MATH 3338 Probability

Wenjiang Fu
wfu2@central.uh.edu

Lecture 1

UNIVERSITY of HOUSTON
DEPARTMENT OF MATHEMATICS

Outline

UNIVERSITY of HOUSTON
DEPARTMENT OF MATHEMATICS

Probability by examples

Example 1. Toss a coin
{HTTHHTTTH...}

Example 2. Roll a dice
{3125326...}

Example 3. Shuffle a deck of cards
...
Other examples: Weather, temperature, body weight, color, stock market, grade, (fail or pass).

Objective vs subjective: Grade.

UNIVERSITY of HOUSTON
DEPARTMENT OF MATHEMATICS

What is Probability?

- Probability is a measure of the likelihood of an event to occur. Many events cannot be predicted with complete certainty, one can figure out only the chance of an event to occur i.e., how likely to happen.
- Probability is a frequency of an event to occur in an infinite repetitions.
  Q: Is it subjective or objective?
- Probability is the science of studying uncertainty, or the randomness, by using distribution and simulation.

UNIVERSITY of HOUSTON
DEPARTMENT OF MATHEMATICS

Uncertainty in a data set

https://www.math.uh.edu/~cathy/Math3339/data/grades.txt

Student	Score	Grade	Tests	Quiz	HW	Session
1	100.707	A	99.233	87.308	101.270	Sp16
2	81.310	B	75	98.231	64.444	Sp16
3	8.194	F	14.667	12.769	3.175	Sp16
4	90.449	A	91.533	77.231	82.222	Sp16
5	68.461	D	65.783	81.769	68.571	Sp16
6	103.955	A	103.32	97.923	101.905	Sp16
7	92.889	A	95.6	85.923	75.556	Sp16
8	84.805	B	83.2	79.385	75.238	Sp16
9	91.640	A	89.967	91.231	85.079	Sp16
10	22.316	F	17.433	40.615	44.444	Sp16
11	98.363	A	94.167	99.231	101.587	Sp16
12	49.250	F	43.917	73.077	78.095	Sp16
13	16.967	F	15.5	20.077	29.841	Sp16
14	50.747	F	45.533	67.385	57.460	Sp16
15	43.184	F	72.983	47.462	38.413	Sp16
16	100.845	A	98.667	96.231	100.317	Sp16
17	84.195	B	77.5	87.154	95.556	Sp16
18	84.400	B	78.733	78.615	82.540	Sp16
19	67.170	D	74.3	68.538	72.063	Fal15
20	87.413	B	92	82.077	77.778	Fal15
21	67.899	D	71.8	71.077	84.127	Fal15
22	74.676	C	70.083	83.308	73.016	Fal15
23	40.054	F	44.133	21.308	33.333	Fal15
24	101.014	A	101.08	98.923	95.873	Fal15
25	11.972	F	17.1	10.385	3.810	Fal15
26	79.831	B	86.233	71.923	46.667	Fal15
27	83.301	B	94.6	69.692	60.317	Fal15
28	72.299	C	64.967	67.615	99.394	Sum16
29	83.821	B	77.2	80.923	83.030	Sum16
30	90.703	A	83.617	87.923	80.000	Sum16

UNIVERSITY of HOUSTON
DEPARTMENT OF MATHEMATICS

- **Setup of Probability**
  1) It is a measure of likelihood (chance) of some event to occur.
  2) Need an event.
  3) Need more characterization of the event, and its similar.

- **Three Components of Probability**
  Sample space is a collection of all possible outcomes of interest.
  It is denoted by Ω, $\Omega = \{\omega_1, \omega_2, ...,\}$
  An event is a set of elements in the sample space.
  A set is a collection of elements in the sample space.
  Probability is a function or a mapping from the set of a sample space to the interval $[0, 1]$.

Probability as a function

- **Definition 1.1** Let X denote the outcome of an experiment, which depends on chance. That is, X takes random values, and is called a random variable. If the sample space Ω is either finite or countably infinite, then X is called a discrete random variable.

  Example. Rolling a dice, $\Omega = \{1, 2, 3, 4, 5, 6\}$
  Define a set $A = \{1, 3, 5\}$. It is defined to be all odd number outcomes in Ω. We can say our random variable (RV) X takes only odd numbers.

