Digital Image Processing

COSC 6380/4393

Midterm Review

Oct 26th, 2023

Mid Term Exam

- Syllabus:
  - Introduction
  - Binary Image Processing
  - Point Operations
  - Discrete Fourier Transform
  - Spatial Filtering

Image Formation

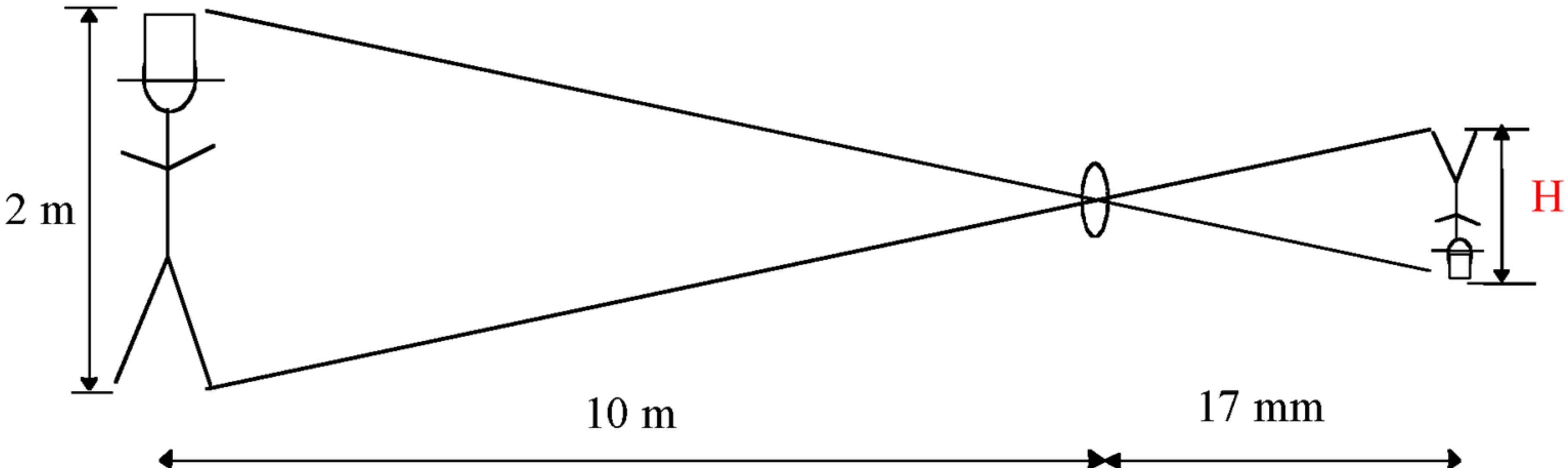
Image Formation: (Projection)

1. A person is standing M meters in front of a camera
   - h is the height of the person
   - X is the focal length of the camera
   - H is the height of the projection of the person on the imaging plane. What is H? (Show steps)

2. A sphere/square is placed at a distance of M meters (m) in front of a camera
   - a/v is the area/volume of the object
   - X is the focal length of the camera
   - A/V is the area of the projection of the object on the imaging plane. What is A/V? (Show steps)

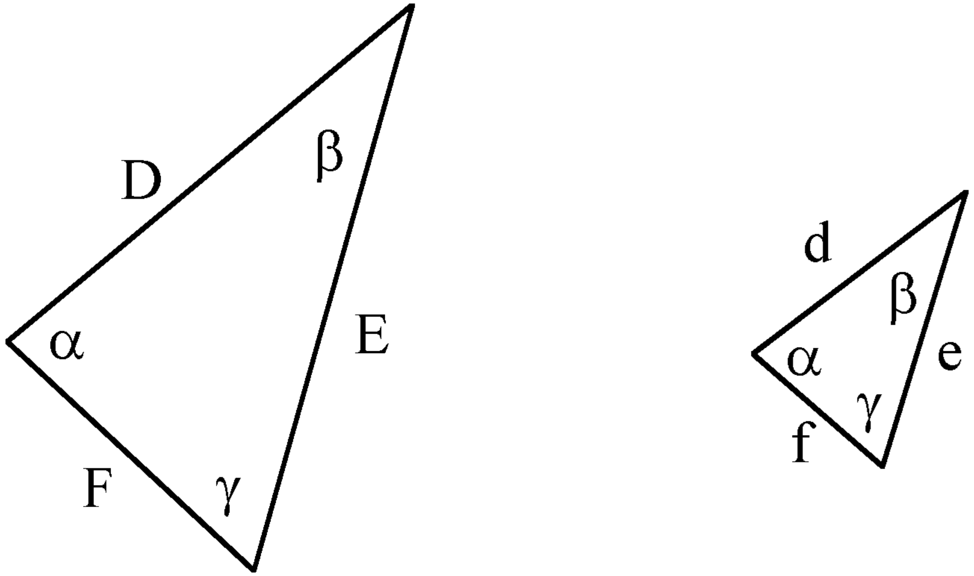
Example: Solution

- There is a man standing 10 meters (m) in front of you
- He is 2 m tall
- The focal length of your eye is about 17 mm
- Question: What is the height H of his image on your retina?



SIMILAR TRIANGLES

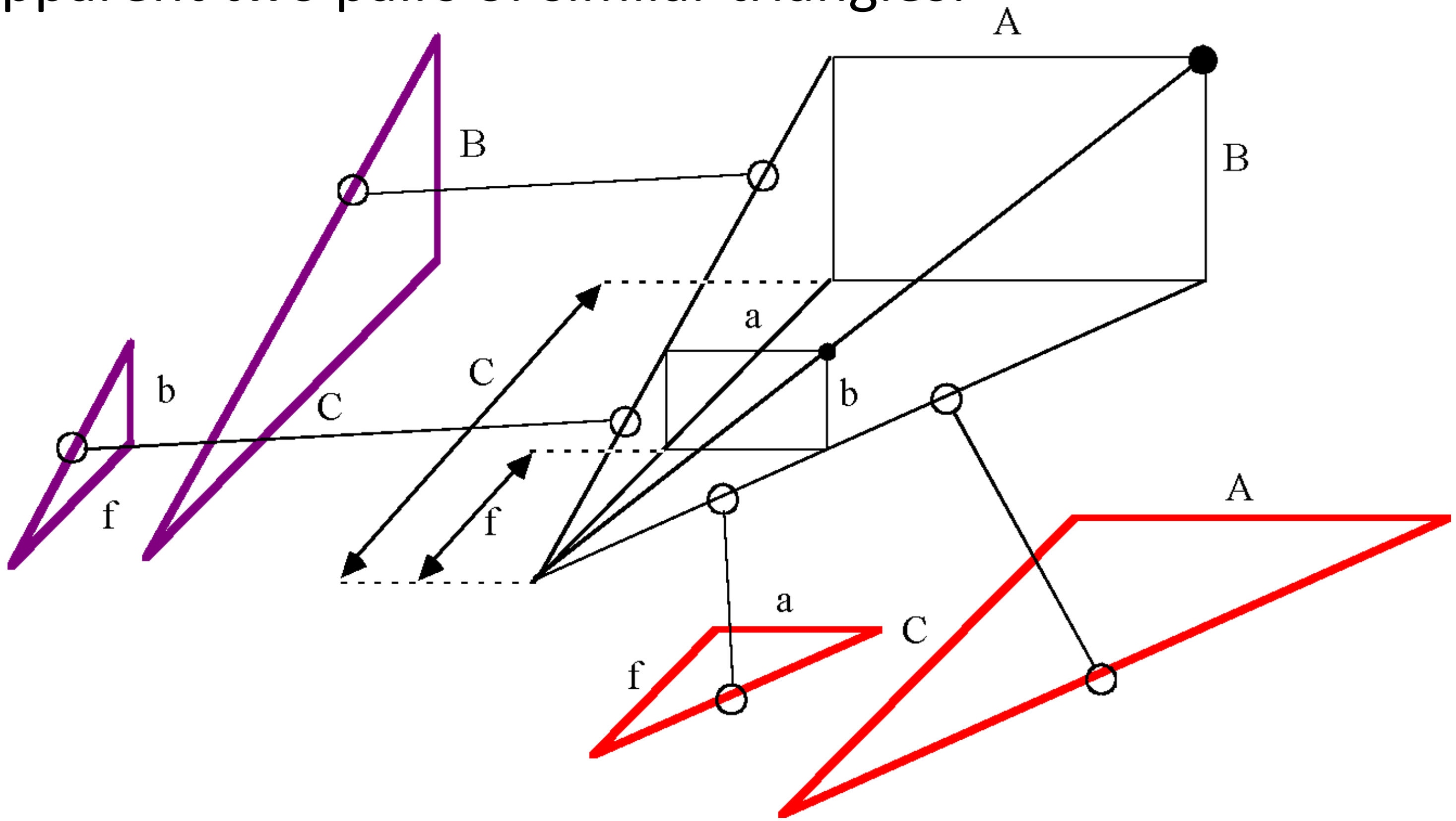
- Similar Triangles Theorem - Similar triangles have their side lengths in the same proportions.



