Digital Image Processing

COSC 6380/4393

Lecture – 24

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Fourier transform of Derivative of f

$$f(x) = \int_{-\infty}^{\infty} F(u) e^{\sqrt{-1} 2\pi u x} du$$

$$f'(x) = \frac{\partial}{\partial x} \int_{-\infty}^{\infty} F(u) e^{\sqrt{-1} 2\pi u x} du$$

Leibniz rule

$$f'(x) = \int_{-\infty}^{\infty} F(u) \frac{\partial}{\partial x} e^{\sqrt{-1} 2\pi u x} du = \int_{-\infty}^{\infty} F(u) \left(\sqrt{-1} 2\pi u\right) e^{\sqrt{-1} 2\pi u x} du$$

$$f''(x) = \int_{-\infty}^{\infty} F(u) (\sqrt{-1} 2\pi u)^2 e^{\sqrt{-1} 2\pi u x} du$$

Fourier transform of Derivative of f

$$f'(x) = \int_{-\infty}^{\infty} F(u)\left(\sqrt{-1}2\pi u\right)e^{\sqrt{-1}2\pi ux}du$$

$$f'(x) \leftrightarrow \left(\sqrt{-1}2\pi u\right)F(u)$$

$$f''(x) = \int_{-\infty}^{\infty} F(u)(\sqrt{-1}2\pi u)^{2}e^{\sqrt{-1}ux}du$$

$$f''(x) \leftrightarrow (\sqrt{-1}2\pi u)^{2}F(u)$$

The Laplacian in the Frequency Domain

- Laplacian

$$\nabla^2 f(t,z) = \frac{\partial^2 f(t,z)}{\partial t^2} + \frac{\partial^2 f(t,z)}{\partial z^2}.$$

$$\Im\left[\frac{\partial^n f(t,z)}{\partial z^n}\right] = (j2\pi v)^n F(\mu,v). \qquad \Im\left[\frac{\partial^m g(t,z)}{\partial t^m}\right] = (j2\pi\mu)^m G(\mu,v).$$

$$\begin{aligned}
\Im\left[\nabla^2 f(t,z)\right] &= \Im\left[\frac{\partial^2 f(t,z)}{\partial t^2}\right] + \Im\left[\frac{\partial^2 f(t,z)}{\partial z^2}\right] \\
&= (j2\pi\mu)^2 F(\mu,v) + (j2\pi v)^2 F(\mu,v) \\
&= -4\pi^2(\mu^2 + v^2)F(\mu,v).
\end{aligned}$$

$$H(u,v) = -4\pi^2(u^2 + v^2)$$

The Laplacian in the Frequency Domain

$$H(u,v) = -4\pi^2(u^2+v^2)$$

$$\begin{aligned} H(u,v) &= -4\pi^2\left[(u-P/2)^2+(v-Q/2)^2)\right] \\ &= -4\pi^2 D^2(u,v) \end{aligned}$$

The Laplacian image

$$\nabla^2 f(x,y) = \Im^{-1}\left\{H(u,v)F(u,v)\right\}$$

Enhancement is obtained

$$g(x,y) = f(x,y) + c\nabla^2 f(x,y) \qquad c = \text{-}1$$

The Laplacian in the Frequency Domain

The enhanced image

$$
\begin{aligned}
g(x,y) &= \Im^{-1}\left\{F(u,v) - H(u,v)F(u,v)\right\} \\
&= \Im^{-1}\left\{\left[1 - H(u,v)\right]F(u,v)\right\} \\
&= \Im^{-1}\left\{\left[1 + 4\pi^2 D^2(u,v)\right]F(u,v)\right\}
\end{aligned}
$$

The Laplacian in the Frequency Domain

a b

FIGURE 4.58 (a) Original, blurry image. (b) Image enhanced using the Laplacian in the frequency domain. Compare with Fig. 3.38(e).

Unsharp Masking, Highboost Filtering and High-Frequency-Emphasis Fitering

$$g_{mask}(x,y) = f(x,y) - f_{LP}(x,y)$$

$$f_{LP}(x,y) = \Im^{-1}\left[H_{LP}(u,v)F(u,v)\right]$$

Unsharp masking and highboost filtering

$$g(x,y) = f(x,y) + k * g_{mask}(x,y)$$

$$g(x,y) = \Im^{-1}\left\{\left[1 + k*\left[1 - H_{LP}(u,v)\right]\right]F(u,v)\right\}$$
$$= \Im^{-1}\left\{\left[1 + k*H_{HP}(u,v)\right]F(u,v)\right\}$$

Selective Filtering

Non-Selective Filters:
operate over the entire frequency rectangle

Selective Filters
operate over some part, not entire frequency rectangle
- **bandreject or bandpass**: process specific bands
- **notch filters**: process small regions of the frequency rectangle

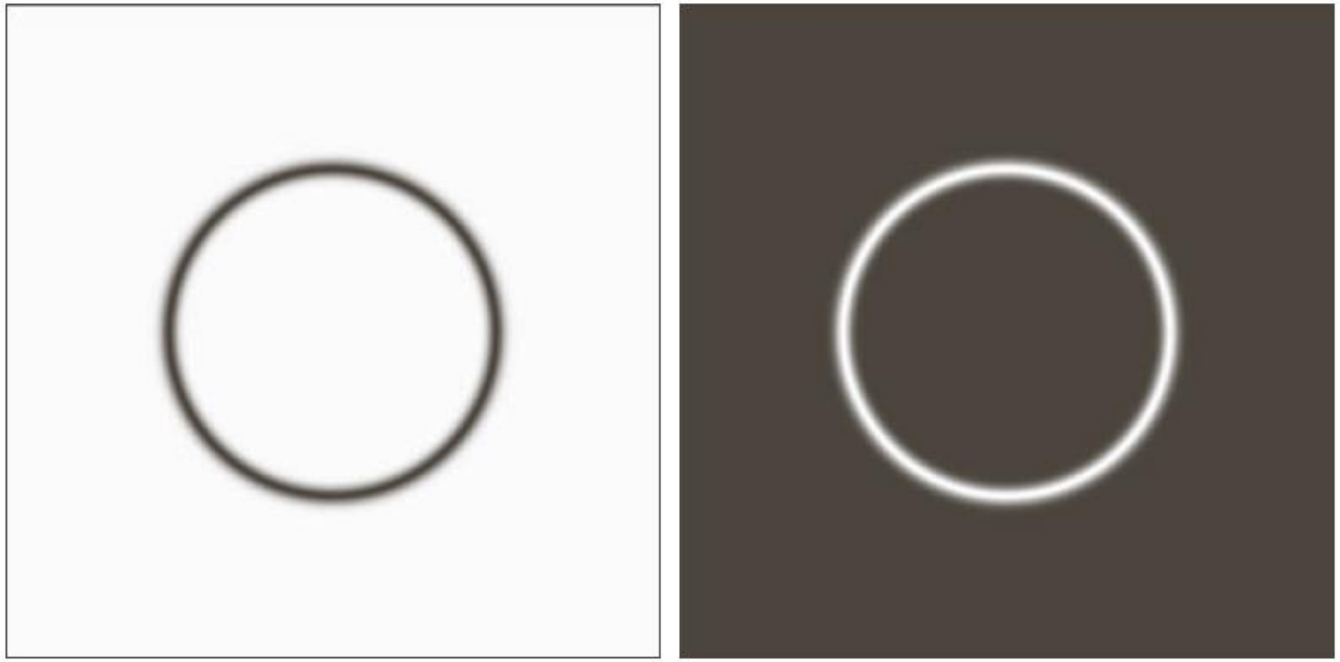
Selective Filtering: Bandreject and Bandpass Filters

TABLE 4.6
Bandreject filters. W is the width of the band, D is the distance $D(u, v)$ from the center of the filter, D_0 is the cutoff frequency, and n is the order of the Butterworth filter. We show D instead of $D(u, v)$ to simplify the notation in the table.

Ideal	Butterworth	Gaussian
$H(u,v) = \begin{cases} 0 & \text{if } D_0 - \frac{W}{2} \le D \le D_0 + \frac{W}{2} \\ 1 & \text{otherwise} \end{cases}$	$H(u,v) = \dfrac{1}{1 + \left[\dfrac{DW}{D^2 - D_0^2}\right]^{2n}}$	$H(u,v) = 1 - e^{-\left[\frac{D^2 - D_0^2}{DW}\right]^2}$

$$H_{BP}(u,v) = 1 - H_{BR}(u,v)$$

Selective Filtering: Bandreject and Bandpass Filters



a b

FIGURE 4.63 (a) Bandreject Gaussian filter. (b) Corresponding bandpass filter. The thin black border in (a) was added for clarity; it is not part of the data.

Image Restoration

- Similar to enhancement, the goal is to improve an image.

Image Restoration

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Enhancement	Restoration
Subjective process	Objective process
Use psychophysical aspects of HVS to manipulate image	Use prior knowledge of degradation to restore image
Manipulate image to please viewer	Recover original image by removing degradation

Image Enhancement

- Enhancement:
  - Point operations
    - Full contrast stretch
    - Histogram equalization
    - Histogram flattening
- Restoration:
  - Removing blur
    - By applying a deblurring function

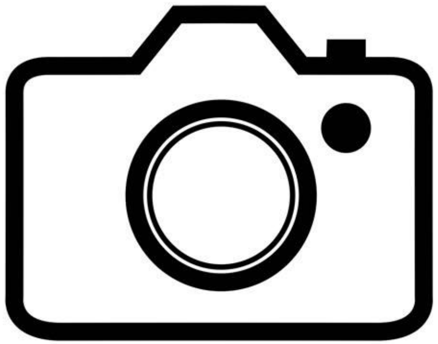
Image Restoration

- Image restoration: recover an image that has been degraded by using a prior knowledge of the degradation phenomenon.