- Probability function
  $m(\cdot) : \Omega \to [0, 1]$, $0 \leq m(\omega) \leq 1$ for any $\omega \in \Omega$

  Let's calculate the probability of the event A in the above example.
  $m(A) = m(\{1, 3, 5\}) = m(1) + m(3) + m(5) = 1/6 + 1/6 + 1/6 = 1/2$

UNIVERSITY of HOUSTON
DEPARTMENT OF MATHEMATICS

Probability as a function

- In general,

$$m(A) = \sum_{\omega \in A} m(\omega)$$

  That means the probability of a set A is equal to the sum of the probabilities of all elements in set A.

- By default, the sample space Ω has the total probability 1.

$$m(\Omega) = \sum_{\omega \in \Omega} m(\omega) = 1.$$

  Hence the probability of a set is always between 0 and 1, i.e.

$$0 \leq m(A) \leq 1$$

  for any set A.

UNIVERSITY of HOUSTON
DEPARTMENT OF MATHEMATICS

Examples of Sample space and Events

Identify the sample space and the event in the following:

- University of Houston is interested in how many foreign students are on campus and their nationality. A list of foreign students of UH by their nationality.
  - Sample space -
  - Events -
- An elementary school is creating a new lunch menu. They send questionnaires to collect student's preference of lunch menu
  - sample space -
  - Events -

Many interesting questions

For the coin tossing example, we may ask many questions.

- Is the coin a fair coin?
- If not, what is the probability to get a head?
- In 100 tosses, what is the probability of getting exact 40 Hs?
- What is the probability of getting exact 10 consecutive Hs in a row?
- Example of Medicine
    - In 1000 diagnosis, what is the chance to give a wrong diagnosis?
    - In 1000 dispense of medication, what is the probability of having a wrong drug?

Two Types of quantities / Variables

Discrete: if the total number of elements in the sample space is finite or infinite but countable.
Examples: number of siblings, number of heart attacks, or episodes of symptoms, total number of traffic accidents in a country in one year.

continuous: Body weight, height. blood pressure, ...

- Discrete values - a countable set of values, often are categorical, or integers.
- Continuous values - those that can take on any values within some interval - representing some continuum.

UNIVERSITY of HOUSTON
DEPARTMENT OF MATHEMATICS

More Examples

Classify the following variables as discrete or continuous.

- Political preference.
- Number of siblings.
- Chance to win a computer game.

Examples of Variables

Classify the following variables as categorical or quantitative. If quantitative, state whether the variable is discrete or continuous.

- Blood type.
- Height of men on a professional basketball team.
- Time it takes to be on hold when calling the IRS right before a tax return filing deadline.
- Blood pressure readings.

Winning the State Lottery

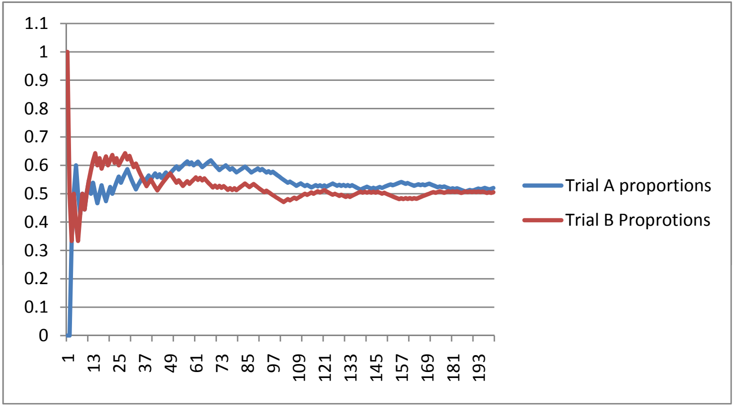
- Suppose a person won the Jackpot of the State Lottery five times in a row.
- What do you think would happen?
- Is it possible for a person to win the state lottery five consecutive times?

UNIVERSITY of HOUSTON
DEPARTMENT OF MATHEMATICS

Randomness and Probability

- We call a phenomenon **random** if individual outcomes are uncertain.
- However, there is a regular distribution of outcomes in a large number of repetitions.
- Chance behaviors unpredictable in the short run but has a regular and predictable pattern in the long run.
- Long-run must be infinitely long to give them frequencies of enough time to even out.