$$\frac{D}{E} = \frac{d}{e} \qquad \frac{E}{F} = \frac{e}{f} \qquad \frac{F}{D} = \frac{f}{d}$$

SOLVING PERSPECTIVE PROJECTION

- Using similar triangles we can solve for the relationship between 3-D coordinates in space and 2-D image coordinates
- Redraw the imaging geometry once more, this time making apparent two pairs of similar triangles:



SOLVING PERSPECTIVE PROJECTION

- By the Similar Triangles Theorem, we conclude that

$$\frac{a}{f} = \frac{A}{C} \quad \text{and} \quad \frac{b}{f} = \frac{B}{C}$$

OR

$$(a, b) = \frac{f}{C} \cdot (A, B) = (fA/C, fB/C)$$

PERSPECTIVE PROJECTION EQUATION

- Thus the following relationship holds between 3-D space coordinates (X, Y, Z) and 2-D image coordinates (x, y) :

$$(x, y) = \frac{f}{Z} \cdot (X, Y)$$

  where f = focal length.

- The ratio f/Z is the magnification factor, which varies with the range Z from the lens center to the object plane.

ANSWER

- By similar triangles,

$$\frac{2\text{ m}}{10\text{ m}} = \frac{H}{17\text{ mm}}$$

$\underline{H = 3.4\text{ mm}}$

Alternatives: Given a circle or square of known area, parallel to the imaging plane.
How do you compute area of the projected image?

Bilinear Interpolation

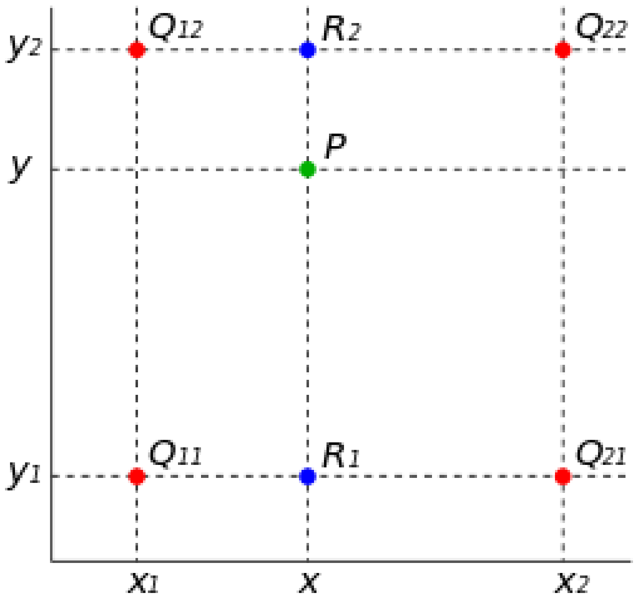
Given the location of four pixels Q_{11}, Q_{12},Q_{21}, Q_{22} and their intensity values I_{11}, I_{12}, I_{21},I_{22}. Assuming that Q_{11}, Q_{12},Q_{21}, Q_{22} are the nearest pixels to P

Estimate the intensity value of pixel located at P using bilinear interpolation?

Bi-Linear Interpolation(2D)

- $Q_{11} = (x_1, y_1)$,
  $Q_{12} = (x_1, y_2)$,
  $Q_{21} = (x_2, y_1)$,
  and $Q_{22} = (x_2, y_2)$
  $f(Q_i) \rightarrow intensity\ at\ Q_i$

