GENERAL ELECTRICAL ENGINEERING LAB 2

The Wheatstone Bridge

CH-211-B

Natural Science Laboratory

Lab Experiment 3

Experiment conducted by: Kuranage R R Ranasinghe and Abigail Christie

Date of Experiment: 08 April 2021

Date of Submission: 18 April 2021

JACOBS UNIVERSITY BREMEN

Author: Kuranage R R Ranasinghe

INTRODUCTION AND THEORY

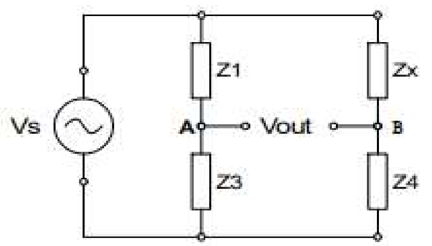


Figure 1: Wheatstone Bridge (taken from Lab Manual)

During this experiment, measurements of impedance and resistance were done using a simple circuit and the systematic and methodical errors associated with this method were dissected. The main circuit used for all parts of the experiment is known as a Wheatstone bridge. A Wheatstone bridge - developed by Charles Wheatstone- is used to accurately measure unknown resistances and impedance values, or to calibrate measuring instruments (voltmeters, ammeters, etc.) using a variable resistance and simple mathematical formulae.

Figure 1 portrays a Wheatstone bridge containing four impedances and a voltage source. When the Wheatstone bridge is 'balanced', V_{out} across A and B becomes 0V. When Z_x is the unknown impedance, this balanced state is attained by adjusting any/all the other impedances until 0V is seen at V_{out}. Next, using all known impedance values, the unknown impedance can be calculated. The Wheatstone bridge can also be utilized as an unbalanced bridge. In this situation, V_{out} is 0V at a known reference point of a transducer which replaces the unknown impedance at Z_x. Any changes in the physical parameter of the transducer (which consecutively changes its impedance) will result in a change at V_{out}.

The calculations for a Wheatstone Bridge with a DC or AC source are done slightly differently. In a DC Bridge, since all impedances are purely resistive, when the bridge is balanced V_{out} = 0V, the value of the unknown resistor can be found using the following formula where $R_{1,3,4}$ are known resistors and R_X is the unknown resistor:

$$\frac{R_1}{R_3} = \frac{R_X}{R_4} \leftrightarrow R_x = \frac{R_1}{R_3} \times R_4 \qquad (1)$$

Considering the Unbalanced DC Bridge, the resistances values are of less importance. Instead, the change in V_{out} (a direct measure for the transducer's physical parameter, which is temperature in this experiment) is our main concern.

In an AC Bridge, the conditions for (1) still play a part, but since some elements are no longer purely resistive at this point, both magnitude and phase should be considered. Refer to (2) below that further implies (3):

$$(|Z_1| * |Z_4|) * e^{j(\varphi 1+\varphi 4)} = (|Z_3| * |Z_X|) * e^{j(\varphi 3+\varphi x)} \qquad (2)$$

$$|Z_1| * |Z_4| = |Z_3| * |Z_X| \quad \text{and} \quad \varphi 1 + \varphi 4 = \varphi 3 + \varphi x \qquad (3)$$

From (3), it is obvious that to adjust V_{out} to 0V, at least two components must be varied to adjust both the amplitude and the phase.

EXPERIMENTAL SET-UP AND RESULTS

Part 1: Balanced DC Wheatstone Bridge

The experiment was set up as shown in (2) on the breadboard. The exact values of the resistors used (except ones read from the Decade box) were measured using the ELABO multimeter.

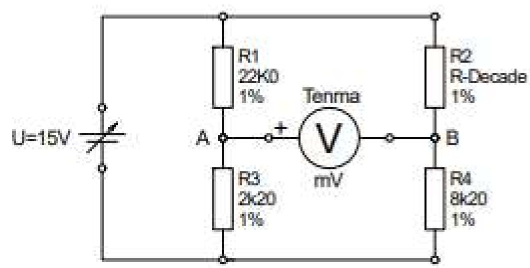


Figure (2): Set-up for Part 1 of experiment (taken from Lab Manual)

The resistance of the R-Decade was adjusted until the final V_{AB}, which was measured in the mV range of the TENMA became 0V. This final V_{AB} was recorded. Additional values for the read resistance from the decade box and the measured resistance across the box from the ELABO multimeter were also recorded.

