Carbon Cycle Simulation Using Numerical Methods – ODE Project

Rayane Dakhlaoui[1], Adam Chgour[1], Alexandre Mallez[1] and Arthur Hottier[1]

Télécom Paris[1]

ODE Project Report
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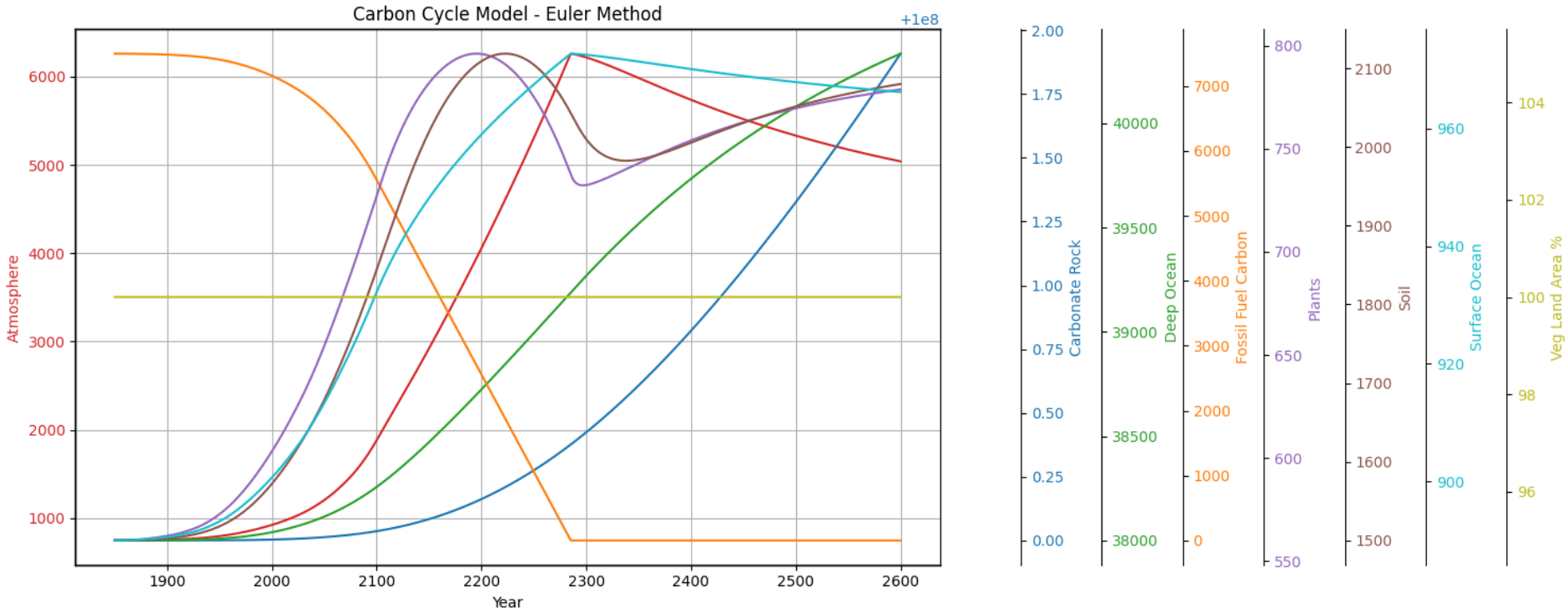
Outline

