2.38 For the adjacency matrix A, determine A^2 and A^3.

$A^2 =$

$$\begin{bmatrix} 2 & 1 & 1 & 1 & 0 \\ 1 & 2 & 1 & 1 & 0 \\ 1 & 1 & 3 & 0 & 1 \\ 1 & 1 & 0 & 2 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix}$$

$A^3 =$

$$\begin{bmatrix} 2 & 3 & 4 & 1 & 1 \\ 3 & 2 & 4 & 1 & 1 \\ 4 & 4 & 2 & 4 & 0 \\ 1 & 1 & 4 & 0 & 2 \\ 1 & 1 & 0 & 2 & 0 \end{bmatrix}$$

2.41

a) Compute BB^t

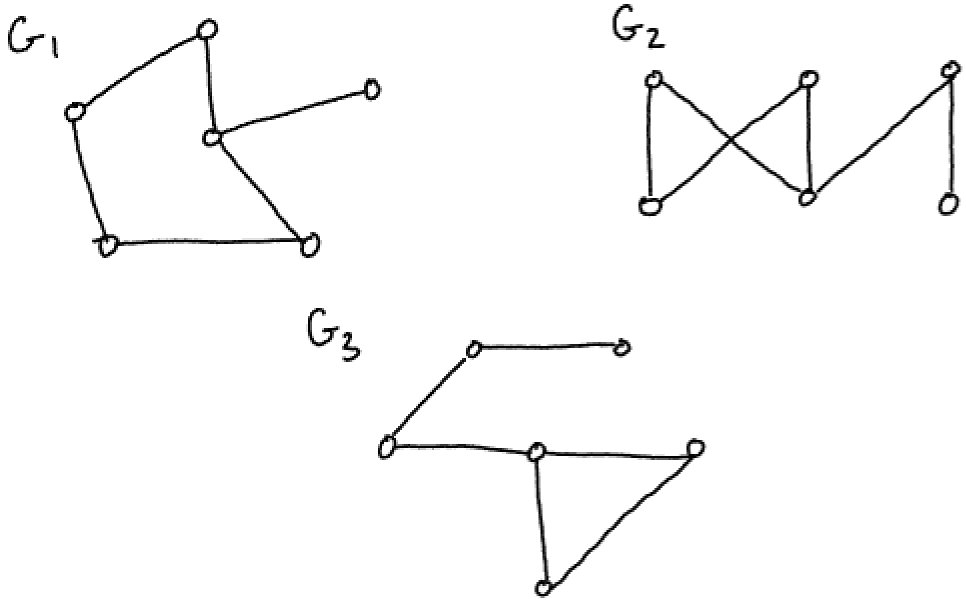
$BB^t =$

$$\begin{bmatrix} 3 & 1 & 1 & 1 & 0 \\ 1 & 2 & 0 & 1 & 0 \\ 1 & 0 & 2 & 1 & 0 \\ 1 & 1 & 1 & 4 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

b) What does the (i, j) entry of BB^t represent?

The i is the vertex and the j is the edge. 0 means there is no connection, 1 means there is a connection. And the diagonal entries represent the degrees for each vertex.

3.2 Give an example of 3 graphs of the same size, order, and degree sequence that are not isomorphic to one another.



Here each graph has order 6, size 6, and degree sequence (3, 2, 2, 2, 2, 1) but they are not isomorphic to one another.

4.2 Prove every connected graph with even degrees contains no bridges.

Proof. Let G be a connected graph with all vertices of even degree. Since all degrees are even, G is not a tree. Because G cannot contain a leaf (vertex of degree 1). Since G is connected, not a tree, and only has even degrees, every edge of G must lie on a cycle. But by Theorem 4.1, no edge of a cycle is a bridge. So G cannot contain a bridge. □

4.4 Let G be a connected graph and let e_1 and e_2 be two edges of G. Prove that G -e_1 -e_2 has three components if and only if e_1 and e_2 are bridges.

Proof. ($\Rightarrow$) Assume G -e_1 -e_2 has three components and e_1 and e_2 have distinct vertices. Since G is connected, removing e_1 and e_2 must have increased the number of components from 1 to 3. So, e_1 and e_2 are bridges.
($\Leftarrow$) Assume e_1 and e_2 are bridges with distinct vertices. Since G is connected and has 2 bridges, removing e_1 and e_2 must increase the number of components from 1 to 3. So, G -e_1 -e_2 has three components. □