Data Obtained from Part 1 of Experiment	
R_1	Resistance: 22.07kΩ Range: 200k Ω
R_3	Resistance: 2.185kΩ Range: 20k Ω
R_4	Resistance: 8.205kΩ Range: 20k Ω
V_{AB}	000.00mV
Read R-Decade	82.6474 kΩ
Measured R-Decade	Resistance: 82.94 kΩ Range: 200 kΩ

Part 2: Unbalanced DC Wheatstone Bridge

For this part of the experiment, the circuit was pre-assembled on a printed circuit board due to the sensitivity of the small sensor. The printed circuit is shown in (3) and the specifications for the R_{PT1000} are as follows:

The function of the PT1000 resistor over temperature:
$R_T = R_0 * (1 + \alpha * \Delta T)$
R_T = Resistance (Ω) at temperature T (◦ C)
R_0 = Resistance at 0◦ C = 1000Ω
T = Temperature of environment in ◦C

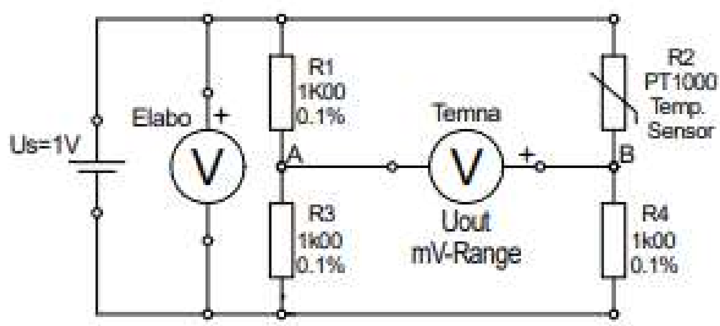


Figure (3): Set-up for Part 2 of experiment (taken from Lab Manual)

The exact value of V_S was measured and recorded using the ELABO multimeter and the voltage of V_{out} using the mV range on the TENMA. V_S was then changed to 10V and after 10 minutes, V_S and V_{out} were recorded again.

Data Obtained from Part 2 of Experiment		
Us=1V	V_s	1.0574 V
	V_{out}	-018.99 mV
Us=10V	V_s	10.109 V
	V_{out}	-206.69 mV

Part 3: Balanced AC Wheatstone Bridge

The circuit for this section was set up as shown in (4) below. The exact values of R1, R3 and R4 were measured and recorded using the ELABO multimeter. The exact component values of Y_1 (for C_{1p} = 1µ F) were measured with the RLC-Meter in the parallel equivalent mode.

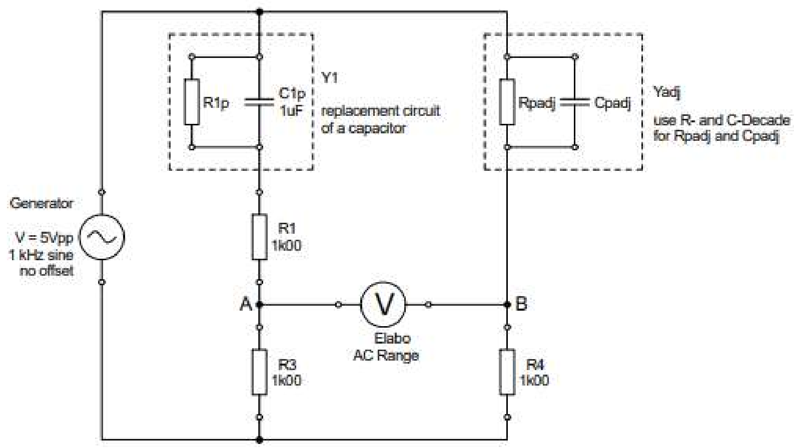


Figure (4): Set-up for Part 3 of experiment (taken from Lab Manual)

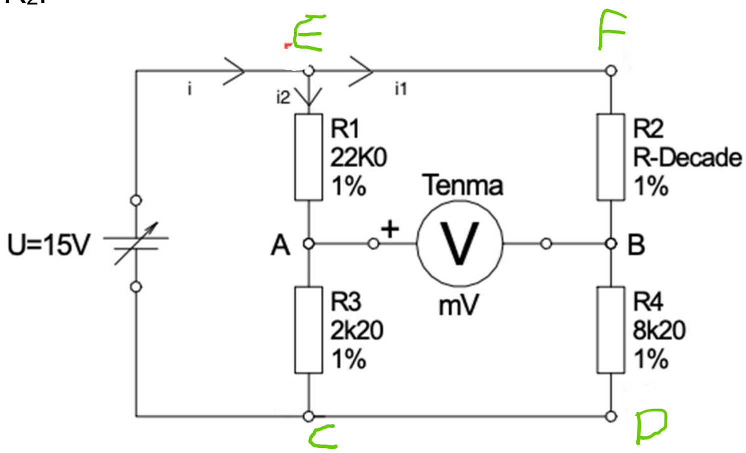
An oscilloscope was used to measure the phase difference with CH1 at node A and CH2 at node B. Both the R and C-decade were adjusted until the bridge was balanced (i.e., obtaining a V_{AB} as close as possible to 0V and a phase of 0˚ on the oscilloscope). To make this process easier, the theoretical values for R_{padj} and C_{padj} were calculated first (calculations shown in Evaluation).

