

# Combinatorial Analysis

Raymond Bian

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### 1 Introduction

## Lecture 1: Syllabus and Review

### 1 Introduction

This course is basically just a second course in Combinatorics, and will cover a range of topics.

**Definition 1. Matroids** are the structures that capture whether or not the greedy algorithm works. They will be covered later in the course.

Now, for some examples and review:

**Definition 2.** We say points are in **convex position** if no point is inside a triangle made by 3 other points.

**Example.** Given a finite set of points on the plane, what is the maximum number of points such that no 3 are on a line, and no 4 are in convex position.

**Proof.** Informally, we know that the “outside” of our points has at most 3 points in the shape of a triangle. We can then place a point in the middle. However, if we try to add another point, then we find that 4 points are in convex position, which is a contradiction. Therefore, 4 points is the maximum size of such a set.

This example is actually part of a more general problem, shown below.

**Theorem 1.** (ES, 1935) The maximum number of points such that no 3 are on a line and no  $n$  are in convex position is  $\leq 4^n$  and  $\geq 2^{n-2}$ .

**Theorem 2.** (Suk, 2017) This number is actually  $\leq 2^{n+o(1)}$

**Notation.** Think of  $o(1)$  as standing for a function  $f(n)$  such that  $\lim_{n \rightarrow \infty} f(n) = 0$ . In other words, for every  $\varepsilon > 0$ , there exists  $n_0$  such that  $|f(n)| < \varepsilon$  for every  $n \geq n_0$ .

**Example.** How many distinct 5-letter words are there on the 26-letter english alphabet?

**Proof.** There are 26 options for each of the 5 slots, so there are  $26^5$  words.

**Example.** What if repetitions aren't allowed?

**Proof.** Each slot you lose an option, so there are  $26 \cdot 25 \cdot 24 \cdot 23 \cdot 22 = \frac{26!}{21!}$  words.

**Example.** How many ways are there to choose 5 students out of 35 to present?

**Proof.** There are  $\binom{35}{5} = \frac{35!}{5! \cdot 30!}$  ways.