UNIVERSITY of HOUSTON

DEPARTMENT OF COMPUTER SCIENCE

COSC 4370 Fall 2023

Interactive Computer Graphics

M & W 5:30 to 7:00 PM

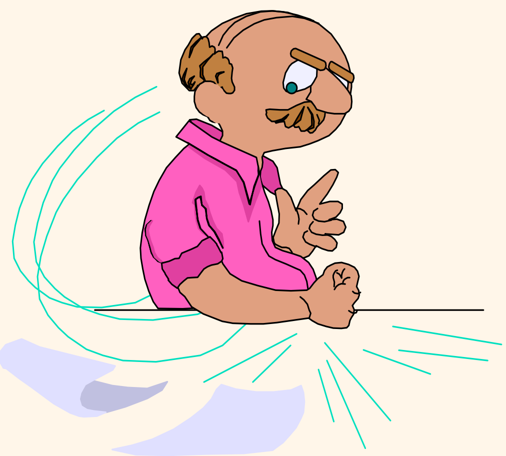
Prof. Victoria Hilford

PLEASE TURN your webcam ON

NO CHATTING during LECTURE

COSC 4370

5:30 to 7



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From 5:30 to 6:30 PM – 60 minutes.

09.11.2023 (M 5:30 to 7) (6)		Math Review 2
09.13.2023 (W 5:30 to 7) (7)	Homework 3	Lecture 4
09.18.2023 (M 5:30 to 7) (8)		PROJECT 1
09.20.2023 (W 5:30 to 7) (9)		EXAM 1 REVIEW
09.25.2023 (M 5:30 to 7) (10)		EXAM 1

COSC 4370 – Computer Graphics

Math Review 2

4

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CLASS PARTICIPATION - 15%

15% of Total

Class PARTICIPATION on Math Review 2

Not available until Sep 11 at 5:30pm | Due Sep 11 at 6:45pm | 100 pts

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LECTURES

MATH REVIEW 2 CLASS PARTICIPATION

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For the Grader: if the student did not submit, please skip and do not assign a ZERO (MISSING).

Points	100
Submitting	a file upload

Due	For	Available from	Until
Sep 11 at 6:45pm	Everyone	Sep 11 at 5:30pm	Sep 11 at 8:45pm

4.2 REVIEW OF VECTORS

> Not only Newton's laws, but also the other laws of physics, so far as we know today, have the two properties which we call invariance under translation of axes and rotation of axes. These properties are so important that a mathematical technique has been developed to take advantage of them in writing and using physical law . . . called vector analysis.
>
> *Richard Feynman*
> *(1918–1988)*

- A **vector v** has *length* and *direction*, but not position (relative to a coordinate system). It can be moved anywhere.

- A point P has *position* but not length and direction (relati to a coordinate system).

- A scalar **α** has only *size* (a number).

4.2.3 The Magnitude of a Vector and Unit Vectors

If a vector a is represented by the n coefficients (a_1, a_2, … a_n)

$$a = a_1 + a_2 + \ldots + a_n$$

The magnitude (length or size) of a is represented by |a| and it represents the distance from its tail to its head.

Based on the Pythagorean theorem it's magnitude |a| and it represents the is:

$$\boxed{|a| = \sqrt{a^2{}_1 + a^2{}_2 + \ldots + a^2{}_n}}$$

$a = (4, -2)$

$|a| = \sqrt{16 + 4} = \sqrt{20}$

$a = (1, -3, 2)$

$|a| = \sqrt{1 + 9 + 4} = \sqrt{14}$

If w is the vector from point A to point B, then |w| is the distance from A to B.

It is useful to scale a vector so that the result has unit length.

This is called normalizing a vector and the result is known as a unit vector.

$\mathbf{a} = (3, -4)$

We form the normalized version of a by scaling it with the value 1/ |a|.

$$\boxed{\hat{a} = a / |a|}$$

Denote it as a normalized vector by placing a ^ caret over the vector's name to make it more recognizable.

Clearly this is a unit vector: $\boxed{|\hat{a}| = 1}$

$|a| = \sqrt{9 + 16} = \sqrt{25} = 5$

$\hat{a} = a / |a| = a / 5 = (3/5, -4/5)$

$|\hat{a}| = \sqrt{(3/5)^2 + (-4/5)^2} = \sqrt{9/25 + 16/25} = \sqrt{25/25} = \sqrt{1} = 1$

$a = (1, -3, 2)$

$|a| = \sqrt{1 + 9 + 4} = \sqrt{14}$

$\hat{a} = a / |a| = (1, -3, 2) / \sqrt{14}$

$|\hat{a}| = \sqrt{(1/14 + 9/14 + 4/14)} = \sqrt{14/14} = 1$

$a = a_1 + a_2 + \ldots + a_n$

$$\boxed{|a| = \sqrt{a^2_1 + a^2_2 + \ldots + a^2_n}}$$

$$\boxed{\hat{a} = a \, / \, |a|}$$

$$\boxed{|\hat{a}| = 1}$$

We refer to a unit vector as a direction.

Any vector can be written as its magnitude times its direction.

If $\hat{a}$ is the normalized version of a vector a, vector a may always be

$$\boxed{a = |a| \, \hat{a}}$$

Normalize vector a = (8, 6)

$\hat{a}$

CLASS PARTICIPATION 1!
(Next Slide)

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Name: ______________________

Total score: [ ]

Class PARTICIPATION on Math Review 2.doc **ANSWER SHEET**

(Out of 100 points. Please record your own total score!)

(Attach as score.doc!)

1. (20 points)

Normalize vector **a** = (8, 6).

$\hat{a}$

Self Graded - correctly

Solution:

$|a| = 10$

(8/10, 6/10)

$\hat{a} = a \, / \, |a|$

EXAM format: (x/c, y/c)

4.3 THE DOT PRODUCT •

We could use up two Eternities in learning all that is to be learned about our own world and the thousands of nations that have arisen and flourished and vanished from it. Mathematics alone would occupy me eight million years.