The final values for R_{padj} and C_{padj} were recorded; first taken from reading the decade box and then using the Agilent RLC-Meter.

Data Obtained from Part 3 of the Experiment		
R1		0.9934 kΩ
R3		0.9945 kΩ
R4		0.9947 kΩ
Y1	C1p R1p	1.0507µF 36.428kΩ
V_{AB}		0.99mV
Theoretical	Cpadj Rpadj	25.02nF 1019.1Ω
Read Yadj	Cpadj Rpadj	28.4nF 1017Ω
Measured Yadj	Cpadj Rpadj	28.524nF 1.0214kΩ

Part 1: Balanced DC Wheatstone bridge

- Formula for R_2:



For loop ABFE (using KVL):

$-R_1 * i_2 + 0 + R_2 * i_1 = 0$

$$R_2 = \frac{R_1 \,.\, i_2}{i_1} \qquad (1)$$

For loop ABDC (using KVL):

$R_3 * i_2 = R_4 * i_1$

$$i_1 = \frac{R_3 \,.\, i_2}{R_4} \qquad (2)$$

Using (1) and (2):

$$R_2 = \frac{R_1 \,.\, R_4}{R_3}$$

-Calculation for R_2

With the nominal resistor values:

$R_1 = 22k\Omega$, $R_3 = 2.20k\Omega$ and $R_4 = 8.20k\Omega$

$$R_2 = \frac{R_1 \,.\, R_4}{R_3}$$

$$R_2 = \frac{22000 \,.\, 8200}{2200}$$

$R_2 = 82k\Omega$

With the measured values from R_1, R_3, and R_4:

R_1 = 22.07kΩ, R_3 = 2.185kΩ and R_4 = 8.205kΩ

$$R_2 = \frac{R_1 \,.\, R_4}{R_3}$$

$$R_2 = \frac{22.07 * \;\; 8.205}{2.185}$$

R_2 = 82.88kΩ

-Maximum relative error for R_2 using:

-Values read from the resistor decade box

Maximum Relative Error = Tolerance of the resistor

Relative Error = ±1%

-Values directly measured with the ELABO multimeter

Absolute Error in ELABO = (0.06% * f.value) + (0.01% * f.range) + 50 $m\Omega$
where f.value = 82.94kΩ and f.range = 200kΩ

Therefore,

$$Absolute\ Error\ (AE) = \frac{0.06}{100} * 82.94 * 10^3 + \frac{0.01}{100} * 200 * 10^3 + 50 * 10^{-3}$$

AE = 69.81Ω

$$Relative\ Error = \frac{AE}{f.value} * 100\% = \frac{69.81}{82.94 * 10^3} * 100\%$$

Relative Error = ±0.08%

-Values calculated from the nominal resistor values

Error for R_1 (ΔR_1) = 1% * 22kΩ
ΔR_1 = 220Ω
Error for R_3 (ΔR_3) = 1% * 2.2kΩ
ΔR_3 = 22Ω
Error for R_4 (ΔR_4) = 1% * 8.2kΩ
ΔR_4 = 82Ω

$$R_2 = \frac{R_1 \,.\, R_4}{R_3}$$

$$\Delta R_2 = \left|\frac{\delta(\frac{R_1 \,.\, R_4}{R_3})}{\delta R_1} * \Delta R_1\right| + \left|\frac{\delta(\frac{R_1 \,.\, R_4}{R_3})}{\delta R_3} * \Delta R_3\right| + \left|\frac{\delta(\frac{R_1 \,.\, R_4}{R_3})}{\delta R_4} * \Delta R_4\right|$$

$$\Delta R_2 = \left|\frac{R_4}{R_3} * \Delta R_1\right| + \left|\frac{R_1 \,.\, R_4}{{R_3}^2} * \Delta R_3\right| + \left|\frac{R_1}{R_3} * \Delta R_4\right|$$

