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The field of statistics is a broad and inclusive field. This form of mathematics can be used to study relationships between data in many applications in everyday life. When we begin with analyzing discrete random variables, we must study not only their probability distributions, but also their expected values and variances in order to fully see the relationship. These proofs show how the definition of variance is related to the computational form and how when taking the variance of constants multiplied and added to the random variable we get a certain formula to work with.

To understand what is happening in these proofs on variance, it may help to understand what variance and expected value truly mean. A simple definition of variance is that it is a sample of measurements x1, x2, ..., xn which is the sum of the square of the differences between the measurements and their mean divided by n – 1. Symbolically it appears as I - )2. We also see that variance is the square of the standard deviation. The simple definition of expected value is that it is the summation of the outcome times the probability of the outcome. It may be written as E(X) = I p(xi).

In our math classes at JSU, we almost always work independently on our proofs and other assignments. So, both proofs I have detailed were done completely on my own. Not working with anyone else allows me to work at my own pace and forces me to learn all of the material by myself. Both proofs are from MS 304 Math Stats I, which I took last semester. I am currently taking MS 404 Math Stats II, and actually feel as this assignment gave me the opportunity to go back and do an in-depth review of the concepts of variance and expected value. With math, everything builds off basic concepts, so it is mandatory to understand these proofs to be able to proceed with the class. Also, then next two paragraphs do not contain the proofs, they are simply an elaboration on the proofs given in separate files.

In Proof #1, we show that the definition for of variance, which is given to us, is E[]. We show that it will be equivalent to the computational form of variance which is also given to us as . Through this proof we are capable of seeing the relationship between variance and expected value, meaning that expected value is the building block of variance. To begin the proof, we started with the definition form of variance, which we stated was E[]. From that point it was necessary to implement basic algebra to expand the inside function to get E[]. The next step was to use theorem 3.5, which deals with linearity, to simplify. To elaborate on the proof, theorem 3.5 states that E[ g1(y) + g2(y)] = E[g1(y)] + E[g2(y)]. This means that the expected value of two functions added together is the same as the expected value of a function added to the expected value of another function. By implementing this we then have . From there we are given that is a constant, because the expected value is only taken by the random variable, constants do not affect them and are therefore pulled out of the function giving us . We are also told that E(X) = , because the expected value is equal to the mean of all the random variables. Therefore, anywhere that we see E(X) we substitute with  and then simplify to get . Now we simply use this step in reverse and substitute E(X) for , and we end up with the computational form, , which completes our proof #1.

Now that we have proved the definition of variance will be equal to the computational form of variance we have a better idea of how variance and expected value are related. We may now start proof #2. Here we are showing the relationship between the function of variance of the random variable and the constants. We again, start with the computational form of variance. This time though, because we are taking the variance of aX + b we must substitute that into our expected values in the computational form. So we see that V(aX + b) = E(() - . We then use algebra to expand what we have wrote inside the functions to get E( + 2baX + ) – []. From there we used theorem 3.5. As stated above, theorem 3.5 says that E[ g1(y) + g2(y)] = E[g1(y)] + E[g2(y)]. This means that the expected value of two functions added together is the same as the expected value of a function added to the expected value of another function. So we again are able to simplify by separating our functions to get . From here we simply simplify and then we factor out a2 from what we get from that operation to end up with . We see that what is inside of those brackets is what we have previously stated was the computational form of variance. Therefore we see that we now have that V(aX + b) = .

So we have proved the relationship between the variance function and the constants.