Problem Sheet 2

1. Draw the solution sets of the following, and name the shape:
a) $y^2 - x^2 = 1$ b) $25x^2 + 36y^2 = 900$ c) $8x = y^2$ d) $xy = -1$

2. a) If $f(x) = x^2 - x$ show that $f(x+1) = f(-x)$

b) if $f(x) = 1/x$ show that $f(a) - f(b) = f\left(\frac{ab}{b-a}\right)$

3. Give a formula for a function f whose graph is equal to the solution set of:

a) $x^5 y + 4x - 2 = 0$ b) $x = \frac{2+y}{2-y}$

4. What are the domains and ranges of the following:

a) $f(x) = \begin{cases} x+2 & if\ -1 < x < 0 \\ x & if\ 0 \le x < 1 \end{cases}$ b) $h(x) = \begin{cases} \frac{x^2-4}{x-2} & if\ x \neq 2 \\ 4 & if\ x = 2 \end{cases}$

5. Give a function whose domain and range are:
a) $(0,2)$ and $(1,7)$ respectively
b) $(0,1)$ and $(1,\infty)$ respectively

6. Evaluate the following limits:
a) $\lim_{x\to\infty} \frac{2x+3}{4x-5}$ b) $\lim_{x\to\infty} \frac{2x^2+1}{6+x-3x^2}$ c) $\lim_{x\to\infty} \frac{x}{x^2+5}$

7. a) What does it mean for the function *f* to be continuous at the point x=c?
b) What is a precise formulation of the condition in a?