Euler

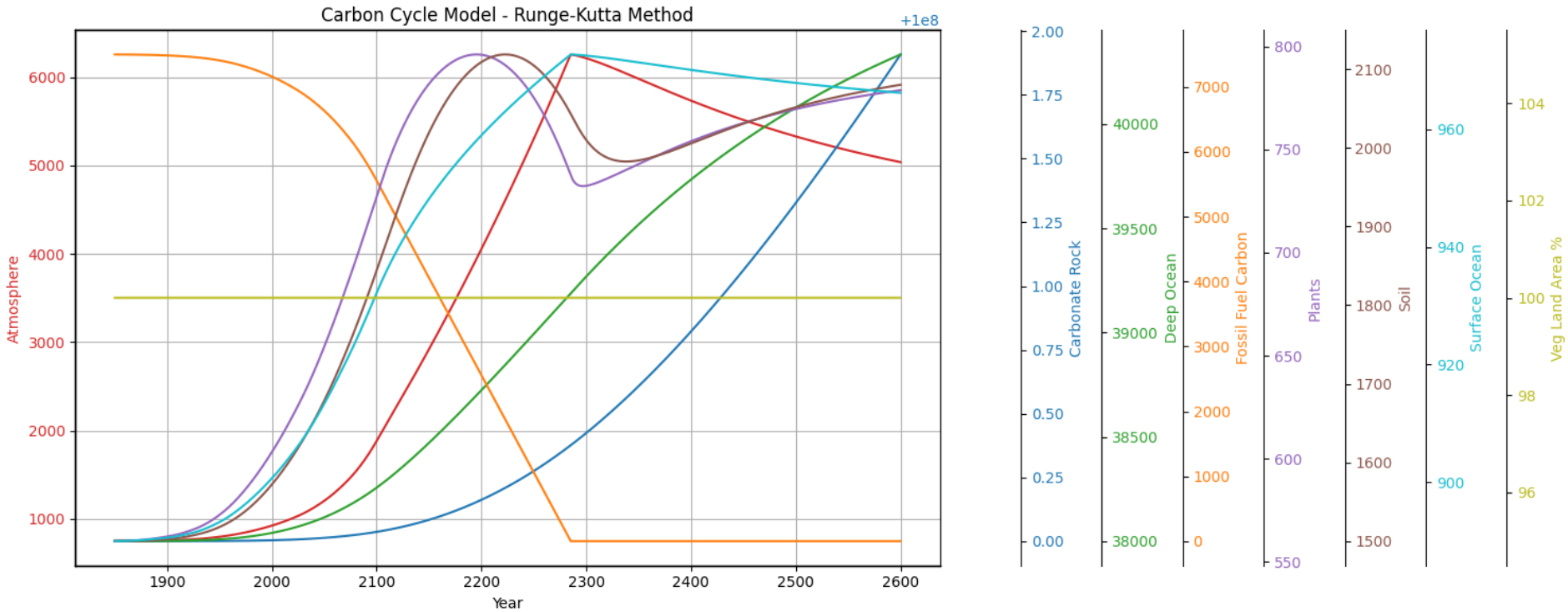
$$y_{n+1} = y_n + h_n f(t_n, y_n)$$



Runge-Kutta 4 (RK4)

$$k_1 = f(t_n, y_n), \quad k_2 = f\left(t_n + \frac{h_n}{2}, y_n + \frac{h_n}{2}k_1\right), \quad k_3 = f\left(t_n + \frac{h_n}{2}, y_n + \frac{h_n}{2}k_2\right), \quad k_4 = f(t_n + h_n, y_n + h_n k_3)$$

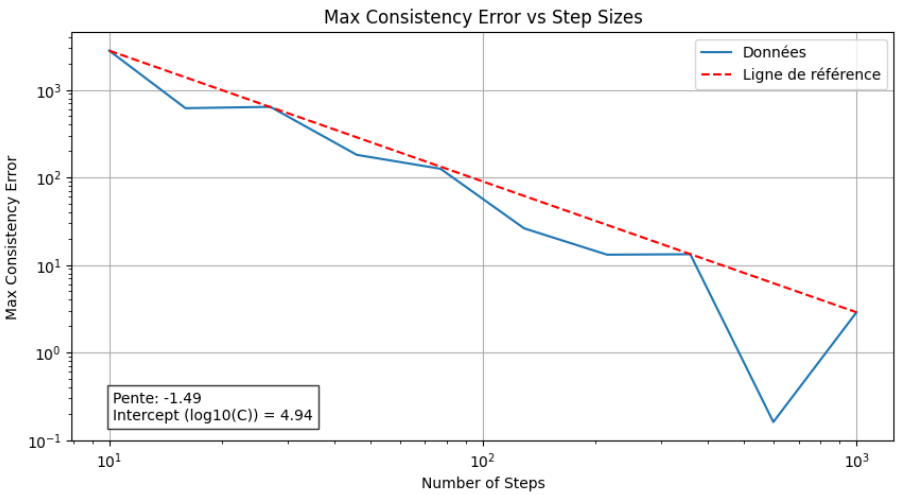
$$y_{n+1} = y_n + \frac{h_n}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right)$$



