GENERAL ELECTRICAL ENGINEERING LAB 2

FILTER

CH-211-B

Natural Science Laboratory

Lab Experiment 5

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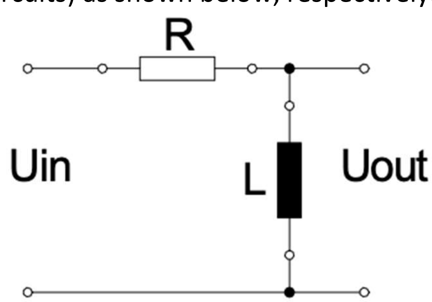
INTRODUCTION AND THEORY

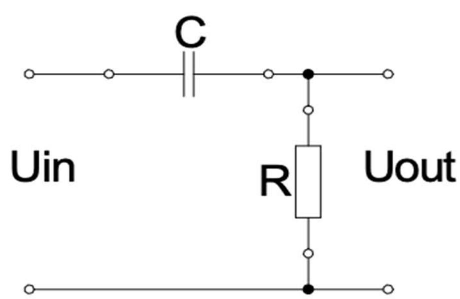
A filter is a circuit/device that removes a selected characteristic (most likely a frequency/frequency range) from a signal. In most cases, filters are used to select a frequency/frequency range from an incoming signal (most likely an AC voltage), while rejecting all the other frequency components present in the signal. There are two common ways to build a filter. It could be achieved using active components like transistors or operational amplifiers together with networks of resistors, capacitors, and inductors or it could be done using digital signal processors (DSP's) together with analog to digital and digital to analog convertors (ADC's and DAC's respectively).

In this experiment, the simplest type of filter is constructed using a passive network of resistors, capacitors, or inductors. In general, there are four types of filters that can be built in this simple manner, i.e., high pass, low pass, band pass and notch filters.

1. High Pass Filters

   A high pass filter consists of a circuit which lets through signals of high frequencies unchanged, while low frequency signals are attenuated significantly with a positive phase shift (i.e., the output signal advances the input signal). There are two ways in which a high pass filter can be constructed; using an RL circuit or an RC circuit (both passive components circuits) as shown below, respectively.





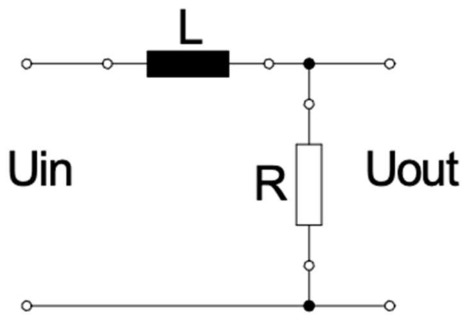
   For this experiment, the high pass filter was constructed using RC components. The magnitude and phase shift are calculated as follows:

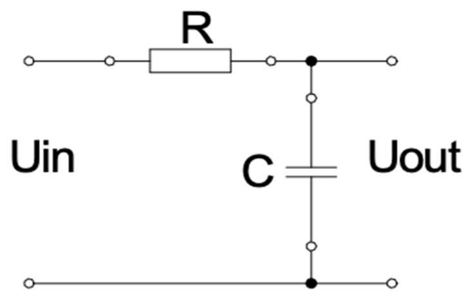
$$|\underline{A}| = \frac{1}{\sqrt{1 + 1/(\omega RC)^2}} \quad and \quad \varphi = \arctan\left(\frac{1}{\omega RC}\right)$$

$$f_{-3dB} = \frac{1}{2\pi RC}$$

2. Low Pass Filters

   A low pass filter consists of a circuit which lets through signals of low frequencies unchanged, while high frequency signals are attenuated significantly with a negative phase shift (i.e., the output signal lags behind the input signal). There are two ways in which a low pass filter can be constructed; again, using an RL circuit or an RC circuit (both passive components circuits) as shown below, respectively.





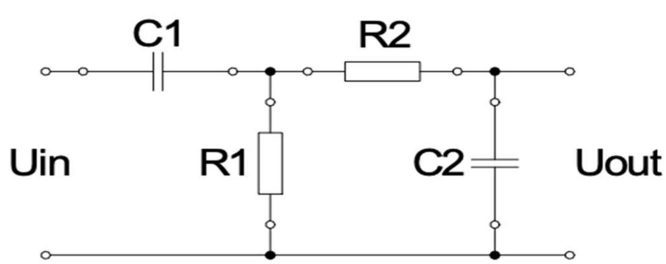
The magnitude and phase shift are calculated as follows:

$$| A | = \frac{1}{\sqrt{(1+(\omega RC)^2}}$$

$$f_{-3dB} = \frac{1}{2\pi RC}$$

$$\emptyset = -arctan\,(\omega RC)$$

3. Band Pass Filters
   This is a special type of filter which only allows a certain band of frequency to pass through unaltered. In a general situation, this frequency band lies between the high and low frequency range and hence, this filter blocks both high and low frequency components in signals. As logic follows, this filter is made combining both a high and low pass filter. The schematic for a band pass filter is given below.



The magnitude and phase difference can be calculated as follows:

$$\underline{A_{HP}}\ (\mathrm{j}\omega) = \frac{\underline{U_{out-HP}}}{\underline{U_{in}}}$$

$$\underline{A_{LP}}\ (\mathrm{j}\omega) = \frac{\underline{U_{out-LP}}}{\underline{U_{out-HP}}}$$

Now, the magnitude of Band pass Filter is given by:

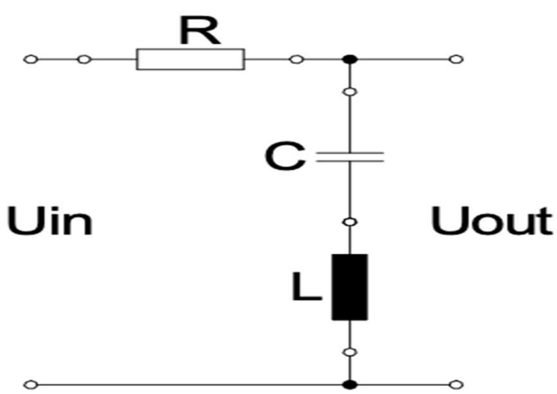
$$\underline{A_{BP}}\ (\mathrm{j}\omega) = \underline{A_{HP}}\ (\mathrm{j}\omega) \cdot \underline{A_{LP}}\ (\mathrm{j}\omega) = \frac{\underline{U_{out-LP}}}{\underline{U_{in}}}$$

Also the phase shift of band pass is given by:

$$\emptyset_{BP} = \emptyset_{LP} + \emptyset_{HP}$$

4. <u>Notch Filters</u>

A notch filter, also known as a Band Stop Filter, is one which lets through most frequencies unaltered, but attenuates some in a specific range to very low levels. This functionality is opposite to that of a band pass filter. An RLC circuit is implemented in the setup of this circuit as shown below.



In this experiment, a Notch filter is constructed. For these:

$$\underline{A}(j\omega) = \frac{\underline{V}_{out}}{\underline{V}_{in}} = \frac{j\left(\omega L - \frac{1}{(\omega C)}\right)}{R + j\left(\omega L - \frac{1}{\omega C}\right)} = \frac{1}{1 - j\left(\frac{R}{\omega L - \frac{1}{\omega C}}\right)}$$

$$|\underline{A}| = \frac{1}{\sqrt{1 + \left(\frac{R}{\omega L - \frac{1}{\omega C}}\right)^2}} \quad and \quad \varphi = \arctan\left(\frac{R}{\omega L - \frac{1}{\omega C}}\right)$$

$$f_{cf} = \frac{1}{2\pi\sqrt{LC}}$$

$$\omega_{cut\_low} = \frac{-RC \pm \sqrt{(RC)^2 + 4LC}}{2LC}$$

$$\omega_{cut\_high} = \frac{RC \pm \sqrt{(RC)^2 + 4LC}}{2LC}$$

Some important properties of filters are listed and explained below:

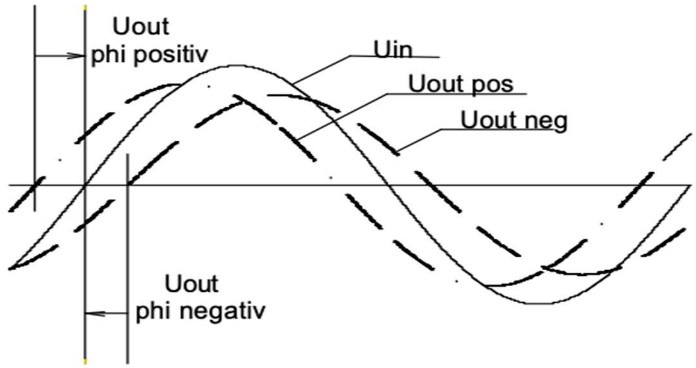
1. **Cut-off Frequency**
   This frequency, also known as the corner frequency, is the frequency either above which or below which the output power of the filter is half the power of the passband, and since voltage is proportional to power P, $Vout$ is $\frac{1}{\sqrt{2}}$ of the $Vout$ in the passband. The cut-off frequency exists close to the magnitude of -3 decibels. Hence, this point is also called the -3dB point. A bandpass filter and a notch filter each have two cut-off frequencies. Their geometric mean is the center frequency. The geometric mean of two numbers is given by: $f_{bw} = \sqrt{f1 \cdot f2}$.

2. **Bandwidth**
   The bandwidth for a band pass filter is the difference between the upper and lower cut-off frequencies.

   Bandwidth (BW) = f_{c1}- f_{c2},

   where f_{c1} is the upper cut off frequency and f_{c2} is the lower cut off frequency.