  Find the value at P



https://en.wikipedia.org/wiki/Bilinear_interpolation

Example: Linear Interpolation

Solve for I

$$I = \frac{I_1(x_2 - x)}{(x_2 - x_1)} + \frac{I_2(x - x_1)}{(x_2 - x_1)}$$

$$I = \frac{10(1 - 0.3)}{(1 - 0)} + \frac{15(0.3 - 0)}{(1 - 0)}$$

$$I = 7 + 4.5 = 11.5$$

Example

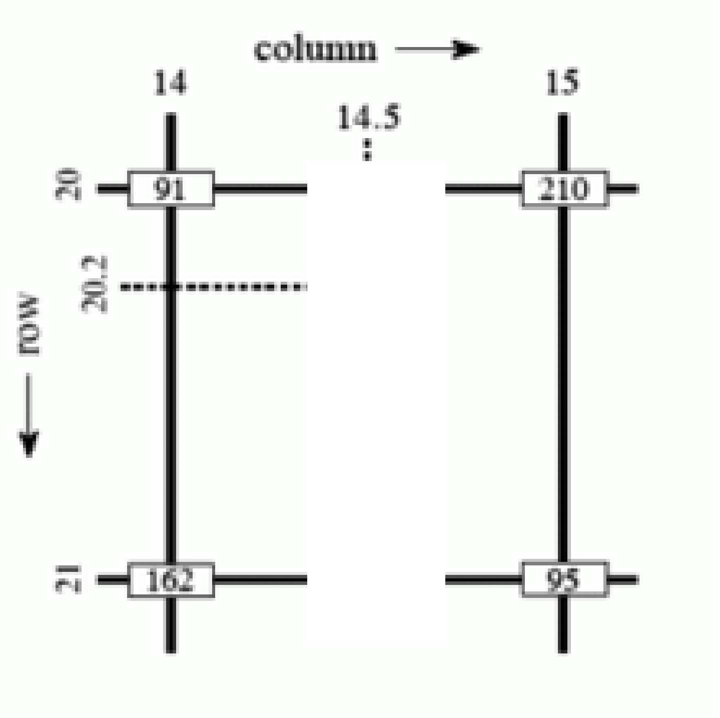
$I(21,14) = 162,$

$I(21,15) = 95,$

$I(20,14) = 91,$

$I(20,15) = 210$

$I(20.2, 14.5) = ?$



https://en.wikipedia.org/wiki/Bilinear_interpolation

Example

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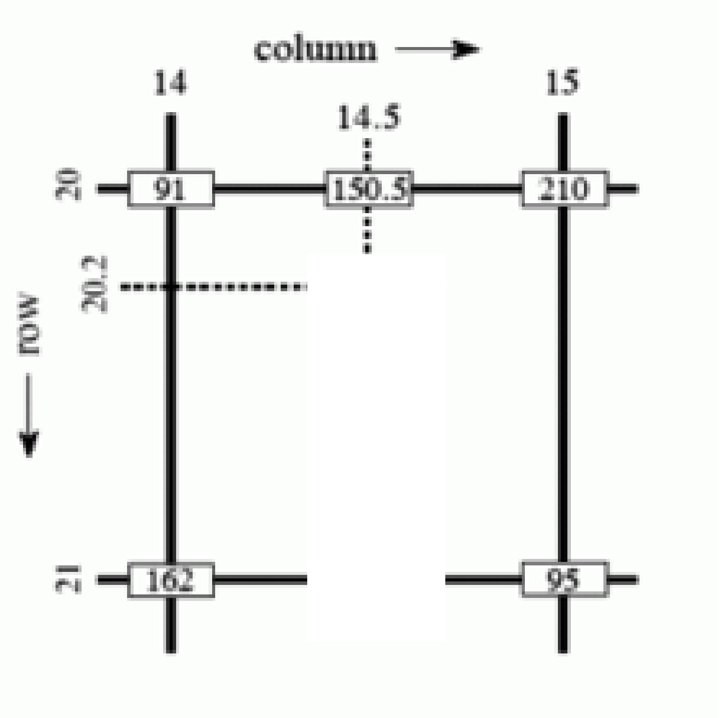
$$I_{20,14.5} = \frac{15-14.5}{15-14} \cdot 91 + \frac{14.5-14}{15-14} \cdot 210 = 150.5,$$



https://en.wikipedia.org/wiki/Bilinear_interpolation

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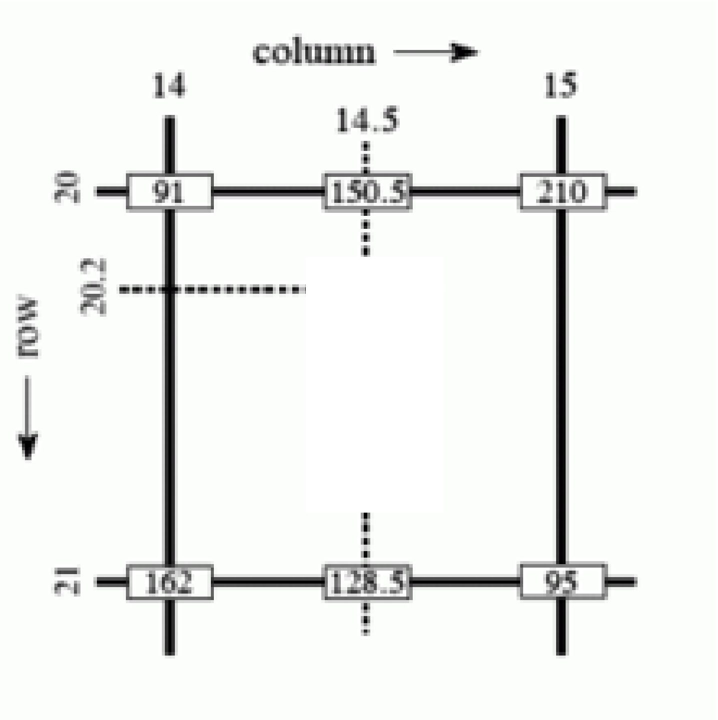
$$I_{21,14.5} = \frac{15-14.5}{15-14} \cdot 162 + \frac{14.5-14}{15-14} \cdot 95 = 128.5,$$



https://en.wikipedia.org/wiki/Bilinear_interpolation

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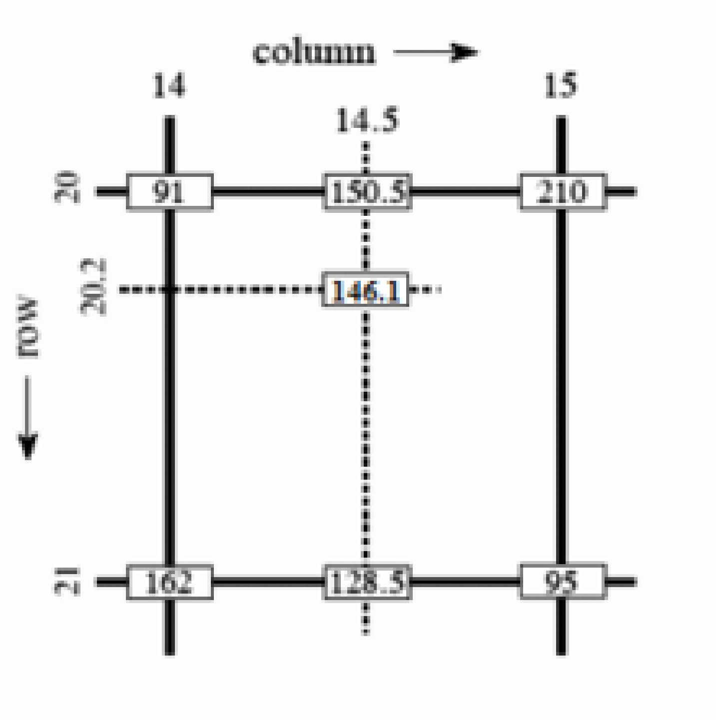


$$I_{20.2,14.5} = \frac{21-20.2}{21-20} \cdot 150.5 + \frac{20.2-20}{21-20} \cdot 128.5 = 146.1.$$

https://en.wikipedia.org/wiki/Bilinear_interpolation

Binary Image Processing

Binary Image Logical Operations

- Given an acquired binary images I and a model binary Image M below. Generate a third binary image D representing the unmatched pixels in the acquired image compared to the model image.
- Since binary operations are quicker, you are allowed to use only binary operators (And, OR and NOT) or a combination of these on binary image M and I to accomplish this task.