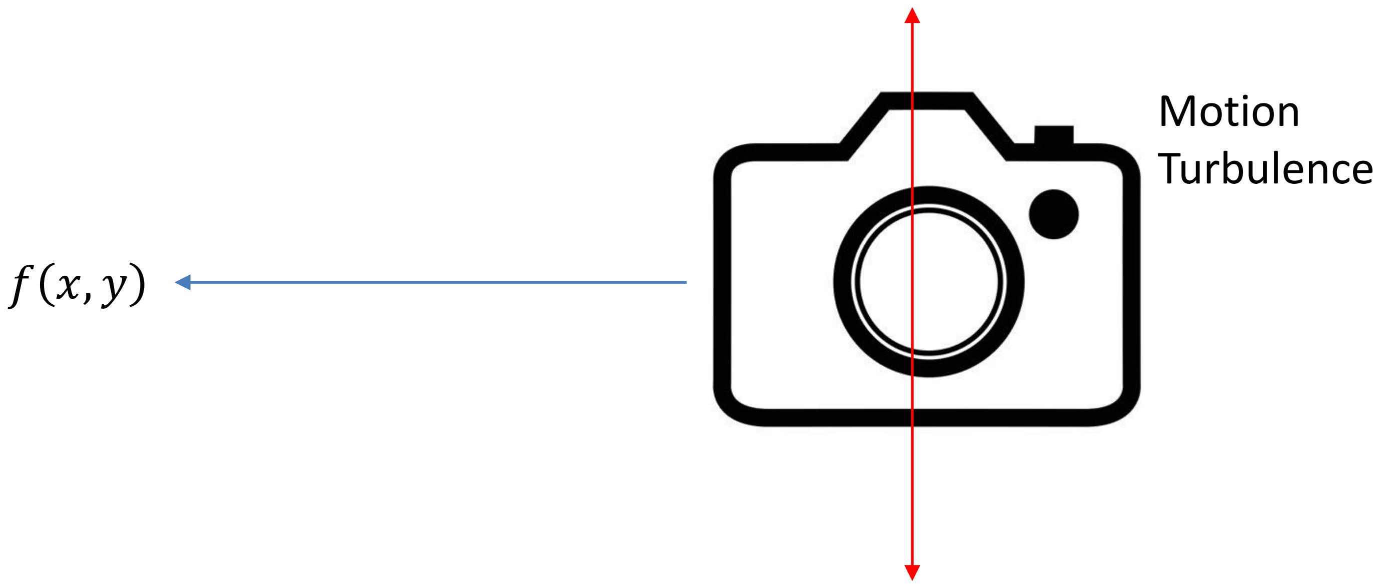
- Model the degradation and applying the inverse process in order to recover the original image.

A Model of Image Degradation/Restoration Process

$f(x, y)$

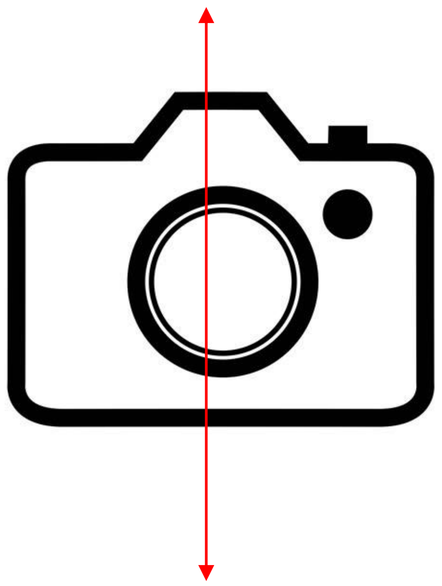


A Model of Image Degradation/Restoration Process



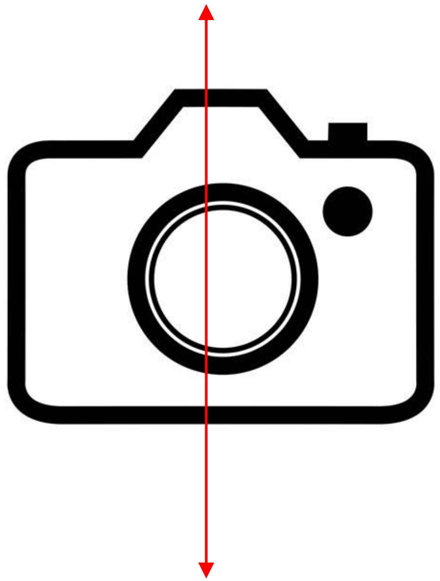
A Model of Image Degradation/Restoration Process

$h(x,y) \otimes f(x,y)$



A Model of Image Degradation/Restoration Process

$h(x,y) \otimes f(x,y)$



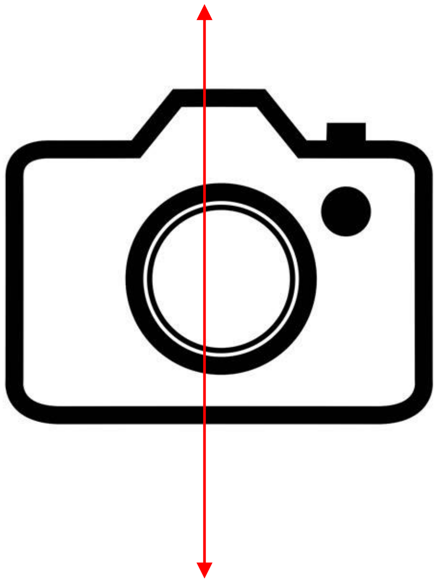
Motion
Turbulence

Noise:
Image acquisition: light, temperature, etc.

Transmission: atmospheric , electronic disturbance

A Model of Image Degradation/Restoration Process

$h(x, y) \otimes f(x, y)$



Motion
Turbulence

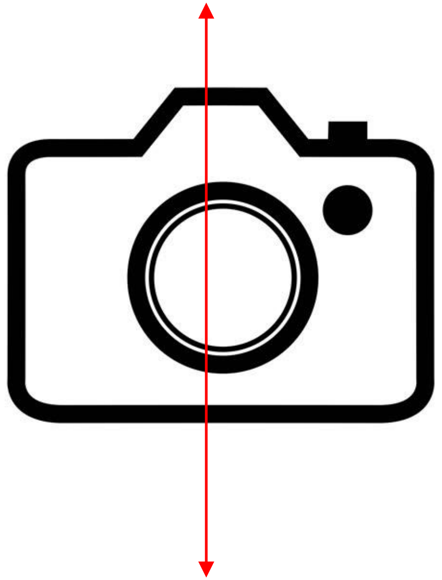
Noise:
Image acquisition: light, temperature, etc.

Transmission: atmospheric , electronic disturbance

Additive noise

$\eta(x, y)$

A Model of Image Degradation/Restoration Process



Motion
Turbulence

$h(x,y) \otimes f(x,y)$

Noise:
Image acquisition: light, temperature, etc.

Additive noise

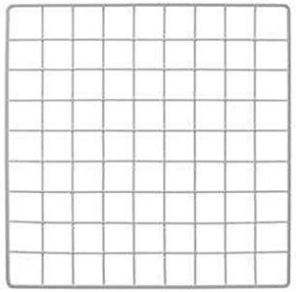
$\eta(x,y)$

Transmission: atmospheric , electronic disturbance

$$\boxed{g(x,y) = h(x,y) \otimes f(x,y) + \eta(x,y)}$$

A Model of Image Degradation/Restoration Process

$h(x,y) \otimes f(x,y)$



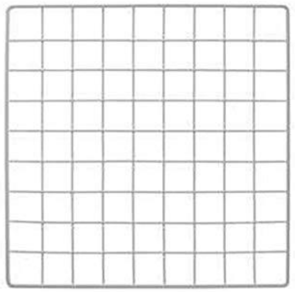
Ideal image

$\eta(x,y)$

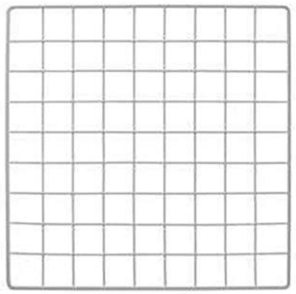
$$\boxed{g(x,y) = h(x,y) \otimes f(x,y) + \eta(x,y)}$$

