Cohomology of Shimura Curves in quasi-linear time

Assumed $\mathfrak{h}$, $\mathrm{PSL}_2(\mathbb{R})$.

1 Preliminaries

Let G be a topological group and X be a topological space. We say a left action of G on X is

- **Discontinuous** if G is endowed with the discrete topology.
- **Proper** if for any compact sets K and K' in X, the set of $g \in G$ such that the intersection $K \cap g.K'$ is nonempty is compact.
- **Properly discontinuous** if it is both discontinous and proper.
- **Totally discontinuous** if it is properly discontinous and free.

Remarque 1. *One can also define a proper action of G on X to be a G-action on X such that the map $G \times X \to X \times X$ defined by $(g, x) \mapsto (x, g.x)$ is proper as a continuous map.*

La prop clé ici c'est que $\mathfrak{h}$ est haussdorf localement compact.

An idiomatic example of a proper action is given by the action of $\mathrm{PSL}_2(\mathbb{R}) := \mathrm{SL}_2(\mathbb{R})/\{\pm I_2\}$ on the upper-half plane $\mathfrak{h} \subset \mathbb{C}$ by homographies. Further, discrete subgroups of $\mathrm{PSL}_2(\mathbb{R})$ or so-called Fuchsian groups yield properly discontinuous on $\mathfrak{h}$.

1.1 Fuchsian groups

$\mathrm{Isom}(\mathfrak{h}) = \mathrm{Isom}^+(\mathfrak{h}) \cup \tau\mathrm{Isom}^+(\mathfrak{h})$ avec $\tau\colon z \mapsto -\bar{z}$.

Proposition 1. *$PSL_2(\mathbb{R})$ acts properly on $\mathfrak{h}$.*

As a corollary, discrete subgroups of $\mathrm{PSL}_2(\mathbb{R})$ act properly discontinuously on $\mathfrak{h}$.

1.2 Arithmetic fuchsian groups

1.3 Fundamental groups of surfaces