Notebook #22, *Spring 1883–Sept. 1884*
Mark Twain
(1835–1910),

The dot product • of 2 vectors **v** and **w** is

$$\mathbf{v} \bullet \mathbf{w} = v_1 w_1 + v_2 w_2 + \ldots + v_n w_n$$

- is commutative: $\mathbf{v} \bullet \mathbf{w} = \mathbf{w} \bullet \mathbf{v}$
- is distributive: $(\mathbf{a} \pm \mathbf{b}) \bullet \mathbf{c} = \mathbf{a} \bullet \mathbf{c} \pm \mathbf{b} \bullet \mathbf{c}$
- is associative over multiplication by a **scalar s**: $(s\mathbf{a}) \bullet \mathbf{b} = s(\mathbf{a} \bullet \mathbf{b})$
- of a vector **b** with itself is it's *magnitude* squared: $\mathbf{b} \bullet \mathbf{b} = |\mathbf{b}|^2$

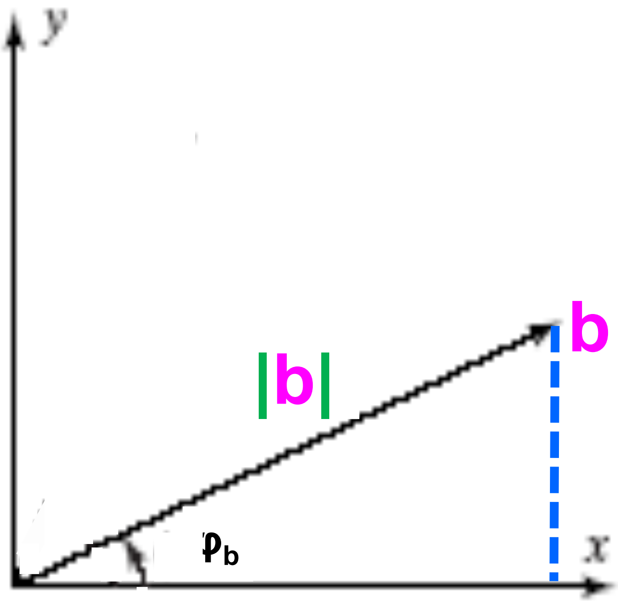
Vectors

Φ, φ phi fie

b_y

$\sin \varphi_b = b_y / |b|$

$b_y = |b| \sin \varphi_b$



$\cos \varphi_b = b_x / |b|$

$b_x = |b| \cos \varphi_b$

b_x

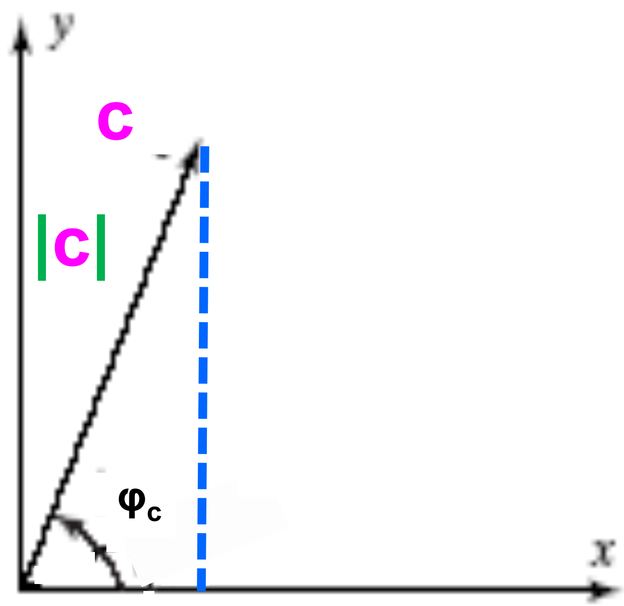
$$\mathbf{b} = (|\mathbf{b}|\cos \varphi_b,\ |\mathbf{b}|\sin \varphi_b)$$

b_x b_y

Vectors

Φ, φ phi fie

c_y



$\sin \varphi_c = c_y / |c|$

$c_y = |c| \sin \varphi_c$

$\cos \varphi_c = c_x / |c|$

$c_x = |c| \cos \varphi_c$

c_x

$$\mathbf{c} = (|\mathbf{c}|\cos \varphi_c, |\mathbf{c}|\sin \varphi_c)$$

c_x c_y

vector-vector Multiplication

vector * vector = **scalar**

- dot product, aka inner product

$$\mathbf{u} \bullet \mathbf{v}$$

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \bullet \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = (u_1 * v_1) + (u_2 * v_2) + (u_3 * v_3)$$

$\mathbf{b} = (\mathbf{b_x}, \mathbf{b_y})$ $\mathbf{c} = (\mathbf{c_x}, \mathbf{c_y})$

$$\mathbf{b} \bullet \mathbf{c} = \mathbf{b_x\, c_x + b_y\, c_y}$$

Applications: Angle Between 2 Vectors

$$\mathbf{b} = (\overbrace{|\mathbf{b}| \cos \varphi_b}^{b_x}, \overbrace{|\mathbf{b}| \sin \varphi_b}^{b_y}), \text{ and } \mathbf{c} = (\overbrace{|\mathbf{c}| \cos \varphi_c}^{c_x}, \overbrace{|\mathbf{c}| \sin \varphi_c}^{c_y})$$

$$\mathbf{b} \bullet \mathbf{c} = |\mathbf{b}||\mathbf{c}| \overbrace{\cos \varphi_c}^{c_x} \overbrace{\cos \varphi_b}^{b_x} + |\mathbf{b}||\mathbf{c}| \overbrace{\sin \varphi_c}^{c_y} \overbrace{\sin \varphi_b}^{b_y}$$

$$= |\mathbf{b}||\mathbf{c}| \cos (\varphi_c - \varphi_b) = |\mathbf{b}||\mathbf{c}| \cos \theta,$$

$$\cos(\alpha \pm \beta) = \cos\alpha \cos\beta \mp \sin\alpha \sin\beta$$

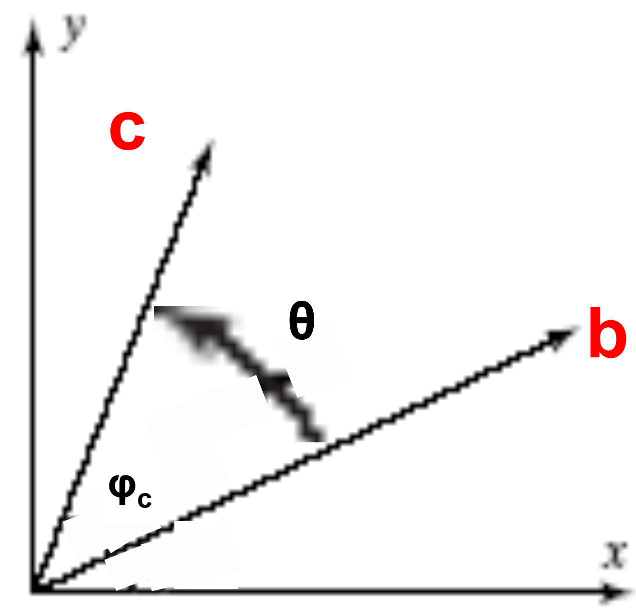
where $\theta = \varphi_c - \varphi_b$ is the smaller angle between **b** and **c**:

$$\mathbf{b} \bullet \mathbf{c} = |\mathbf{b}||\mathbf{c}| \cos \theta$$

$$\mathbf{b}/|\mathbf{b}| \bullet \mathbf{c}/|\mathbf{c}| = \cos \theta$$

$$\boxed{\hat{\mathbf{a}} = \mathbf{a} / |\mathbf{a}|}$$

$$\cos(\theta) = \hat{\mathbf{b}} \bullet \hat{\mathbf{c}}$$



Angle Between 2 Vectors

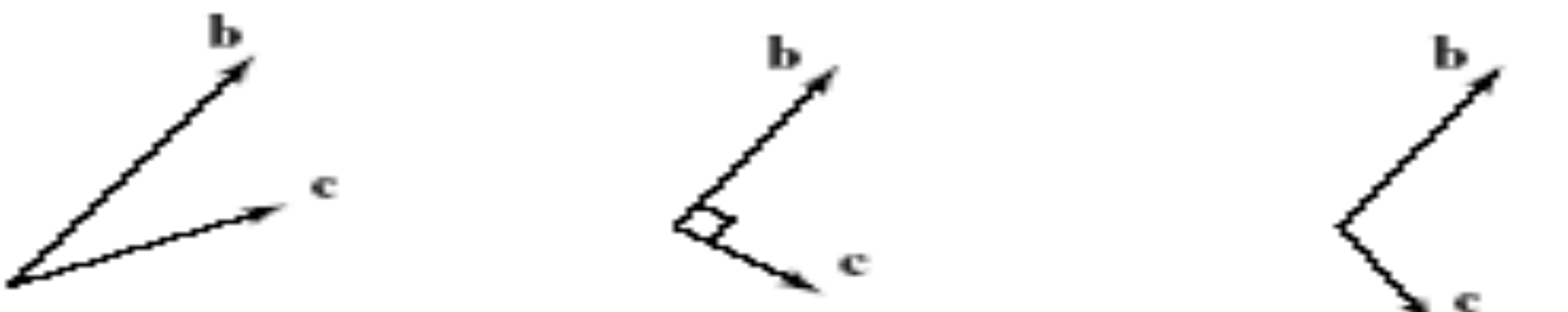
$$\mathbf{b} \bullet \mathbf{c} = |\mathbf{b}||\mathbf{c}| \cos\theta$$

The cosine is

positive if $|\theta| < 90^{o}$,
zero if $|\theta| = 90^{o}$,
negative if $|\theta| > 90^{o}$.

$$\cos(\theta) = \hat{\mathbf{b}} \bullet \hat{\mathbf{c}}$$

Vectors **b** and **c** are **perpendicular (orthogonal, normal)**
if $\mathbf{b} \bullet \mathbf{c} = 0$



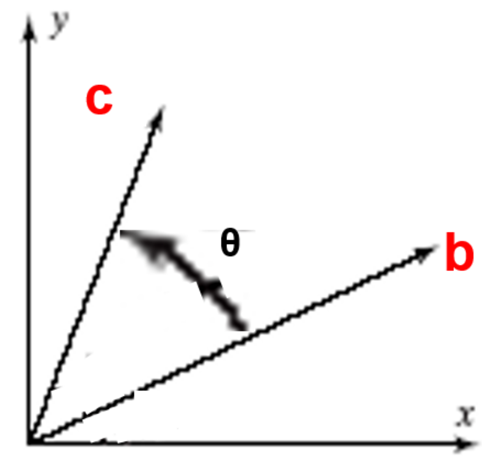
$\mathbf{b} \bullet \mathbf{c} > 0$ $\mathbf{b} \bullet \mathbf{c} = 0$ $\mathbf{b} \bullet \mathbf{c} < 0$

Example

$\mathbf{b} \bullet \mathbf{c} = |\mathbf{b}||\mathbf{c}| \cos \theta$

$\cos \theta = (\mathbf{b} \bullet \mathbf{c}) / |\mathbf{b}||\mathbf{c}|$

$\cos \theta = \hat{\mathbf{b}} \bullet \hat{\mathbf{c}}$



Find the angle between vectors b and c where

b = (4, 3) and c = (-4, 3)

CLASS PARTICIPATION 2!
(Next Slide)

Inverse Cosine Calculator arccos(x) (dqydj.com)

The Un

2. (20 points) (Dot product)

$\mathbf{b} \bullet \mathbf{c} = |\mathbf{b}||\mathbf{c}| \cos\theta$

$\boxed{\cos\theta = (\mathbf{b} \bullet \mathbf{c}) / |\mathbf{b}||\mathbf{c}|}$

$\cos\theta = \mathbf{b} \bullet \mathbf{c}$

Find the angle between vectors **b** and **c** where

b = (4, 3) and **c** = (-4, 3)

Answer:

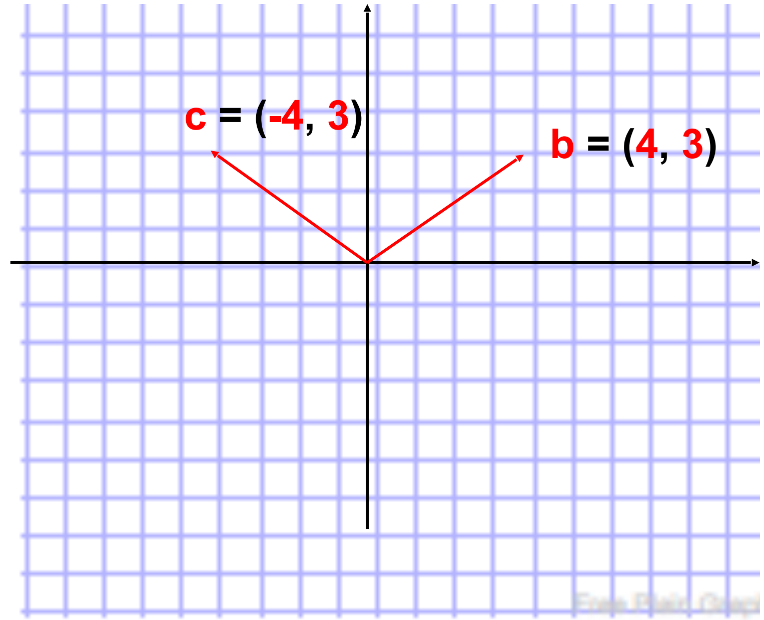
$|b| = 5 \quad |c| = 5$

$b \bullet c = 4 * -4 + 3 * 3 = -7$

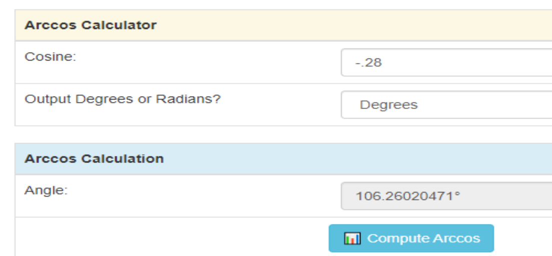
$\cos\theta = -7 / 25 = -0.28$

negative if $|\theta| > 90^0$

$\theta \sim= 100$



Inverse Cosine Calculator



Self Graded - correctly

Standard Unit Vectors

In 3D **they are**

i = (1, 0, 0), **j** = (0, 1, 0), and **k** = (0, 0, 1)