A Model of Image Degradation/Restoration Process

$h(x,y) \otimes f(x,y)$



$\otimes$



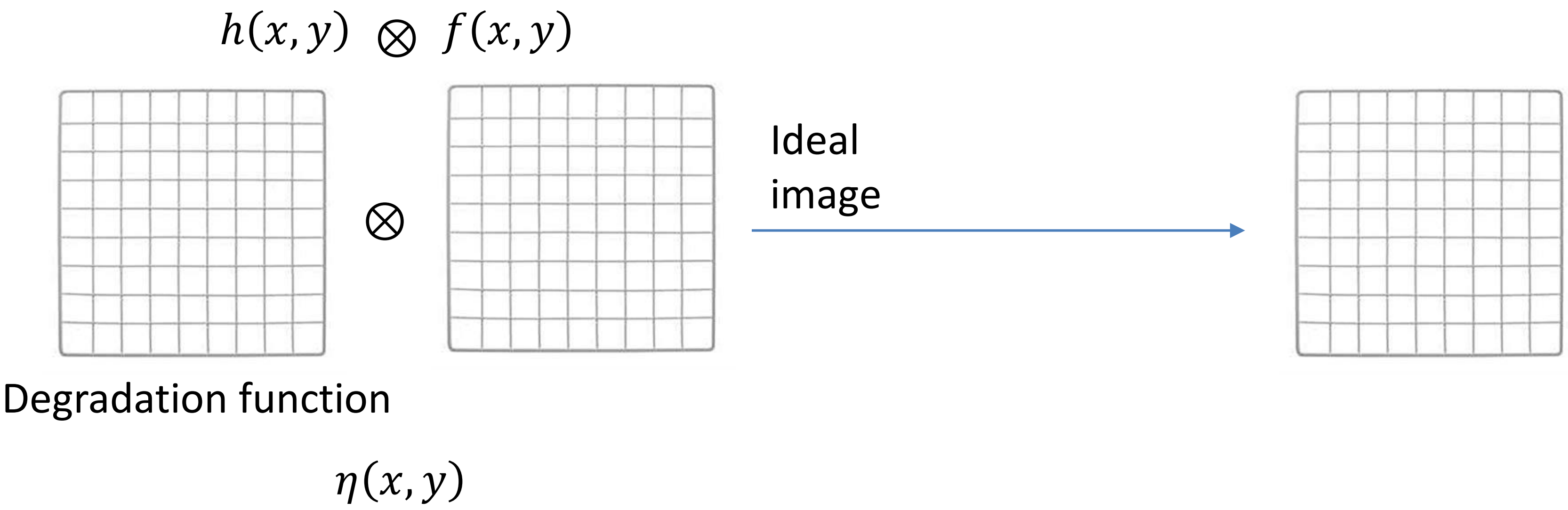
Ideal image

Degradation function

$\eta(x,y)$

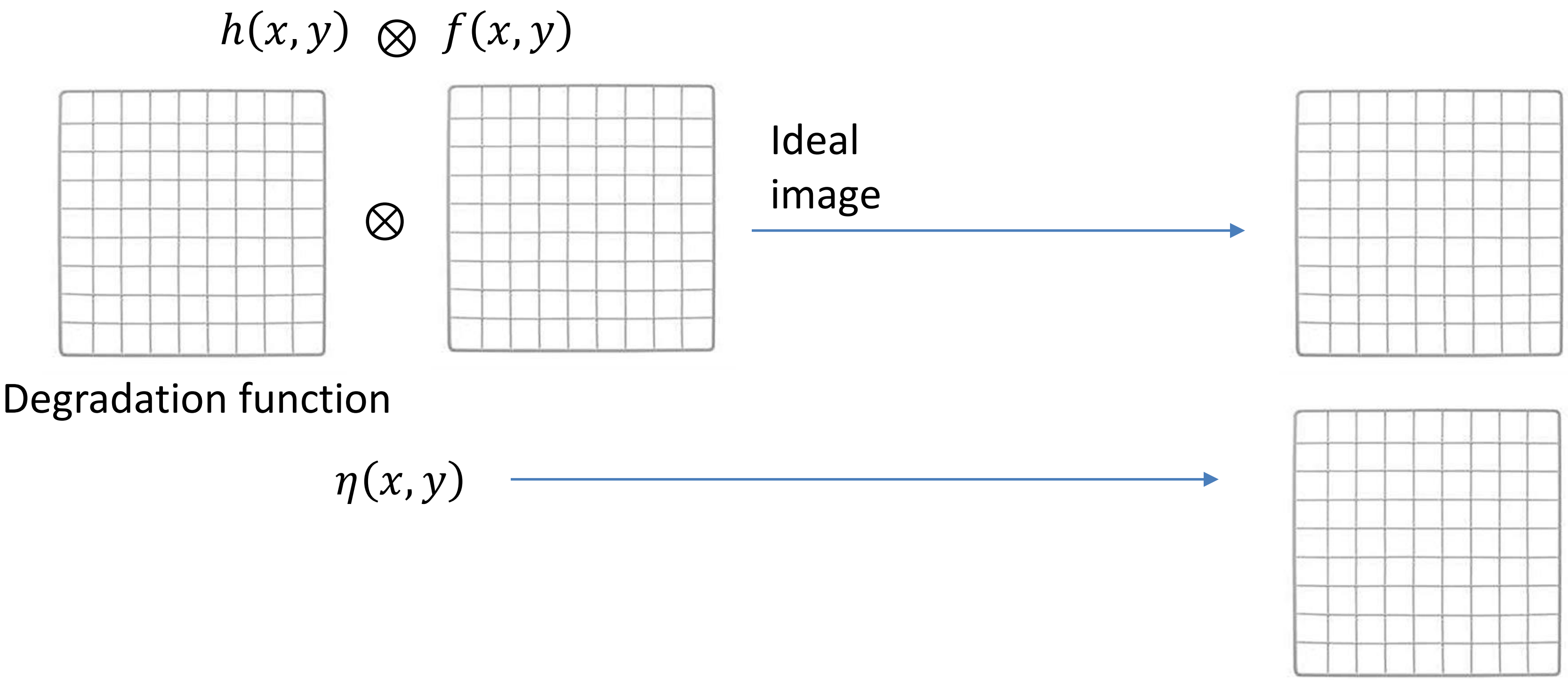
$$\boxed{g(x,y) = h(x,y) \otimes f(x,y) + \eta(x,y)}$$

A Model of Image Degradation/Restoration Process



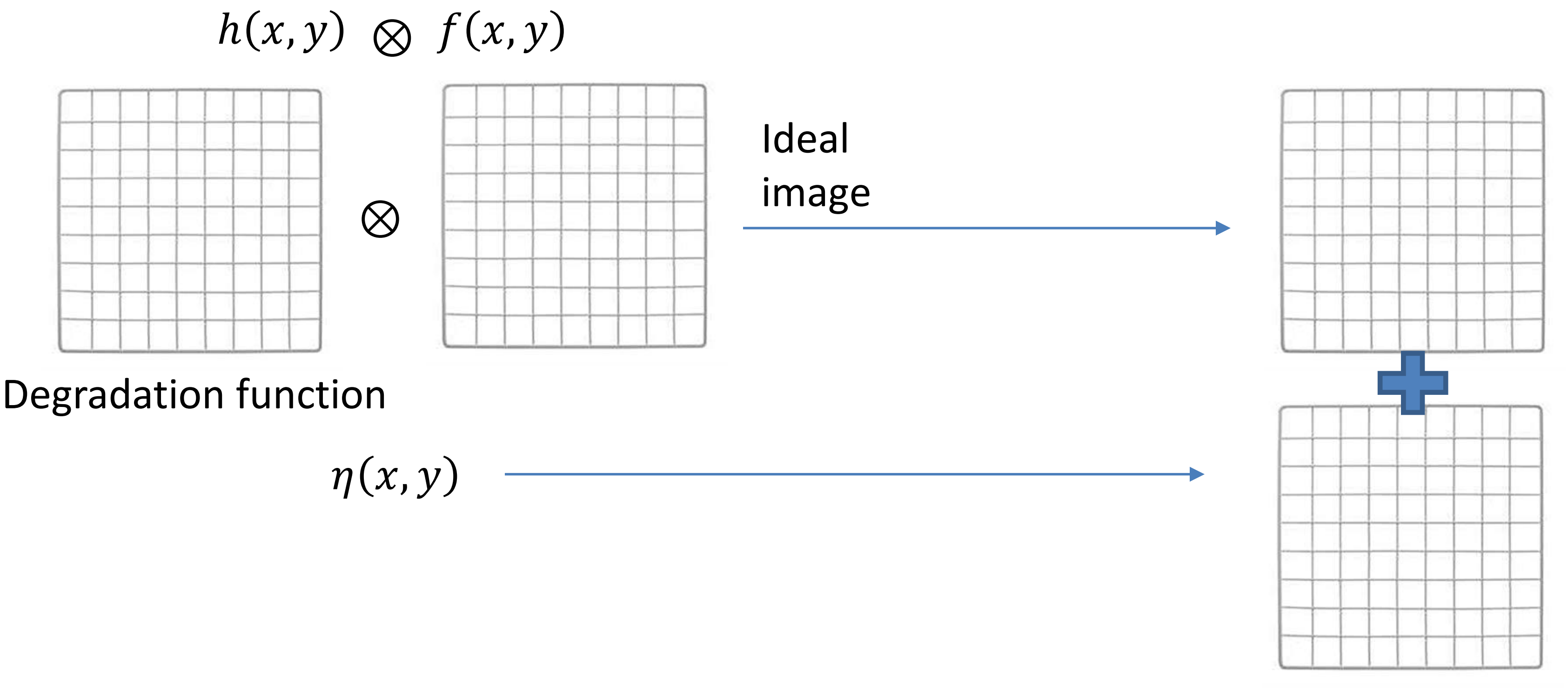
$$\boxed{g(x,y) = h(x,y) \otimes f(x,y) + \eta(x,y)}$$

A Model of Image Degradation/Restoration Process



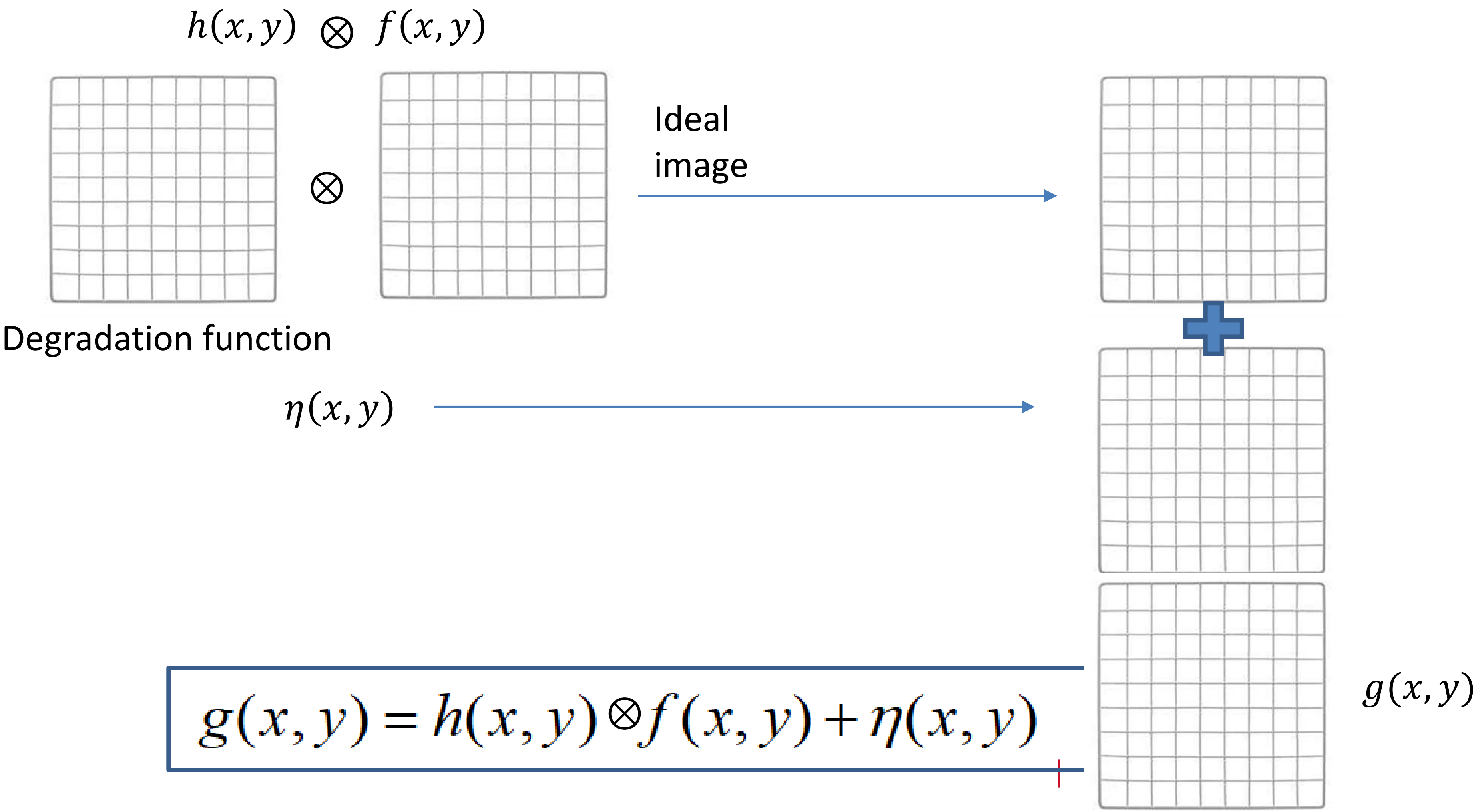
$$\boxed{g(x,y) = h(x,y) \otimes f(x,y) + \eta(x,y)}$$