Max Consistency Error

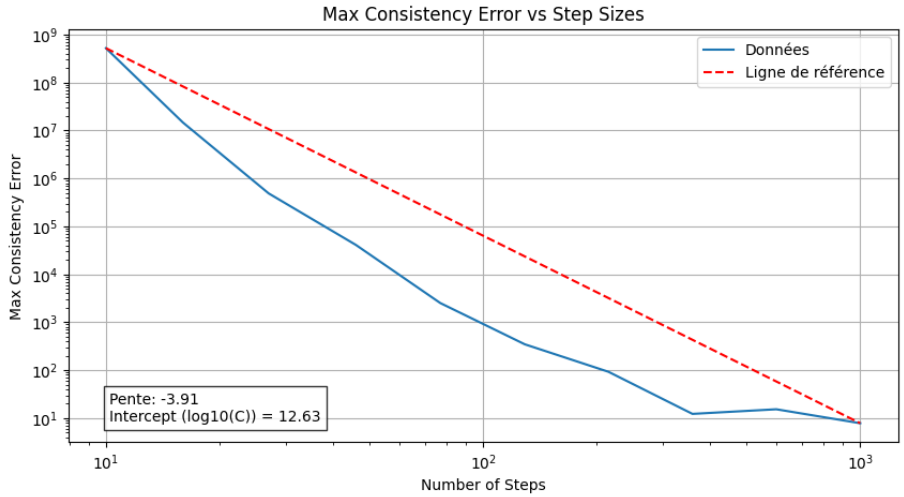
$$\varepsilon_{\max} = \max_{h_n} \|e(t_n, y_n, h_n)\| \xrightarrow[\text{if of order } p]{} \exists\, C \text{ independant of } n \text{ s.t} : \varepsilon_{\max} \leq C\, h_n^{p+1}$$

Euler



$p \sim 1 \quad C \sim 10$

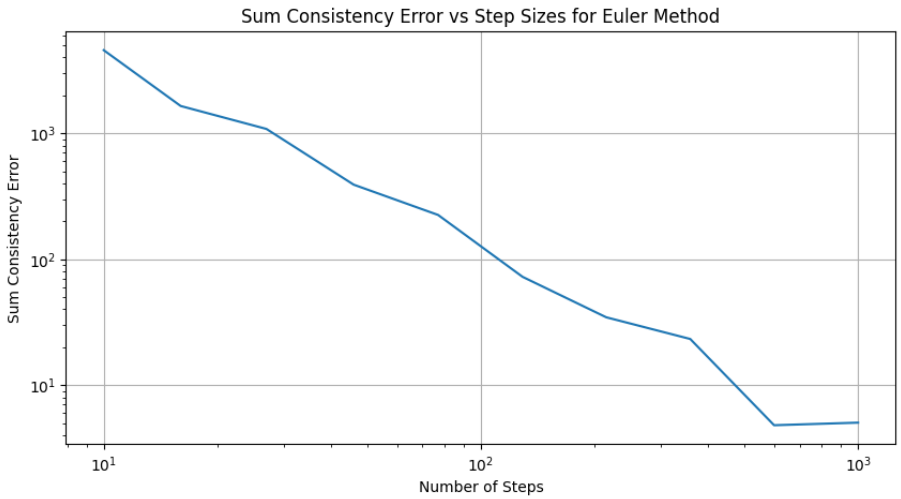
Runge-Kutta 4 (RK4)