k always points in the positive z direction

In **2D** **they are**

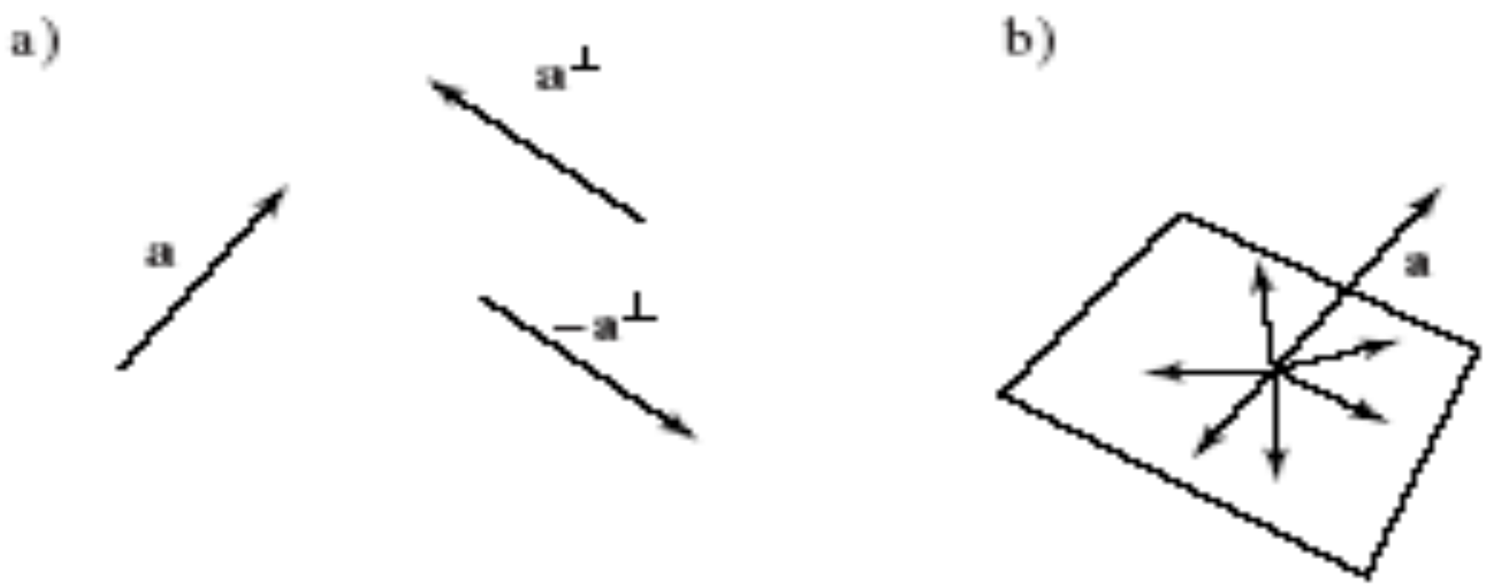
i = (1, 0) and **j** = (0, 1)

The unit vectors are orthogonal.

Finding a 2D "Perpendicular" Vector

If vector **a** = (a_x, a_y), then the vector perpendicular to **a** in the *counterclockwise* sense is $a^{\perp}$ = ($-a_y$, a_x), and in the *clockwise* sense it is $-a^{\perp}$.

In 3D, any vector in the plane perpendicular to **a** is a "perpendicular" vector.



Properties of $\perp$

$(\mathbf{a} \pm \mathbf{b})^{\perp} = \mathbf{a}^{\perp} \pm \mathbf{b}^{\perp}$;

$(s\mathbf{a})^{\perp} = s(\mathbf{a}^{\perp})$;

$(\mathbf{a}^{\perp})^{\perp} = -\mathbf{a}$

$\mathbf{a}^{\perp} \cdot \mathbf{b} = -\mathbf{b}^{\perp} \cdot \mathbf{a} = -a_y b_x + a_x b_y$;

$\mathbf{a}^{\perp} \bullet \mathbf{a} = \mathbf{a} \bullet \mathbf{a}^{\perp} = 0$;

$|\mathbf{a}^{\perp}| = |\mathbf{a}|$;

Example

Are **b** = (3, 4, 1) and **c** = (0, -1, 4) perpendicular?

CLASS PARTICIPATION 3!
(Next slide)

The Un

3. (20 points) (Dot product)

Are **b** = (3, 4, 1) and **c** = (0, -1, 4) perpendicular?

Answer:

$$\mathbf{b} \bullet \mathbf{c} = 0 \quad ?????$$

Self Graded - correctly

$$\mathbf{b} \bullet \mathbf{c} = (3, 4, 1) \bullet (0, -1, 4) = 0 - 4 + 4 = 0$$

YES

Example

Given **b** = (3, 4) and **c** = (2, 1) compute:

$\mathbf{b} \bullet \mathbf{c}$

$\mathbf{b}^{\perp} \bullet \mathbf{c}$?????

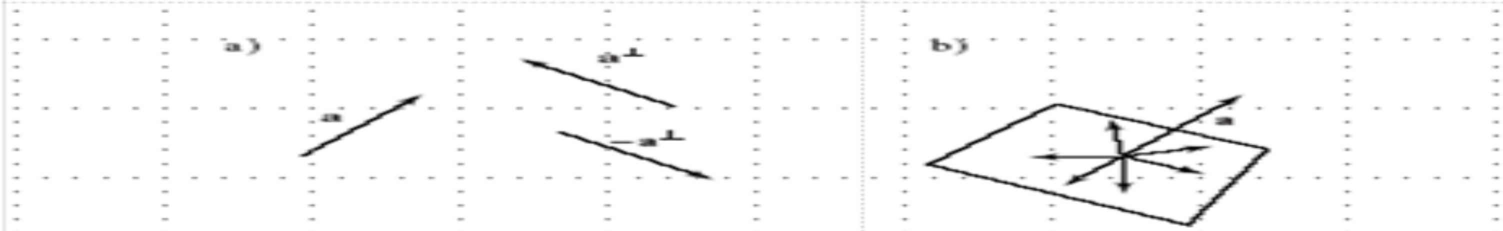
CLASS PARTICIPATION 4!
(Next slide)

The Un

Finding a 2D "Perpendicular" Vector

If vector **a** = (a_x, a_y), then the vector **perpendicular** to **a** in the *counterclockwise* sense is $a^{\perp}$ = ($-a_y$, a_x), and in the *clockwise* sense it is $-a^{\perp}$.