$$\Delta R_2 = \left|\frac{8200}{2200} * 220\right| + \left|\frac{22000 \,.\, 8200}{2200^2} * 22\right| + \left|\frac{22000}{2200} * 82\right|$$

$$\Delta R_2 = 2460\Omega$$

$$Relative\ Error = \frac{\Delta R_2}{R} * 100\%$$

$$Relative\ Error = \frac{2460}{82000} * 100\%$$

Relative Error = $\pm$ 3%

-Values calculated from the measured resistor values

Error in R_1 (ΔR_1) = (0.06% * f.value) +(0.01% * f.range) + 50 $m\Omega$

ΔR_1 = (0.06% * 22.07kΩ) + (0.01% * 200kΩ) + 50 $m\Omega$

ΔR_1 = 33.292Ω

Error in R_3 (ΔR_3) = (0.06% * f.value) + (0.01% * f.range) + 50 $m\Omega$

ΔR_3 = (0.06% * 2.185kΩ) + (0.01% * 20kΩ) + 50 $m\Omega$

ΔR_3 = 3.361Ω

Error in R_4 (ΔR_4) = (0.06% * f.value) + (0.01% * f.range) + 50 $m\Omega$

ΔR_4 = (0.06% * 8.205kΩ) + (0.01% * 20kΩ) + 50 $m\Omega$

ΔR_4 = 6.973Ω

$$R_2 = \frac{R_1 \,.\, R_4}{R_3}$$

$$\Delta R_2 = \left|\frac{\delta(\frac{R_1 \,.\, R_4}{R_3})}{\delta R_1} * \Delta R_1\right| + \left|\frac{\delta(\frac{R_1 \,.\, R_4}{R_3})}{\delta R_3} * \Delta R_3\right| + \left|\frac{\delta(\frac{R_1 \,.\, R_4}{R_3})}{\delta R_4} * \Delta R_4\right|$$

$$\Delta R_2 = \left|\frac{R_4}{R_3} * \Delta R_1\right| + \left|\frac{R_1 \,.\, R_4}{{R_3}^2} * \Delta R_3\right| + \left|\frac{R_1}{R_3} * \Delta R_4\right|$$

$$\Delta R_2 = \left|\frac{8205}{2185} * 33.292\right| + \left|\frac{22070 * \ 8205}{2185^2} * 3.361\right| + \left|\frac{22070}{2185} * 6.973\right|$$

$$\Delta R_2 = 323\Omega$$

$$Relative\ Error = \frac{\Delta R_2}{R} * 100\%$$

$$Relative\ Error = \frac{323}{82876} * 100\%$$

Relative Error = $\pm$ 0.39%

-Table with all the calculated and measured R_2 values with their respective errors:

Measurements	R_2 (kΩ)	Relative Error
Decade Box	82.67	1%
ELABO Multimeter	82.94	0.08%
Nominal Values	82.00	3%
Measured Values	82.88	0.39%

According to our calculations, the value of R_2 which was read from the ELABO multimeter has the smallest relative error and hence, is the best measurement.

Discussing error sources:

While using a decade dox, the error is the tolerance of the decade and the fact that exact values cannot be entered into the decade directly only limits the getting of an accurate measurement further.

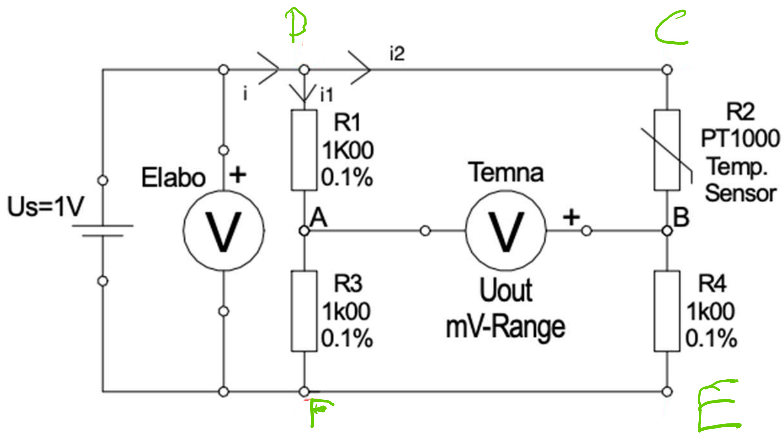
When nominal values were used to calculate R_2, the errors in each resistor (which are comparatively prominent) contributed towards the error during the error calculation done using partial derivatives.