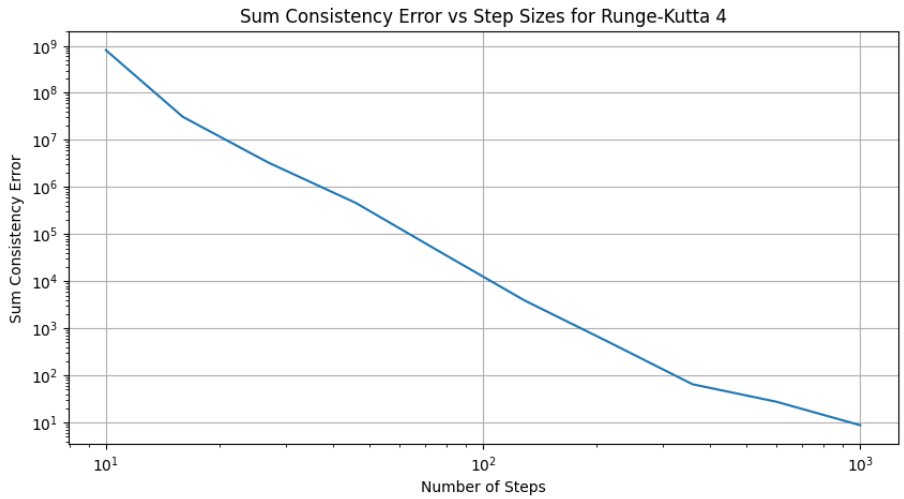


$p \sim 4 \quad C \sim 14$

Global Consistency Error

$$h_{\max} = \max_n h_n \longrightarrow \lim_{h_{\max} \to 0} \sum_{n=1}^{N-1} ||e(t_n, y_n, h_n)|| = 0$$