3. Phase Shift
   This is the variation in phase between the input and output signal and is represented as φ. It is measured relative to the input signal (which is considered to have a phase of 0°) and is positive when the output signal is ahead of the input signal (and negative when the output signal lags behind the input signal). A diagram describing phase shift is given below.



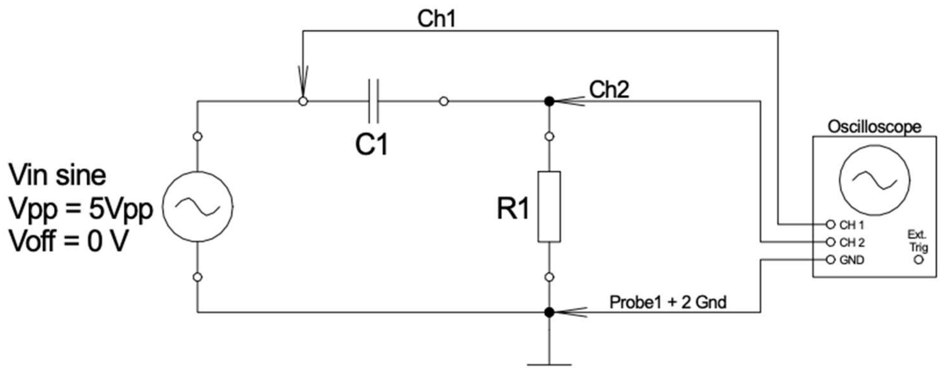
For the analysis of filter characteristics, two kinds of graphs are plotted.

1. **Bode Plots**
   There are two types of Bode plots, i.e., the Bode Magnitude plot and the Bode Phase plot. A Bode magnitude plot is a graph of magnitude in the dB scale against a frequency on a logarithmic scale while the Bode phase plot is a graph of phase difference against a frequency on a logarithmic scale. It is often used in signal processing to show the transfer function of a system. The magnitude-frequency plot can often be approximated to straight lines in a Bode plot. The magnitude of a bode plot is measured in the decibel scale. It is calculated using the formula:
   $$A = 20 \cdot log\left(\frac{Uout}{Uin}\right)$$

2. Nyquist Plots
   It is the general plot of a transfer function in the cartesian plane. In this plot, the real part of the transfer function is plotted in the x-axis of the cartesian plane while the imaginary part is plotted on the y-axis. It provides information about the shape and information on the number of poles and zeros in the transfer function. It also helps determine the stability of a system.

Part 1: Hi-Pass

The High Pass filter was constructed with R_1 = 1kΩ and C_1 = 100nF, signal generator connected via BNC-to-Kleps cable as follows:



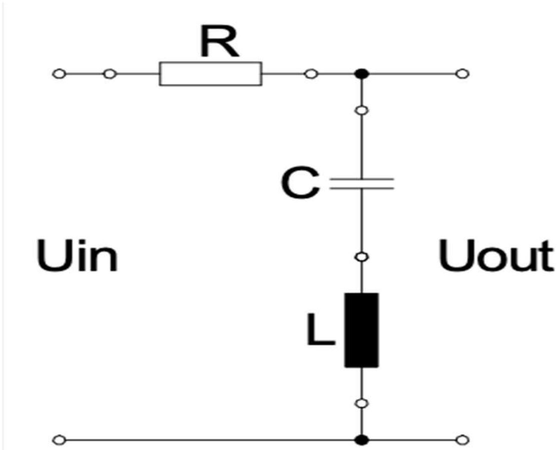
$$C_1 = 100n\,\mathrm{F} \text{ and } R_1 = 1K\Omega$$

The frequencies produced by the signal generator were varied from 50Hz to a 100kHz in 1,2,5, …. steps and the corresponding input and output voltages with their phase shifts were recorded using the oscilloscope. The results obtained are tabulated below:

f (Hz)	U_{in}(V)	U_{out}(V)	Ø (˚)
50	10.10	0.316	88.2
100	10.10	0.62	86.4
200	10.10	1.23	83.5
500	10.10	2.94	72.7
1000	10.10	5.20	58.3
2000	9.84	7.52	38.3
5000	9.68	9.12	17.6
10000	9.68	9.44	9.01
20000	9.68	9.52	3.74
50000	9.68	9.60	2.88
100000	9.68	9.60	1.44

Part 2: Notch

For this part of the experiment, a notch filter circuit was constructed with R = 2.7kΩ and C = 2.2nF, L = 10mH. Again, the signal generator was connected to the circuit via the BNC-to-Kleps cable, CH-1 of oscilloscope was connected to the input signal and CH-2 to the output signal using voltage probes as shown below:



For this part, the center frequency and cut-off frequencies were calculated from the R, L and C values. The center frequency was also determined experimentally taking into consideration the fact that the phase shift between the output and the input voltage at center frequency is 0˚. Again, frequencies from the signal generator were varied from 10kHz to 100kHz including the cut-off and center frequencies and the corresponding input and output voltages including the phase shifts were recorded and tabulated as follows using the oscilloscope:

- **Calculated values**
  f_{cf} = 33931.95Hz
  f_{low} = 18676.52Hz
  f_{high} = 61648.36Hz

- **Measured value for center frequency**
  f_{cf} = 33830Hz when ø = 0.0˚

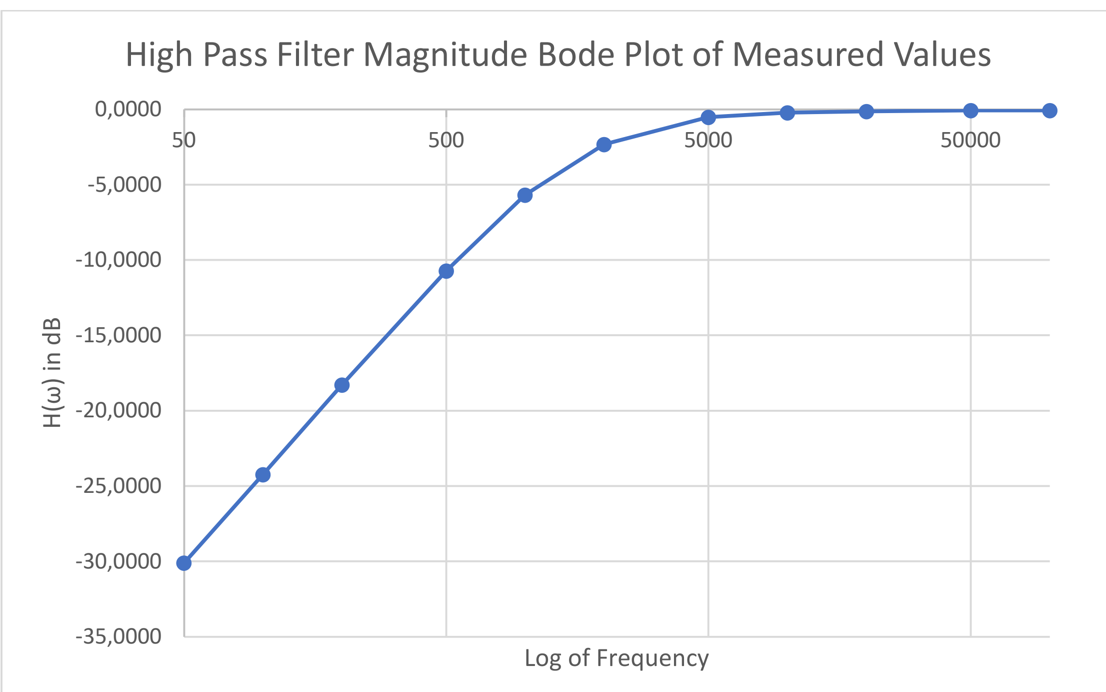
- **Tabulated results**

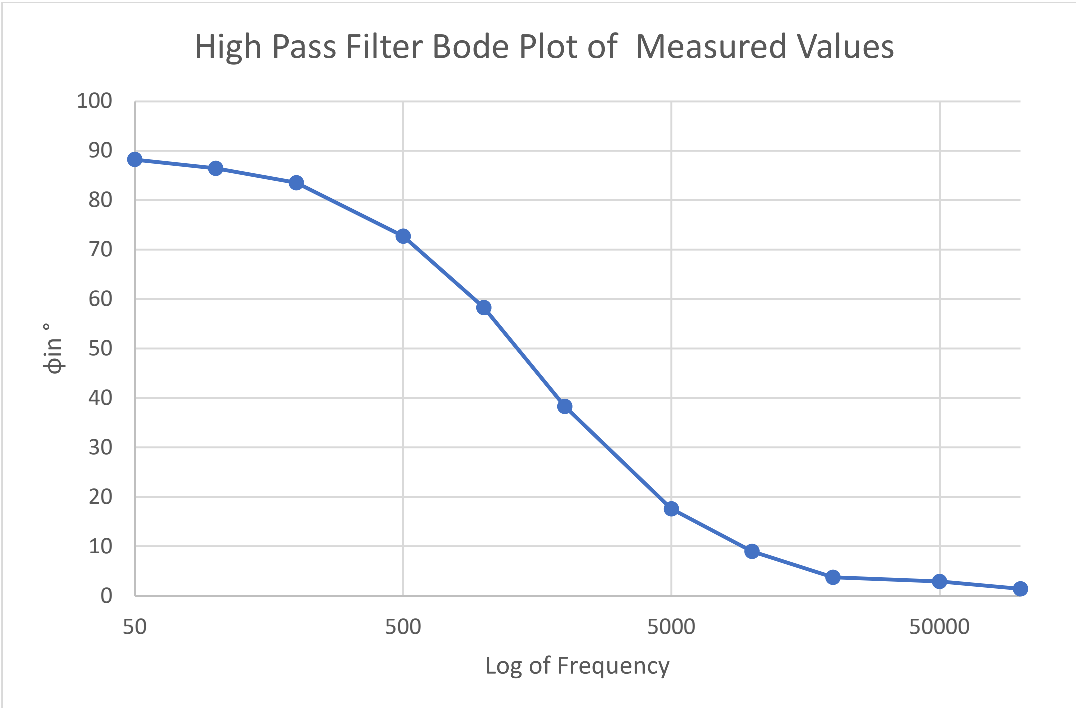
f (Hz)	V_{in} (V)	V_{out}(V)	Ø (˚)
10000	10.00	9.28	-21.9
12500	10.00	8.80	-27.4
17500	10.00	7.44	-41.3
19390	10.00	6.76	-45.2
22000	10.00	5.76	-53.2
24000	10.00	4.84	-58.4
26000	10.00	3.92	-63.9
28000	10.00	2.92	-68.1
30000	9.92	1.90	-72.5
33830	9.92	0.192	-0.00
35000	9.92	0.584	69.0
40000	10.00	2.66	71.4
45000	10.00	4.36	62.8
50000	10.00	5.56	54.7
55000	10.00	6.52	48.7
57800	10.00	6.92	44.9
72000	10.00	8.40	32.6
86000	10.10	9.12	26.6
100000	10.10	9.44	20.1

