

Carbon Cycle Simulation Using Numerical Methods – ODE Project

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ODE Project Report
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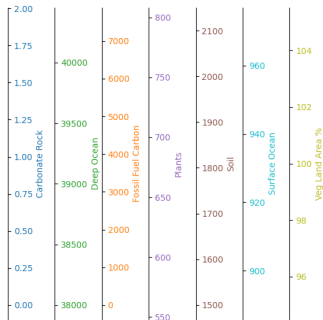
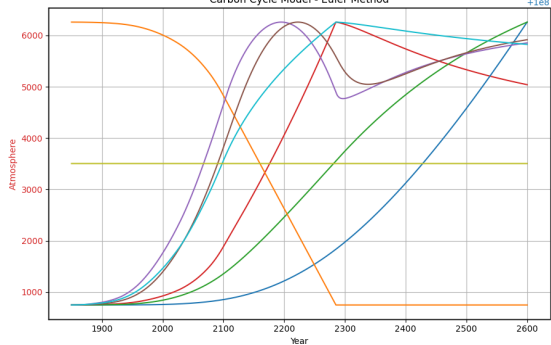
Outline

- 1 Implemented Numerical Methods
 - Euler
 - Runge-Kutta 4 (RK4)
- 2 Consistency Error
 - Max Consistency Error
 - Global Consistency Error
- 3 Stability
- 4 Global Error and Step Size
- 5 Execution Time

Euler

$$y_{n+1} = y_n + h_n f(t_n, y_n)$$

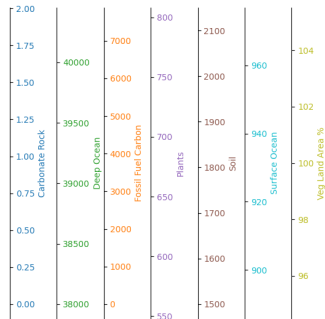
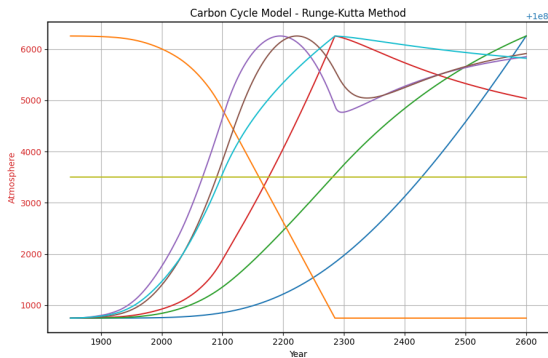
Carbon Cycle Model - Euler Method



Runge-Kutta 4 (RK4)

$$k_1 = f(t_n, y_n), \quad k_2 = f\left(t_n + \frac{h_n}{2}, y_n + \frac{h_n}{2} k_1\right), \quad k_3 = f\left(t_n + \frac{h_n}{2}, y_n + \frac{h_n}{2} k_2\right), \quad k_4 = f(t_n + h_n, y_n + h_n k_3)$$

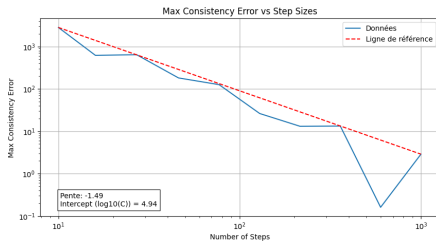
$$y_{n+1} = y_n + \frac{h_n}{6} (k_1 + 2k_2 + 2k_3 + k_4)$$



Max Consistency Error

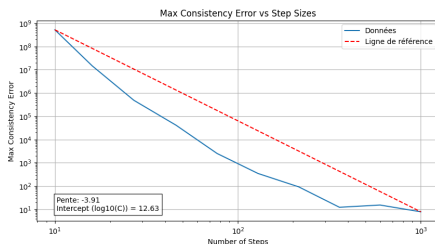
$$\varepsilon_{\max} = \max_{h_n} \|e(t_n, y_n, h_n)\| \xrightarrow[\text{if of order } p]{\quad} \exists C \text{ independent of } n \text{ s.t. : } \varepsilon_{\max} \leq C h_n^{p+1}$$

Euler



$$p \sim 1 \quad C \sim 10$$

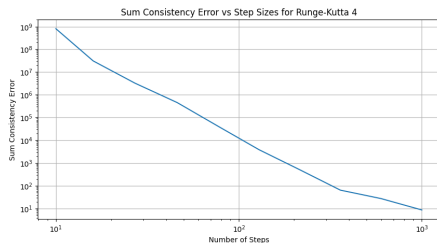
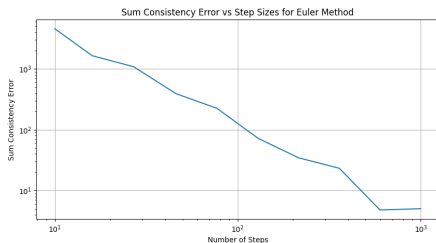
Runge-Kutta 4 (RK4)



$$p \sim 4 \quad C \sim 14$$

Global Consistency Error

$$h_{\max} = \max_n h_n \longrightarrow \lim_{h_{\max} \rightarrow 0} \sum_{n=1}^{N-1} \|e(t_n, y_n, h_n)\| = 0$$



