$u \to v$ any walk contains a $u \to v$ path

Proof:— Let, w be a $u \to v$ walk in G. we proof the theorem by induction on the length of w if w is of length 1 or 2 then it is easy to see that w must be a path. For the induction hypothesis suppose the result is true for all walk of length less than k. And suppose,
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w has length k. So that w is ______.

$$w = u = w_0, w_1, w_2, w_3 \ldots w_k = v$$

where w is not distinct. If the vertices are infact distinct, then w itself is the desired path from u to v. If not, then let j be the smallest integer such that —

$$w_j = w_r \text{ for some } r > j.$$ Let w_1 be the walk as

$$w_1 = u = w_0, w_1, w_2, w_j, w_{r+1} \ldots w_k = v$$

this walk length strictly less ~~that~~ than k. There induction hypothesis implies.

w_1 contain u to v path. This means that w contain u to v path & hence the result.