Part 1: Hi-Pass

-Bode magnitude and phase plot for measured values (magnitude of transfer function was calculated in a decibel scale):

Table Showing Results of Calculations					
Frequency (Hz)	U_{in} (V)	U_{out} (V)	Ø (˚)	H(ω) (Vout/Vin)	20log(H(ω)) (dB)
50	10.10	0.316	88.2	0.03129	-30.0927
100	10.10	0.62	86.4	0.06139	-24.2386
200	10.10	1.23	83.5	0.12178	-18.2883
500	10.10	2.94	72.7	0.29109	-10.7195
1000	10.00	5.20	58.3	0.52000	-5.6799
2000	9.84	7.52	38.3	0.76423	-2.3355
5000	9.68	9.12	17.6	0.94215	-0.5176
10000	9.68	9.44	9.01	0.97521	-0.2181
20000	9.68	9.52	3.74	0.98347	-0.1448
50000	9.68	9.60	2.88	0.99174	-0.0721
100000	9.68	9.60	1.44	0.99174	-0.0721





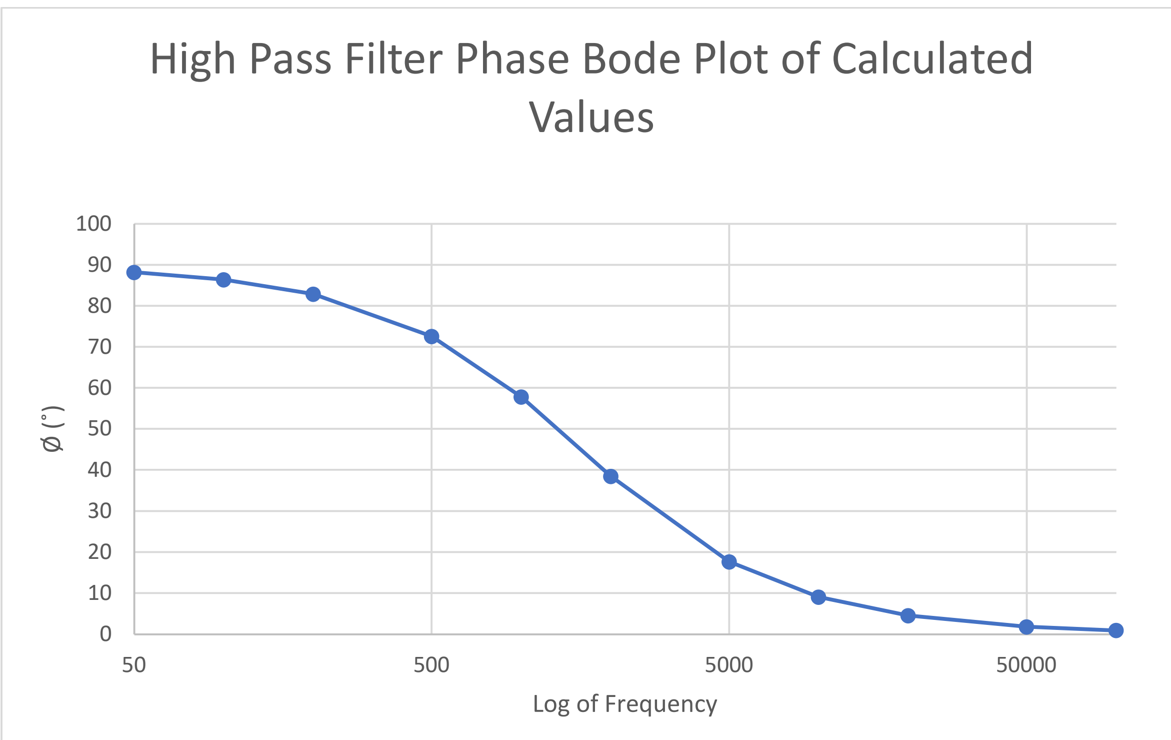
-Bode magnitude and phase plot for nominal values (using formulae given in the theory section) and comparison of plots

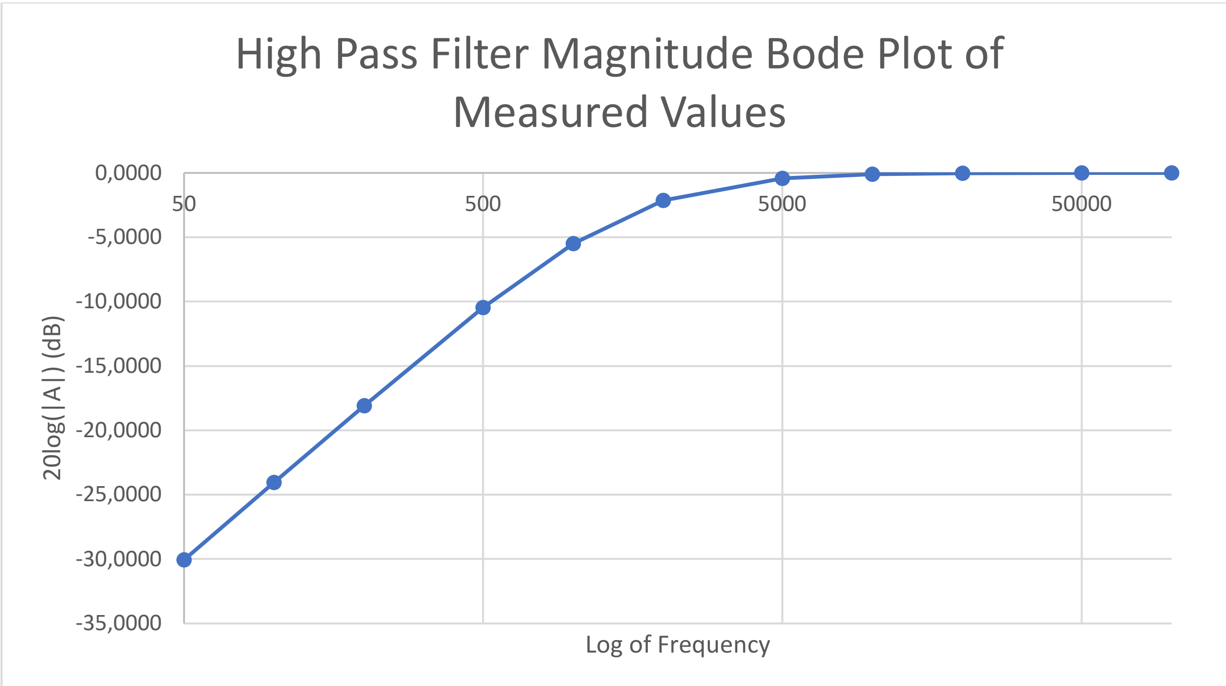
$$|A| = \frac{1}{\sqrt{1 + \frac{1}{(\omega RC)^2}}}$$

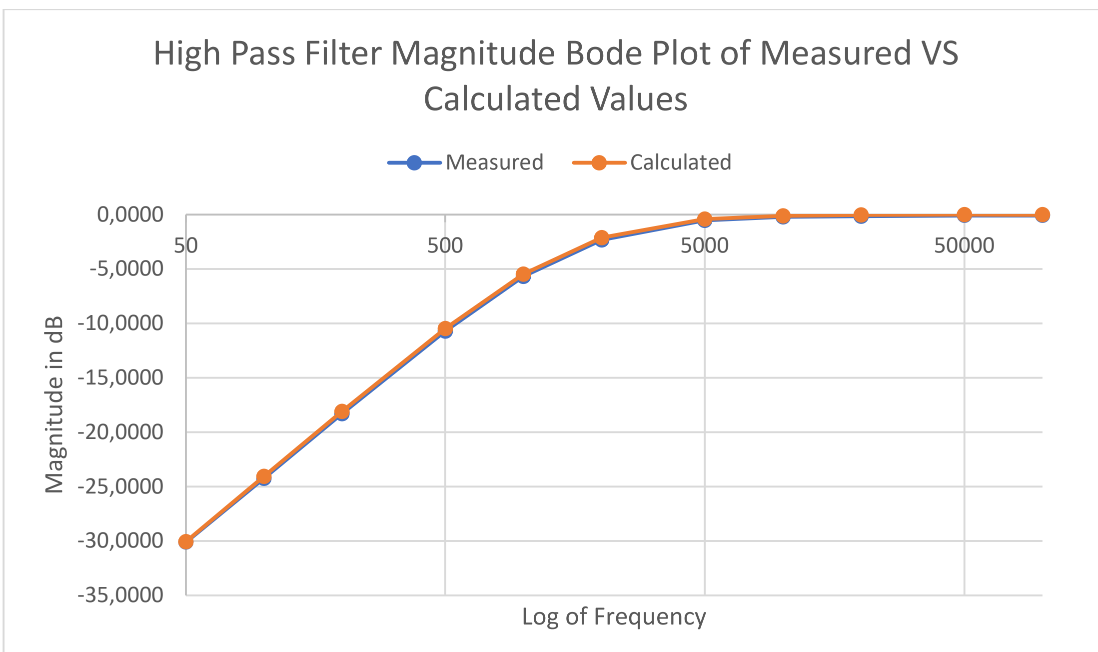
$$\text{Ø} = \arctan\left(\frac{1}{\omega Rc}\right)$$