Finally, in the case where measured values were used, the error calculated was comparatively lesser than those from the decade box and nominal values since the resistances of the resistors were measured using an ELABO multimeter (which contributes only a comparatively small error during error calculation).

-From the measurements, it can indeed be seen that the bridge does not have the best accuracy. One of the main ways to improve the accuracy of the bridge could have been to measure the resistance of the decade box using the ELABO multimeter rather than using the value provided since the ELABO contributes the minimum error during measurements. Another method to minimize errors and improve the accuracy would be to use better resistors with low error values since the errors of individual resistors also play a part when it comes to the accuracy of the bridge as a whole.

Part 2: Unbalanced DC Wheatstone bridge

-Calculation of R_{PT1000} for the two cases using the given component values and measured voltages:



For loop ABCD (using KVL):

$R_1 * i_1 - V_{out} - R_2 * i_2 = 0$

$$R_2 = \frac{R_1 \, . \, i_1 - V_{out}}{i_2} \qquad (3)$$

For loop ABEF (using KVL):

$R_3 * i_1 - R_4 * i_2 + V_{out} = 0$

$$i_2 = \frac{R_3 \, . \, i_1 + V_{out}}{R_4} \qquad (4)$$

From (3) and (4):

$$R_2 = \frac{(R_3 \, . \, i_1 - V_{out}) * R_4}{R_3 \, . \, i_1 + V_{out}} \qquad (5)$$

But $V_{DF} = V_{Elabo}$:

$$V_{Elabo} = i_1 * (R_1 + R_3) \qquad (6)$$

For $V_{Elabo} = 1.0574V$ and $V_{out} = -18.99mV$

From (6):

$$i_1 = \frac{1.0574}{1000 + 1000} = 5.2870 * 10^{-4} A$$

From (5):

$$R_2 = \frac{(1000 * 5.2870 * 10^{-4} + 18.99 * 10^{-3}) * 1000}{(1000 * 5.2870 * 10^{-4}) - (18.99 * 10^{-3})}$$

$R_{PT1000} = 1.075k\Omega$

Next, for V_{Elabo} = 10.109V and V_{out} = -209.69mV

From (6):

$$i_1 = \frac{10.109}{1000 + 1000} = 5.0545 * 10^{-3}A$$

From (5):

$$R_2 = \frac{(1000 * 5.0545 * 10^{-3} + 209.69 * 10^{-3}) * 1000}{(1000 * 5.0545 * 10^{-3}) - (209.69 * 10^{-3})}$$

R_{PT1000} = 1.087kΩ

-Conversion of the values to temperatures:

For R_{PT1000} = 1.075kΩ

$R_T = R_0 * (1 + \alpha * \Delta T)$
$1.075 * 10^3 = 1000 * (1 + 3.850 * 10^{-3} * \Delta T)$
ΔT = 19.48˚C
$T = \Delta T + T_0$ = 19.48 + 0 = 19.48˚C
T = 19.48˚C

For R_{PT1000} = 1.087kΩ

$R_T = R_0 * (1 + \alpha * \Delta T)$
$1.087 * 10^3 = 1000 * (1 + 3.850 * 10^{-3} * \Delta T)$
ΔT = 22.60˚C
$T = \Delta T + T_0$ = 22.60+ 0 = 22.60˚C
T = 22.60˚C

-One of the main sources of error during the calculation of temperature values was that when a high potential passes through a device, the device itself gets heated up. This could have caused a significant error in our calculations since the values recorded would have been slightly different. Another source of error could have been the fluctuation of temperature in the environment around the sensor, and an error in the R_0 value and internal resistances associated with it (which were not considered) could also have had an impact on the final calculated temperatures.

-It can be observed that i_2 flows through R_{PT1000}.

Using (4):

$$i_2 = \frac{R_3 \cdot i_1 + V_{out}}{R_4}$$

For V_{Elabo} = 1.0574V:

$$i_2 = \frac{1000 * 5.2870 * 10^{-4} - 18.99 * 10^{-3}}{1000} = 5.0971 * 10^{-4} A$$

R_{PT1000} = 1.075kΩ

So, E = 0.2˚C/mW = 200˚C/W

Power consumed (P) = $(i_2)^2 * R_{PT1000} = (5.0971*10^{-4})^2 * 1.075*10^3$

= $2.793 * 10^{-4}$ W

Additional Temperature (T_1) = P*E = $2.793 * 10^{-4} * 200$

T_1 = 0.06˚C

For V_{Elabo} = 10.109V:

$$i_2 = \frac{1000 * 5.0545 * 10^{-3} - 209.69 * 10^{-3}}{1000} = 4.845 * 10^{-3} A$$

