal and noise

- Analogue:
  - continuous, infinite range
- Transmission:
  - subject to noise in media interference, e.g. radio
  - hard to determine what is noise and what is signal.

- Digital:
  - discrete binary codes
- Transmission:
  - only have to transmit and receive ones and zeros
  - noise rejection is easier up to some level of noise





Advantages of digital signal

- ✓ High fidelity:
  - ✓ better error control (detection and correction)
- ✓ Source independence:
  - ✓ “anything” (audio, video etc.) can be digitised
- ✓ Time independence (storage is much easier):
  - ✓ transmission rate $\neq$ recording/capture rate
- ✓ Encoding
  - ✓ encryption and compression
- ✘ More complex and expensive
  - ✘ Particularly for real-time communication

Communication networks

- The earliest networks were digital
- – Telegraph systems encoded with Morse code
    - Discrete pulses either long (“dash”) or short (“dot”)
- Then we had many analogue systems

1. Bell invented the telephone by designing microphones and speakers
    - Sound waves represented by analogous electrical waves
2. Public service telephone network (PSTN) aims to carry speech frequencies ranging from 400Hz to 3400 Hz
    - Other frequencies are deliberately filtered out
3. Traditional radio and TV
    - Frequencies carried are much higher than the PSTN

Modern digital networks

- Digitised analogue systems
  - e.g. the core of PSTN becoming digital, where analogue voice signals are **encoded** to a stream of bits at 64,000 bps and the corresponding **decoding** restores the analogue signal before the final hop (the “local loop”) into a subscriber's home.
- Data communications between computers
  - computers process and store information digitally
  - ethernet, Internet, GSM mobile phone networks
- Digital networks can be general-purpose
  - possible to represent any kind of signal digitally

Channels

- Analogue channels – optimised for analogue signals
  - PSTN, analogue radio and TV, connecting speakers
- Digital channels – designed to transfer bits accurately
  - ISDN, Local Area Networks, digital radio and TV, GSM mobile phone system
- Often one is built on another

Digital ⇔ Analogue conversion

- Analogue-to-digital converter (ADC)
- Digital-to-analogue converter (DAC)
- Modem:
  - **mo**dulator/ **dem**odulator (DAC/ADC)
  - digital signal ⇔ analogue signal (analogue network)
  - tends to be hardware (software possible)
  - e.g. fax, dial-up to Internet using modem over PSTN
- Codec:
  - **co**der/**dec**oder (ADC/DAC)
  - analogue signal ⇔ digital signal (digital network/ computer)
  - can be software, hardware has better performance
  - e.g. mobile phone, Internet phone, video conference