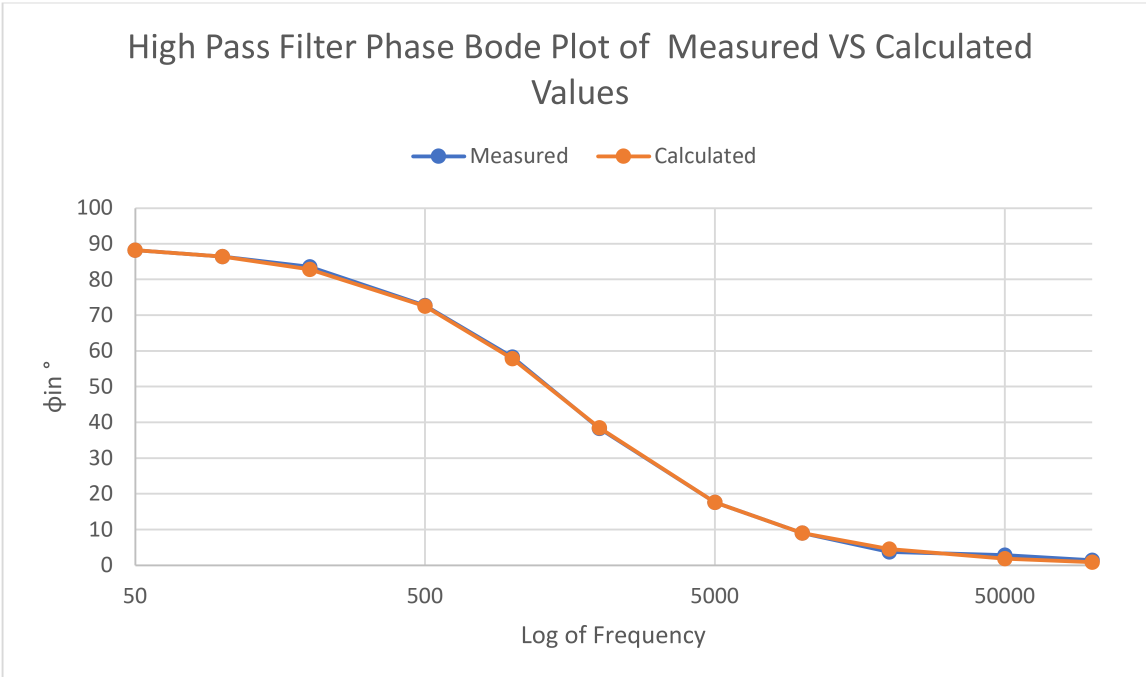
Where,
ω=2πf
R= 1kΩ
C=100nF

Table Showing Calculated Values				
Frequency (Hz)	ω (rads-1)	Ø (˚)	|A|	20log(|A|) (dB)
50	314.159	88.2	0.03140	-30.0613
100	628.319	86.4	0.06271	-24.0535
200	1256.637	82.8	0.12468	-18.0838
500	3141.593	72.6	0.29972	-10.4658
1000	6283.185	57.9	0.53202	-5.4815
2000	12566.371	38.5	0.78248	-2.1305
5000	31415.927	17.7	0.95289	-0.4191
10000	62831.853	9.0	0.98757	-0.1086
20000	125663.706	4.5	0.99685	-0.0274
50000	314159.265	1.8	0.99949	-0.0044
100000	628318.531	0.9	0.99987	-0.0011




High Pass Filter Magnitude Bode Plot of Measured Values
0,0000
-5,0000
-10,0000
-15,0000
-20,0000
-25,0000
-30,0000
-35,0000
50
500
5000
50000
20log(|A|) (dB)
Log of Frequency


High Pass Filter Magnitude Bode Plot of Measured VS Calculated Values
Measured
Calculated
0,0000
-5,0000
-10,0000
-15,0000
-20,0000
-25,0000
-30,0000
-35,0000
50
500
5000
50000
Magnitude in dB
Log of Frequency



The two graphs directly above display the Bode plots of magnitude and phase of the high pass filter comparing measured values to theoretical values, respectively. Initially, taking the magnitude plot into consideration, it could be observed that both the theoretical and measured values align almost perfectly with the calculated value being just slightly higher than the measured at all points in the graph. Moving onto the phase plot, plot both the theoretical and measured values coincided almost perfectly with very little variance except at the highest frequencies where the overlap is slightly less. These slight differences between the measured and calculated values were most probably because the latter were calculated using nominal values and not the actual measured impedance values used in the circuit which have a direct impact on the other measured parameters.

-Calculation for the -3dB frequency:

Using theoretical values and formulae

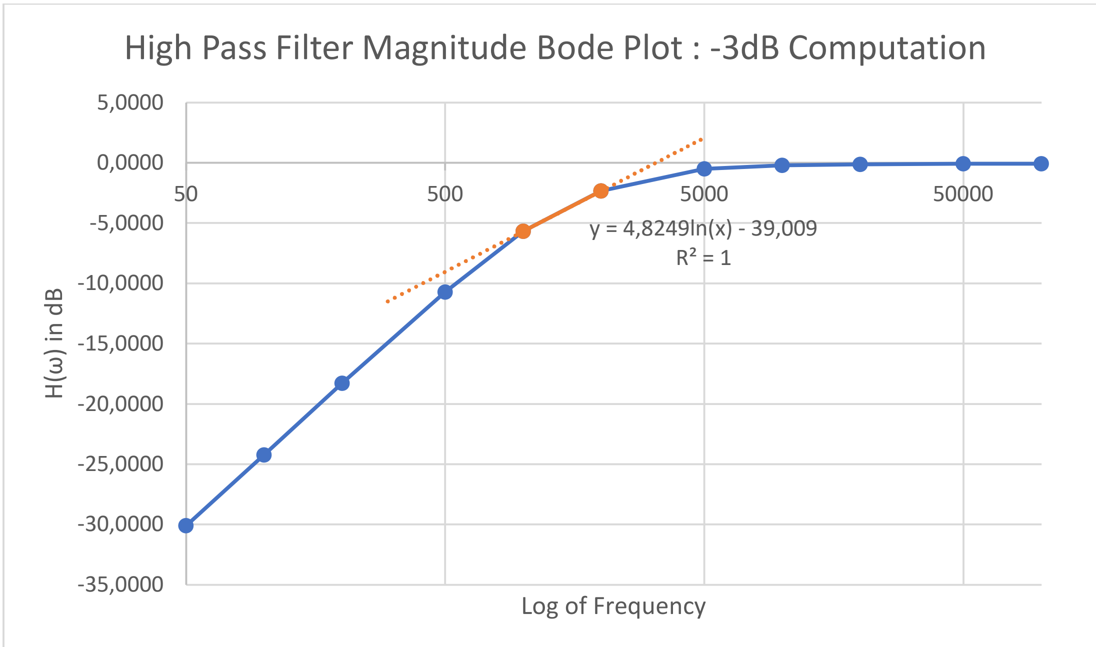
$$f_{-3dB} = \frac{1}{2\pi RC}$$

$$f_{-3dB} = \frac{1}{2\pi(1000)(100 * 10^{-9})}$$

$$f_{-3dB} = 1591.55Hz$$

Where,
R= 1kΩ
C=100nF

Approximating from measured values



The approximate straight-line plot around the region of -3dB gives the equation of the line:

$$y = 4.8249\ln(x) - 39.0090$$

Where, y=-3 & x= f_{-3dB}

$$\ln f_{-3dB} = \frac{-3 + 39.0090}{4.8249} = 7.46315$$

$$f_{-3dB} = e^{7.46315} = 1\,742.65\ Hz$$

Values obtained for frequency at -3dB (Hz)	
Calculated	1591.55
Estimation from graph	1742.65

When comparing the calculated value and approximated value from the graph, they do not fall very close to each other and this could be explained by several factors. One major reason could have been that nominal values were used to work out the calculated value (no actual impedances were used). Another reason is that the value obtained from the plot was done using a straight-line approximation across a fairly large range (1000-2000 Hz). Hence, there could be some errors associated with this value. A final point to also note would be that the plot of the transfer function itself was not smooth, further affecting the straight-line approximation (the gradient and y-intercept).

