5.4 Sound Measurement

The intensity of a wave is the power transmitted per unit area (or the rate at which energy flows through an area). This is proportional to the *amplitude* of the wave *squared*: $I \propto A^2$. For sound, a scale of *sound level* β has been defined relative to a reference intensity, and measured in decibels (dB). We define:

$$\beta = 10 \log_{10}(I/I_0) \tag{203}$$

with $I_0 = 10^{-12}$ W/m^2, a reference intensity defined as the threshold of hearing (though of course the threshold will vary from person to person and with age). Various tables of sound levels can be found; for instance, a pneumatic drill (or jackhammer) at 1m is 130 dB, a vacuum cleaner at 1m is 70 dB, conversation in a room is 50 dB and a whisper at 1m is 30 dB. The sound level for any source will decrease with distance; if the power is emitted uniformly in all directions then the intensity will decrease with at least the square of the distance.

Notice that the scale is logarithmic (and uses logarithms in base 10, *not* natural logarithms) and is defined relative to a reference. An increase in sound level of 1dB is equivalent to an intensity about 1.3 times as large, while an increase of 3dB is about twice the intensity. If we have a sound with intensity $I/I_0 = 10^b$ then it would be b bels louder than I_0. But bels are too coarse a unit, so it is scaled by 10 and decibels (dB) are used. The threshold of pain is around 120-130 dB (or 1 W/m^2).

Two sound levels can be compared: if $I_2 = 10^{n/10} I_1$ then I_2 is n dB louder than I_1. Another way of writing this is to say that I_2 is n dB stronger than I_1 if $n = 10\log_{10}(I_2/I_1)$. We can see this by writing:

$$\begin{aligned}
\beta_2 &= 10\log_{10}(I_2/I_0) && (204)\\
\beta_1 &= 10\log_{10}(I_1/I_0) && (205)\\
\beta_2 - \beta_1 &= 10\log_{10}(I_2/I_0) - 10\log_{10}(I_1/I_0) && (206)\\
&= 10\left[\log_{10}(I_2) - \log_{10}(I_0) - (\log_{10}(I_1) - \log_{10}(I_0))\right] && (207)\\
&= 10\left[\log_{10}(I_2) - \log_{10}(I_0) - \log_{10}(I_1) + \log_{10}(I_0)\right] && (208)\\
&= 10\left[\log_{10}(I_2) - \log_{10}(I_1)\right] = 10\log_{10}(I_2/I_1) && (209)
\end{aligned}$$

For example, if I increase a sound *intensity* by a factor of 100, what is the change in sound *level* ? We have that $I_2/1_1 = 100$, so $\log_{10}(I_2/I_1)$ is 2 and the sound level change is 20 dB.

Sounds have pitch, loudness and timbre or quality. Pitch is the same as frequency—for instance, middle C is about 260 Hz. The range of audible frequencies is about 20–20,000Hz though this varies with person and age. Loudness is a quality *perceived* by ear, and is not fully understood. A rough rule of thumb is that loudness doubles with increase in intensity of a factor of 10. Timbre is related to the harmonics excited, the shape of a pulse (the attack and decay and any vibrato or variation).

6 Doppler Effect [1]

Waves will have both sources and observers (or, if you prefer, detectors). If the source or the observer are moving, then the frequency perceived by the observer will change relative to the frequency of the source. (It is important to realise that the definition of a moving source or moving observer is only unambiguous if the wave propagates through a medium; in this case, the motion we are concerned about is *relative to the medium*, while for electromagnetic waves in a vacuum the relative motion of the source and observer is all that matters.) This change in the frequency is known as the *Doppler Effect* or *Doppler Shift*, and it applies to all types of waves. The standard (or Newtonian) Doppler effect requires us to consider three different cases: the source moving; the observer moving; and both moving. The waves will be emitted with frequency f, wavelength λ and speed $v = \lambda f$; this is illustrated in Fig. 11(a) for a stationary observer.