Part 2

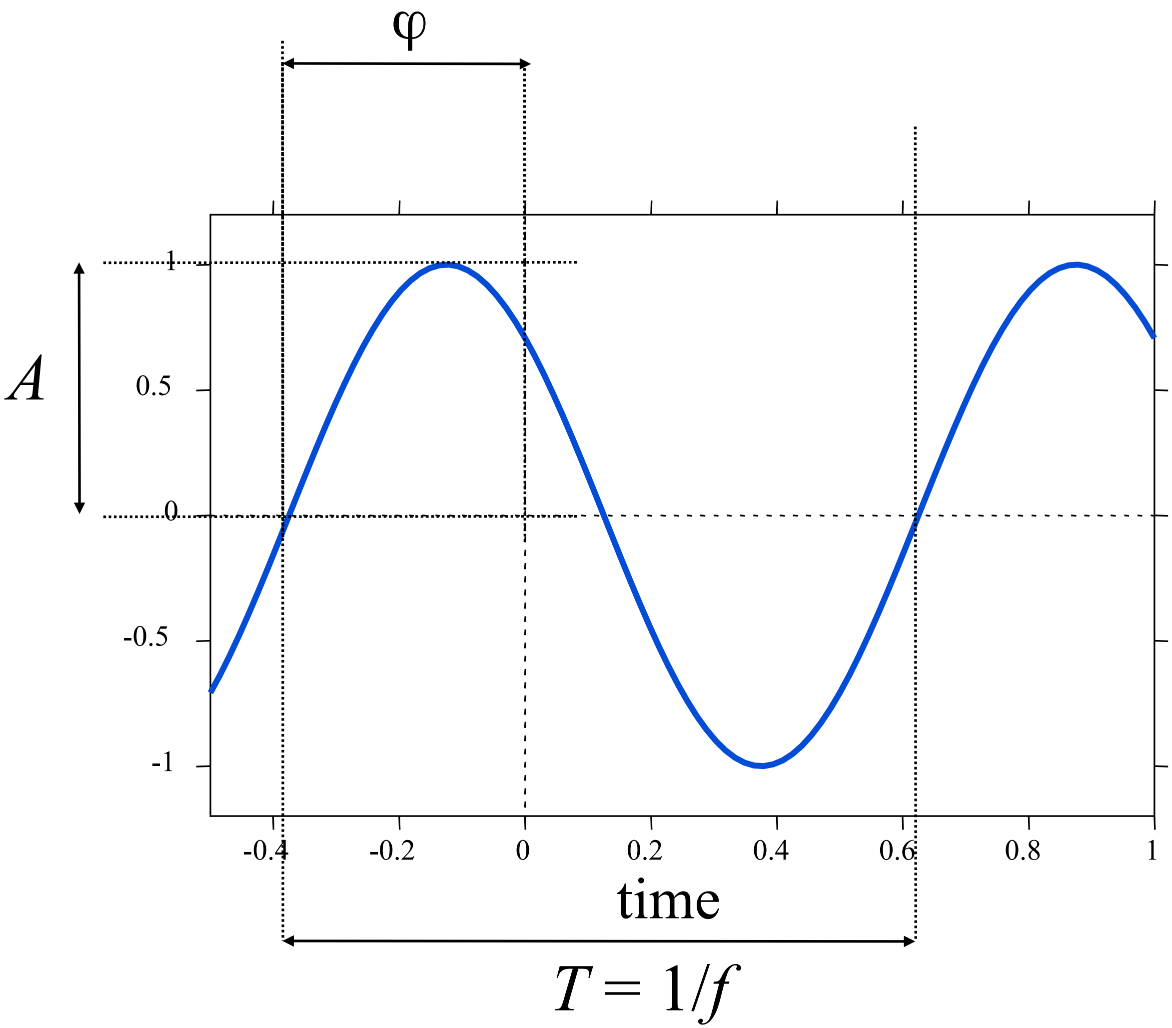
Fourier analysis and channel bandwidth

Description of signals

- Time domain
  - variation of signal in time
- Frequency domain
  - spectrum analysis, describing frequency content
  - **Fourier Theorem**: Any periodic signal can be shown to be composed of sinusoidal components of varying amplitude and frequency.

- We look at two simple examples
  - Analogue signal: sinusoidal wave
  - Digital signal: square wave