Proportion of Heads in Long Run



UNIVERSITY of HOUSTON
DEPARTMENT OF MATHEMATICS

Probability

- "A **probability** is a numerical value that measures the likelihood that an uncertain event occurs." *Business Statistics: Communicating with Numbers, Jaggia and Kelly, pg 96*
- The **probability** of any outcome of a random phenomenon is the proportion of times the outcome would occur in a very long series of repetitions.

UNIVERSITY of HOUSTON
DEPARTMENT OF MATHEMATICS

Why Study Probability ?

- Scientific Idea: To better understand the real world with some or full uncertainty. Rare event rule for inferential statistics - meaningful.
- It's extremely unlikely to win a lottery, but some do.

Random Experiments - simulations

In order to study probability, especially some difficult ones, we need know how random experiments work, and conduct simulations. Let the results or data to give us the answer through a scientific experiment, better than a random guess.

- In the old days, people used coin tossing, dice, etc.
  Today, we can use computer programming to generate many random series and perform calculation based a large number of such experiments.

A random experiment has the following two characteristics:

1. The experiment can be replicated an indefinite number of times under essentially the same experimental conditions.
2. There is a degree of uncertainty in the outcome of the experiment. The outcome may vary from replication to replications even though the experimental conditions are the same.

Three components of probability

- **Sample Space**
  A collection of elements (all possible outcomes).
  Example 1 Toss a coin: {HT}
  Example 2 Roll a dice: {123456}
  Example 3 Student grade: {A, A-, B+, B, B-, C+, C, C- ...}

- An **event** is a subset of the sample space that will occur
  A **set** is a collection of objects.

- The items that are in a set called **elements.**

- We typically denote a set by capital letters of the English alphabet.
  Usually, E_i

- Examples: $E_1 = \{knife, spoon, fork\}$, $E_2 = \{2, 4, 6, 8\}$.

- The set E_2 could also be written as
  $E_2 = \{x | x \text{ are even whole numbers between 0 and 10}\}$.

Notations of Sets

Notation	Description
$a \in A$	The object a is an element of the set A.
$A \subseteq B$	Set A is a subset of set B. That is every element in A is also in B.
$A \subset B$	Set A is a proper subset of set B. Every element that is in set A is also in set B and there is at least one element in set B that is not in set A.
$A \cup B$	A set of all elements that are in A **or** B.
$A \cap B$	A set of all elements that are in A **and** B.
Ω	Called the **universal set**, all elements we are interested in.
$^{\sim}A$	The set of all elements that are in the universal set but are not in set A.
$\bigcup_i E_i$	$E_1 \cup E_2 \cup \ldots$, the union of multiple sets
$\bigcap_i E_i$	$E_1 \cap E_2 \cap \ldots$, the intersection of multiple sets

UNIVERSITY of HOUSTON
DEPARTMENT OF MATHEMATICS

Examples

The following are sets: $\Omega = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $E_1 = \{1, 2, 3, 4, 5, 6, 9, 10\}$, $E_2 = \{3, 4, 7, 8\}$, and $E_3 = \{2, 3, 9, 10\}$

UNIVERSITY of HOUSTON
DEPARTMENT OF MATHEMATICS

Examples

The following are sets: $\Omega = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$,
$E_1 = \{1, 2, 3, 4, 5, 6, 9, 10\}$, $E_2 = \{3, 4, 7, 8\}$, and $E_3 = \{2, 3, 9, 10\}$

UNIVERSITY of HOUSTON
DEPARTMENT OF MATHEMATICS

More Examples

The following are sets: $\Omega = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$,
$E_1 = \{1, 2, 3, 4, 5, 6, 9, 10\}$, $E_2 = \{3, 4, 7, 8\}$, and $E_3 = \{2, 3, 9, 10\}$

