Computational Physics - Assignment 6

The logistic map is a one-dimensional discrete-time map that, despite its formal simplicity, exhibits an unexpected degree of complexity.
This assignment dealt with plots of logistic maps and other functions. Another subsection which was covered was the Lyapunov exponent in the final part of the assignment.

The logistic map is defined by the following equation:

$$x_{n+1} = \lambda x_n(1 - x_n) \quad \text{with} \quad n = 0, 1, 2, 3...$$

This equation was implemented and a logistic map was generated.

Part A

An algorithm was implemented which iterates the logistic map. This algorithm works by taking in some initial parameters for x0 and r. Then, x0 is assigned as the first value to an empty array and the equation above is implemented which fills up the rest of the array according to the equation. This iteration occurs 201 times (n), which means the total length of the x array was also n. Then x is plotted against n and this gives us our logistic map. When the code 6A.py is run, it will generate a pdf which includes all the logistic maps for the specified initial parameters.

—–Refer to 6A.pdf file included for the detailed plots—-

Yes, the results were in agreement with the general statements discussed in class, since as seen in the pdf, the plots show a

change in behavior according to their initial conditions and value of r.

Part B

The same iterative for loop was implemented with the function changed as in the question. Run codes 6B1.py and 6B2.py to obtained detailed pdfs giving the plots for the functions. 6B1.pdf shows the maps for an exponential function while 6B2.pdf shows the maps for a sine function.

——Refer to 6B1.pdf and 6B2.pdf files included for the detailed plots—-

Part C

Here, first the same logistic map was obtained as in the first problem for the specified parameters of r=0.92 and x0=0.6. Next, another logistic map was created with the parameters set to r=0.92 and x0=6.0001. The difference between these two arrays was computed and this difference was plotted as the difference between 2 trajectories adhering to the equation given in the question. Run code 6C.py to get a detailed pdf of this difference.

——Refer to 6C.pdf file included for the detailed plot—-

Part D

First, three different functions were defined for the three different maps required (logistic, exponential and sine).
Next, a function Lyapunov was defined which:

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The function below first computes the derivatives
of the generated map array passed into the function
from one of the functions above.
Next, the sum of the derivatives is calculated and then
divided by the size of the derivative array to get
the Lyapunov value, which is returned.
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Following this, some parameters were defined (refer to 6D.py) which ran the algorithm and generated arrays for Lyapunov's exponent depending on the r value. These arrays were then plotted (m against r).

——Refer to 6D.pdf file included for the detailed plot—-

References:

https://www.complexity-explorables.org/flongs/logistic/