0	1
1	0

M

0	1
0	1

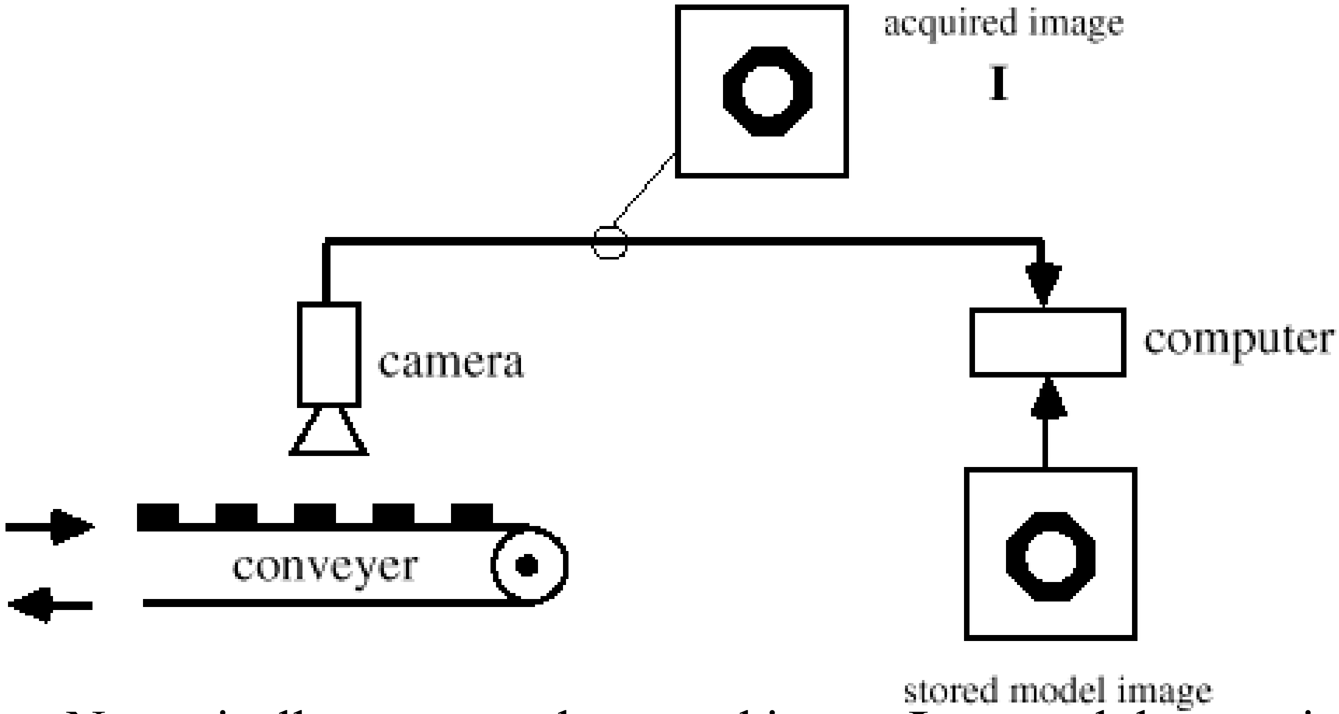
I

0	0
1	1

D

3. Image difference

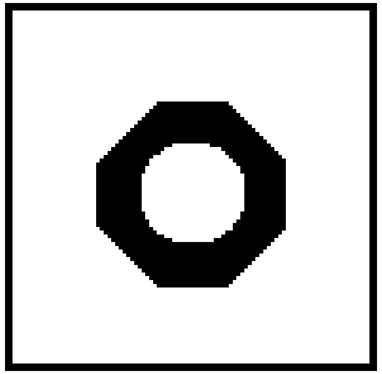
- An assembly-line image inspection system. Similar to many marketed by industry:



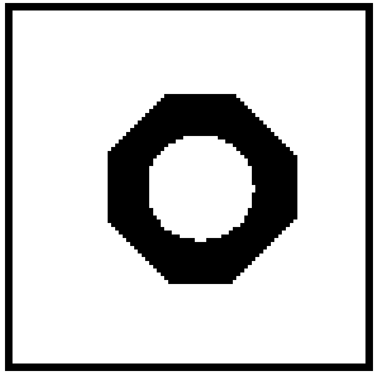
- **Objective**: Numerically compare the stored image $\mathbf{I}_{model}$ and the acquired image $\mathbf{I}$

EXAMPLE

- Observe that the object in **I** has been shifted very slightly



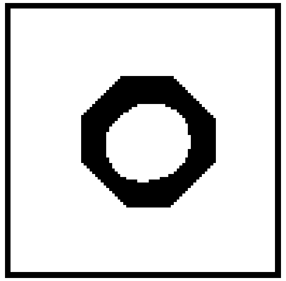
I model



I

Logical AND

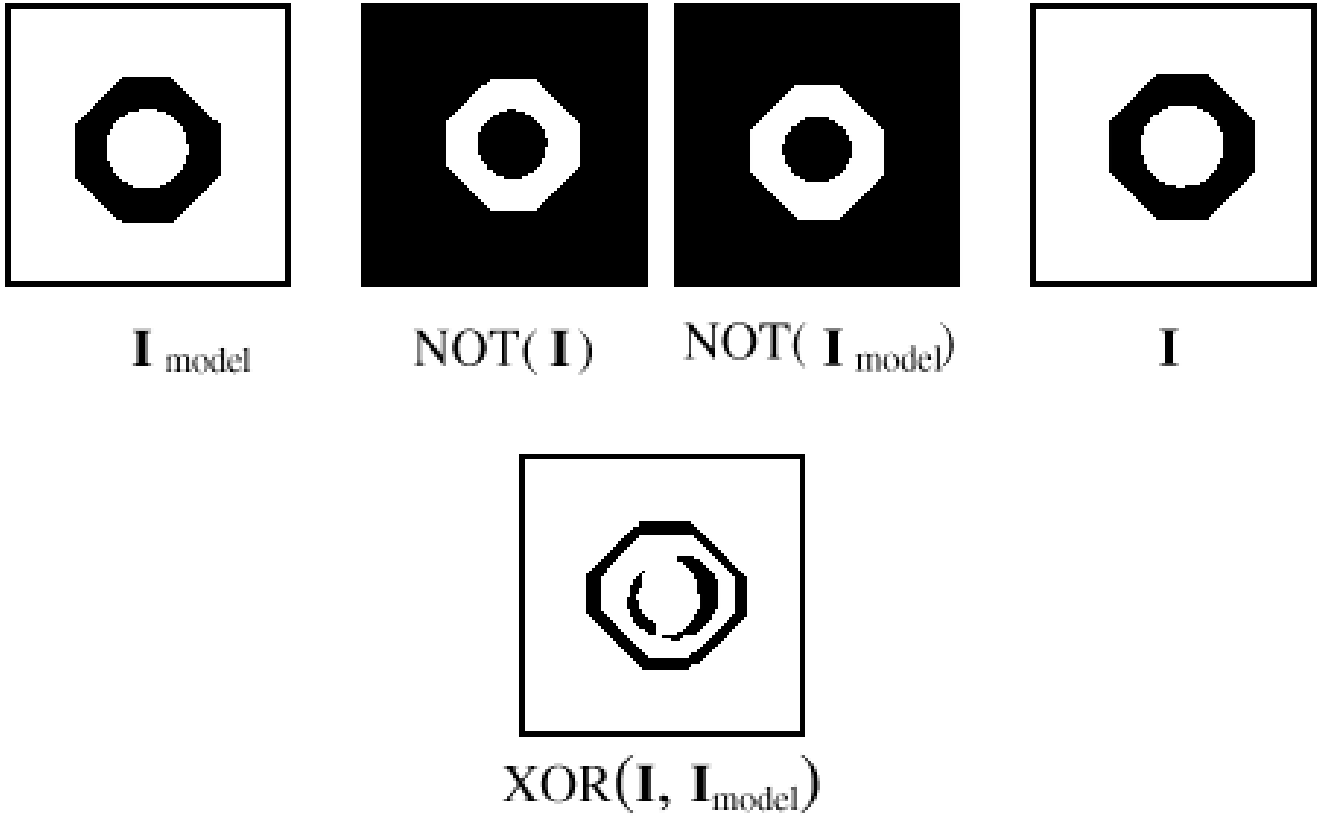
- The logical AND conveys the **overlap**