A Model of Image Degradation/Restoration Process



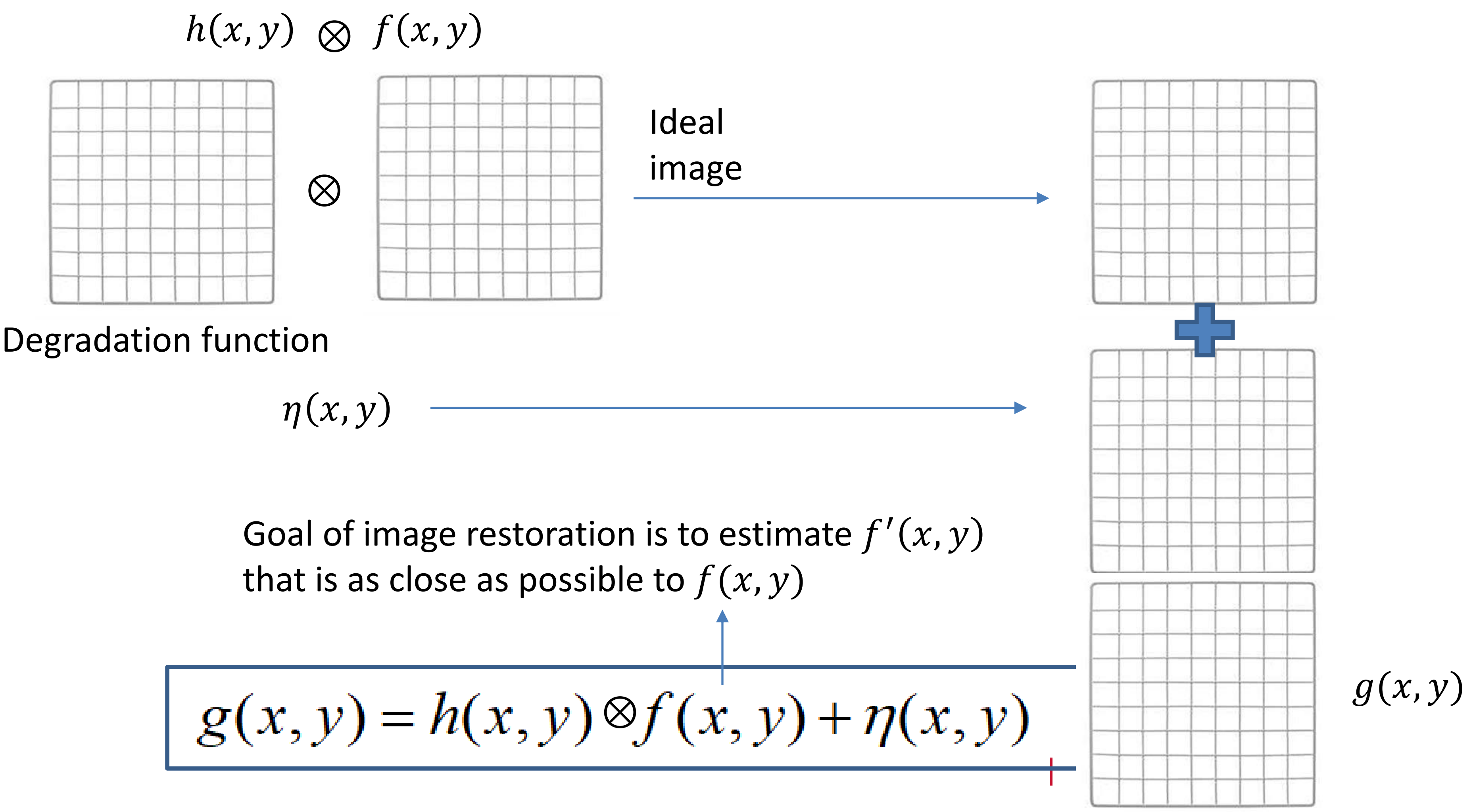
$$\boxed{g(x,y) = h(x,y) \otimes f(x,y) + \eta(x,y)}$$

A Model of Image Degradation/Restoration Process



$$g(x,y) = h(x,y) \otimes f(x,y) + \eta(x,y)$$

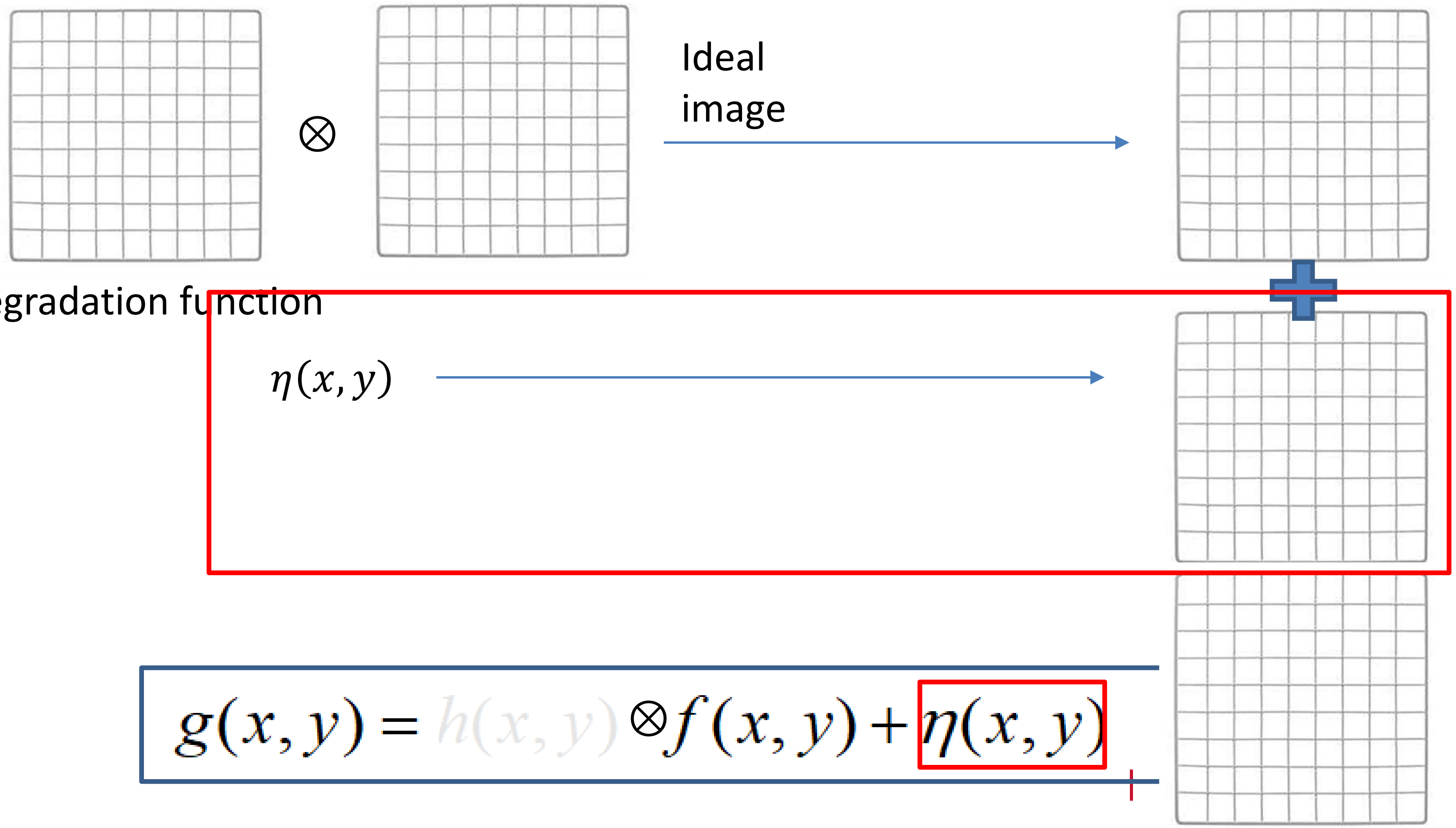
A Model of Image Degradation/Restoration Process



Goal of image restoration is to estimate $f'(x,y)$ that is as close as possible to $f(x,y)$

$$\boxed{g(x,y) = h(x,y) \otimes f(x,y) + \eta(x,y)}$$

A Model of Image Degradation/Restoration Process



$$g(x,y) = h(x,y) \otimes f(x,y) + \boxed{\eta(x,y)}$$

Noise

- Source
- Types of Noise
- Estimation of Noise
- Image Restoration

Noise Sources

- The principal sources of noise in digital images arise during **image acquisition and/or transmission**

✓ Image acquisition (CCD cameras)

e.g., light levels, sensor temperature, etc.