Time domain: Sinusoidal wave



$s(t) = A \sin(\omega t+\varphi)$

$= A \sin(2\pi f t+\varphi)$

A - amplitude

φ - phase

ω - frequency

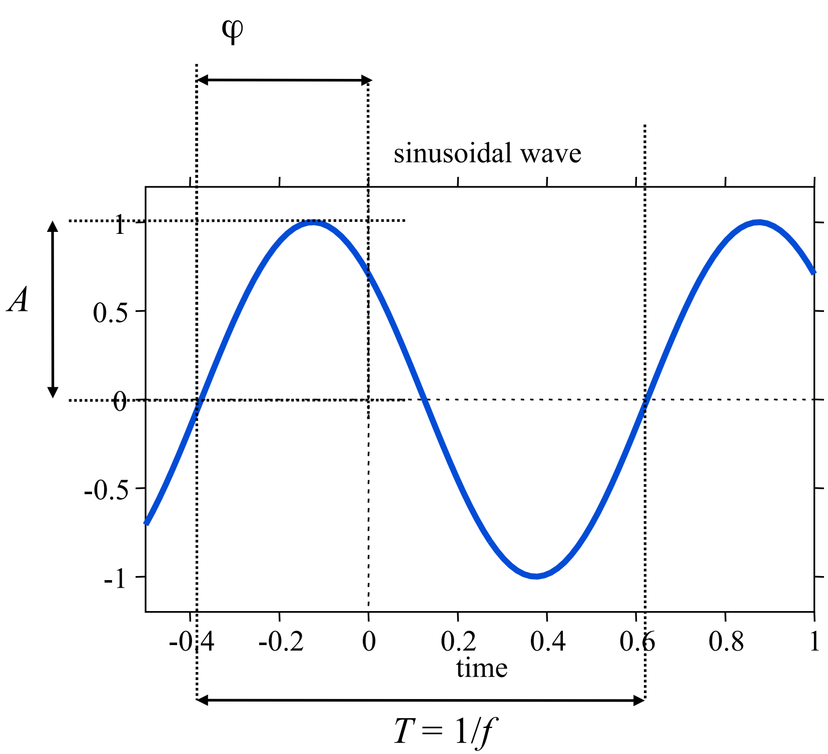
T - period

f - frequency

$f = 1/T$, Hertz (Hz)

