Combinatorial Analysis

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Lecture 1: Syllabus and Review

1 Introduction

This course is basically just a second course in Combinatorics, and will cover a range of topics.

Definition 1. **Matroids** are the structures that capture whether or not the greedy algorithm works. They will be covered later in the course.

Now, for some examples and review:

Definition 2. We say points are in **convex position** if no point is inside a triangle made by 3 other points.

Example. Given a finite set of points on the plane, what is the maxmimum number of points such that no 3 are on a line, and no 4 are in convex position.

Proof. Informally, we know that the "outside" of our points has at most 3 points in the shape of a triangle. We can then place a point in the middle. However, if we try to add another point, then we find that 4 points are in convex position, which is a contradiction. Therefore, 4 points is the maximum size of such a set.

This example is actually part of a more general problem, shown below.

Theorem 1. (ES, 1935) The maxmimum number of points such that no 3 are on a line and no n are in convex position is $\leq 4^n$ and $\geq 2^{n-2}$.

Theorem 2. (Suk, 2017) This number is actually $\leq 2^{n+o(1)}$

Notation. Think of $o(1)$ as standing for a function $f(n)$ such that $\lim\limits_{n\to\infty} f(n) = 0$. In other words, for every $\varepsilon > 0$, there exists n_0 such that $|f(n)| < \varepsilon$ for every $n \geq n_0$.

Example. How many distinct 5-letter words are there on the 26-letter english alphabet?

Proof. There are 26 options for each of the 5 slots, so there are 26^5 words.

Example. What if repetitions aren't allowed?

Proof. Each slot you lose an option, so there are $26 \cdot 25 \cdot 24 \cdot 23 \cdot 22 = \frac{26!}{21!}$ words.

Example. How many ways are there to choose 5 students out of 35 to present?

Proof. There are $\binom{35}{5} = \frac{35!}{5! \cdot 30!}$ ways.

Lecture 2: Review of Proofs

We will now review the types of proofs covered in Math-3012, as well as guidelines for writing them in this class.

Proposition 1. Let N be an n-element set and M be an m-element set. Then, there are m^n mappings (or functions) from N to M.

Proof. (Inductive) "We go by induction on n. For the base case $n = 0$, we consider the empty set $\varnothing$ to be a mapping from the empty set to M. $\square$

Notation. If F is a mapping from N to M, we write $F : N \to M$.