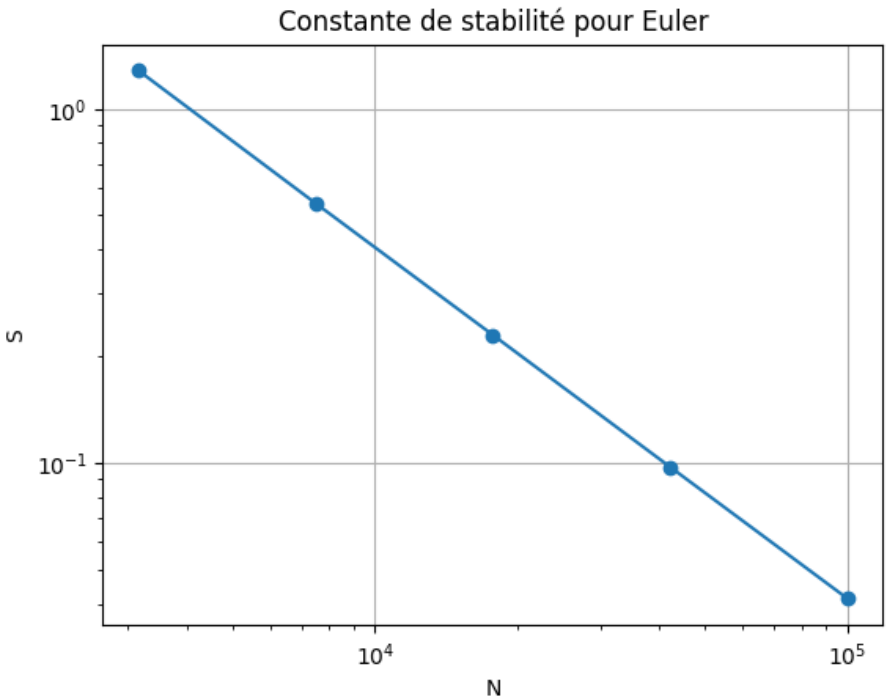


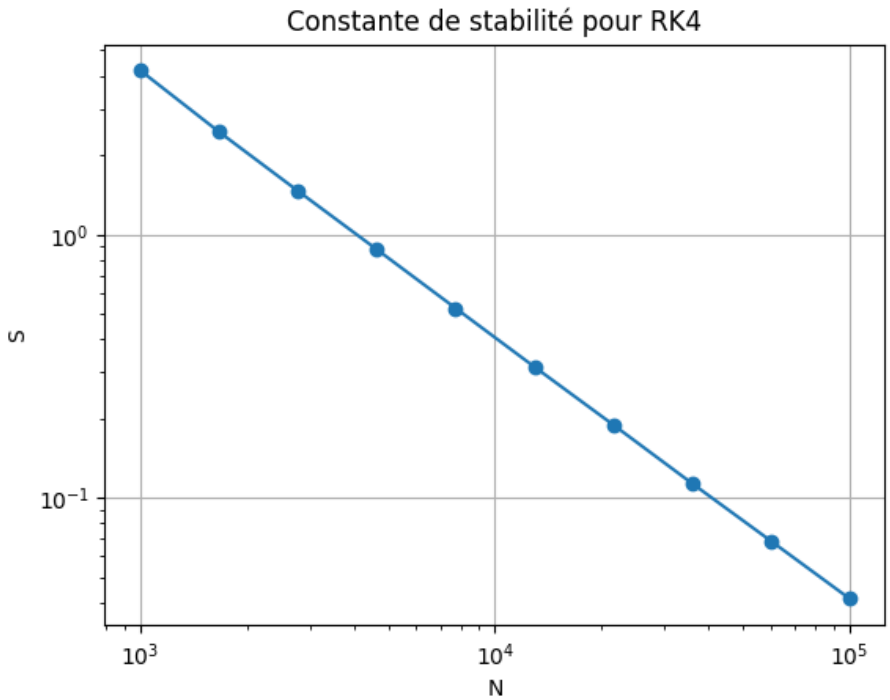


Both methods are consistent

Stability

$$S = \frac{\max_n \|\tilde{y}_n - y_n\|}{\|\tilde{y}_0 - y_0\| + \sum_n \|\epsilon_n\|} \quad \text{with} \quad \varepsilon_n \sim \mathcal{N}(0, 0.01)$$

