AOA Final Exam 2

1. (20 points) The following statements may or may not be correct. In each case, either prove it or give a counter example.
   (a) Give a counter example to the conjecture that if there is a path from u to v in a directed graph G and if d[u] < d[v] in a dfs search of G, then v is a descendant of u in the depth first forest produced.
   (b) Given an undirected graph G = (V,E), $log(E) = O(log(V))$ is always true under which condition
   (c) In a directed graph, define a vertex as source if it has indegree zero. Also, define a vertex as sink if it has out-degree zero. Given a directed acyclic graph G = (V,E). If discovery time and finishing time are obtained by applying standard DFS, which is the property of vertex that is guaranteed to be a source?

2. (20 points) You are given a binary tree T = (V,E) in adjacency list format along with the designated root node r ∈ V. Recall that u is said to be ancestor of v in rooted tree, if the path from r to v in T passes through u. You are asked to design an algorithm to preprocess the tree so that queries of the form "is u the ancestor of v?" can be answered in constant time. The preprocessing itself should take linear time.

3. (20 points) Here's a proposal for how to find length of shortest cycle in an undirected graph with unit edge lengths.

   When a back edge, say(v,w) is encountered during a depth first search, it forms a cycle with the tree edges from w to v. The length of cycle is level[v] - level[w] + 1, where the level of vertex is its distance in DFS tree from root vertex.

   This suggests the following algorithm:

   (a) Do a depth-first search, keeping track of level of each vertex.
   (b) Each time a back edge is countered, compute the cycle length and save it if it is smaller than the shortest one previously seen.

   Show that this strategy does not always work by providing a counter example as well as explanation.

4. (20 points) The square of directed graph G = (V,E) is the graph G^2 = (V,E^2) such that (u,w) ∈ E^2 if and only if for some v ∈ V, both (u,v) ∈ E and (u,w) ∈ E. That is, G^2 contains an edge between u and w whenever G contains a path with exactly two edges between u and w. Describe efficient algorithms for computing G^2 from G for both the adjacency-list and adjacency-matrix representations of graph. Analyze running time of your algorithms.

5. (20 points) Milk packing is such a low margin business It is Important to keep the price of of your product as low as possible help Mary milk makers get the milk they have to sell cheapest possible manner. Merry milk makers company has several farmers from which they may buy milk and each one has potentially different price at which they sell to milk producing plant. Moreover, as cow can only produce so much milk in a day, the farmers only have so much milk to sell per day. Each day Merry milk makers can buy an integral amount of milk from each farmer less than or equal to farmer's limit.

   Given the Merry milk makers daily requirement of milk along with the cost per gallon and amount of available milk for each farmer, calculate the minimum amount of money that it takes to fulfil Merry milk makers requirement.

   Note: The total milk produced by farmers will be enough to meet demands of Merry Milk Makers

6. (20 points) We studied coin change problem in class and found a greedy algorithm to solve it. That greedy approach works for special denominations and may not work for some bizarre denominations. For example, consider making change for 30 given coins of denominations 1, 10, 20, 25. The greedy algorithm produces 25, 1, 1, 1, 1, 1 but the optimal solution is 20, 10. Design a DP solution to this problem.