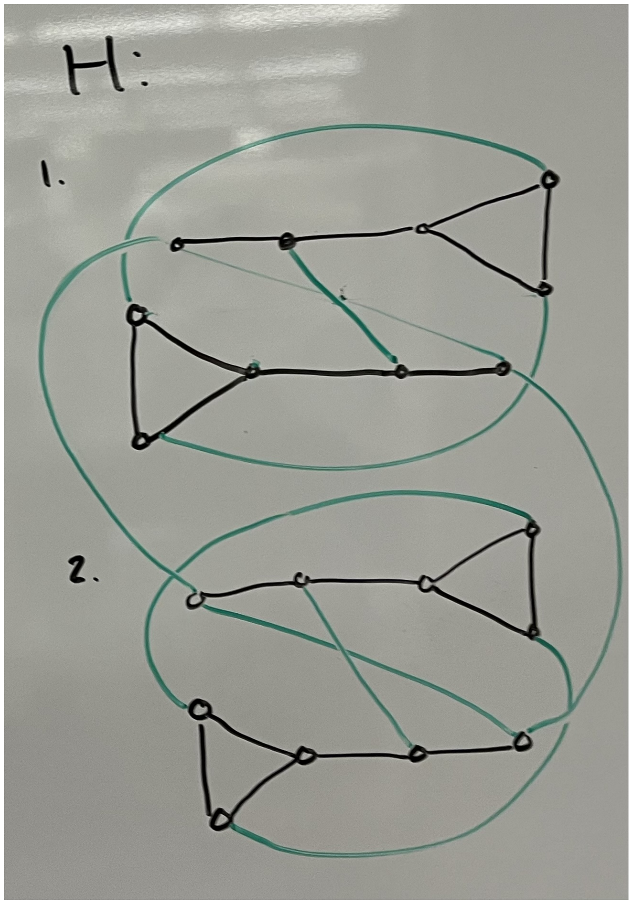
2.20 Show that if G is a connected graph that is not regular, then G contains adjacent vertices u and v such that deg(u) > deg(v).

2.22 

order = 20

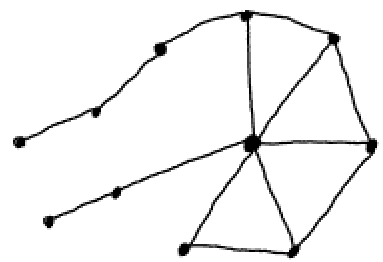
2.26

a Show that a graph G is regular if and only if its complement is regular.

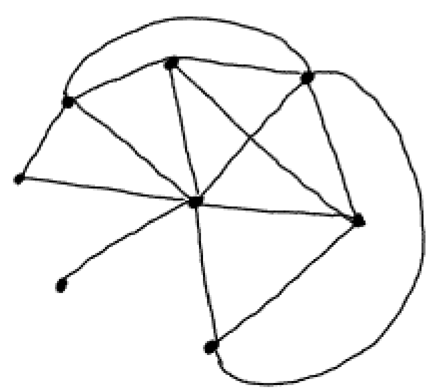
b Show that if G and $\overline{G}$ are r-regular for some nonnegative integer r, then G has odd order.

2.32 Determine if the following sequences are graphic. If so, construct a graph.

b S_1: 6,3,3,3,3,2,2,2,2,1,1
S_2: 2,2,2,2,2,2,1,1,1,1
S_3: 2,2,2,1,1,1,1,1,1
S_4: 1,1,1,1,1,1,1,1. Here, we have an even number of 1s, and can recognize this is graphic.



d S_1: 7,5,4,4,4,3,2,1
S_2: 4,3,3,3,2,1,0
S_3: 2,2,2,1,1
S_4: 1,1,1,1. Here, we have an even number of 1s, and can recognize this is graphic.



2.33 Prove that for every integer x with $0 \leq x \leq 5$, the sequence x,1,2,3,5,5 is not graphical.

Proof. The sum of all degrees must be even. Here, the sum is x + 16. So x must be even. Therfore, x $\neq$ 1, 3, 5. That leaves 0, 2, and 4. We can use Theorem 2.10 to show each case is not graphical.
x = 0: S_1: 1,2,3,5,5 or 5,5,3,2,1. This is not graphical because there is a vertex of degree 5, but only 4 other vertices.
x = 2: S_1: 2,1,2,3,5,5 or 5,5,3,2,2,1
S_2: 4,3,1,1,0. This is not graphical because there is a vertex of degree 4, but only 3 other vertices.
x = 4:

S_1: 4,1,2,3,5,5 or 5,5,4,3,2,1
S_2: 4,3,2,1,0. This is not graphical because there is a vertex of degree 4, but only 3 other vertices.
Therefore, for every integer x with $0 \leq x \leq 5$, the sequence x,1,2,3,5,5 is not graphical.
$\square$