✓ Transmission (interference on channel)

e.g., lightning or other atmospheric disturbance in wireless network

Types of Noise

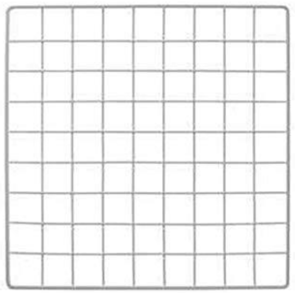
- Spatially independent
- Spatially dependent

Types of Noise

- Spatially independent

The noise at location (x, y), $\eta(x, y)$ is defined by a function H that is not dependent on the (x, y)

$$\eta(x, y) \rightarrow H$$



- Spatially dependent

Types of Noise

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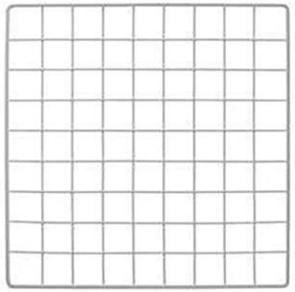
The noise at location (x, y), $\eta(x, y)$ is defined by a function H that is not dependent on the (x, y)

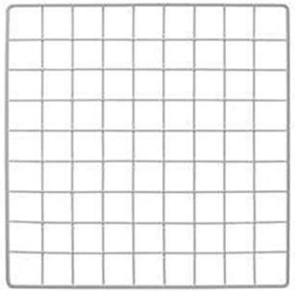
$$\eta(x, y) \rightarrow H$$



- Spatially dependent

The noise at location (x, y), $\eta(x, y)$ is defined by a function H that is dependent on the location (x, y)

$$\eta(x, y) \rightarrow H(x, y)$$



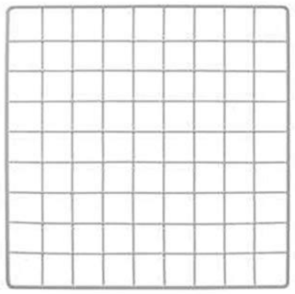
Types of Noise

- Spatially independent

The noise at location (x, y), $\eta(x, y)$ is defined by a function H that is not dependent on the (x, y)

$$\eta(x, y) \rightarrow H$$

Statistical noise

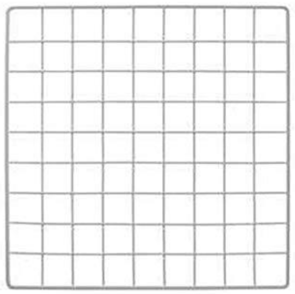


- Spatially dependent

The noise at location (x, y), $\eta(x, y)$ is defined by a function H that is dependent on the location (x, y)

$$\eta(x, y) \rightarrow H(x, y)$$

Periodic noise

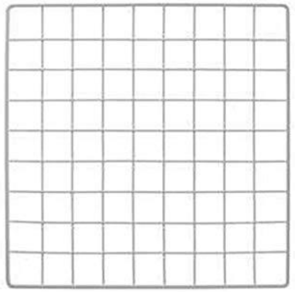


Types of Noise

- Spatially independent

The noise at location (x, y), $\eta(x, y)$ is defined by a function H that is not dependent on the (x, y)

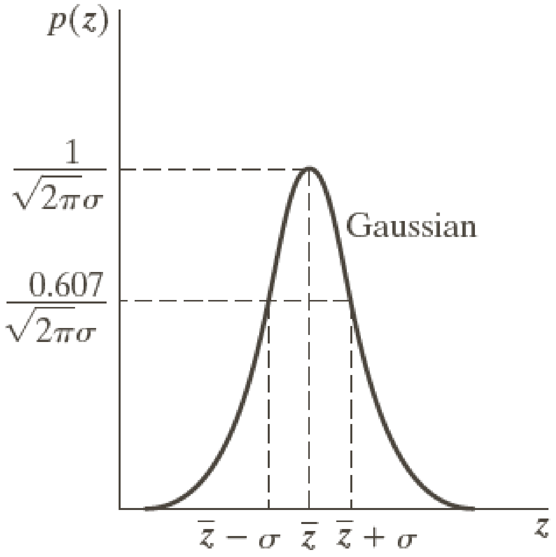
$$\eta(x, y) \rightarrow H$$



- Statistical noise:
  - Defined by the function H (which usually is a probability distribution)
  - Different H arise in various aspects of imaging.
  - H is parametric in nature

Gaussian Noise

- Electronic circuit noise
- Sensor noise due to poor illumination
- Sensor noise due to high temperature



a	b	c
d	e	f

FIGURE 5.2 Some important probability density functions.

Gaussian Noise

The PDF of Gaussian random variable, z, is given by

$$p(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(z-\bar{z})^2/2\sigma^2}$$

where, z represents intensity

$\bar{z}$ is the mean (average) value of z

σ is the standard deviation

Gaussian Noise

The PDF of Gaussian random variable, z, is given by

$$p(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(z-\bar{z})^2/2\sigma^2}$$

- 70% of its values will be in the range

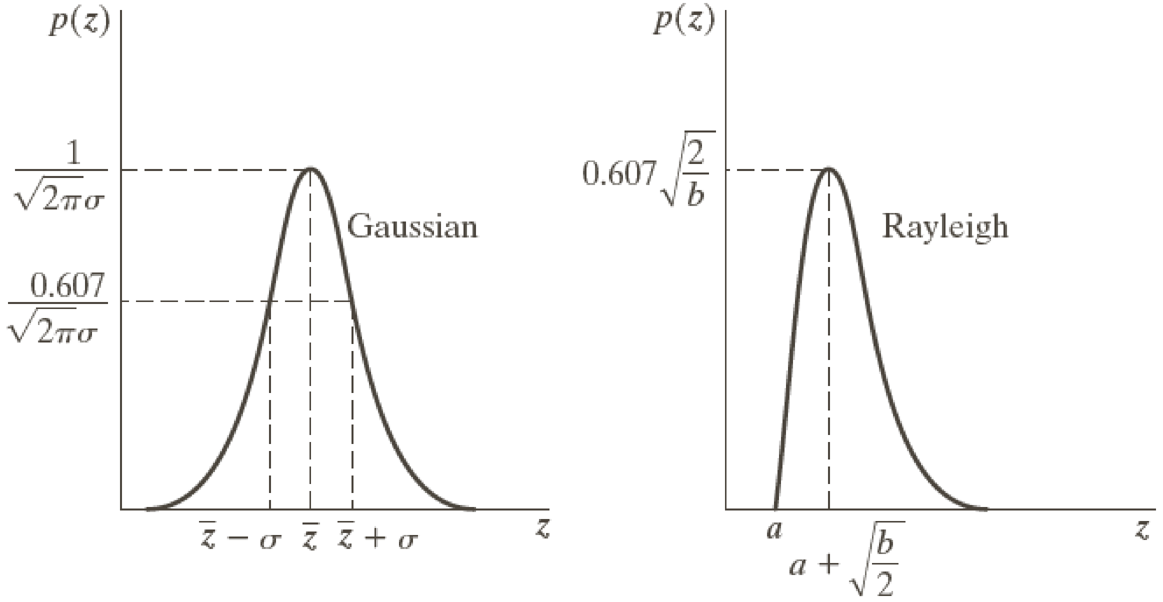
$$[(\mu - \sigma), (\mu + \sigma)]$$

- 95% of its values will be in the range

$$[(\mu - 2\sigma), (\mu + 2\sigma)]$$

Rayleigh Noise

- Range imaging: Image showing the distance to points in a scene from a specific point (Radar).



a	b	c
d	e	f

FIGURE 5.2 Some important probability density functions.

Rayleigh Noise

The PDF of Rayleigh noise is given by

$$p(z) = \begin{cases} \frac{2}{b}(z-a)e^{-(z-a)^2/b} & \text{for } z \geq a \\ 0 & \text{for } z < a \end{cases}$$

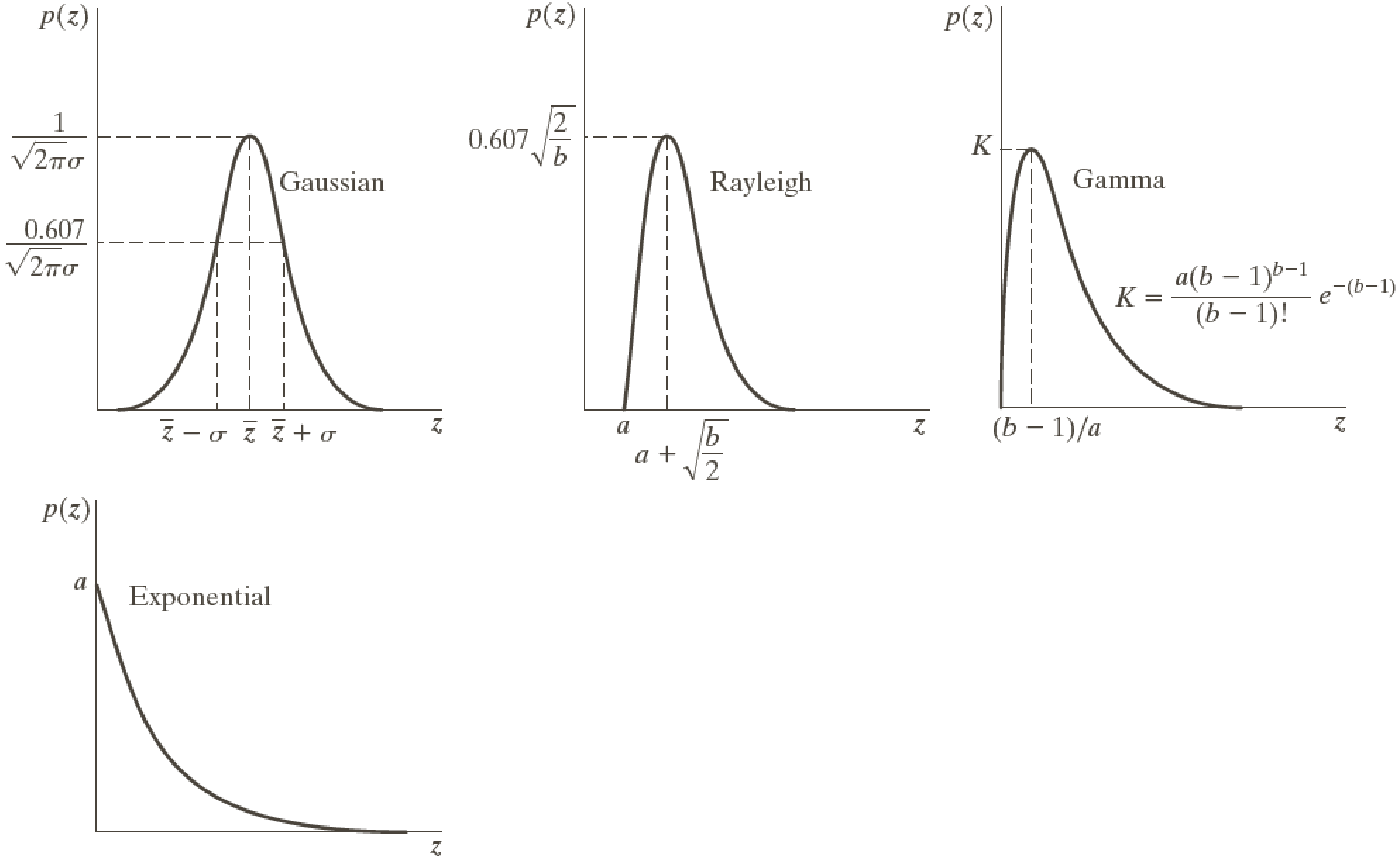
The mean and variance of this density are given by

$$\bar{z} = a + \sqrt{\pi b / 4}$$

$$\sigma^2 = \frac{b(4-\pi)}{4}$$

Erlang (gamma) and Exponential Noise

- Laser imaging: The Laser images are used to get 3D images with the help of laser. The images are captured by sensors which are mounted on laser.



a	b	c
d	e	f

FIGURE 5.2 Some important probability density functions.