6.1 Moving Observer

If the observer moves with speed v_O, then the apparent speed of the waves will change to $v' = v + v_O$ (we assume that the sign of v_O is such that if the observer moves *towards* the source it is positive). Thus the observed frequency will change:

$$\begin{aligned}
f' &= \frac{v'}{\lambda} = \frac{v+v_O}{\lambda} && (210)\\
&= \frac{v+v_O}{\lambda}\frac{v}{v} = \frac{v+v_O}{v}\frac{v}{\lambda} && (211)\\
&= \left(\frac{v+v_O}{v}\right) f = \left(1 + \frac{v_O}{v}\right) f && (212)
\end{aligned}$$

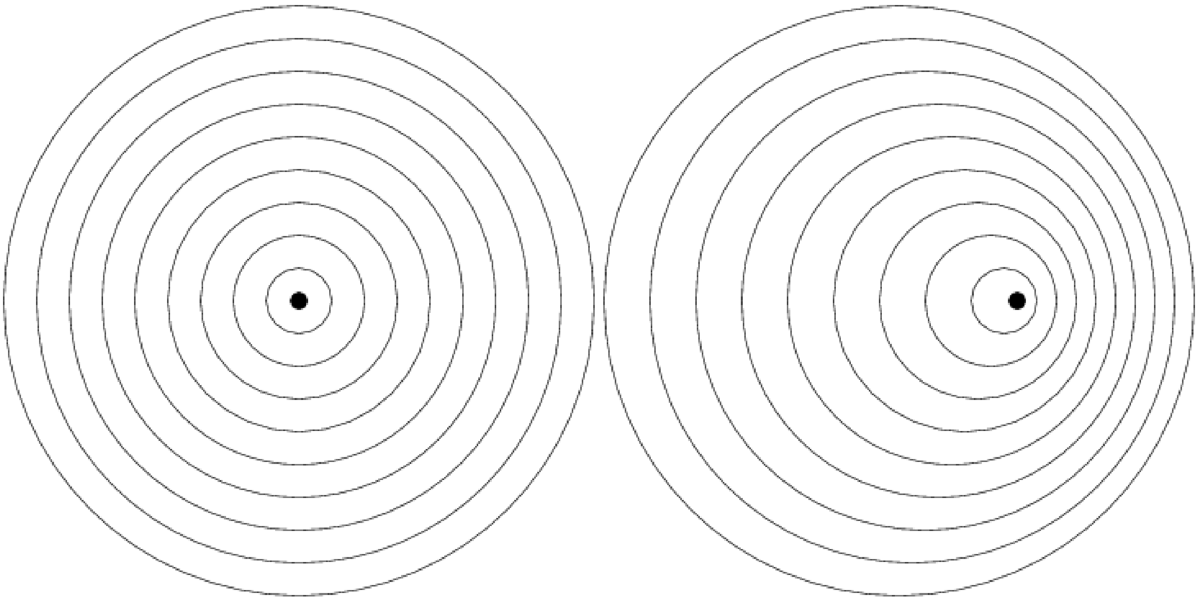

Figure 11: (a) Waves emitted by a stationary source at a fixed frequency. (b) Waves emitted by a moving source at the same frequency.

Physically, an observer moving towards the source will encounter the wave peaks (or troughs) *more* frequently than a stationary observer, but leaving the . Similarly an observer moving away from the source will encounter the wave peaks *less* frequently; these changes in frequency are reflected in the formula just given.

6.2 Moving Source

If the source moves with speed v_S then the spacing between waves will change (decreasing in the direction the source moves and increasing in the opposite direction) as illustrated in Fig. 11(b). This will lead to a change in the measured wavelength, and as the velocity of the waves is unchanged, the frequency will change. In one period, the source will move $v_S T$ which is also the change in wavelength, $\Delta\lambda = v_s T$; if the source moves *towards* the observer the wavelength *decreases*. Then we can find the changed frequency:

$$\begin{aligned} \lambda' &= \lambda - \Delta\lambda = \lambda - v_s T && (213) \\ &= \lambda - \frac{v_s}{f} = \frac{v}{f} - \frac{v_s}{f} && (214) \\ f' &= \frac{v}{\lambda'} = \frac{v}{\frac{1}{f}(v - v_S)} && (215) \\ &= f\left(\frac{v}{v - v_S}\right) = f\left(\frac{1}{1 - v_s/v}\right) && (216) \end{aligned}$$