In **3D**, any vector in the **plane perpendicular** to **a** is a "**perpendicular**" vector.



4. (20 points) (Dot product)

Given **b** = (3, 4) and **c** = (2, 1) compute:

$$\boxed{\begin{array}{l} b \bullet c \\ b^{\perp} \bullet c \end{array}}$$

Answer:

$b^{\perp}$ = (-4, 3)

Self Graded - correctly

$b \bullet c = (3, 4) \bullet (2, 1) = 6 + 4 = 10$

$b^{\perp} \bullet c = (-4, 3) \bullet (2, 1) = -8 + 3 = -5$

4.5 REPRESENTATIONS OF KEY GEOMETRIC OBJECTS

All that transcends geometry, transcends our comprehension.

Blaise Pascal
(1623–1662)

- **Lines** and **planes** are essential to graphics, and we must learn how to represent them

  – i.e., how to find an equation or function that **distinguishes points on the line** or **plane** **from points off the line** or **plane**.

Lines

slope-intercept form ????

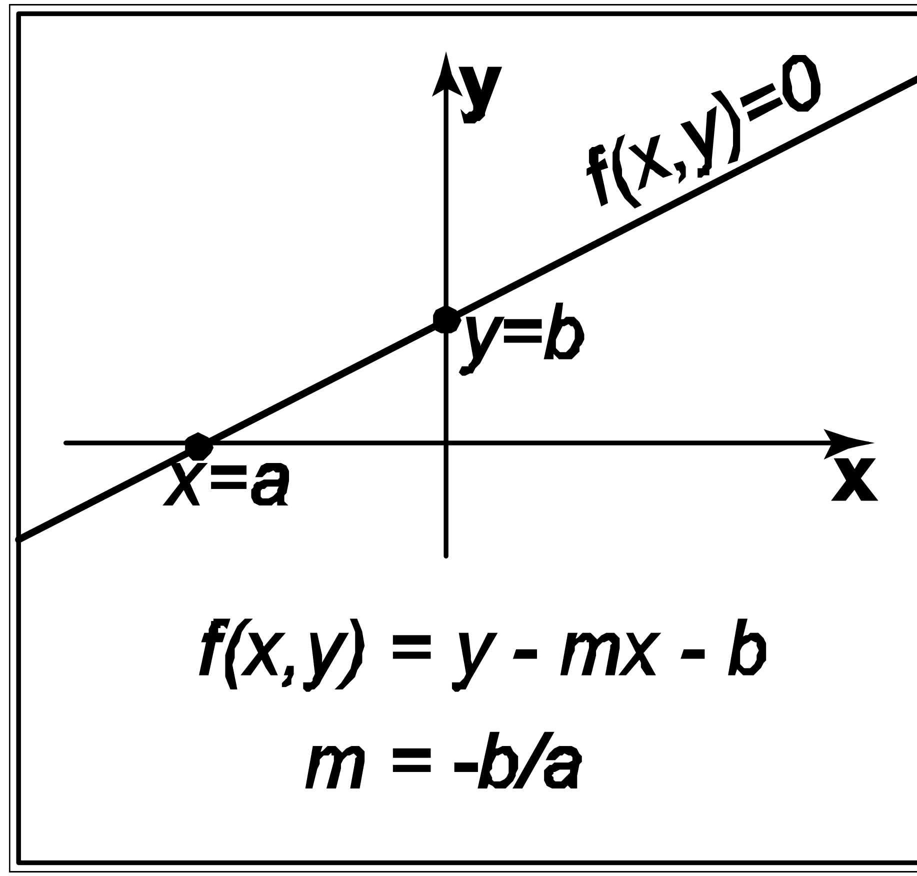
$$y = mx + b$$

implicit form ????

$$y - mx - b = 0$$

$$Ax + By + C = 0$$

$$f(x,y) = 0$$



2D Parametric Lines

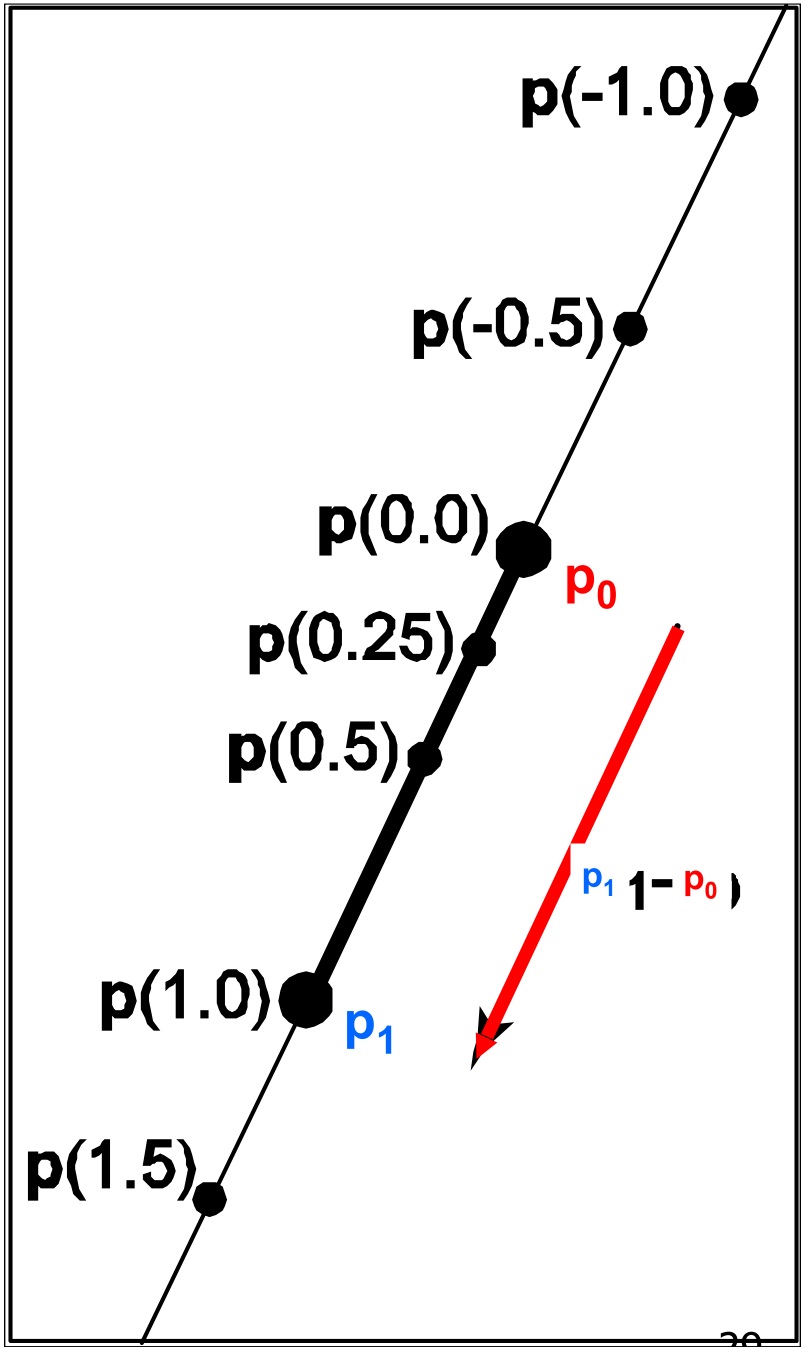
????