Erlang (Gamma) Noise

The PDF of Erlang noise is given by

$$p(z) = \begin{cases} \dfrac{a^b z^{b-1}}{(b-1)!} e^{-az} & \text{for } z \geq 0 \\ 0 & \text{for } z < a \end{cases}$$

The mean and variance of this density are given by

$$\bar{z} = b / a$$

$$\sigma^2 = b / a^2$$

Exponential Noise

The PDF of exponential noise is given by

$$p(z) = \begin{cases} ae^{-az} & \text{for } z \geq 0 \\ 0 & \text{for } z < a \end{cases}$$

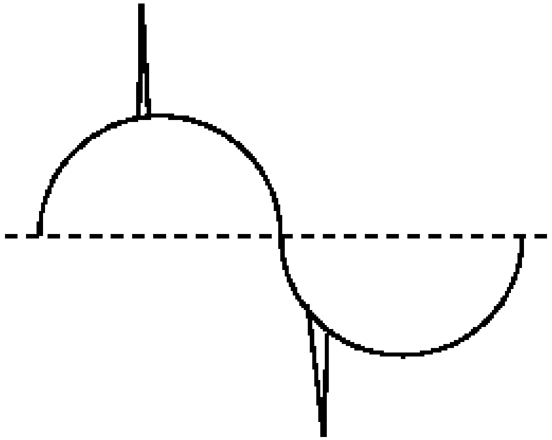
The mean and variance of this density are given by

$$\bar{z} = 1/a$$

$$\sigma^2 = 1/a^2$$

Impulse noise

- Faulty switching results in quick transients



Impulse Noise

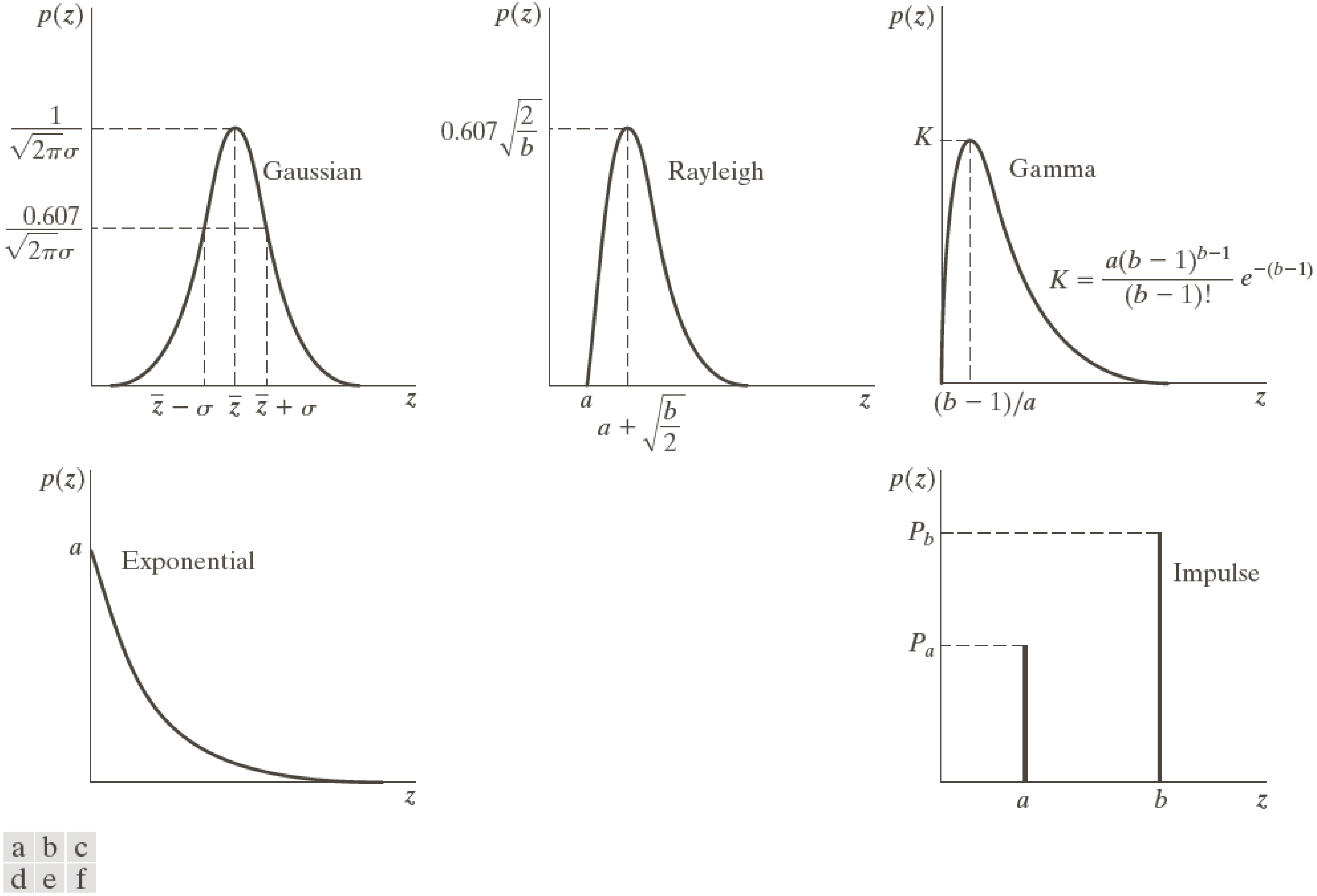


FIGURE 5.2 Some important probability density functions.

Impulse (Salt-and-Pepper) Noise

The PDF of (bipolar) impulse noise is given by

$$p(z) = \begin{cases} P_a & \text{for } z = a \\ P_b & \text{for } z = b \\ 0 & \text{otherwise} \end{cases}$$

if $b > a$, gray-level b will appear as a light dot, while level a will appear like a dark dot.

If either P_a or P_b is zero, the impulse noise is called *unipolar*

Uniform noise

- Not common in practical situations
- Basis for numerous random number generators

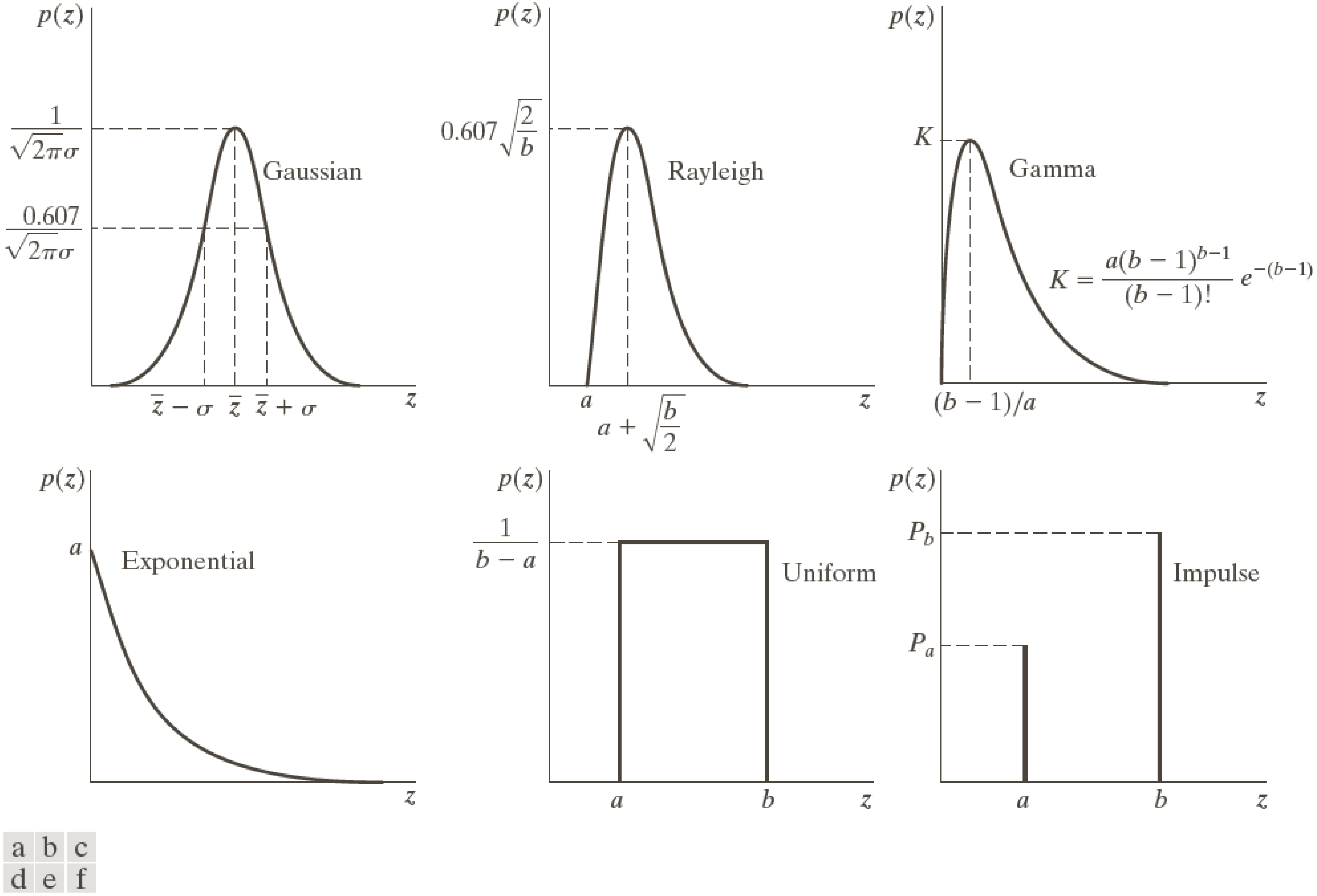


FIGURE 5.2 Some important probability density functions.