$\omega = 2\pi f$, radians per sec

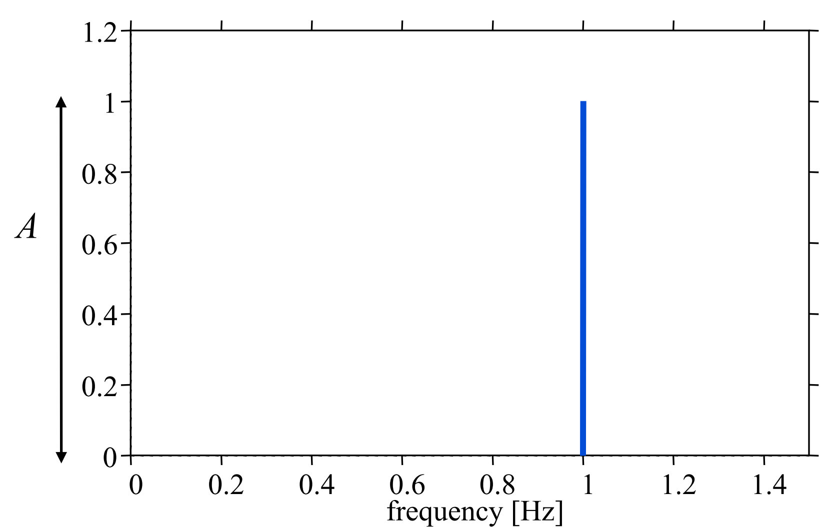
Frequency domain: sinusoidal wave



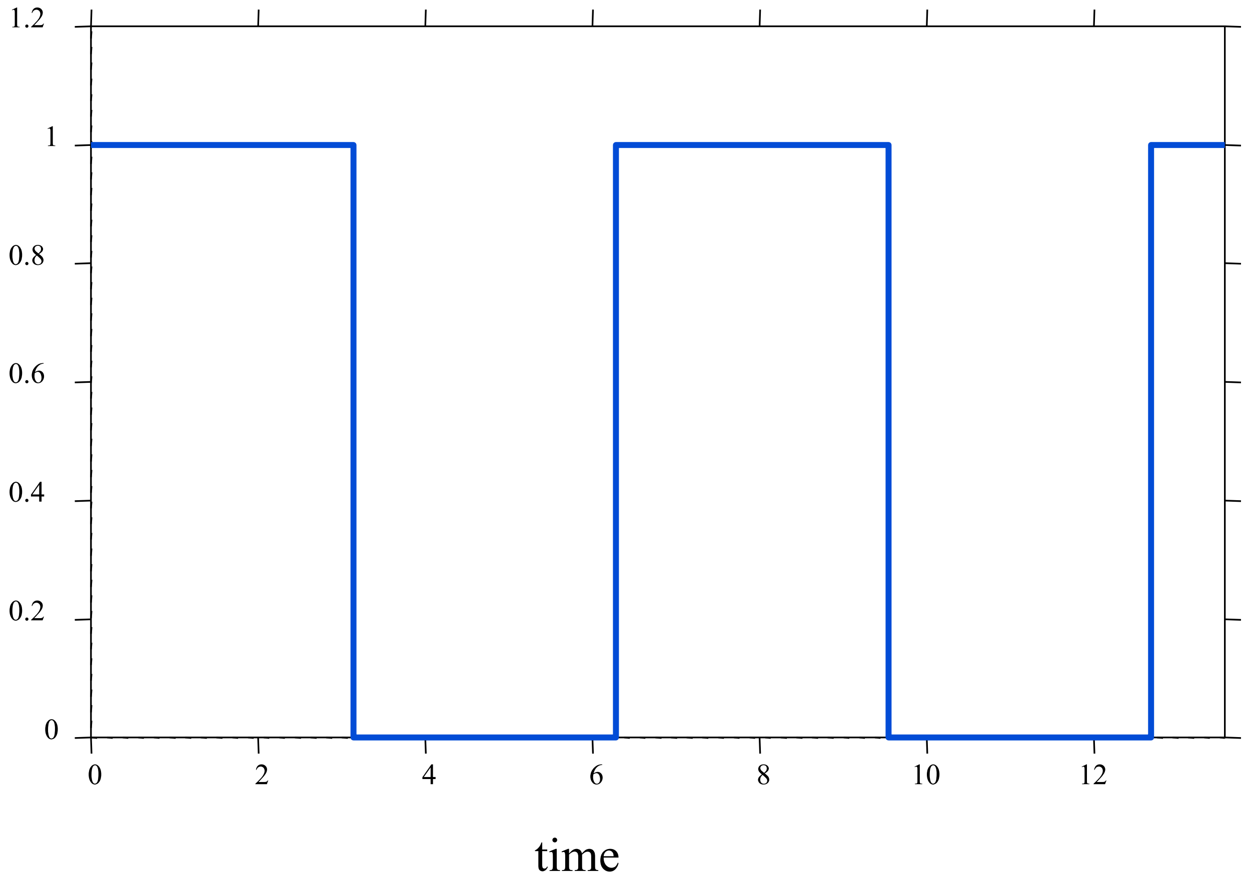
$$s(t) = A\sin(\omega t + \varphi)$$

$$\omega = 2\pi f$$

- Time domain:
  - $S(t)$
- Frequency domain:
  - a single frequency with amplitude 1
  - discrete spectrum
  - zero bandwidth