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x_0 + t(x_1 - x_0) \\ y_0 + t(y_1 - y_0) \end{bmatrix}$$

$$\mathbf{p}(t) = \mathbf{p}_0 + t(\mathbf{p}_1 - \mathbf{p}_0)$$

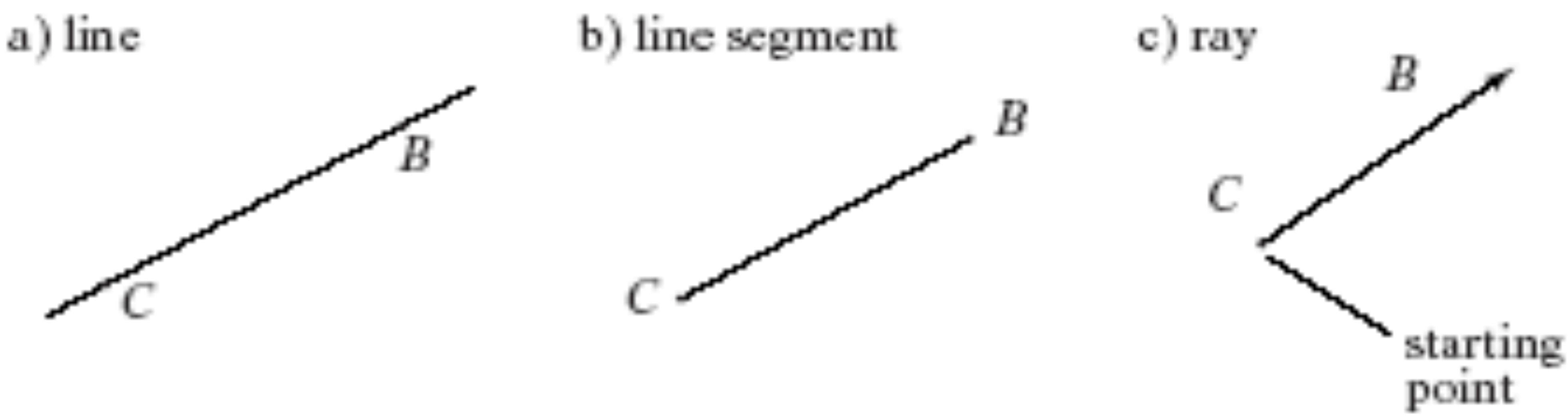
start at point p_0, go towards $\mathbf{p}_1$, according to parameter **t**

$\mathbf{p}(0) = p_0$, $\mathbf{p}(1) = \mathbf{p}_1$



Representing Lines

- A line passes through 2 points and is infinitely long.
- A line segment has 2 endpoints.
- A ray has a single endpoint.



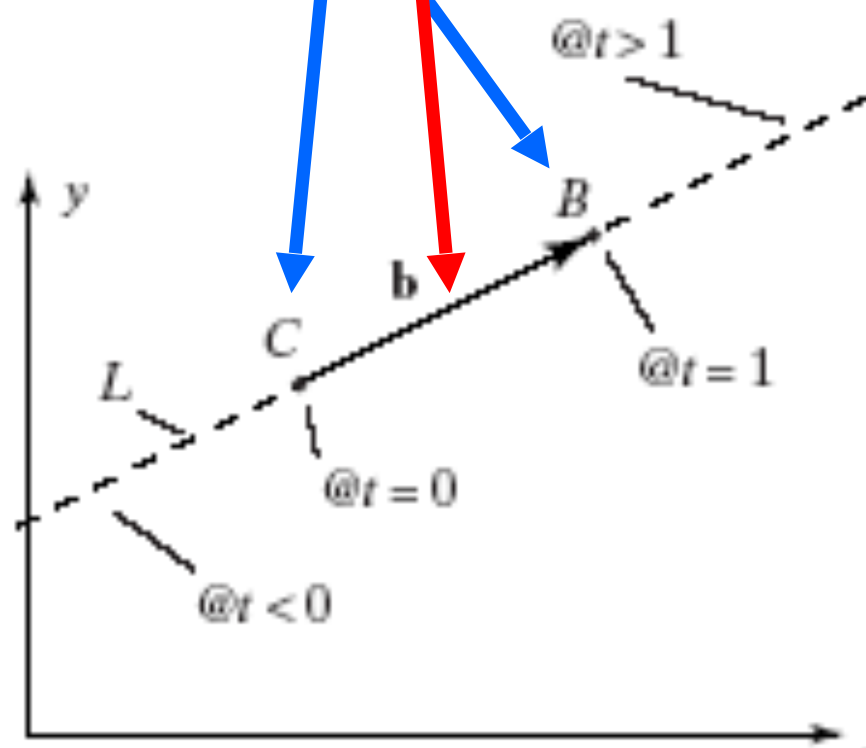
Representing Lines - 1

There are 2 useful line representations:

Parametric form: we have 2 points, B and C, on the line.