R_{PT1000} = 1.087kΩ

So, E = 0.2˚C/mW = 200˚C/W

Power consumed (P) = $(i_2)^2 * R_{PT1000} = (4.845*10^{-3})^2 * 1.087*10^3$

= $25.52 * 10^{-3}$ W

Additional Temperature (T_2) = P*E = $25.52 * 10^{-3} * 200$

T_2 = 5.10˚C

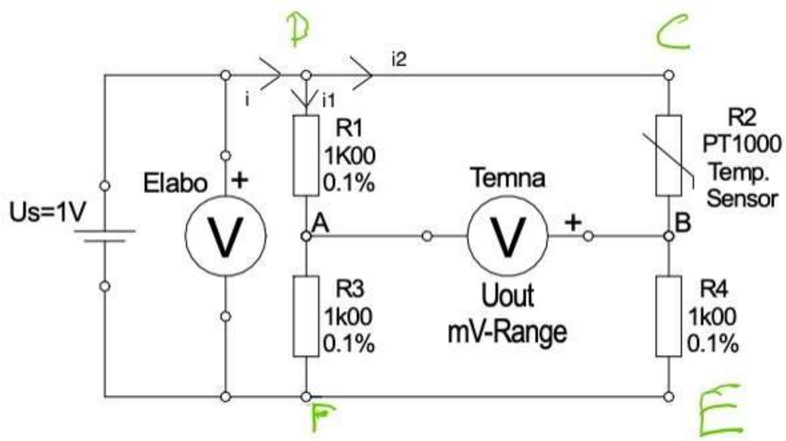
For the voltage of 1.0574V, the additional increase in temperature was 0.06˚C which is almost negligible compared to the previously calculated environmental temperature of 19.48˚C.
However, for the voltage of 10.109V, the additional increase in temperature was 5.10˚C. This is 20% of the calculated value for the environmental temperature 22.60˚C. Hence, in conclusion, when the supply voltage is lower, as was at 1.0574V, the calculated value for the temperature of the environment is fairly accurate but when a larger supply voltage (10.109V) is used, the sensor by itself contributes significantly towards the temperature measurement giving us an inaccurate measurement of the environmental temperature.

-Now considering the influence of the voltmeter which measures V_{OUT}:

(a) A TENMA in the mV range was used for measuring V_{OUT}. Its internal resistance in the mV range is 2.5GΩ.

(b) V_S = 1V

R_{PT} at 20˚C = $R_0 * (1 + \alpha * \Delta T) = 1000 * (1 + 3.850 * 10^{-3} * 20) = 1077\Omega$



The Thevenin equivalent for the circuit above is calculated as follows:

Across AB:

Equivalent Resistance (R_{eq}) = (R_1 + R_{PT1000}) || (R_3 + R_4)

R_{eq} = (1000 + 1077) || (1000 + 1000)

R_{eq} = 1019Ω

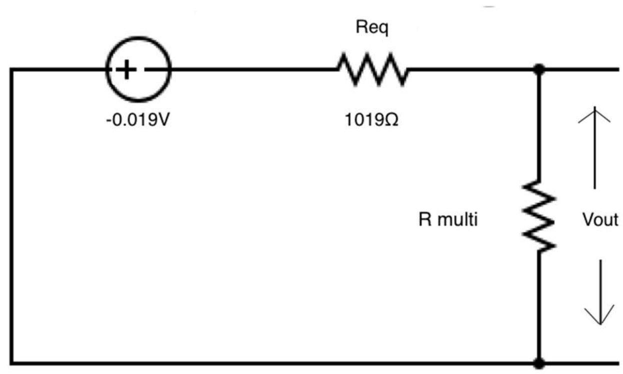
$$V_A = \frac{R_3}{R_1+R_3} * V_S = \frac{1000}{2000} * 1 = 0.5V$$

$$V_B = \frac{R_4}{R_{PT1000} + R_4} * V_S = \frac{1000}{2077} * 1 = 0.481V$$

$V_{AB} = V_B – V_A = 0.481 – 0.5$

= -0.019V

The equivalent Thevenin circuit is:



Now,

$$V_{OUT} = \frac{R_{multi}}{1019 + R_{multi}} * (-0.019V)$$

For a low input impedance multimeter R_{multi} = 10kΩ:

$$V_{OUT} = \frac{10000}{1019 + 10000} * (-0.019V) = -0.01724V$$

Absolute Error (AE) = -0.019V + 0.01724V = -1.76mV

$$Relative\ Error = \frac{AE}{-0.019} * 100\% = \frac{-0.00176}{-0.019} * 100\%$$