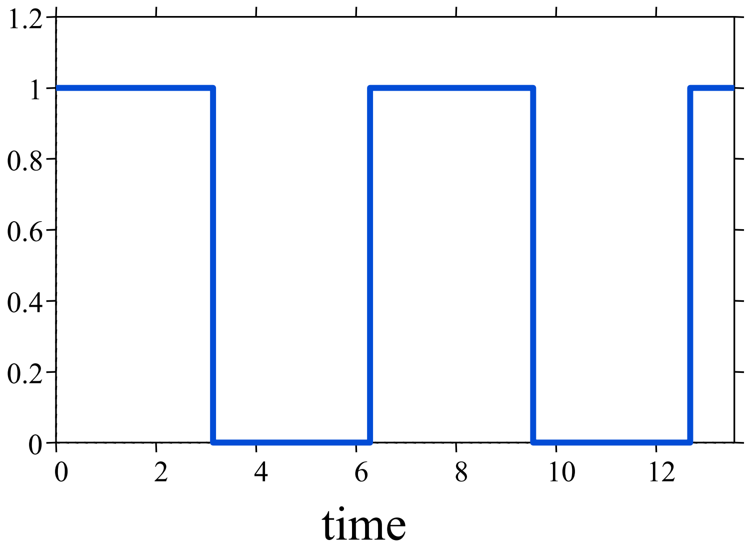


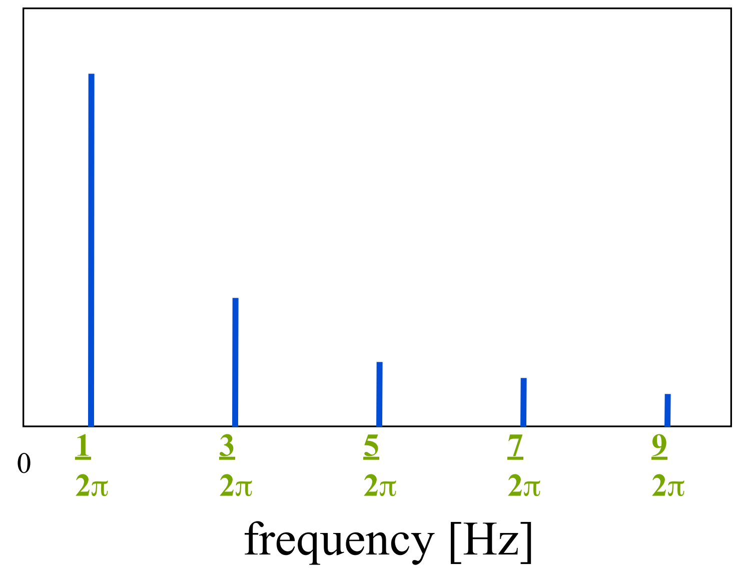
Time domain: Square wave



$$s(t) = 1; \quad 0 \le t < \pi, 2\pi \le t < 3\pi, \ldots$$
$$\quad\;\; = 0; \quad \pi \le t < 2\pi, 3\pi \le t < 4\pi, \ldots$$

Frequency domain: square wave



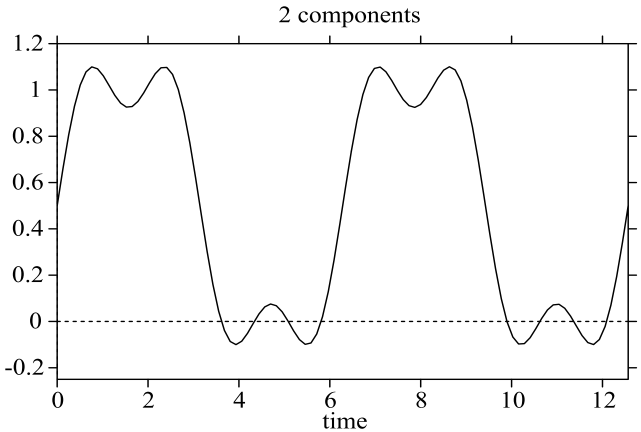


$$
\begin{aligned}
s(t) &= 1; \quad 0 \le t < \pi, 2\pi \le t < 3\pi, \ldots \\
&= 0; \quad \pi \le t < 2\pi, 3\pi \le t < 4\pi, \ldots
\end{aligned}
$$

