Computer Organization and Programming

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Lecture 1: Introduction

1 Big Ideas

Definition 1. Big Idea 1: All computers can compute the same kinds of things. We call this **turing-completeness**.

Definition 2. Big Idea 2: **Abstraction** represents the layers that make the electorns work.

Definition 3. Big Idea 3: **Binary numbers** are better than decimal for electronic computing. Saves energy and is easier to distinguish.

Definition 4. Big Idea 4: Computers store **finite-sized representations** of data and information.

2 Data Types

What does the number 5 mean to us? Does it mean the character 5, the integer 5, or the floating point number 5.0? It could also be represented by roman numberals, etc. These are all many different representations of the idea of "5".

Definition 5. A **data representation** is the set of values from which something can take its value. It also includes the meaning of those values

Example. These representations can be integers, floats, instructions, pointers, addresses, etc. All of these types have various flavors depending on the size, etc.

You can usually expect the processor to deal with data representations gracefully.

Definition 6. A **bit** quite literally, is a charge on the capacitor in the DRAM. It is short for "binary digit" and takes on the value of 0 or 1.

Example. How many bits would we need to store 5 different items?

Proof. 3, as $2^2 \leq 5 \leq 2^3$.

Example. How many different numbers can be represented by 7 bits?

Proof. $2^7 = 128$ numbers.

Definition 7. **Unsigned integers** are represented in bits as a base 2 number mathematically.

For binary numbers, addition and subtraction are defined in exactly the same way. However, you have to carry the bits more often.

Lecture 2: Two's Complement

Multiplication in binary can be done like normal multiplication, but it is a lot faster. Note that we don't even need a microprocessor to add numbers. It can be done purely with transistors.

Definition 8. The **not** operator, denoted ¬, toggles the value of a variable.

2.1 Two's Complement

How do we represent negative numbers? We could have signed magnitude, where 1 bit is used on the left. We could also use 1's complement, in which the negative number is just the negated bit value of the positive.

> **Definition 9. Two's Complement** wraps back around after the maxmimum number to the least negative number. You can find the negative value of a number by negating it and adding 1.

> **Example.** Sum of 101111_2 and 001010_2 is 111001_2.

What should we do when we overflow? When we add two positive numbers, and get a negative result, we have probably overflowed. When we add two negative numbers, and get a positive number, we have probably underflowed.

2.1.1 Sign extension

To extend the side of a number, we fill in to the left copies of the leading bit. If the number is negative, this bit will be 1.

2.1.2 Doubling

When we double a number, we just shift the number to the left. When quadrupling, we can shift the number twice to the left. The operator for this in many languages is «.

2.2 Fractional Binary Numbers

Fractional binary numbers can be denoted by a place for the decimal point, and bits for the number itself otherwise.

3 Logical Operators

There are many binary logical operators, including AND, OR, NOT, XOR, NAND, NOR, and the shift operators, etc.

> **Definition 10.** The **AND** operator results in 1 only if both are 1.

> **Definition 11.** The **OR** operator results in 1 if either are 1.

> **Definition 12.** The **XOR** operator results in 1 if exactly 1 input bit is 1.

> **Definition 13.** The **NOT** operator results in 0 if the single input bit is 1, and vice versa.

Other logical operators can be done by adding the not operator to previous operators.

Lecture 3: Hexadecimal and Octal

Octal is just like binary, but in base 8 as well. However, grouping bits in 3 is kind of weird. So instead, we use hexadecimal, which is base 16 (1...F). This way, we can group bits into 4.

We can define constant hex numbers with 0x, oct numbers with 0, and binary numbers (sometimes) with 0b.

3.1 Bit Vectors

To clear a bit vector, we can & it will all 0s. To set a bit vector, we can | it with 1s. To test a certain bit, you can mask the other bits away, and check the singular bit.

To put 1 in any bit position n in a mask, shift left by n.

> **Example.** $1 << 2 = 0100_2$

3.2 ASCII

> **Example.** How do we transform 'P' to 'p'?

> **Proof.** We can do 'P' + 32, 'P' + 0x20, 'P' | 0b00010000, etc.

3.3 Unicode

Now, we give other languages characters as well. Each character get a number ("code point"), and there are now 1,112,064 code points. It is also backwards compatible with ASCII.

3.4 Floating Point

The first bit is the sign. The exponent gets 8 bits. And finally, the mantissa (1.) has 23 bits.

If A is a floating point NaN, then $A \neq B$ is always true, $A \neq A$ is true, and $A < B, A > B, A = B$ are all false.

Lecture 4: Logic Circuits

Diodes, relays, and logic gates can all be made with electricity. These, made from transistors, form the basis of all modern computers.

> **Definition 14.** A **N-type** transistor is normally ope**n** and propagates a strong 0 signal. The diagram does **n**ot have a bubble and has the drain on top and is connected to grou**n**d.

> **Definition 15.** A **P-type** transistor is normally closed and propagates a strong 1 signal. It is normally connected to **p**ower and has the drain on bottom and is connected to **p**ower.

> **Definition 16.** The **gate** is the input to a particular transistor.

Note that a transistor cannot output many values, instead the output value will be indeterminate.

Lecture 5: Combinational Logic

The process for making circuits is this: Once we have a working circuit, we use logic gates to implement it. Once that is finished, we can use boolean algebra to simplify our logic gate.

Here are some non-traditional rules for simplification where $+$ is OR and $\times$ is AND:

1. $A + AB = A$

2. $A + \overline{A}B = A + B$

3. $(A + B)(A + C) = A + BC$

DeMorgan's Law can be combined with double negation to transform circuits into some with less transistors.

To simplify a circuit, OR together all cases which result in 1. Then, simplify it with boolean algebra.