Relative Error = 9.26%

(c) Considering our experiment with R_{multi} = 2.5GΩ:

$$V_{OUT} = \frac{R_{multi}}{1019 + R_{multi}} * (-0.019V) = \frac{2.5 * 10^9}{1019 + 2.5 * 10^9} * (-0.019V) = -0.0190V$$

Absolute Error (AE) = -0.019V + 0.019V = 0mV

$$Relative\ Error = \frac{AE}{-0.019} * 100\% = \frac{0}{-0.019} * 100\%$$

Relative Error = 0%

Hence, for the voltmeter we used, the relative error is approximately 0%. Hence, in conclusion, our measurement had a high accuracy with respect to this factor and the measured output voltage was once again, approximately equal to the true value.

Part 3: Balanced AC Wheatstone bridge

-For this final part of the experiment, an AC source was used. In our circuit for this bridge, both a capacitor and a resistor connected in parallel were present, which leads to the introduction of both real and imaginary components to the impedance of the bridge. Hence, to balance this Wheatstone bridge, we need both real and imaginary components such that both components of the impedance are balanced, respectively. Therefore, in our case, the requirement was fulfilled with two components, both a capacitor and a resistor, which helped us balance the bridge.

- Calculations for the values of R_{padj} and C_{padj} respectively:

f = 1000Hz, $\omega = 2\pi f = 2000\pi$

$$Y_1 = \frac{1}{R_{1p}} + j\omega C_{1p}$$

Using nominal value for C_{1p} (R_{1p} was ignored),

$$Y_1 = 2000\pi * 1 * 10^{-6}j = 6.283 * 10^{-3}j$$

$$Z_1 = \frac{1}{Y_1} = \frac{1}{6.283 * 10^{-3}j} = -159.16j$$

For the balanced bridge where $R_1 = 993.4\Omega$, $R_3 = 994.5\Omega$, $R_4 = 994.7\Omega$,

$$(Z_1 + R_1) * R_4 = \frac{1}{Y_{adj}} * R_3$$

$$(-159.16j + 993.4) * 994.7 = \frac{1}{Y_{adj}} * 994.5$$

$$\frac{1}{Y_{adj}} = 993.60 - 159.19j$$

$$Y_{adj} = (9.813 * 10^{-4}) + (1.572 * 10^{-4})j$$

Comparing the value with $Y_{adj} = \frac{1}{R_{padj}} + j\omega C_{padj}$

$R_{padj} = 1019.1\Omega$ and $C_{padj} = 25.02nF$

- Calculations for Y_1 (from the measured values of R_1, R_3, R_4, and Y_{adj}), R_{1P} and C_{1P}:

$$Y_{adj} = \frac{1}{R_{padj}} + j\omega C_{padj} = (9.813 * 10^{-4}) + (1.572 * 10^{-4})j$$

$$Z_{adj} = \frac{1}{Y_{adj}} = 993.60 - 159.19j$$

For the balanced bridge:

$$Z_{adj} * R_3 = (\frac{1}{Y_1} + R_1) * R_4$$

$$(993.60 - 159.19j) * 994.5 = (\frac{1}{Y_1} + 993.4) * 994.7$$

$$Y_1 = 8.731 * 10^{-9} + 6.283 * 10^{-3} j$$

Comparing the value with $Y_1 = \frac{1}{R_{1p}} + j\omega C_{1p}$

R_{1p} = 114534kΩ and C_{1P} = 1.000μF

-Comparison of calculated and measured values of R_{1P} and C_{1P}:

Components	Measured	Calculated
R_{1P} (kΩ)	36.428	119760
C_{1P} (μF)	1.051	1.000

As seen, the value of C_{1P} in both situations (measured and calculated) are very close to each other with a difference of less than 10%. However, there is a large difference in the value of R_{1P} in the different instances (measured and calculated). There are several reasons why this could be the case. First, during the calculation for R_{1P}, the phase angle was extremely close to 90˚(As seen in a previous experiment, when the phase angle gets close to 90˚, the calculations become extremely sensitive and even a small change results in a larger difference in values) resulting in a greater error in the calculated value. Another source of error which could have contributed to this could be the resistances in the RLC meter which could have induced some error in the measured values (which could have amplified during calculations), hence changing the result.