Uniform Noise

The PDF of uniform noise is given by

$$p(z) = \begin{cases} \dfrac{1}{b-a} & \text{for a} \le z \le b \\ 0 & \text{otherwise} \end{cases}$$

The mean and variance of this density are given by

$$\bar{z} = (a+b)/2$$

$$\sigma^2 = (b-a)^2/12$$

Examples of Noise: Original Image

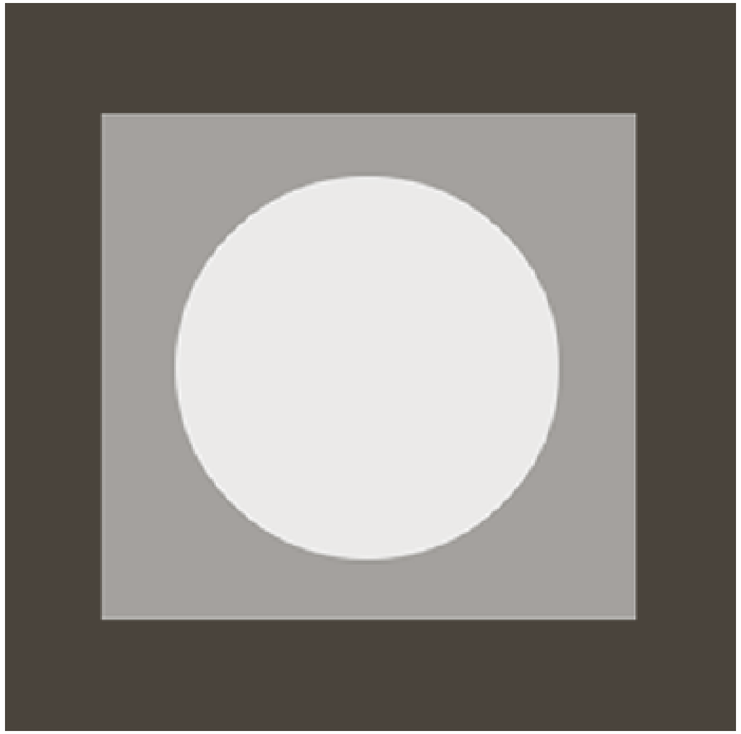


FIGURE 5.3 Test pattern used to illustrate the characteristics of the noise PDFs shown in Fig. 5.2.

Examples of Noise: Noisy Images(1)

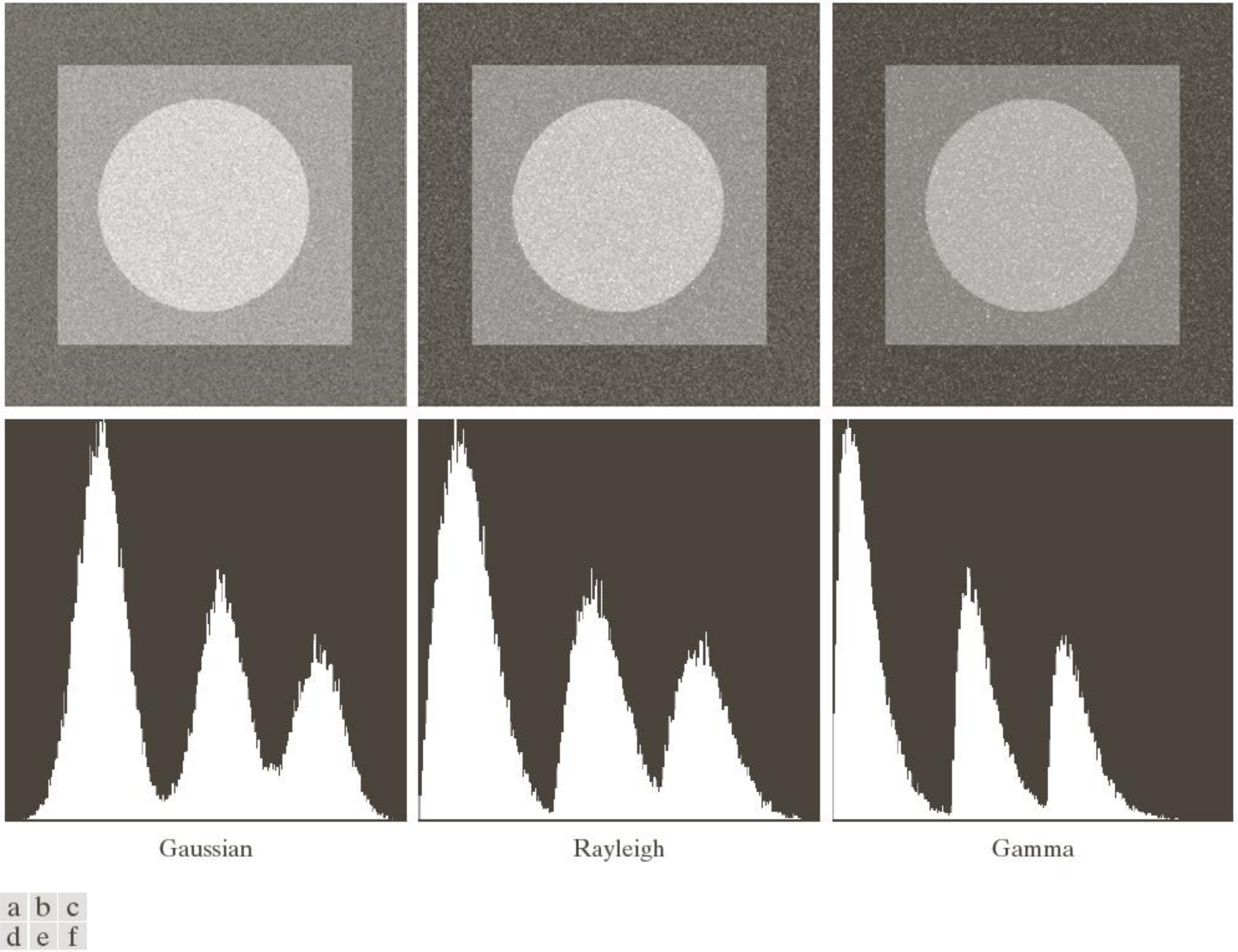


FIGURE 5.4 Images and histograms resulting from adding Gaussian, Rayleigh, and gamma noise to the image in Fig. 5.3.

Examples of Noise: Noisy Images(2)

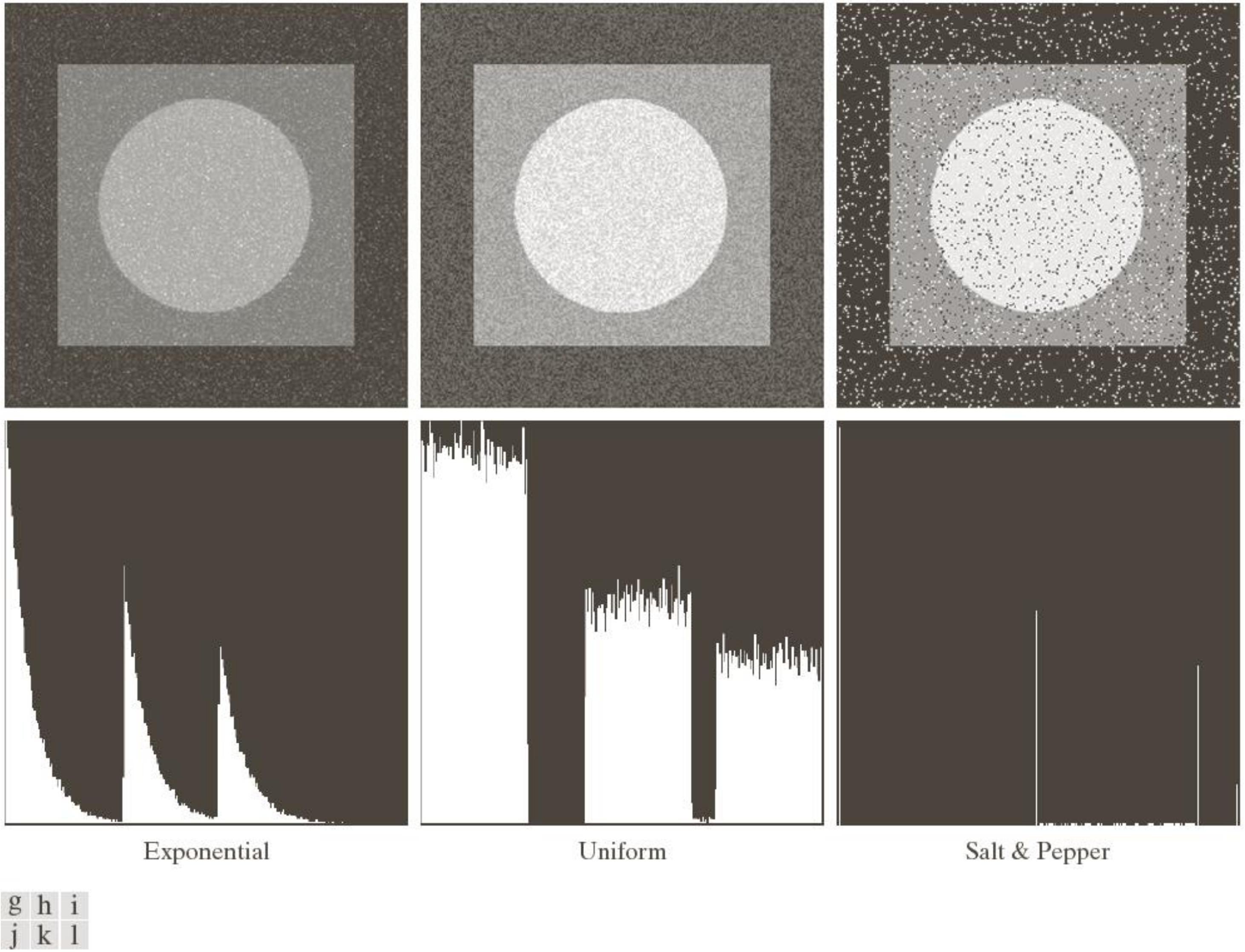


FIGURE 5.4 *(Continued)* Images and histograms resulting from adding exponential, uniform, and salt and pepper noise to the image in Fig. 5.3.