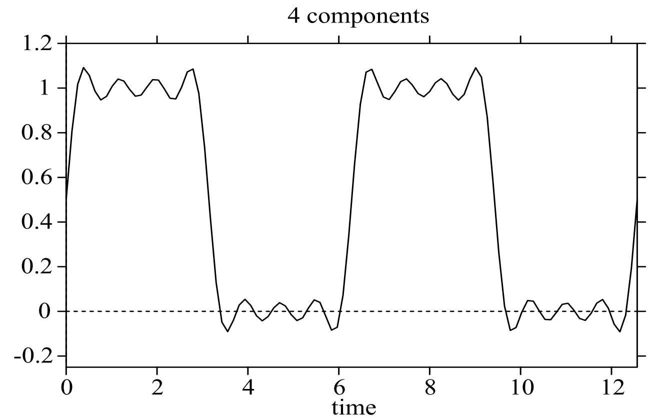
$$
s(t) = \frac{1}{2} + \frac{2}{\pi}\begin{bmatrix} \sin(t) + \frac{1}{3}\sin(3t) + \frac{1}{5}\sin(5t) + \\ \frac{1}{7}\sin(7t) + \frac{1}{9}\sin(9t) + \ldots \end{bmatrix}
$$

- **Frequency domain**
  - an infinite number of sinusoidal waveforms
  - pulse train, infinite bandwidth
  - the fundamental frequency of the first component is 1/2π Hz
    - same as the square wave's frequency
  - the next component has a frequency 3 times as much and the next 5 times as much.

Fourier Analysis



$$s(t) = \frac{1}{2} + \frac{2}{\pi}\left[\sin(t) + \frac{1}{3}\sin(3t)\right]$$



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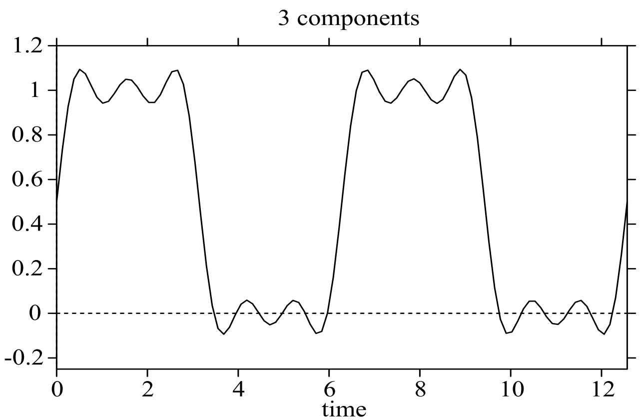

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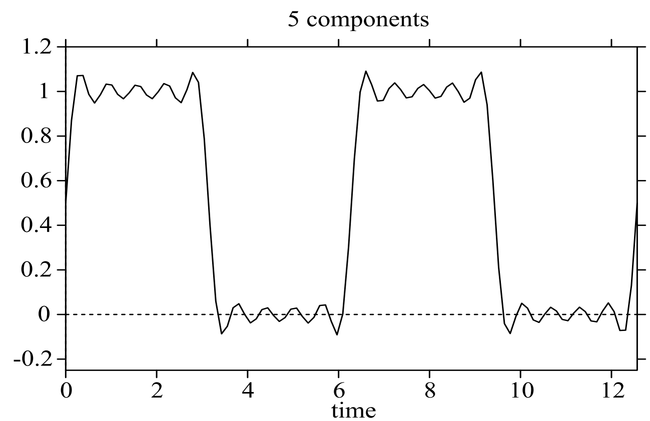


$$s(t) = \frac{1}{2} + \frac{2}{\pi}\left[\sin(t) + \frac{1}{3}\sin(3t) + \frac{1}{5}\sin(5t) + \frac{1}{7}\sin(7t) + \frac{1}{9}\sin(9t)\right]$$

Fourier Analysis [2]

- Lower frequency components are responsible for the basic shape and amplitude
  - most of signal power is in the first component with the fundamental frequency (the lowest frequency)
- Higher frequency components add detail
- When electrical force propagates through an electrical circuit, the electron movement is sinusoidal in nature.
  - like doing a Fourier analysis of the input signal
  - treats each frequency separately
  - modify amplitude of each frequency by a different amount
    - attenuation and distortion