-Calculation for the -3dB phase shift:

Using theoretical values and formulae

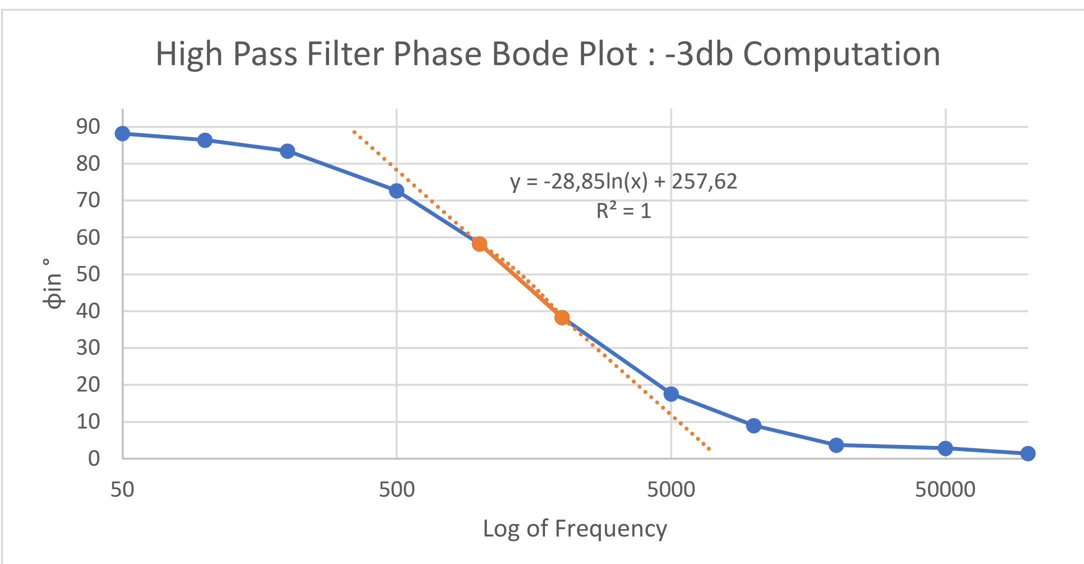
$$\omega_{-3dB} = 2\pi f_{-3dB} = 10000\ rad/s$$

$$\text{Ø}_{-3dB} = \arctan\left(\frac{1}{\omega_{-3dB}RC}\right)$$

$$\text{Ø}_{-3dB} = 45°$$

Where,
R= 1kΩ
C=100nF
F= 1591.55Hz
(from previous calculation)

Approximating from measured values



The approximate straight-line plot around the region of -3dB yields the equation:

$$y = -28.85 \ln(x) + 257.62$$

Where, y=ϕ-3dB & x= f-3dB=1591.55Hz

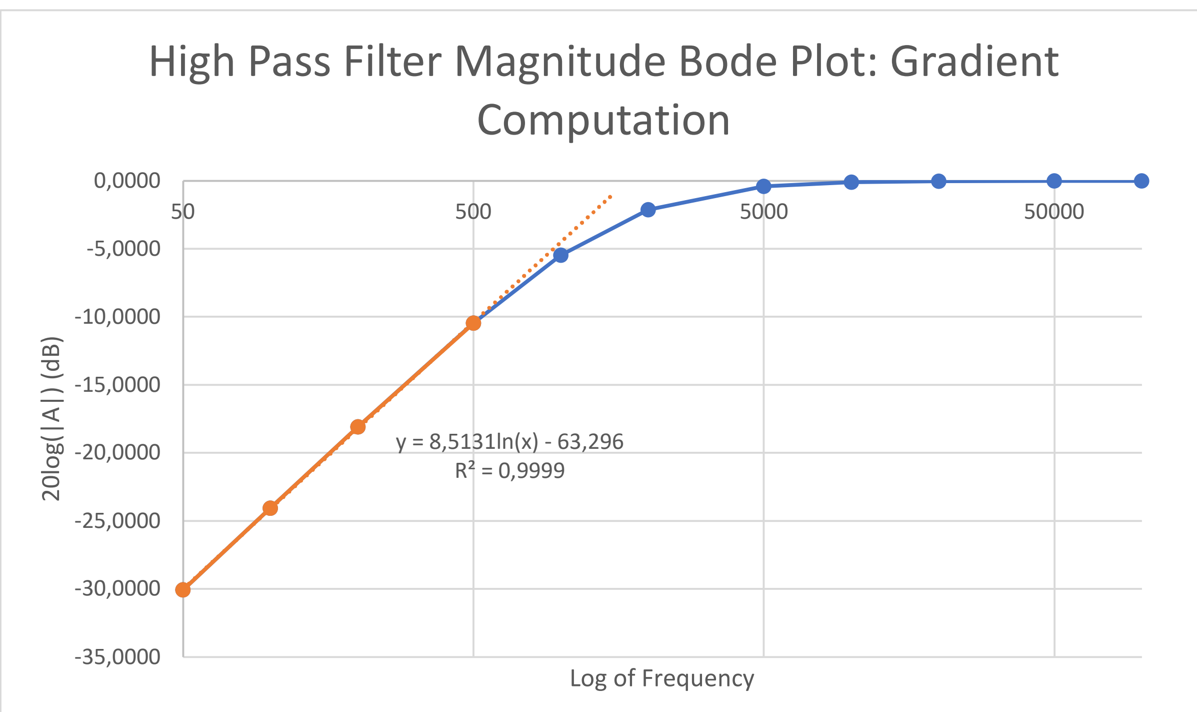
$$\text{Ø}_{-3dB} = -28.85 \ln(1591.55) + 257.62$$

$$\text{Ø}_{-3dB} = 44.9°$$

Values obtained for frequency at -3dB (Hz)	
Calculated	45°
Estimation from graph	44.9°

The calculated and measured values fall very close within each other with a difference of only 0.1°. This small difference could have occurred due to approximating using a straight-line function when the graph is not linear, but since the data in the region over in which the graph is being approximated resembles a linear plot, the difference is kept at a minimal level which explains the slight difference in values almost perfectly.

-Gradient of $|A|$ per decade (unit= dB/Decade)



Note: For a log(f) higher than around 4000, the slope is approximately 0db/decade.

Considering the first decade of values displayed, the gradient of $|A|$ per decade was found using the straight-line approximation. The trend line gives the following equation:

$$y = 8.5131\ln(x) - 63.296$$
$$y = 8.5131\left(\frac{\log_{10} x}{\log_{10} e}\right) - 63.296$$
$$y = 19.46\log_{10} x - 63.296$$

Where,

$$\ln(x) = \frac{\log_{10} x}{\log_{10} e}$$

The equation of the plot is 20log(|A|) =m log(*f*) +c, hence, by comparison, the gradient of $|A|$ per decade is **19.46 dB/decade.**

- Limits of the amplitude ratio in dB and the phase of the High Pass when,

Note (for a High Pass Filter)

$$|A| = \frac{1}{\sqrt{1 + \frac{1}{(\omega RC)^2}}}$$

Taking 20log on both sides

$$20\log|A| = -10\log\left(1 + \frac{1}{(\omega RC)^2}\right)$$

Phase Shift

$$\phi = \tan^{-1}\left(\frac{1}{\omega RC}\right)$$

a) $\boldsymbol{f \ll f_{-3dB}}$

Magnitude Limit:

$$\lim_{\omega\to 0}(20\log|A|) = \lim_{\omega\to 0}\left(-10log\left(1 + \frac{1}{(\omega RC)^2}\right)\right)$$

as ω tends to 0, $\frac{1}{(\omega RC)^2} \gg 1$ so,

$$\lim_{\omega \to 0}(20\log|A|) = -10log\left(\frac{1}{(\omega RC)^2}\right) = -20log\left|\frac{1}{\omega RC}\right|$$

$$\boldsymbol{A_{dB} = -20log\left|\frac{1}{\omega RC}\right|}\textbf{dB}$$

Phase Shift Limit:

$$\lim_{\omega \to 0^+}(\text{ø}) = \lim_{\omega \to 0^+}\left(\tan^{-1}\left(\frac{1}{\omega RC}\right)\right) = 90^\circ$$

$$\boldsymbol{\phi = 90^\circ}$$

b) $\boldsymbol{f \gg f_{-3dB}}$

Magnitude Limit:

$$\lim_{\omega \to \infty}(20\log|A|) = \lim_{\omega \to \infty}\left(-10log\left(1 + \frac{1}{(\omega RC)^2}\right)\right)$$

$$\lim_{\omega \to \infty}(20\log|A|) = -10log(1+0) = 0$$

$AdB\boldsymbol{A_{dB}}$**=0dB**

Phase Shift Limit:

$$\lim_{\omega \to \infty^+}(\text{ø}) = \lim_{\omega \to \infty^+}\left(\tan^{-1}\left(\frac{1}{\omega RC}\right)\right) = 0^\circ$$

$\boldsymbol{\phi}$**=0°**

c) $\boldsymbol{f = f_{-3dB}}$

So, $\frac{\omega}{2\pi} = \frac{1}{2\pi RC}$ $\quad\quad \therefore \omega = \frac{1}{RC}$

Magnitude Limit:

$$20\log|A| = -10\log\left(1 + \frac{1}{\left(\frac{1}{RC}RC\right)^2}\right) = 10\log(2) = 3.01$$

$AdB\boldsymbol{A_{dB}}$**3.01dB**

Phase Shift Limit:

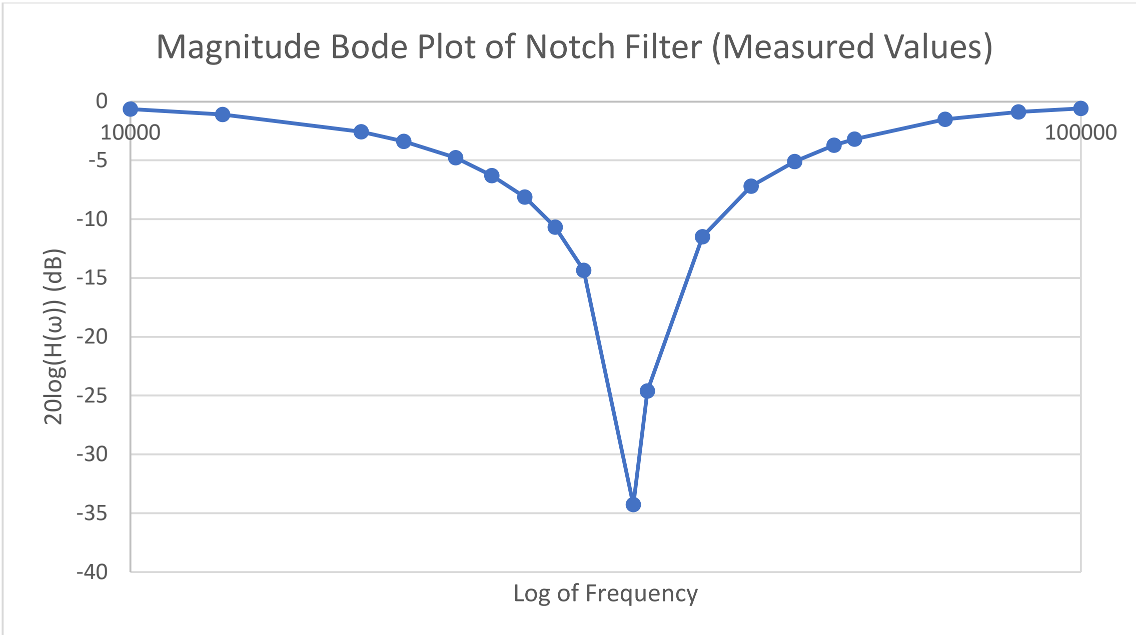
$$\lim_{\omega \to \infty^+}(\emptyset) = \lim_{\omega \to \infty^+}\left(\tan^{-1}\left(\frac{1}{\frac{1}{RC}RC}\right)\right) = 45^\circ$$

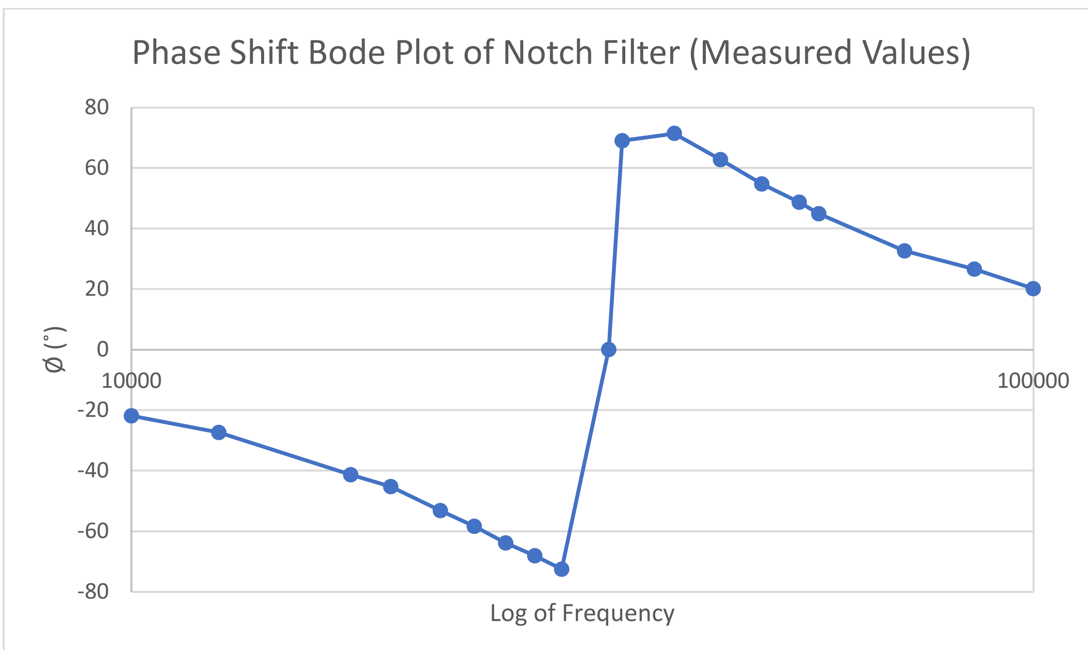
ϕ=45°

Part 2: Notch

-Bode magnitude and phase plot from measured values:

Table Showing Data and Calculation Results for Notch Filter Bode Plots of Measured Values					
Frequency (Hz)	U_{in} (V)	U_{out} (V)	Ø (˚)	H(ω) (Vout/Vin)	20log(H(ω)) (dB)
10000	10	9.28	-21.9	0.928	-0.649
12500	10	8.8	-27.4	0.880	-1.110
17500	10	7.44	-41.3	0.744	-2.569
19390	10	6.76	-45.2	0.676	-3.401
22000	10	5.76	-53.2	0.576	-4.792
24000	10	4.84	-58.4	0.484	-6.303
26000	10	3.92	-63.9	0.392	-8.134
28000	10	2.92	-68.1	0.292	-10.692
30000	9.92	1.9	-72.5	0.192	-14.355
33830	9.92	0.192	0	0.019	-34.264
35000	9.92	0.584	69	0.059	-24.602
40000	10	2.66	71.4	0.266	-11.502
45000	10	4.36	62.8	0.436	-7.210
50000	10	5.56	54.7	0.556	-5.099
55000	10	6.52	48.7	0.652	-3.715
57800	10	6.92	44.9	0.692	-3.198
72000	10	8.4	32.6	0.840	-1.514
86000	10.1	9.12	26.6	0.903	-0.887
100000	10.1	9.44	20.1	0.935	-0.587





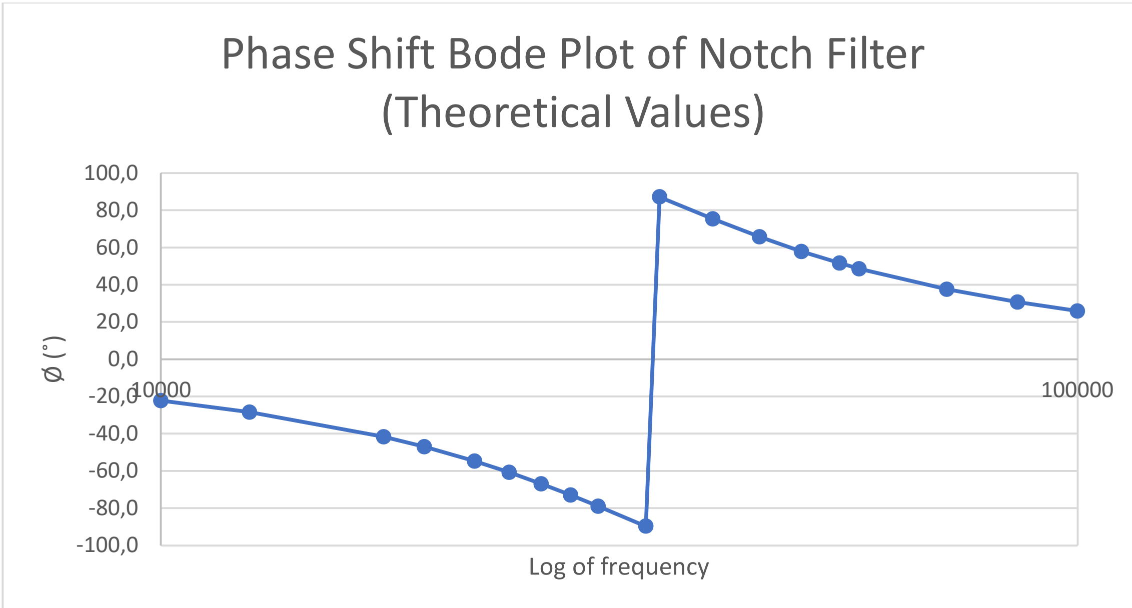
-Theoretical Bode magnitude and phase plot using nominal values of the elements together with the measured values:

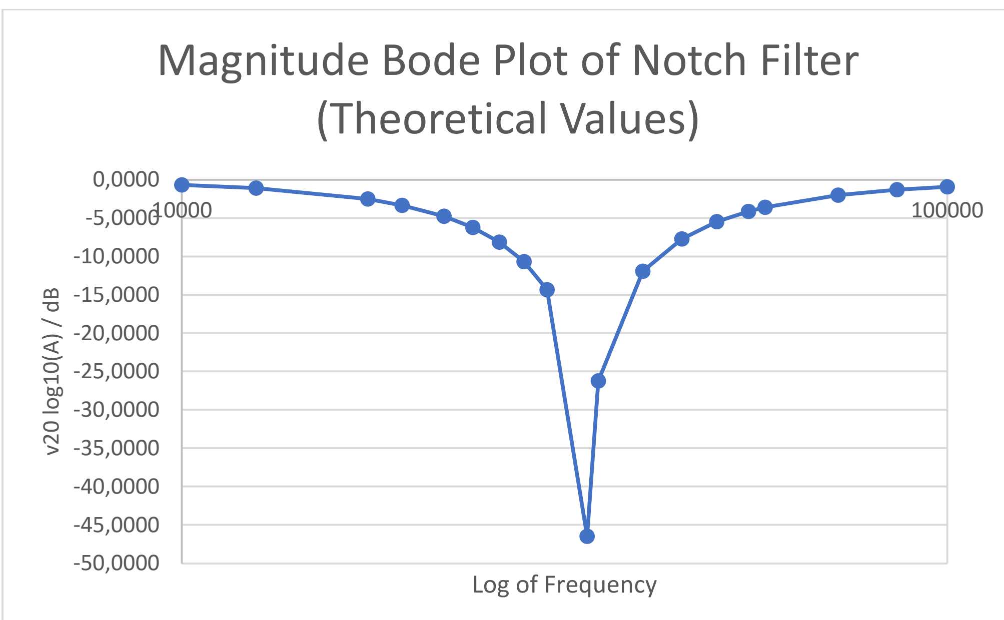
$$|A| = \frac{1}{\sqrt{1 + \frac{1}{\left(\frac{R}{\omega L - \frac{1}{\omega C}}\right)^2}}}$$