1. What elements are in $\sim E_2 \cap \sim E_3$?

2. What elements are in $E_2 \cap \sim E_1$?

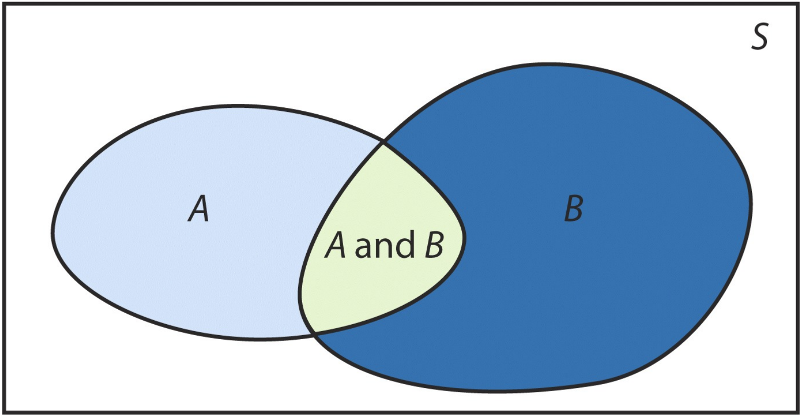
UNIVERSITY of HOUSTON
DEPARTMENT OF MATHEMATICS

Definitions

- A **Venn diagram** is a very useful tool for showing the relationships between sets.
- Venn diagrams consist of a rectangle with one or more shapes (usually circles) inside the rectangle.
- The rectangle represents all of the elements that we are interested in for a given situation. This set is the universal set.

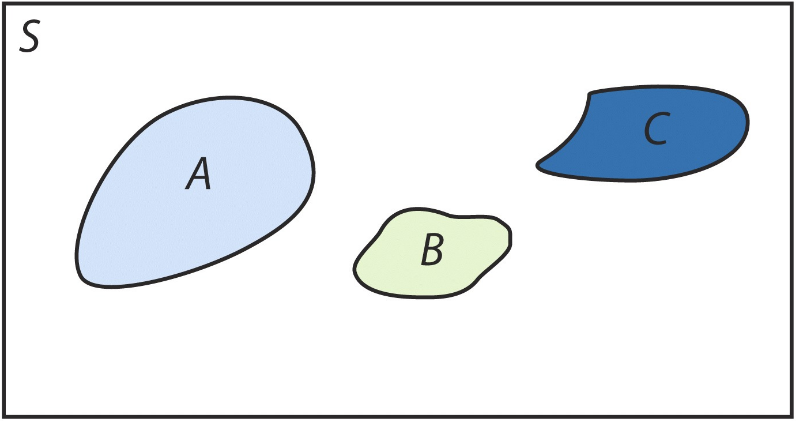
UNIVERSITY of HOUSTON
DEPARTMENT OF MATHEMATICS

Graph of Venn Diagrams



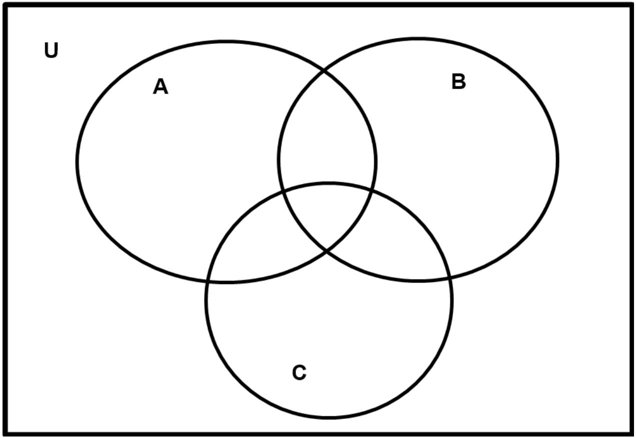
UNIVERSITY of HOUSTON
DEPARTMENT OF MATHEMATICS

Graph of Disjoint Events



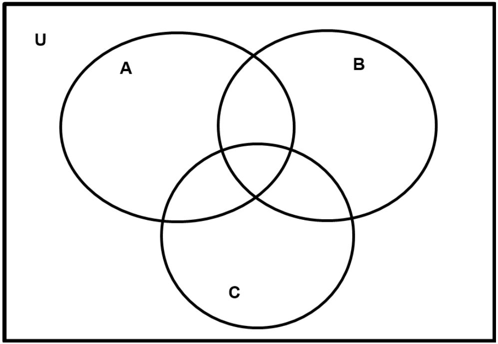
UNIVERSITY of HOUSTON
DEPARTMENT OF MATHEMATICS

$A \cap B \cap C$



Soft Drink Preference

A group of 100 people are asked about their preference for soft drinks. The results are as follows: 55 like Coke, 25 like Diet Coke, 45 like Pepsi, 15 like Coke and Diet Coke, 5 like all 3 soft drinks, 25 like Coke and Pepsi, 5 only like Diet Coke (nothing else). Fill in the the Venn diagram with these numbers.



UNIVERSITY of HOUSTON
DEPARTMENT OF MATHEMATICS