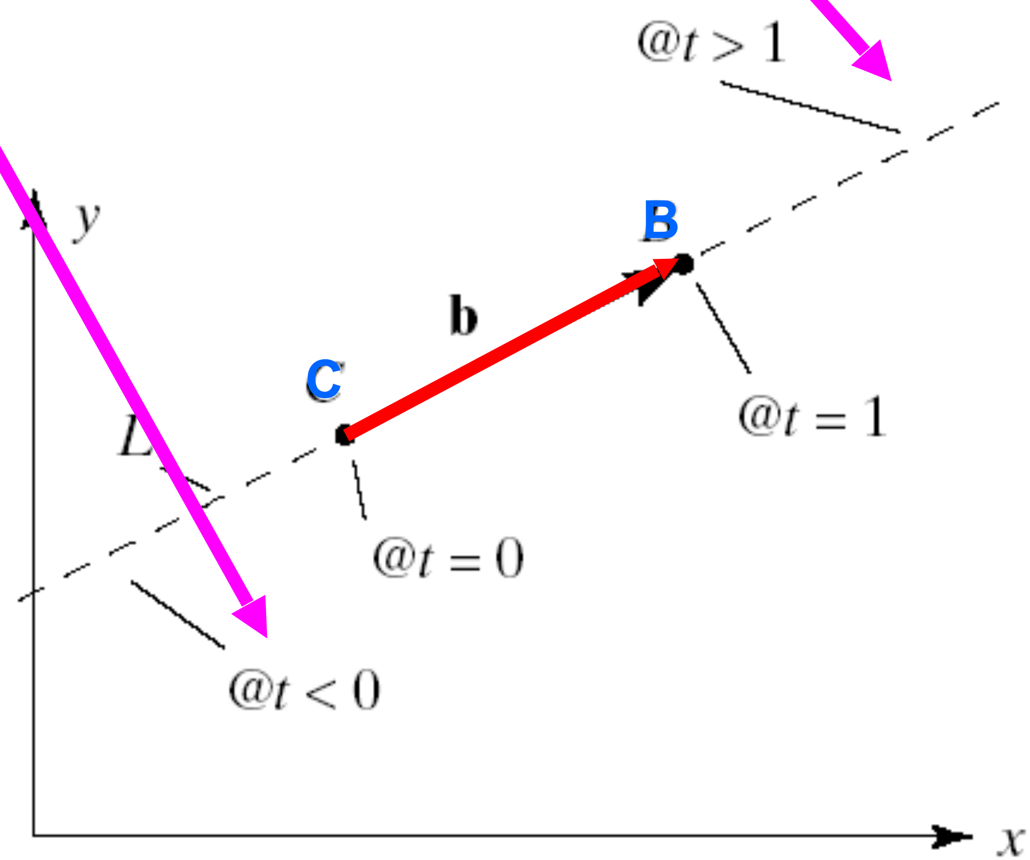
P(x, y) is on the line when P = C + **b**t, where **b** = B – C.

- $0 \le t \le 1$: line segment;
- $-\infty \le t \le \infty$: line;
- $-\infty \le t \le 0$ or $0 \le t \le \infty$: ray.



Representing Lines

As t varies so does the position of $L(t)$ along the line. (Let t be time)
If $t = 0$, $L(\mathbf{0}) = C$ so at $t = 0$ we are at point C.
At $t = 1$, $L(\mathbf{1}) = C + (B - C) = B$.
If $t > 1$ this point lies somewhere on the opposite side of B from C, and when $t < 0$ it lies on the opposite side of C from B.



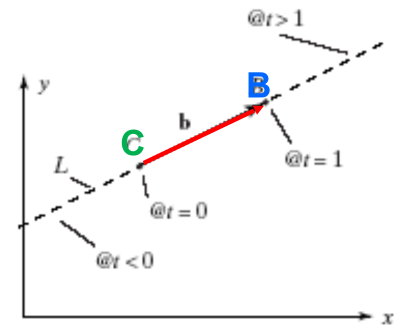
$L(t) = C + t\,(B - C)$

$L(t) = C + t\,\mathbf{b}$

$L(t) = C + t\,(B - C)$

$L(t) = C + t\,\mathbf{b}$

Representing Lines



$L(t)$ lies fraction t of the way between C and B when t lies between 0 and 1.

when $t = 1/2$ the point $L(0.5)$ is the **midpoint** between C and B, and

when $t = 0.3$ the point $L(0.3)$ is **30%** of the way from C to B:

$|L(t) - C| = |\mathbf{b}|\,|t|$ and $|B - C| = |\mathbf{b}|$, so the value of $|t|$ is the ratio of the distances $|L(t) - C|$ to $|B - C|$.

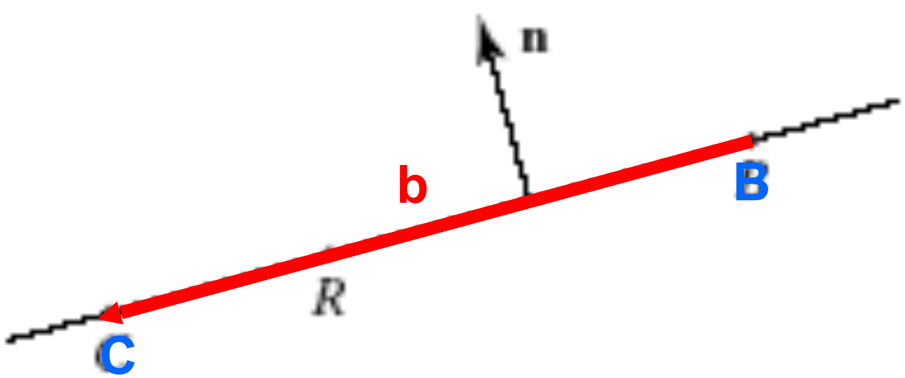
Representing Lines - 2

Point-normal (implicit) form: (this works in **2D only**; the **3D** version requires 2 equations.)

- $fx + gy = 1$ gives $(f, g) \bullet (x, y) = 1$.

Given B and C on the line, **b** = B – C gives $\mathbf{b}^{\perp} = \mathbf{n}$, which is perpendicular to R – C. (R is any point (x, y) on the line.)

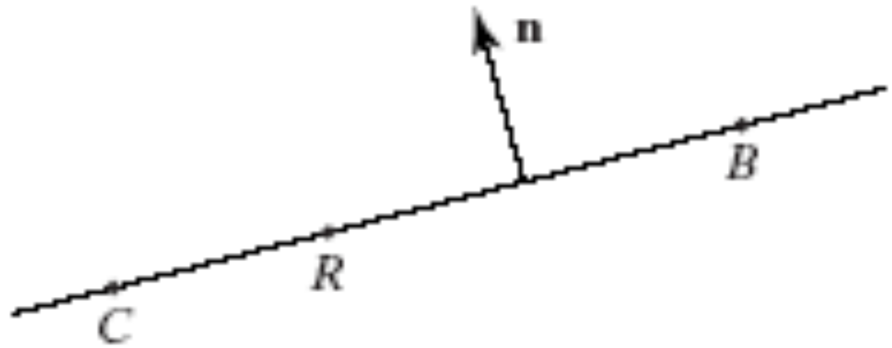
The equation is $\mathbf{n} \bullet (R - C) = 0$.