Bandwidth

- Some circuits attenuate high frequencies strongly
  – smoothing sharp changes in the waveforms
  – equivalent to truncating the Fourier series
- Some circuits attenuate both low and high frequencies
  – e.g. standard telephone wires transmit frequencies between 300 and 3400 Hz

- Bandwidth (b/w) of a circuit is the range of frequency it can transmit
  – an important property of a circuit
  – physical constraint

Effect of channel bandwidth

- Analogue signals
  - CD-quality audio 20Hz to 12000Hz
    - Bandwidth 11980Hz
  - PSTN line 300Hz-3400Hz
    - Bandwidth 3100Hz
  - You cannot get CD-quality from a phone line!
- Digital signals (very high, infinite bandwidth)
  - Usually, high-frequency components go missing
  - Signal is distorted
    - but 0/1 bits may still be recoverable
  - Increasing the fundamental frequency means more components are lost
  - Bandwidth affects capacity!

Part 3

Channel capacity

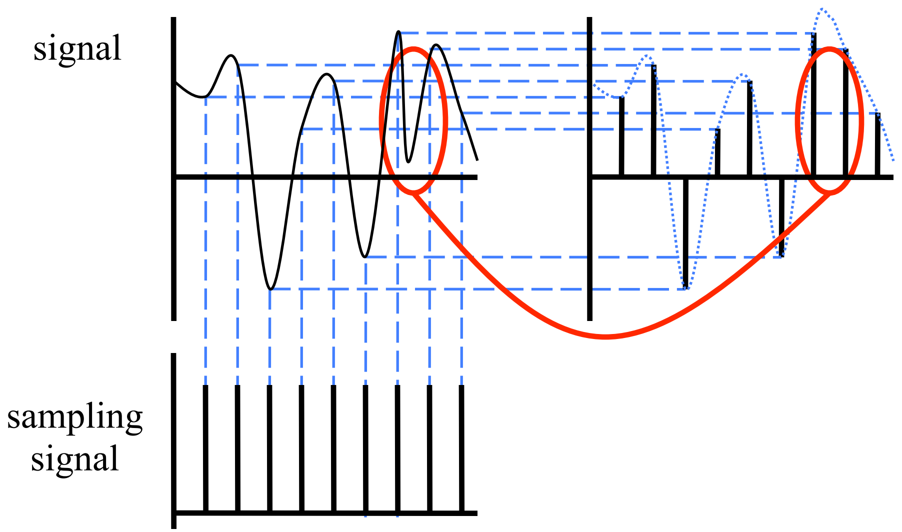
Bandwidth and capacity

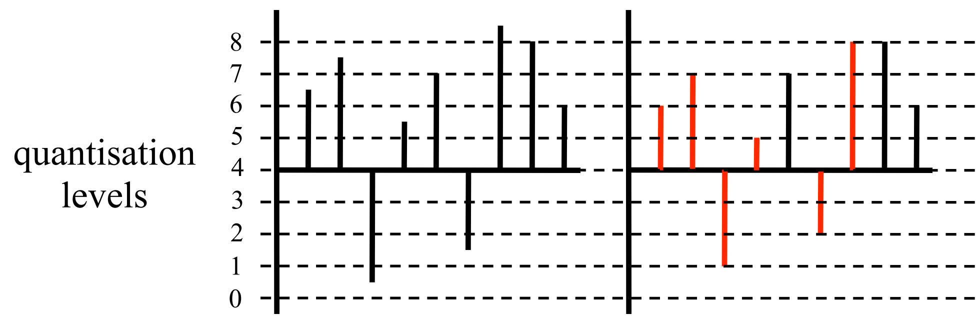
- Bandwidth
  - Difference between highest and lowest frequencies carried by a channel
  - Cycles/sec or Hertz (Hz)
- Capacity
  - Maximum rate at which information can pass through a channel
  - bits/sec (bps), signals/sec (baud rate)
    - if signal is binary, bits/sec ≡ signals/sec
    - if each signal is M bits, bits/sec ≡ M·signal/sec
  - Bandwidth is one of main determinants of capacity
  - ‘Bandwidth’ is often used (loosely and inaccurately) to mean ‘capacity’