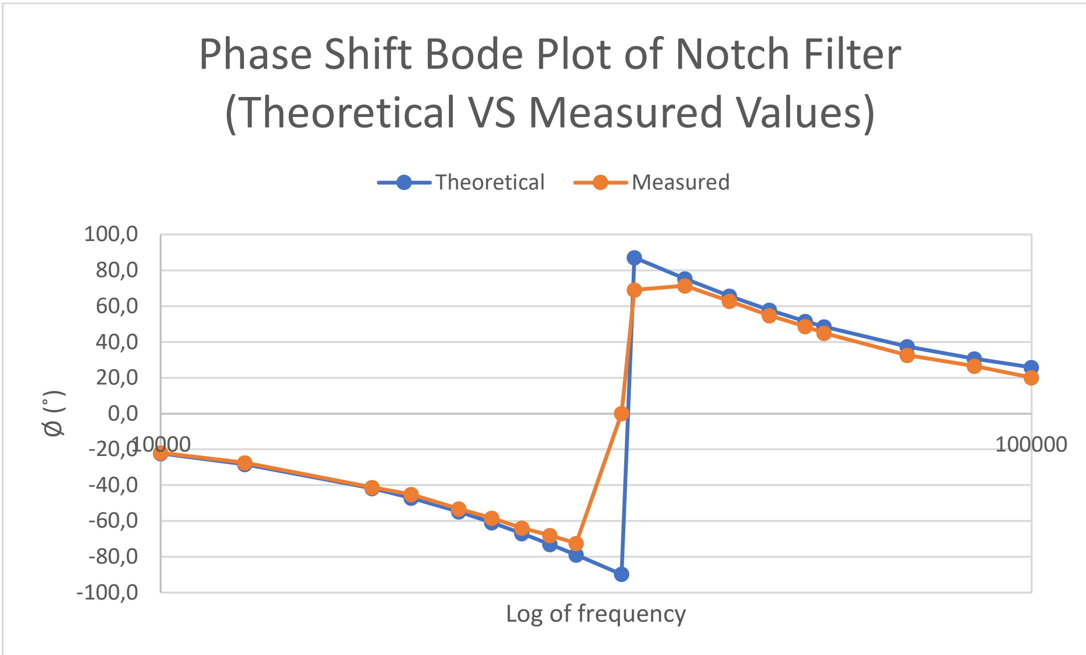
$$\emptyset = \arctan\left(\frac{R}{\omega L - \frac{1}{\omega C}}\right)$$

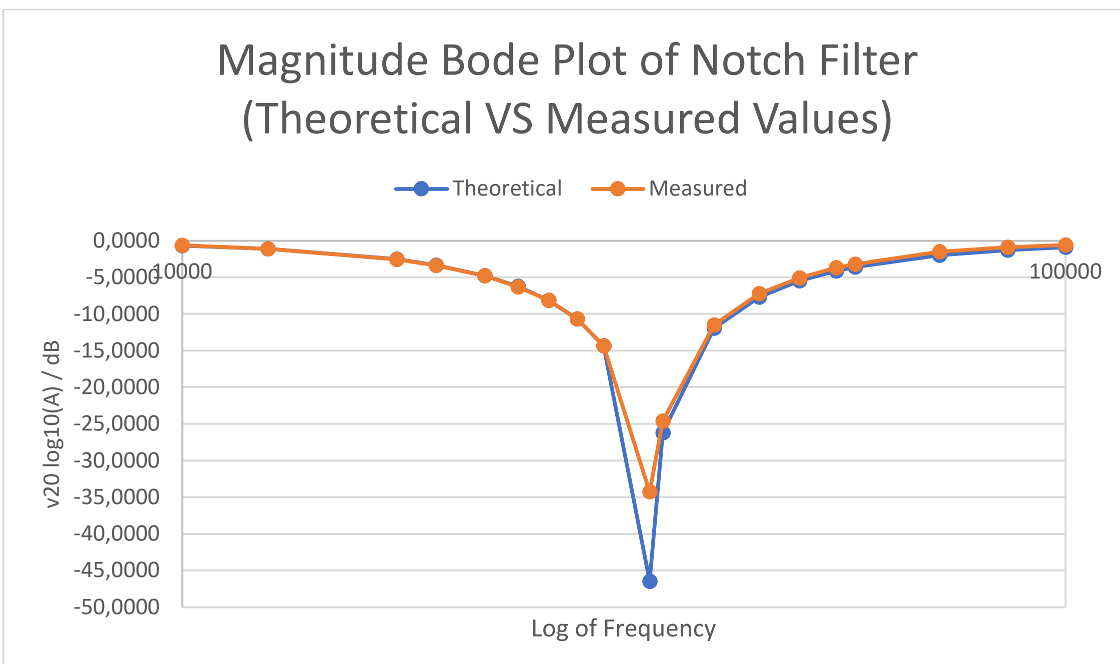
Where,
ω=2πf
R= 2.7kΩ
C=2.2nF
L=10mH

Table Showing Calculations for Notch Filter Bode Plots of Theoretical Values				
f (Hz)	ω (rad/s)	Ø (˚)	|A|	20 log10(A) / dB
10000	62831.8531	-22.2	0.925667	-0.6709
12500	78539.8163	-28.4	0.879987	-1.1105
17500	109955.743	-41.7	0.747066	-2.5328
19390	121830.963	-47.1	0.681251	-3.3339
22000	138230.077	-54.8	0.576712	-4.7808
24000	150796.447	-60.8	0.487208	-6.2457
26000	163362.818	-67.0	0.391516	-8.1450
28000	175929.189	-73.0	0.292022	-10.6917
30000	188495.559	-79.0	0.191389	-14.3617
33830	212560.159	-89.7	0.004752	-46.4625
35000	219911.486	87.2	0.048892	-26.2152
40000	251327.412	75.4	0.252538	-11.9534
45000	282743.339	65.7	0.411714	-7.7081
50000	314159.265	57.9	0.531629	-5.4878
55000	345575.192	51.6	0.621224	-4.1350
57800	363168.111	48.6	0.661265	-3.5925
72000	452389.342	37.5	0.793388	-2.0103
86000	540353.936	30.6	0.860590	-1.3041
100000	628318.531	25.9	0.899537	-0.9196


Phase Shift Bode Plot of Notch Filter
(Theoretical Values)
100,0
80,0
60,0
40,0
20,0
0,0
-20,0
-40,0
-60,0
-80,0
-100,0
Ø (˚)
10000
100000
Log of frequency


Magnitude Bode Plot of Notch Filter
(Theoretical Values)
0,0000
-5,0000
-10,0000
-15,0000
-20,0000
-25,0000
-30,0000
-35,0000
-40,0000
-45,0000
-50,0000
v20 log10(A) / dB
10000
100000
Log of Frequency


Phase Shift Bode Plot of Notch Filter
(Theoretical VS Measured Values)
Theoretical
Measured
100,0
80,0
60,0
40,0
20,0
0,0
-20,0
-40,0
-60,0
-80,0
-100,0
10000
100000
Ø (˚)
Log of frequency


Magnitude Bode Plot of Notch Filter
(Theoretical VS Measured Values)
Theoretical
Measured
0,0000
-5,0000
-10,0000
-15,0000
-20,0000
-25,0000
-30,0000
-35,0000
-40,0000
-45,0000
-50,0000
10000
100000
v20 log10(A) / dB
Log of Frequency

-Calculation of center-frequency, cutoff frequencies, and bandwidth from the given components

$$\mathrm{f}_{cf} = \frac{1}{2\pi\sqrt{\mathrm{LC}}} = \frac{1}{2\pi\sqrt{(0.01)(2.2*10^{-9})}} = 33931.94787\ \mathrm{Hz}$$

Center Frequency: 33 931.95Hz

$$\omega_{\mathrm{cut\_low}} = \frac{-\mathrm{RC} \pm \sqrt{(\mathrm{RC})^2 + 4\mathrm{LC}}}{2\mathrm{LC}}$$

$$\omega_{\mathrm{cut_{low}}} = \frac{-(2700)*(2.2*10^{-9}) \pm \sqrt{(2700*2.2*10^{-9})^2 + 4*0.01*2.2*10^{-9}}}{2*0.01*2.2*10^{-9}}$$

$$\mathrm{f}_{\mathrm{cut_{low}}} = \frac{\omega_{\mathrm{cut\_low}}}{2\pi} = +18676.52\mathrm{Hz\ and} - 61{,}648.36\mathrm{Hz}$$

Taking the positive value:

$$\mathbf{f_{cut_{low}} = 18676.52Hz}$$

$$\omega_{\mathrm{cut\_high}} = \frac{\mathrm{RC} \pm \sqrt{(\mathrm{RC})^2 + 4\mathrm{LC}}}{2\mathrm{LC}}$$

$$\omega_{\mathrm{cut\_high}} = \frac{(2700)*(2.2*10^{-9}) \pm \sqrt{(2700*2.2*10^{-9})^2 + 4*0.01*2.2*10^{-9}}}{2*0.01*2.2*10^{-9}}$$

$$\mathrm{f}_{\mathrm{cut\_high}} = \frac{\omega_{\mathrm{cut\_low}}}{2\pi} = -18676.52\mathrm{Hz\ and} + 61{,}648.36\mathrm{Hz}$$