$\mathbf{I}_{model} \wedge \mathbf{I}$

- A measurement of the **displacement** is given by:
- XOR($\mathbf{I}$, $\mathbf{I}_{model}$) = OR{ AND[$\mathbf{I}_{model}$, NOT($\mathbf{I}$)], AND[NOT($\mathbf{I}_{model}$), $\mathbf{I}$]}

DISPLACEMENT



4. Morphological Operations

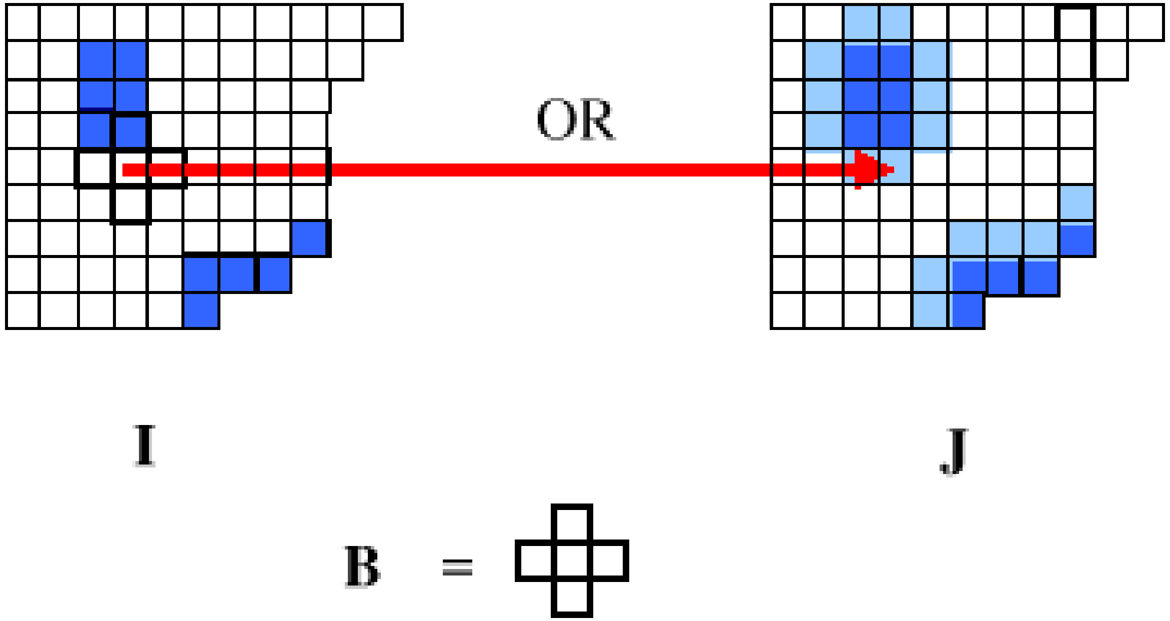
1. Perform a morphological operation using a window of your choice to remove the small object represented by 1 at pixel (2,4) in the image below.

0	0	0	0	0
1	1	0	1	0
1	1	0	0	0
0	1	0	0	0
0	0	0	0	0

Operations to cover holes, remove peninsula, etc.?

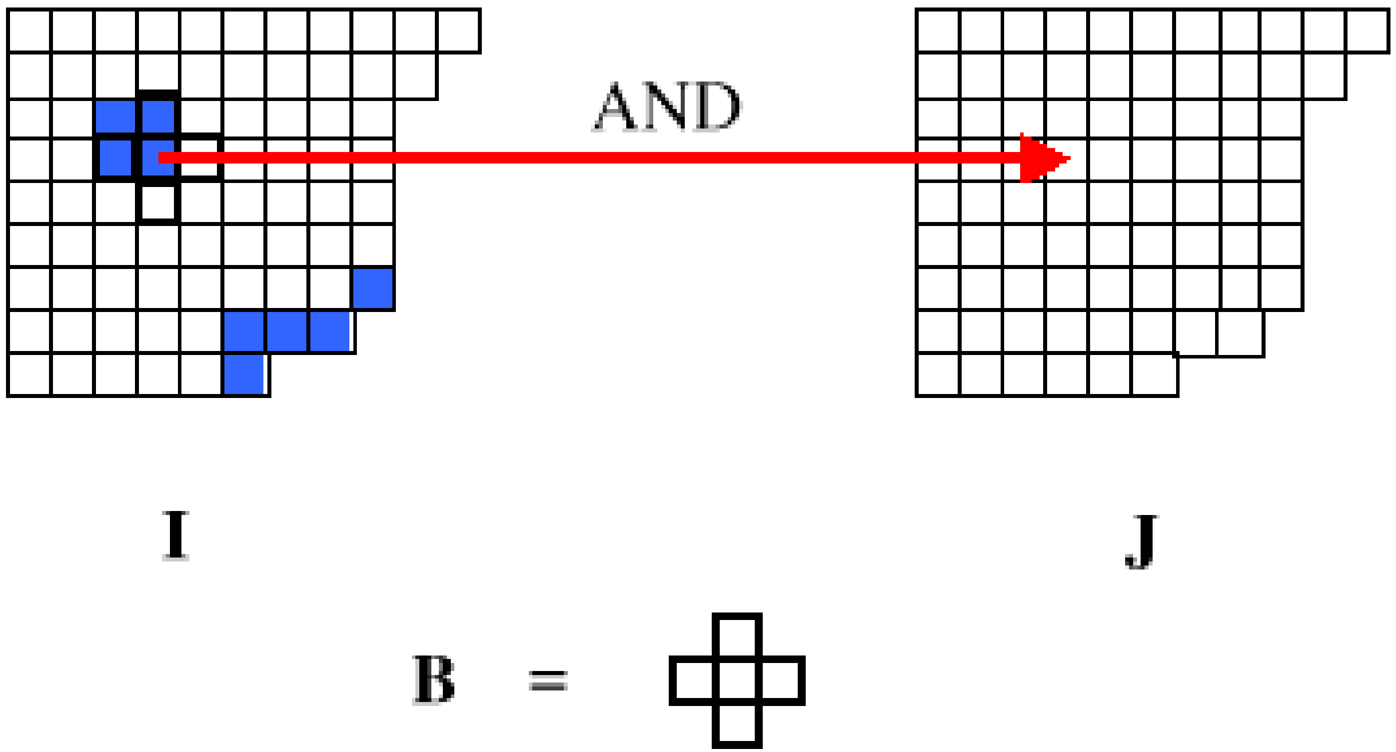
DILATION

- So-called because this operation **increases** the size of BLACK objects in a binary image
- Local Computation: **J** = DILATE(**I**, **B**)