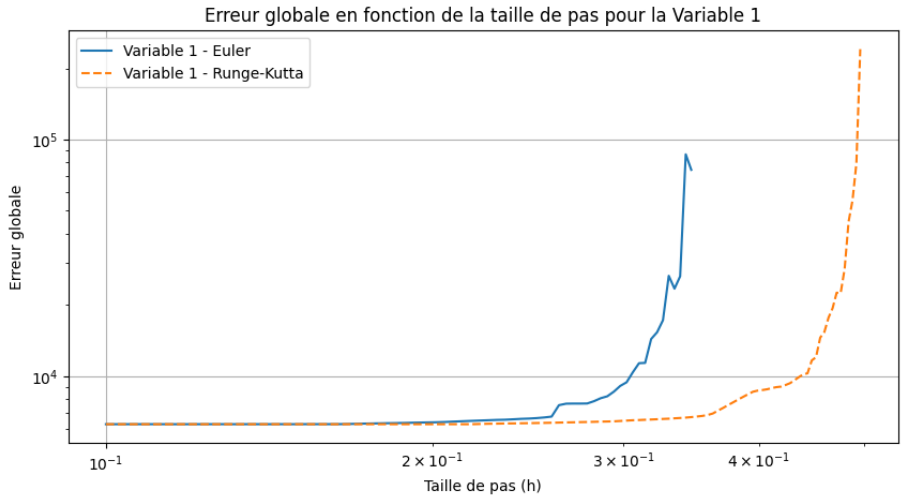




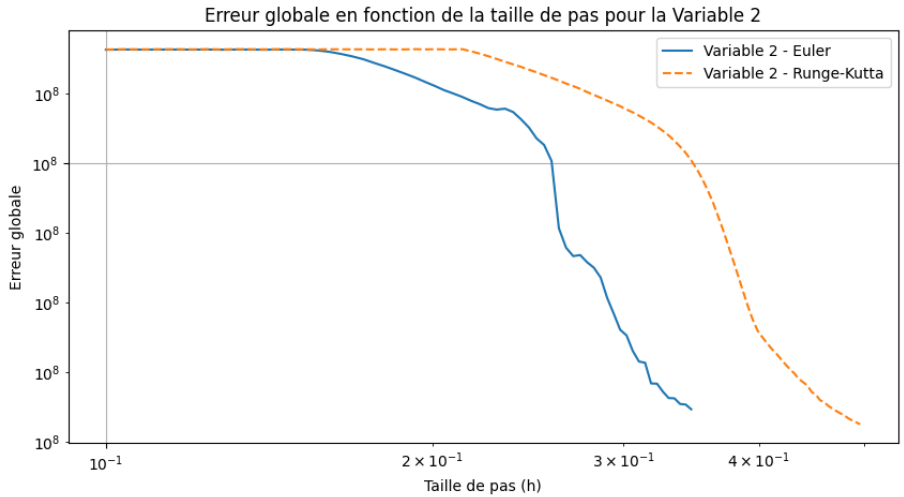
$S = 1.02$ $S = 1.28$

Both methods are stable

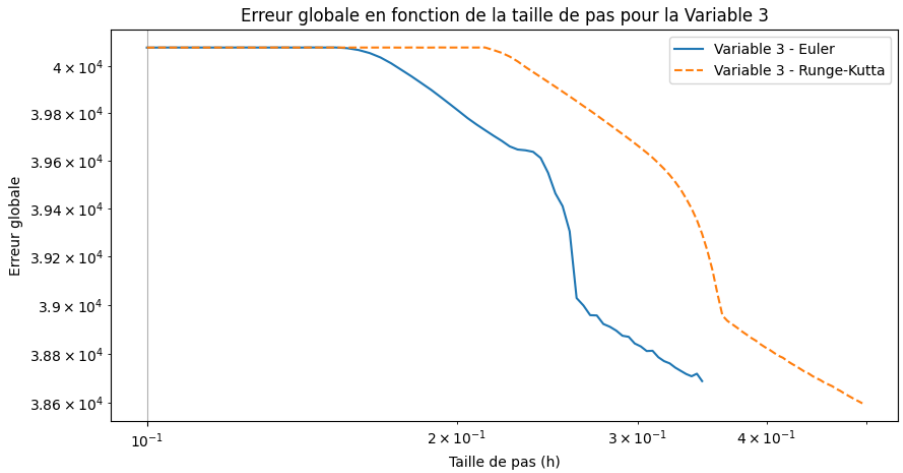
Global Error and Step Size (1)