Taking the positive value:

$$\boldsymbol{f_{cut\_high} = 61648.36Hz}$$

$$Bandwidth = \mathrm{f}_{\mathrm{cut\_high}} - \mathrm{f}_{\mathrm{cut_{low}}} = 61648.36 - 18676.52$$

$$\boldsymbol{Bandwidth = 42,971.84\ Hz}$$

-<u>Comparison and discussion for differences in the two graphs</u>

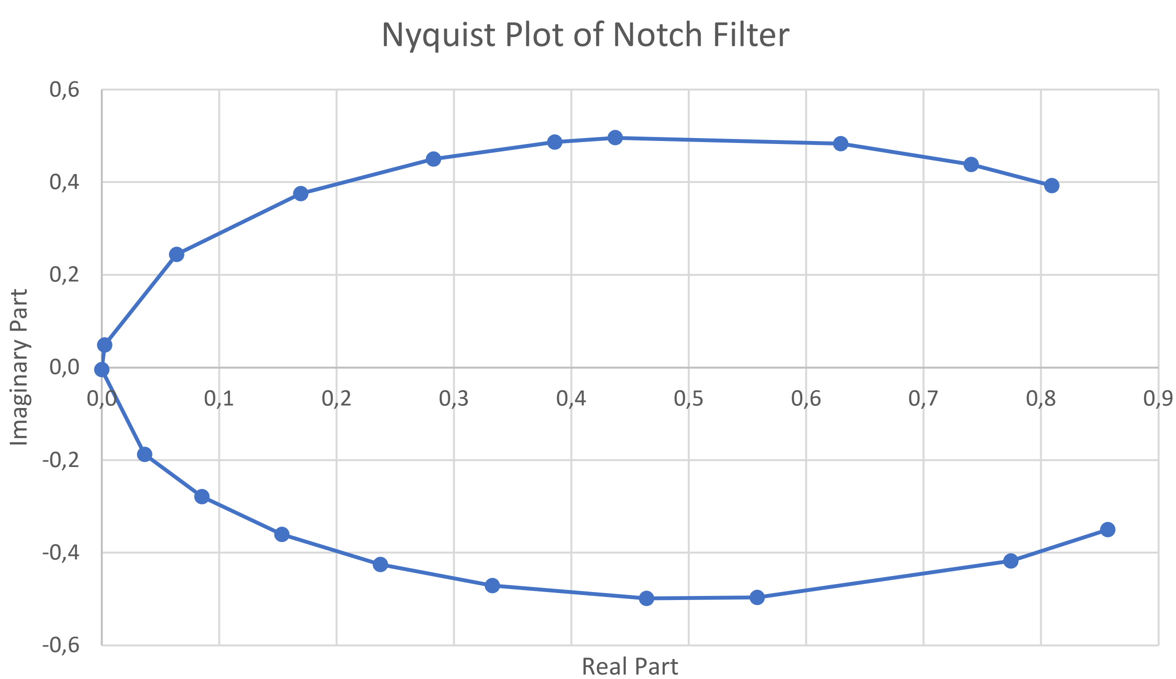
The two graphs given above displays the comparison of the theoretical and measured values of the phase and the magnitude bode plots for the Notch filter in the experiment. Considering both plots, it's clear that for high and low frequencies, the graphs align very closely. However, about the middle values of frequency (around approximately 30 000Hz), the theoretical and measured values vary significantly. There could be several reasons for as to why this occurred. One of the main reasons for this difference could be that since middle values operate at frequencies which correlate to a phase shift close to 90°, at these points, the measurements are very sensitive and fluctuate frequently. Another point is that when the theoretical values for the Bode Plots were calculated, nominal values which do not exactly reflect the actual impedances in the circuit were used and it could be this difference that is reflected in the final plots for measured values. Considering the difference in values of the Bode phase shift plot, a reason for this could be the fact that the internal impedance of the oscilloscope was not taken into account while phase measurements were being made.

-Nyquist plot of $\underline{\mathbf{u}}_{\mathbf{LC}} = \mathbf{f(f)}$ using the measured voltage and phase shift values:

$$\underline{A}(j\omega) = \frac{\underline{V}_{out}}{\underline{V}_{in}} = \frac{j\left(\omega L - \frac{1}{(\omega C)}\right)}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$$

$$H(j\omega) = \frac{1}{1 - j\left(\frac{R}{\omega L - \frac{1}{\omega C}}\right)}, \text{ where } \omega = 2\pi f$$

Data for Nyquist Plot				
f (Hz)	ω (rads–1)	H(jw)	Real	Imaginary
10000	62831.85	0.85686-0.35022j	0.85686	-0.35022
12500	78539.82	0.77438-0.41799j	0.77438	-0.41799
17500	109955.7	0.55811-0.49661j	0.55811	-0.49661
19390	121831	0.4641-0.49871j	0.46410	-0.49871
22000	138230.1	0.3326-0.47114j	0.33260	-0.47114
24000	150796.4	0.23737-0.42547j	0.23737	-0.42547
26000	163362.8	0.15328-0.36026j	0.15328	-0.36026
28000	175929.2	0.08528-0.27929j	0.08528	-0.27929
30000	188495.6	0.03663-0.18785j	0.03663	-0.18785
33830	212560.2	0.00002-0.00475j	0.00002	-0.00475
35000	219911.5	0.00239+0.04883j	0.00239	0.04883
40000	251327.4	0.06378+0.24435j	0.06378	0.24435
45000	282743.3	0.16951+0.3752j	0.16951	0.37520
50000	314159.3	0.28263+0.45028j	0.28263	0.45028
55000	345575.2	0.38592+0.48681j	0.38592	0.48681
57800	363168.1	0.43727+0.49605j	0.43727	0.49605
72000	452389.3	0.62947+0.48295j	0.62947	0.48295
86000	540353.9	0.74062+0.4383j	0.74062	0.43830
100000	628318.5	0.80917+0.39296j	0.80917	0.39296



CONCLUSION

This lab dealt with the concept of a filter using simple passive RLC networks and elaborated on how these components could be combined to create different types of filters (circuits used to filter out signals of a certain frequency/frequency range while attenuating all other frequencies). The focus of this experiment were two types of filters, i.e., the high pass and notch filters.

During the first experiment where the characteristics of a high pass filter (a filter in which signals with high frequencies could pass nearly unchanged while signals with low frequencies were attenuated) were analyzed, Bode plots of both magnitude and phase shift were used to achieve this. Through a comparison between the calculated and measured plots, it was determined that although there was an agreeable relationship between the two, there were also some errors involved.

Further investigation into these errors bring out some distinctly present ones which could have been a cause for the irregularities in the experiment. Firstly, the V_{PP} value seen at the signal generator used throughout all the experiments might not have been the actual value of voltage supplied to the circuit and its own internal impedance could have also had an influence over the final voltage supplied to the circuit, resulting in some errors. Another erroneous source could have been that the reading of frequency (which was once again altered using the signal generator, which in turn gave rise to changes in voltage and phase in the circuit) seen was not accurate, hence giving rise to more fluctuations within the circuit. Now considering the oscilloscope used to measure the phase shifts, the value of the phase fluctuated constantly during the readings even at a constant frequency which forced the use of the run/stop function on the oscilloscope to get an acceptable reading. This procedure could have introduced more errors into the system and combined with the internal impedance of the oscilloscope, which was not considered, could have added to the uncertainty in the values significantly, leading to more errors overall. Another factor could have been the use of the RLC meter to measure the impedance values of the RC components which is known to be error prone since there is a slight difference between nominal values and measured values.

For the next part of the experiment where the features a notch filter (a filter which lets most frequencies pass unaltered but attenuates those in a specific range to very low levels) were analyzed, both Bode plots of magnitude and phase shift and a Nyquist plot were implemented. The Nyquist plot assisted with the disclosure of poles and zeros of the transfer function i.e., H (ω) and the comparison of measured and calculated values was carried out using the Bode plots. Once again, the plots were of a similar shape but some deviations in the values (in a frequency range) hinted the presence of errors.

For this part of the experiment, the fluctuation of the phase reading at the oscilloscope for some values created a problem with measuring the exact point at which the phase is 0˚. An additional error contribution to also consider with respect to the first experiment would be the addition of an inductor here which could have influenced the values.

In conclusion, a thorough understanding of the different characteristics and behavior of some simple filters was obtained. The analysis also revealed some error conditions which must be considered for a fair experiment satisfying both theoretical knowledge and experimental results.

REFERENCES

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2. Charles K. Alexander, Fundamentals of Electric Circuits
3. Tektronix Oscilloscope TBS Series Manual URL: http://www.faculty.jacobs-university.de/upagel/01.0.generaleelab/01.5.0.instrument_manuals/01.5.5.tbs1000b_oscilloscope/index.html

Appendix

Data collected from Operational Amplifier Lab, done on: April 16, 2021

Part 1:

R= 10kΩ			
f (kHz)	Vin (mV)	Vout(mv)	phase
1	492	492	-180
2	492	496	-179
5	492	496	179
10	492	496	178
20	492	500	177
50	496	500	172
100	496	496	165
200	496	472	150
500	500	320	105
1000	496	160	63.3
2000	500	64.8	34.5
5000	520	15.2	-42.8

R=22kΩ			
f (kHz)	Vin (mV)	Vout(mv)	phase
1	496	228	179
2	496	228	179
5	492	228	179
10	492	230	179
20	492	230	177
50	496	230	174
100	496	230	170
200	496	228	159
500	496	210	122
1000	496	124	68.7
2000	500	47.2	21
5000	520	9.6	-40.3

R=1KΩ			
f (kHz)	Vin (mV)	Vout(mv)	phase
1	480	4840	179
2	480	4840	178
5	484	4840	176
10	480	4800	171
20	480	4680	162
50	484	3640	134
100	492	2080	110
200	496	1060	94.1
500	500	420	77.7
1000	500	194	59.4
2000	500	81.6	68.2
5000	520	27.2	-1.44

Part 2: Inverting Integrator

f(Hz)	Vin (mV)	Vout(mv)	phase
200	960	3520	91.3
500	960	1400	92.8
1000	960	704	92
2000	960	356	91
5000	960	148	90.7
10000	952	76	89.4
20000	960	38.4	91.4

Part 3: Differential Amplifier

Ammeter	95.63mA
Ammeter drop	127.84V
U+	4.935v
U-	5.081V
Vout	-1.2114