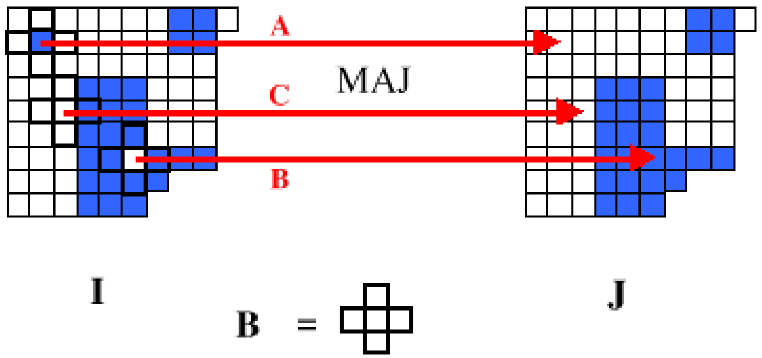
EROSION

- So-called because this operation **decreases** the size of BLACK objects in a binary image
- Local Computation: **J** = ERODE(**I**, **B**)



MEDIAN

- Actually **majority**. A special case of the gray-level **median filter**
- Possesses qualitative attributes of both dilation and erosion, but does not generally change the **size** of objects or background
- Local Computation: **J** = MEDIAN(**I, B**)



- The median removed the small **object** A and the small **hole** B, but did not change the boundary (**size**) of the larger region C

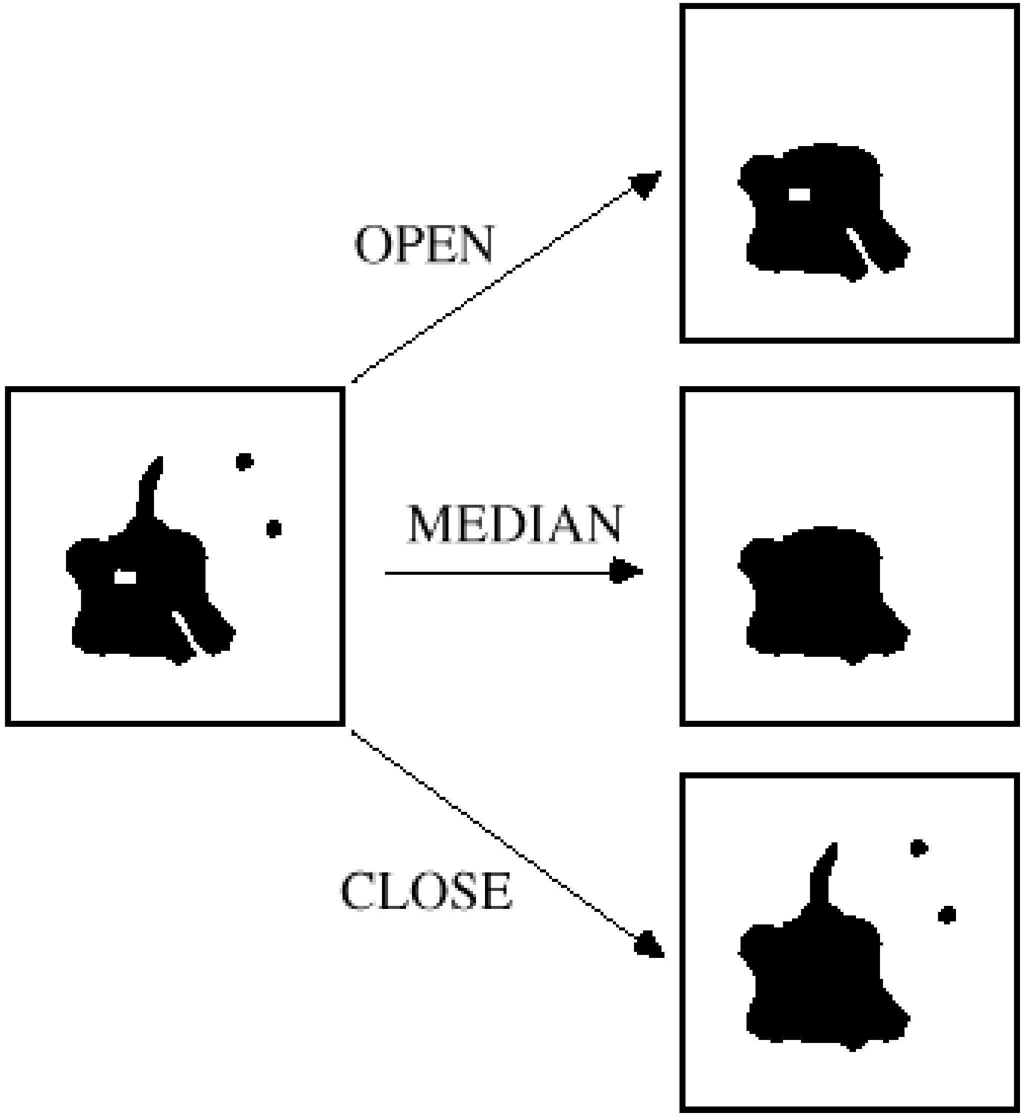
OPENing and CLOSing

- We can define **new** morphological operations by performing the basic ones in sequence
- Given an image **I** and window **B**, define

  OPEN(**I**, **B**) = DILATE [ERODE(**I**, **B**), **B**]

  CLOSE(**I**, **B**) = ERODE [DILATE(**I**, **B**), **B**]
- In other words,
- OPEN = erosion (by **B**) followed by dilation (by **B**)
- CLOSE = dilation (by **B**) followed by erosion (by **B**)

EXAMPLES



4. Morphological Operations

1. Perform a morphological operation using a window of your choice to remove the small object represented by 1 at pixel (1,3) in the image below.

0	0	0	0	0
1	1	0	1	0
1	1	0	0	0
0	1	0	0	0
0	0	0	0	0

1. Operations to cover holes, remove peninsula?

Solution

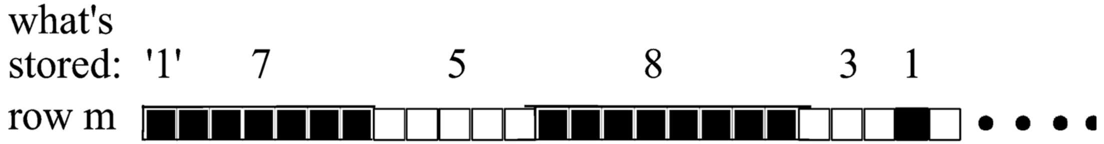
- A possible solution can be using the median operation with a filter (3,1)

0	0	0	0	0
1	1	0	1	0
1	1	0	0	0
0	1	0	0	0
0	0	0	0	0

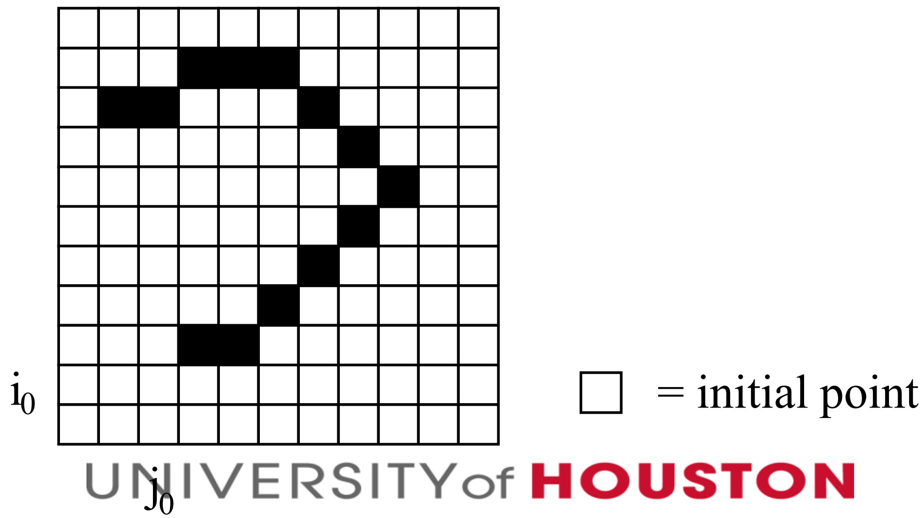
0	0	0	0	0
1	1	0	0	0
1	1	0	0	0
0	1	0	0	0
0	0	0	0	0