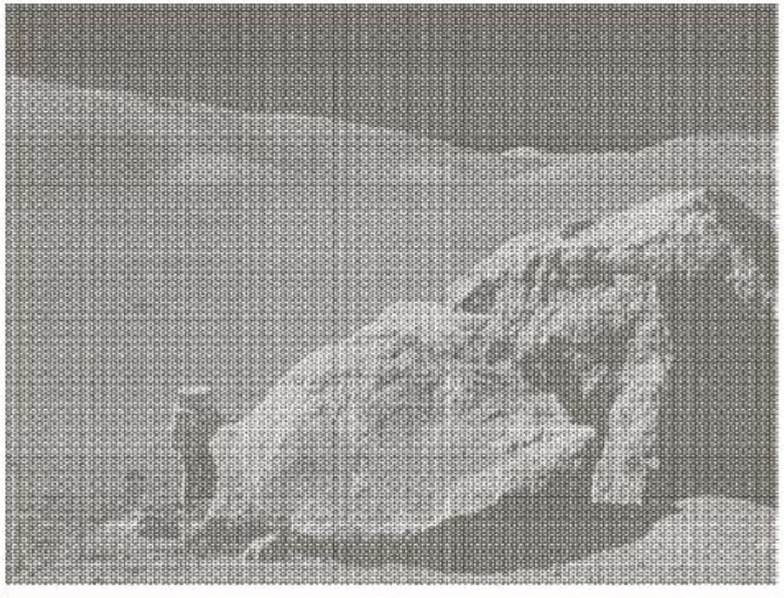
Periodic Noise

- Periodic noise in an image arises typically from electrical or electromechanical interference during image acquisition.

- It is a type of spatially dependent noise

- Periodic noise can be reduced significantly via frequency domain filtering

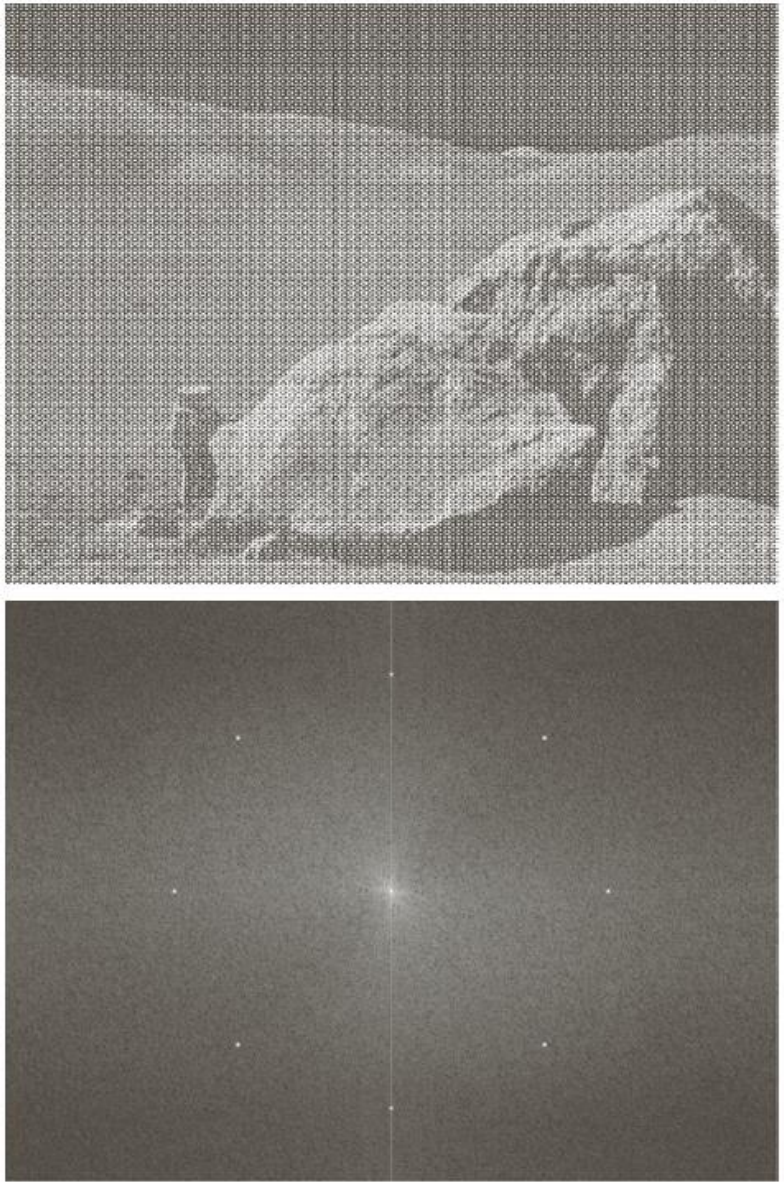
An Example of Periodic Noise



a
b

FIGURE 5.5
(a) Image corrupted by sinusoidal noise. (b) Spectrum (each pair of conjugate impulses corresponds to one sine wave). (Original image courtesy of NASA.)

An Example of Periodic Noise



a
b

FIGURE 5.5 (a) Image corrupted by sinusoidal noise. (b) Spectrum (each pair of conjugate impulses corresponds to one sine wave). (Original image courtesy of NASA.)

Estimation of Noise

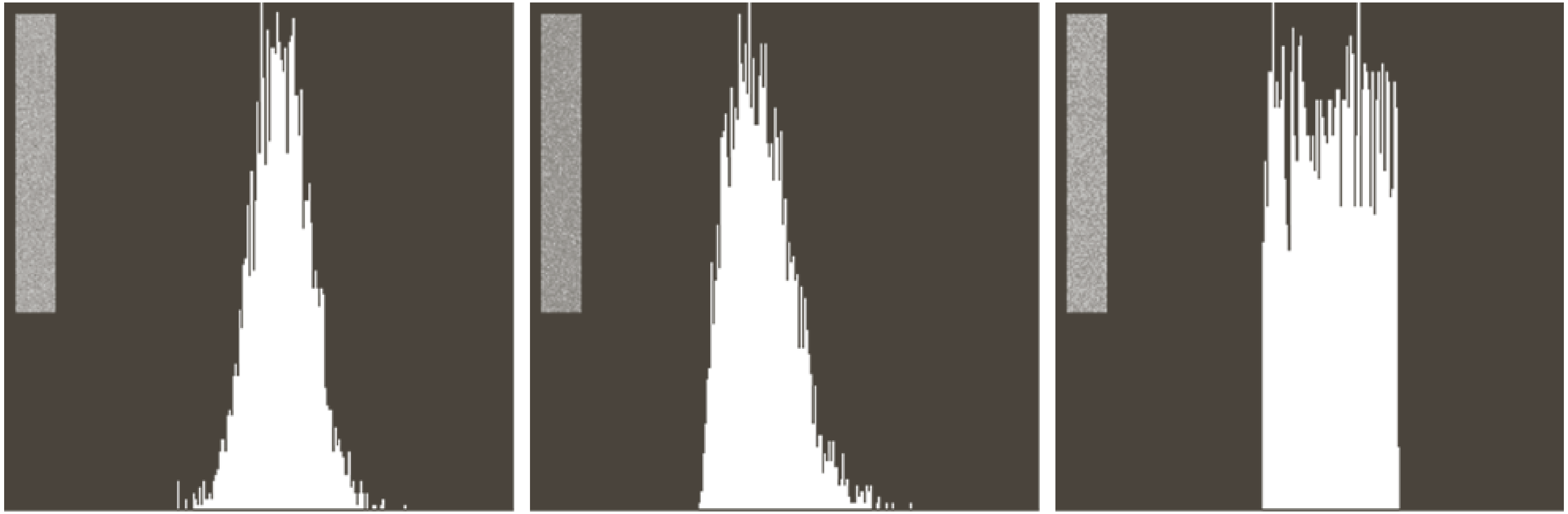
- Periodic noise:
  - Produces frequency spikes
  - Estimated by inspecting the Fourier spectrum.
  - Can be automated if the spikes are pronounced
  - Infer periodicity directly from images (only simplistic cases)

Estimation of Noise

- Statistical noise:
  - Known partially from the sensor specifications
  - If imaging system is available, capture a set of flat images (solid grey board) and study the histogram
  - If only images are available, select an area of image with reasonably constant background and study the histogram.

Estimation of Noise Parameters

The shape of the histogram identifies the closest PDF match



a b c

FIGURE 5.6 Histograms computed using small strips (shown as inserts) from (a) the Gaussian, (b) the Rayleigh, and (c) the uniform noisy images in Fig. 5.4.

Rayleigh Noise

The PDF of Rayleigh noise is given by

$$p(z) = \begin{cases} \dfrac{2}{b}(z-a)e^{-(z-a)^2/b} & \text{for } z \geq a \\ 0 & \text{for } z < a \end{cases}$$

The mean and variance of this density are given by

$$\bar{z} = a + \sqrt{\pi b / 4}$$

$$\sigma^2 = \frac{b(4-\pi)}{4}$$

Estimation of Noise Parameters

Consider a subimage denoted by S, and let $p_s(z_i)$, $i = 0,\ 1,\ \ldots,\ L\text{-}1$, denote the probability estimates of the intensities of the pixels in S. The mean and variance of the pixels in S:

$$\bar{z} = \sum_{i=0}^{L-1} z_i p_s(z_i)$$

and

$$\sigma^2 = \sum_{i=0}^{L-1} (z_i - \bar{z})^2 p_s(z_i)$$