

This process is terminated after  $n-2$  steps. When only two vertices are left. The tree defines the sequence

\* Theorem: — The number of vertices of all degree in graph is always even.

Proof: — Let us consider a graph  $G=(V,E)$  with vertices  $V = V_1, V_2, \dots, V_n$  and  $E = \{e_1, e_2, \dots, e_n\}$ . Since each edge contributes two degrees. Therefore the sum of the degree of all vertices in  $G$  is twice the number of edges in  $G$ .

$$\text{i.e. } \sum_{i=1}^n \deg(V_i) = 2e = 2|E| \rightarrow \textcircled{1}$$

If we consider the vertices odd & even degrees separately, then the quantity in the left hand side of eqn  $\textcircled{1}$  can be express as the sum of two sums each taken over vertices of even and odd degrees respectively as follows: —

$$\sum_{i=1}^n \deg(V_i) = \sum_{\text{even}} \deg(V_j) + \sum_{\text{odd}} \deg(V_k) \rightarrow \textcircled{2}$$

Since the left ~~and~~ hand side in eqn  $\textcircled{2}$  is even and the first expression on R.H.S is even (being a sum of even nos.) then the second expression must also be even.

$$\text{i.e. } \sum_{\text{odd}} \deg(V_k) = \text{an even number} \rightarrow \textcircled{3}$$

Because the eqn  $\textcircled{3}$  if  $\deg(V_k)$  is odd the total no. of terms in the sum must be even to make the sum and even numbers. Hence the theorem.