Hand in your answers by **Monday 1 February**, *at the lecture or Dr Bowler's pigeonhole in Physics. Put your name and your academic tutor's name on your answers and* **STAPLE** *sheets together. Marks per section are shown in square brackets.*

1. (a) Consider adding together two simple harmonic oscillations at the same frequency $A_1 e^{i(\omega t+\phi_1)}$ and $A_2 e^{i(\omega t+\phi_2)}$ to give a single oscillation $Ae^{i(\omega t+\phi)}$. Use complex exponentials to show that: [4]

$$A = A_1^2 + A_2^2 + 2A_1 A_2 \cos(\phi_2 - \phi_1)$$
$$\phi = \tan^{-1} \frac{A_1 \sin\phi_1 + A_2 \sin\phi_2}{A_1 \cos\phi_1 + A_2 \cos\phi_2}$$

   (b) Draw phasor diagrams for and calculate the sums of the following pairs of oscillations:

   (i) $A_1 = 2, \phi_1 = 0, A_2 = 2, \phi_2 = \pi/3$ [3]

   (ii) $A_1 = 3, \phi_1 = 5\pi/4, A_2 = 2, \phi_2 = \pi/3$ [3]

   (c) A harmonic system vibrates with the following sum of two oscillations:

$$7.5\cos(6.28t + 27^\circ) - 7.5\sin(6.20t - 120^\circ)$$

   where time is measured in seconds. Find the frequency of the net motion, and the time interval separating successive beats. [4]

2. (a) Consider a damped, driven oscillator with mass m =0.01 kg, stiffness s =36 N/m and damping coefficient b =0.5 kg/s driven by a harmonic force with amplitude F_0=3.6N. Use formulae from the notes to find the amplitude and phase constant of the resulting *steady state* motion when:

   (i) $\omega = 8.0\text{s}^{-1}$ [2]

   (ii) $\omega = 80.0\text{s}^{-1}$ [2]

   (iii) $\omega = 800.0\text{s}^{-1}$ [2]

   (b) Two oscillators both with stiffness s and mass m are joined by a spring with stiffness K. At $t = 0$ the first oscillator (displacement ψ_1) is displaced by $\sqrt{2}A_0$ to the right (i.e. the positive ψ_1 direction) while the second oscillator (displacement ψ_2) is held fixed, and then both are released.

   (i) Show that the resulting motion can be written:

$$\psi_1 = \frac{A_0}{\sqrt{2}}\left(\cos(\omega_a t) + \cos(\omega_b t)\right)$$
$$\psi_2 = \frac{A_0}{\sqrt{2}}\left(\cos(\omega_a t) - \cos(\omega_b t)\right)$$

   [Hint: work in terms of q_a and q_b first and calculate what $q_a(0)$ and $q_b(0)$ must be; as the initial velocities are zero, this will allow you to find $A_a, A_b, \phi_a \& \phi_b$; then transform to ψ_1 and ψ_2.] [4]

   (ii) If $s = 81$ N/m and $K = 20$ N/m and the masses are 10 kg, show that after $t = 4.96$s the amplitude of ψ_1 will be zero. What will the amplitude of ψ_2 be ? What type of motion do the oscillators undergo ? [Hint: rewrite the solutions you found in the first part as a product of trigonometrical functions.] [4]

3. (a) A wave of frequency 500 Hz has a velocity of 350 m/s.

   (i) How far apart are two points that differ in phase by $\pi/3$? [2]

   (ii) What is the phase difference between two displacements at a certain point at times 1 ms apart? [2]

   (b) Calculate the tension and mass per unit length of a string which has a characteristic impedance of $Z_0 = 3$ kg/s and phase velocity for waves of $c = 30$ m/s. [2]

4. (a) Two strings with mass 1kg/m and 2kg/m are joined together. If the two are put under a tension of 20 N/m, and a wave pulse of amplitude 1cm is sent down the lighter string towards the join, what will be the amplitude on both strings after the wave pulse reaches the join ? [4]

   (b) If a co-axial extension cable with characteristic impedance 120Ω is joined to an aerial cable with characteristic impedance 75Ω, what amplitude of signal will be received at the end of the extension cable if a signal of 100 μV is received at the aerial ? [Hint: you can treat the voltage as the amplitude of a wave and the impedances in just the same way as the impedance on a string] [2]