6.3 Moving Observer & Source

When both observer and source are moving, then both the effects described above are seen: the observed wavelength changes as does the velocity of the waves relative to the observer. Using the ideas developed above, we can write:

$$\begin{aligned} f' &= \frac{v'}{\lambda'} = \frac{v + v_O}{\frac{1}{f}(v - v_S)} && (217) \\ &= \left(\frac{v + v_O}{v - v_S}\right) f && (218) \end{aligned}$$

6.4 More Advanced Ideas

The basic treatment which we have given is correct, but there are some refinements that we can introduce.

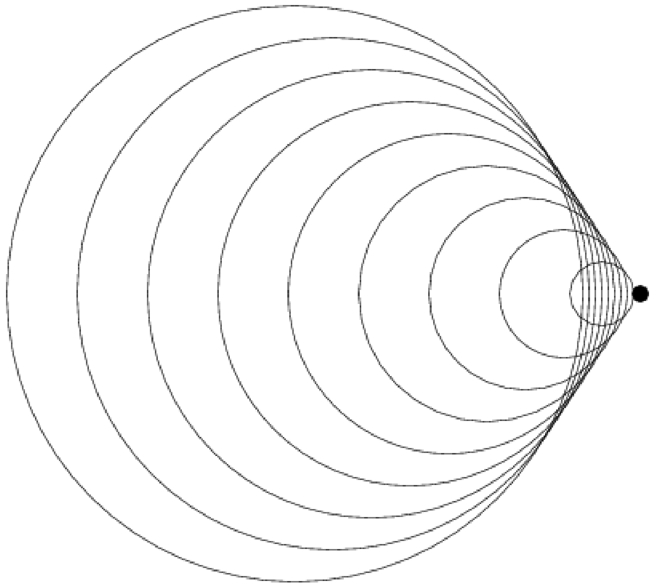

Figure 12: Source moving faster than the wave speed in a medium

6.4.1 Relative Velocities

The derivations given above assumed that the source was travelling directly towards the observer (or vice versa). In this case, the observed frequency will be raised to a constant level when the source is approaching and dropped to a constant level when it is receding. Experience of the Doppler effect (e.g. a siren coming towards us) suggests that when the motion is not directly along the line joining the source and the observer, there is a changing frequency.

The frequency changes because the Doppler shift only takes into account the *component* of the velocity of the source and/or observer along the line joining the two. Let's consider a moving source for definiteness. If the source has velocity $\mathbf{v}_S$ and position $\mathbf{r}_S$, and the observer has position $\mathbf{r}_O$ then we must replace v_s in the formulae above with:

$$\frac{\mathbf{v}_S \cdot (\mathbf{r}_O - \mathbf{r}_S)}{|\mathbf{r}_O - \mathbf{r}_S|} = v_s \cos\theta_{OS}(t) \tag{219}$$

where $\theta_{OS}(t)$ is the angle between the velocity and the line between the observer and source, and depends on time. The same considerations apply to moving observers.

6.4.2 Relativistic Doppler Effect

The Doppler effect can be derived within the framework of special relativity (rather than Newtonian Mechanics as we have just done). This will be covered in detail in PHAS1246; for now, I will just quote the result (which comes from considering the time between wavefronts when the source and observer are moving relative to each other, and transforming from the source's reference frame to the observer's).

$$f_{\mathrm{obs}} = \sqrt{\frac{1 - v/c}{1 + v/c}} f_{\mathrm{source}} \tag{220}$$

where v is the relative speed of the source and observer and c is the speed of light; the signs given here are in line with those used in PHAS1246 and are for a source *receding* from the observer. Note that $\beta = v/c$ is often used here instead.

6.4.3 Shock Waves

If the source is moving *faster* than the velocity of the waves in the medium, then a new phenomenon emerges: a shock wave. This is illustrated in Fig. 12

The angle formed by the wavefronts as they add to each other leads to a shockwave. This is *constructive* interference, and can lead to large pressure variations in a gas.