5. Compression

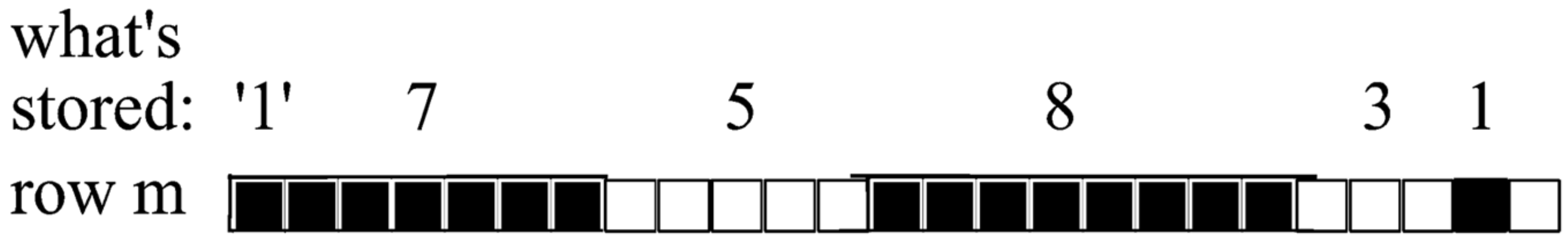
1. Perform compression to represent sequence of pixels in the binary image below



1. Perform compression to represent the contour in the binary image using chain codes



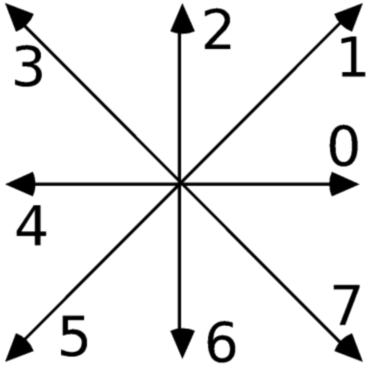
Run Length Encoding



Code: 175831

CHAIN CODE

- We use the following 8-neighbor direction codes:

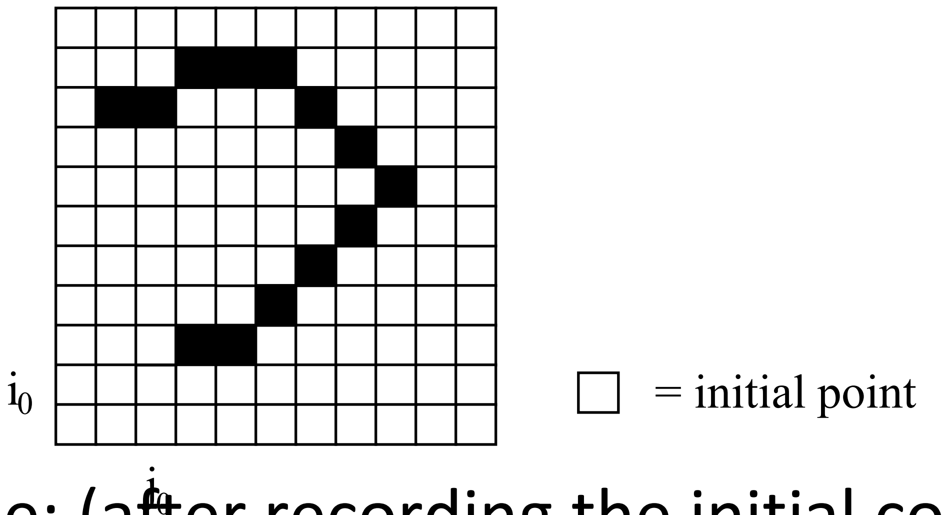


- Since the numbers 0, 1, 2, 3, 4, 5, 6, 7 can be coded by their 3-bit binary equivalents:

  000, 001, 010, 011, 100, 101, 110, 111

  the location of each point on the contour **after** the initial point can be coded by 3 bits.

EXAMPLE



- Its chain code: (after recording the initial coordinate (i0, j0)

1, 0, 1, 1, 1, 1, 3, 3, 3, 4, 4, 5, 4

=

001, 000, 001, 001, 001, 001, 011, 011, 011, 100, 100, 101, 100

Point Operations

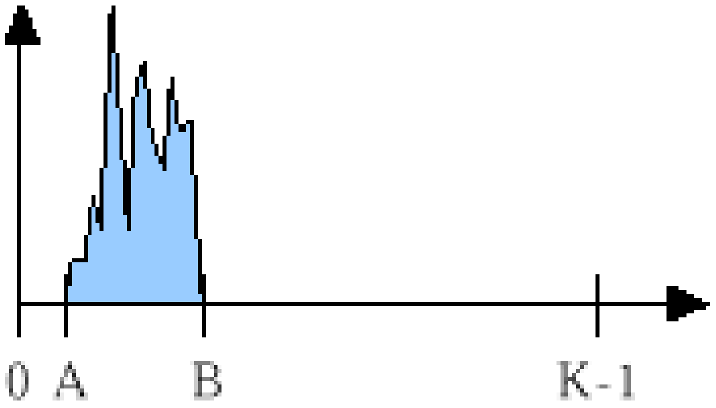
Linear Point Operations

1. Perform a full contrast stretch on the image below assuming that each dynamic range of the image is 0-15
2. Perform histogram flattening, shaping?

$$\mathbf{I} = \begin{array}{|c|c|c|c|} \hline 1 & 1 & 3 & 4 \\ \hline 2 & 5 & 3 & 2 \\ \hline 8 & 1 & 8 & 2 \\ \hline 4 & 5 & 3 & 11 \\ \hline \end{array}$$

6. Full-Scale Contrast Stretch

- The **most common** linear point operation. Suppose **I** has a compressed histogram:



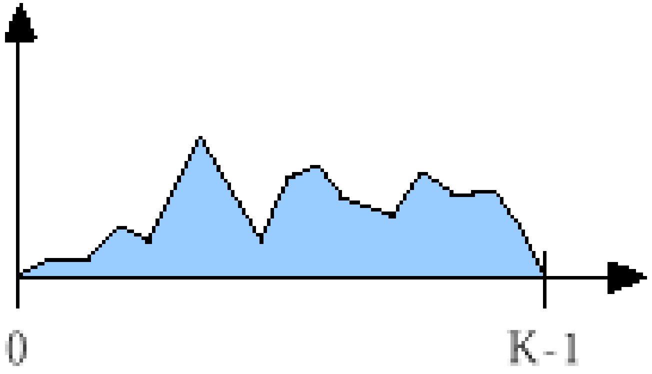
- Let A and B be the min and max gray levels in **I**
- Define

$$J(i, j) = P \cdot I(i, j) + L$$

- such that $P \cdot A + L = 0$ and $P \cdot B + L = (K-1)$

Full-Scale Contrast Stretch

- The result of solving these **2 equations in 2 unknowns** (P, L) is an image **J** with a full-range histogram:



- The solution to the above equations is

$$P = \left\lfloor \frac{K-1}{B-A} \right\rfloor \quad \text{and} \quad L = -A \left\lfloor \frac{K-1}{B-A} \right\rfloor$$

or

$$J(i,j) = \left\lfloor \frac{K-1}{B-A} \right\rfloor [I(i,j) - A]$$

- $B = 11$
- $A = 1$
- $B - A = 10$
- $(K - 1) = 15$

$$\mathbf{I} = \begin{array}{|c|c|c|c|}\hline 1 & 1 & 3 & 4 \\ \hline 2 & 5 & 3 & 2 \\ \hline 8 & 1 & 8 & 2 \\ \hline 4 & 5 & 3 & 11 \\ \hline \end{array}$$

- If I_c is the image with full dynamic range.
- $I_c[0,0] = INT(\frac{15}{10}(I[0,0] - 1) + 0.5)$

$$= INT\left(\frac{15}{10}(1 - 1) + 0.5\right) = 0$$

- $I_c[0,2] = INT(\frac{15}{10}(I[0,2] - 1) + 0.5)$

$$= INT\left(\frac{15}{10}(3 - 1) + 0.5\right) = 3$$

…

- $B = 11$
- $A = 1$
- $B - A = 10$
- $(K - 1) = 15$

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$$= INT\left(\frac{15}{10}(3 - 1) + 0.5\right) = 3$$

…

$\mathbf{I} =$

1	1	3	4
2	5	3	2
8	1	8	2
4	5	3	11

Result:

0	0	3	5
2	6	3	2
11	0	11	2
5	6	3	15

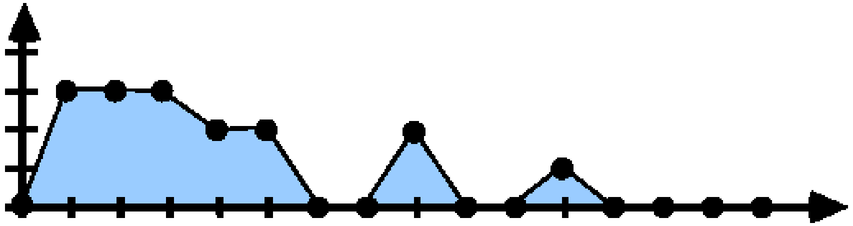
Histogram Flattening

- Given a 4 x 4 image **I** with gray-level range {0, ..., 15} (K-1 = 15):

$$\mathbf{I} = \begin{array}{|c|c|c|c|} \hline 1 & 1 & 3 & 4 \\ \hline 2 & 5 & 3 & 2 \\ \hline 8 & 1 & 8 & 2 \\ \hline 4 & 5 & 3 & 11 \\ \hline \end{array}$$

- It's histogram is

k	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
H(k)	0	3	3	3	2	2	0	0	2	0	0	1	0	0	0	0



Histogram Flattening

- The normalized histogram is

k	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
p(k)	0	$\frac{3}{16}$	$\frac{3}{16}$	$\frac{3}{16}$	$\frac{2}{16}$	$\frac{2}{16}$	0	0	$\frac{2}{16}$	0	0	$\frac{1}{16}$	0	0	0	0

- From which we can compute the intermediate image **J**1 and finally the "flattened" image **J**:

$$\mathbf{J}_1 = \begin{bmatrix} 3/16 & 3/16 & 9/16 & 11/16 \\ 6/16 & 13/16 & 9/16 & 6/16 \\ 15/16 & 3/16 & 15/16 & 6/16 \\ 11/16 & 13/16 & 9/16 & 16/16 \end{bmatrix} \qquad \mathbf{J} = \begin{bmatrix} 3 & 3 & 8 & 10 \\ 6 & 12 & 8 & 6 \\ 14 & 3 & 14 & 6 \\ 10 & 12 & 8 & 15 \end{bmatrix}$$

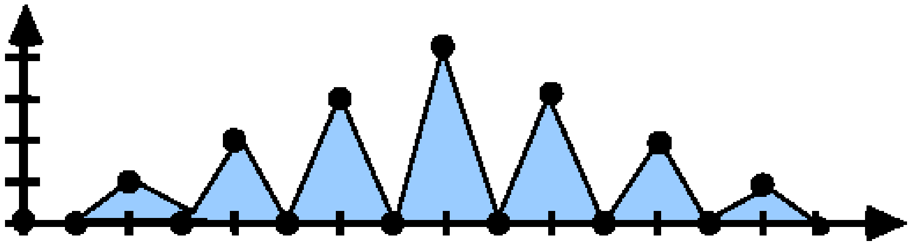
Histogram Shaping

- Consider the same image as in the last example. We had

$$\mathbf{I} = \begin{bmatrix} 1 & 1 & 3 & 4 \\ 2 & 5 & 3 & 2 \\ 8 & 1 & 8 & 2 \\ 4 & 5 & 3 & 11 \end{bmatrix} \qquad \mathbf{J}_1 = \begin{bmatrix} 3/16 & 3/16 & 9/16 & 11/16 \\ 6/16 & 13/16 & 9/16 & 6/16 \\ 15/16 & 3/16 & 15/16 & 6/16 \\ 11/16 & 13/16 & 9/16 & 16/16 \end{bmatrix}$$

- Fit this t

k	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$H_J(k)$	0	0	1	0	2	0	3	0	4	0	3	0	2	0	1	0
$p_J(k)$	0	0	$\frac{1}{16}$	0	$\frac{2}{16}$	0	$\frac{3}{16}$	0	$\frac{4}{16}$	0	$\frac{3}{16}$	0	$\frac{2}{16}$	0	$\frac{1}{16}$	0



Histogram Shaping

- Here's the cumulative (summed) probabilities associated with it:

n	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$\mathbf{P_J}(n)$	0	0	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{3}{16}$	$\frac{3}{16}$	$\frac{6}{16}$	$\frac{6}{16}$	$\frac{10}{16}$	$\frac{10}{16}$	$\frac{13}{16}$	$\frac{13}{16}$	$\frac{15}{16}$	$\frac{15}{16}$	$\frac{16}{16}$	$\frac{16}{16}$

- **Careful** visual inspection of $\mathbf{J}_1$ let's us form the new image:

$$\mathbf{J} = \begin{array}{|c|c|c|c|} \hline 4 & 4 & 8 & 10 \\ \hline 6 & 10 & 8 & 6 \\ \hline 12 & 4 & 12 & 6 \\ \hline 10 & 10 & 8 & 14 \\ \hline \end{array}$$

HISTOGRAM MATCHING

- Just a special case of histogram shaping.
- Difference: the histogram of the original image **I** is matched to that of another image **I**´ .
- Otherwise the procedure is identical, once the cumulative probabilities are computed for the model image **I**´ .
- <u>Useful application</u>: **Comparing** similar images of the same scene obtained under different conditions (e.g., lighting, time of day). Extends the concept of equalizing AOD described earlier.