-Influence of the Voltmeter and Oscilloscope on the circuit

During the experiment, the voltmeter was used to measure voltages and the oscilloscope, the phase difference produced by reactive elements. The balance point on the Wheatstone bridge was found by making both quantities on both instruments 0V and 0˚ respectively. Since the voltmeter used had a large impedance (in the range of GΩ), we can assume that no current passed through, and hence, the contribution of this as an error source is almost negligible. However, there is always a possibility that some current might have passed through and changed our results. The oscilloscope also includes some resistance which is not taken into account during our experiment (an additional impedance could have been the connecting probes) and this could have once again, changed our results slightly. These factors combined also contribute to the slight overall decrease in accuracy of the bridge.

CONCLUSION

This lab dealt with the concept of the Wheatstone bridge and how it was utilized in different cases which involved both DC and AC sources. These were the Balanced and Unbalanced Bridge for DC and the Balanced Bridge for AC.

First, considering the case of the Balanced DC Wheatstone bridge, a resistor decade was used in the place of an unknown resistor. This unknown resistance was read off directly from the decade, measured using the ELABO multimeter and calculated from other known resistances. All these methods had corresponding errors attached to them (refer to Evaluation for a detailed error analysis) all which tied to the fact that the bridge was not perfectly balanced. According to the error calculations, it was concluded that the resistance measured with the ELABO multimeter was the most accurate since it had the least relative error when compared to other methods.

For the next part of the experiment consisting of an Unbalanced DC bridge, the set-up was used to look at temperature using a temperature sensor. The relationship between the output voltage and the temperature read from the sensor was derived using other known resistances. During error analysis, it was observed that with a larger supply voltage, the sensor dissipated power and made a significant difference to the error associated with the measurement. Hence, experimental results were not supported by calculations at higher supply voltages.

Finally, for the last part of the experiment which dealt with a Balanced Wheatstone Bridge, an AC voltage source was implemented with an unknown impedance (a circuit consisting of a capacitor in parallel with a resistor) along with other known impedances. Here, since both real and imaginary components were present, both resistance and capacitance (on separate decades) had to be changed in order to balance the bridge. It was not practically efficient to have an output voltage of 0V, so measurements were made for a value less than 5mV. The phase was also made 0˚ with the help of the oscilloscope. These processes combined also gave rise to some errors discussed in the Evaluation.

In conclusion, these experiments stimulated the understanding of the function of a Wheatstone bridge in balanced and unbalanced cases in both DC and AC. They also developed and explored systematic and the methodical errors common during measurement and calculation of different quantities.

REFERENCES

1. GEE Lab Manual URL:
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2. Charles K. Alexander, Fundamentals of Electric Circuits
3. Tektronix Oscilloscope TBS Series Manual URL:
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Appendix

Data collected from Two Port Network Lab, done on : April 9, 2021

Part 1: Two-port Z/Y Network

Z-Parameters		
Z_{11}	166.12 Ω	
Z_{12}	V_1 =1.0044V I_2 =V/R= 3.0137/100	33.3278 Ω
Z_{21}	V_2 =1.0047V I_1 =V/R= 3.0108/100	33.3698Ω
Z_{22}	166.28Ω	

Y-Parameters		
Y_{11}	2.4813 *10^-3 S	
Y_{12}	V_2 =5.025V I_1 =V/R= -3.0047/330	-1.81197* 10^-3 S
Y_{21}	V_1 =5.027V I_2 =V/R= -0.9092/100	-1.8086 *10^-3 S
Y_{22}	4.0256* 10^-3 S	

Connecting Circuit 1 and 2		
	Circuit 1	Circuit 2
V_1 (V)	5.04	5.04
V_2 (V)	0.869	1.812
I_1 (mA)	30.41	9.248
I_2 (mA)	-0.87	-1.82

Part 2: Interconnection of Two-port Networks

Z-Parameters		
Z_{11}	127.60 Ω	
Z_{12}	V_1 =1.7650V I_2 =V/R= 1.7127/100	37.9847 Ω
Z_{21}	V_2 =1.5029V I_1 =V/R= 2.9664/100+3.2428 /330	38.0585Ω
Z_{22}	108.45Ω	

Connected Circuit to Voltage supply	
V_1 (V)	5.049
V_2 (V)	1.3692
I_1 (mA)	39.915
I_2 (mA)	-1.389

Part 3: Complex Two-port Networks

Measured Impedances		
	R (Ω)	θ
Z1.1	150.01	-90.23
Z2.1	151.86	-89.74
Z3.1	65.349	86.22
Z2.2	151.56	-89.74