Analogue to digital conversion (ADC)

- Step 1: Sampling
  - To measure at regularly spaced instants
  - Sampling frequency, S

- Step 2: Quantisation
  - To convert to discrete numeric values
  - Quantisation levels, M
  - Quantisation error
  - Number of bits required: $b = \log_2(M)$
  - Output data rate is $b \cdot S$

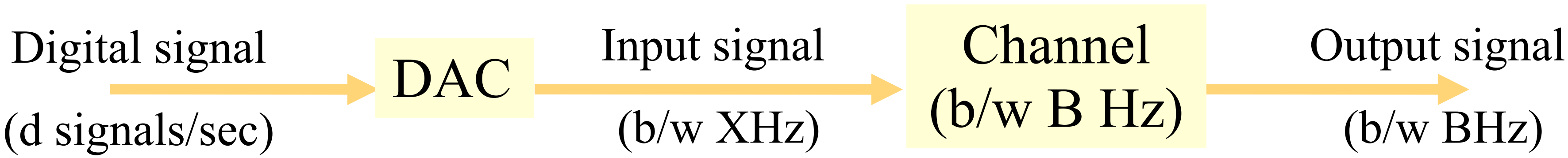




Nyquist-Shannon Theorem

- An analogue signal of bandwidth **B** can be **completely** recreated from its sampled form provided it is sampled at a rate **S** at least twice the signal's bandwidth, i.e. **S≥2B**
  - assume bandwidth B is equal to the highest frequency (i.e. lowest frequency is zero)
  - B can be determined by Fourier analysis
- **Higher sampling rate does not give any advantage**

Channel Capacity



- Original digital signalling rate: d signals/sec
- Channel bandwidth B limits output signal's bandwidth
- Nyquist $\Rightarrow$ no point in sampling output at > 2B Hz
- Sampling at 2B Hz give 2B symbols/sec
- If d > 2B information will be lost.

$$\therefore d \leq 2B$$

- The maximum channel capacity C is 2B signals/sec
- $C = 2B\log_2 M$ bps if each signal represents $\log_2 M$ bits

Hartley-Shannon Theorem

Maximum Channel Capacity (Nyquist)

$$C = 2B\log_2(M)$$

What is the maximum value for M?

- Hartley-Shannon Theorem:
  - signal power, *S*
  - noise power, *N (never zero)*
- Therefore the upper bound on the channel capacity:
  - signal to noise ratio SNR=S/N

$$M_{\max} = \sqrt{\left[\frac{S+N}{N}\right]}$$

$$C = B\log_2\left(\frac{S}{N}+1\right)$$

- For example, PSTN line:

B = 3.4KHz; SNR = 30dB (1000)

$\Rightarrow$ C ≈ 34Kb/s

Channel capacity

- Hartley-Shannon Theorem

$$C = B\log_2\left(\frac{S}{N} + 1\right)$$

C	maximum channel capacity (b/s)
B	channel bandwidth (Hz)
S/N	signal to noise ratio (SNR)

  - it states what is possible, not how it is achieved
- Actual data rate, R, a result of engineering
- Actual information rate, W, a result of source encoding

$$C > R > W$$

- Channel capacity increased by
  - greater (more) bandwidth
    - but bandwidth is closely regulated
  - greater (better) SNR
    - but for a 10 times increase in C, we need to increase the SNR by a power of 10!

Summary

- Analogue vs. digital
- Noise
- Time domain and frequency domain
- Bandwidth
- Sampling – Nyquist-Shannon Theorem
  - maximum rate of change of signal
- Channel capacity – Hartley-Shannon Theorem
  - capacity and bandwidth are directly related

The End