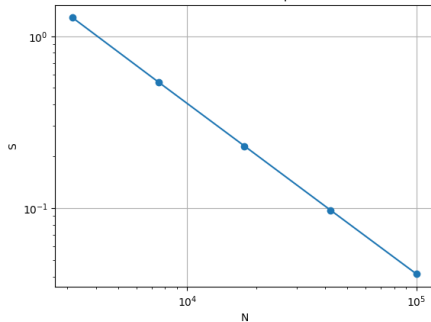
Both methods are consistent

Stability

$$S = \frac{\max_n \|\tilde{y}_n - y_n\|}{\|\tilde{y}_0 - y_0\| + \sum_n \|\epsilon_n\|}$$

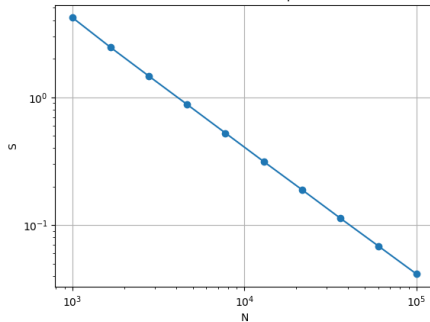
with $\epsilon_n \sim \mathcal{N}(0, 0.01)$

Constante de stabilité pour Euler



$S = 1.02$

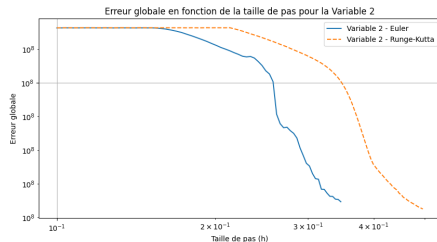
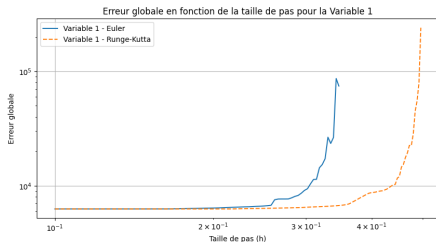
Constante de stabilité pour RK4



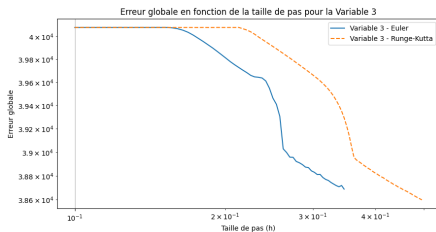
$S = 1.28$

Both methods are stable

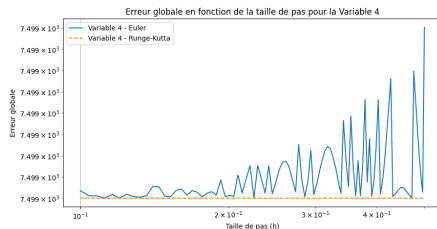
Global Error and Step Size (1)



Atmosphere



CarbonateRock

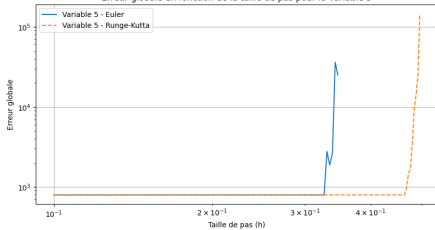


DeepOcean

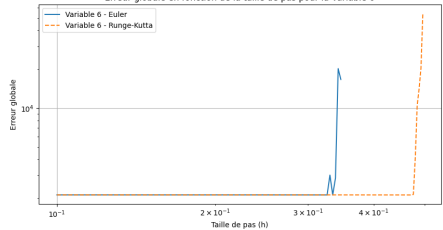
FossilFuel

Global Error and Step Size (2)

Erreur globale en fonction de la taille de pas pour la Variable 5



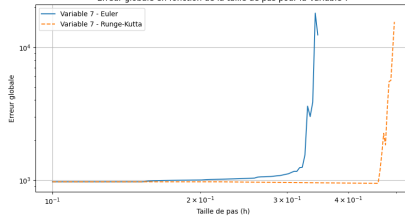
Erreur globale en fonction de la taille de pas pour la Variable 6



Plant

Soil

Erreur globale en fonction de la taille de pas pour la Variable 7



SurfaceOcean

Execution Time

Execution Time Comparison for Euler and RK4