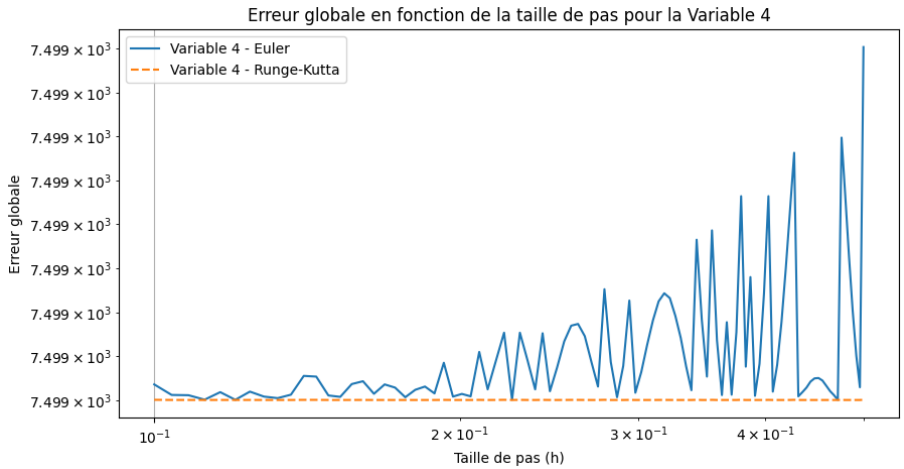
Atmosphere



CarbonateRock

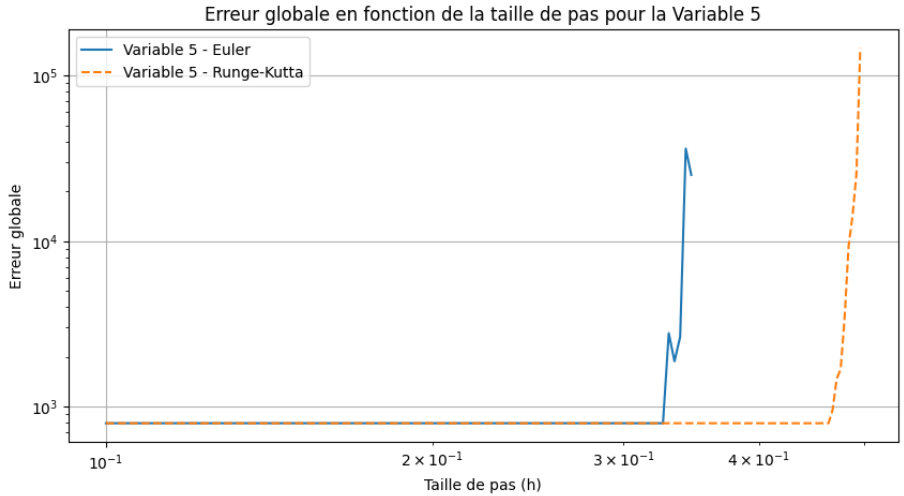


DeepOcean

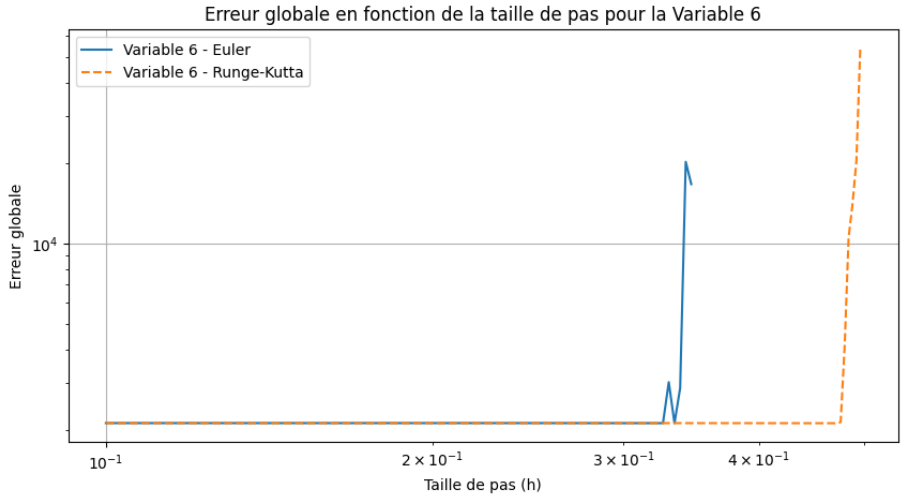


FossilFuel

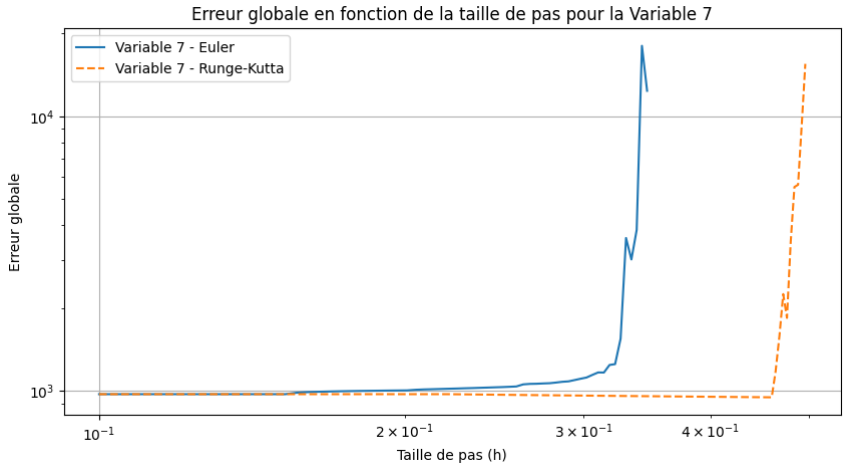
Global Error and Step Size (2)



Plant



Soil



SurfaceOcean

Execution Time