Changing Representations

$L(t) = C + t\,(B - C)$

$L(t) = C + t\,\mathbf{b}$



From parametric form $f\,x + g\,y = 1$ to point-normal form:

Writing

$(f, g) \bullet (x, y) = 1$, $\mathbf{n}$ is (f, g) (or any multiple thereof).

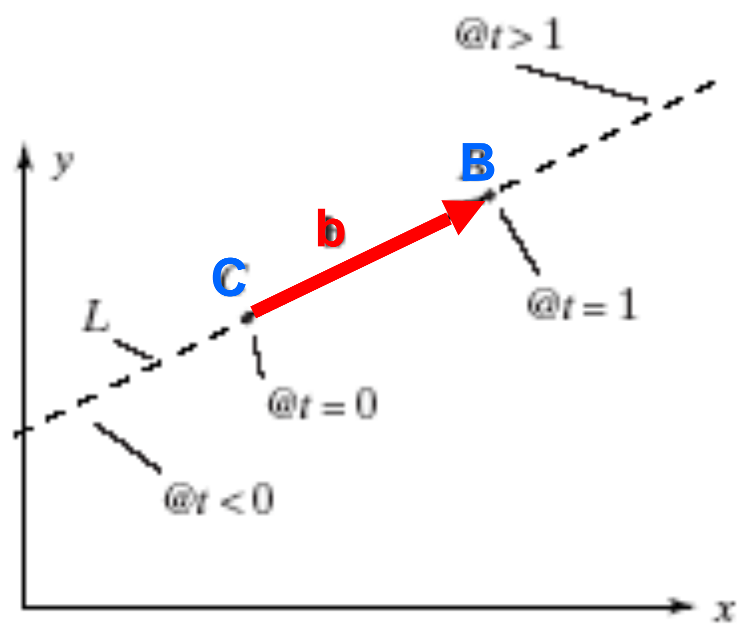
From point-normal form $\mathbf{n} \bullet (R - C) = 0$ to parametric form:

$$L(t) = C + \mathbf{n}^{\perp}\mathbf{t}$$

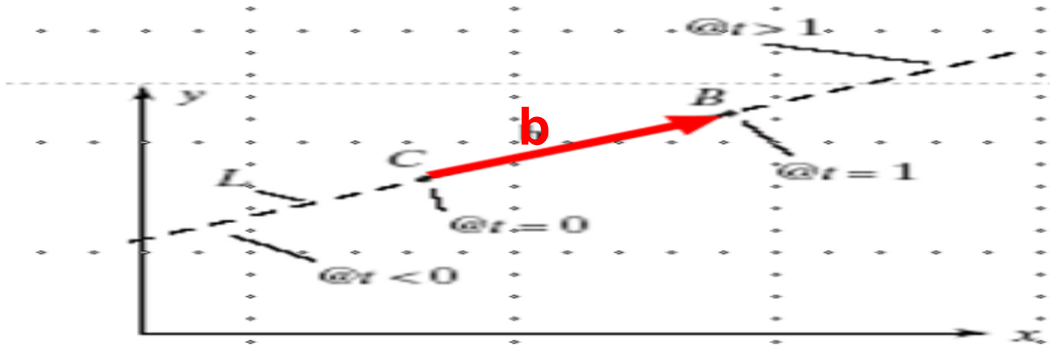
Example

$L(t) = C + \mathbf{b}t = C + (B - C)t$

Find the L(t) parametric form for the line that passes through C = (3, 5) and B = (2, 7)



CLASS PARTICIPATION 5!
(Next slide)



L(t)= C + bt = C + (B-C)t

5. (20 points) (Dot product)

C = (3,5)   B = (2,7)

Find the L(t) parametric form for the line that passes through C = (3, 5) and B = (2, 7)

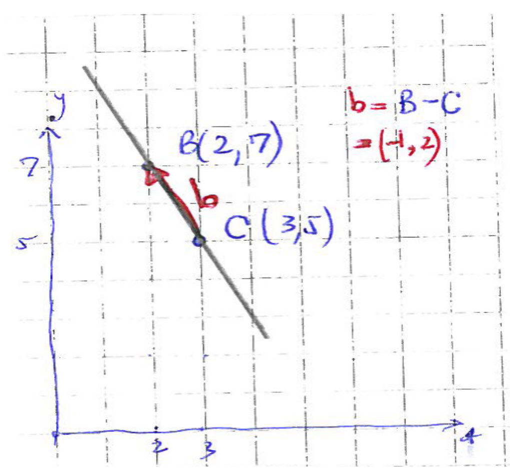
Answer:

Build vector **b** = B – C = (-1, 2) to obtain parametric form

L(t) = C + **b** t   = (3, 5) + **(-1, 2)** t

L(t) = (3 – t, 5 + 2t).

Self Graded - correctly



L(t) = C + bt =

= (3,5) + (-1,2) t

= (3 - t, 5 + 2t)

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Warning:

TA, at random, will inspect the Uploaded document.

If you score is not honestly entered you will get a zero.

Please rename document to score.doc (example 100.doc)
Warning: if you score is not honestly honest you will get a zero.

Total score: ☐

Class PARTICIPATION on Math Review 2.doc **ANSWER SHEET**
(Out of 100 points. Please record your own total score!)
(Attach as score.doc!)

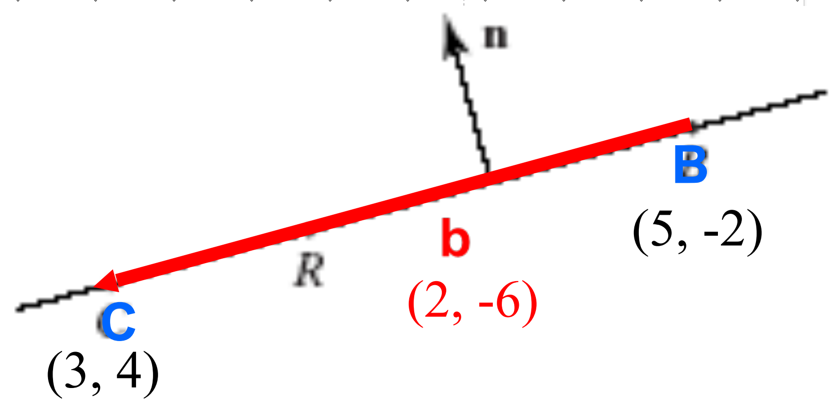
VH, it closes at 6:45 PM

Example

b = **B** – **C**   **n** • $(R - C) = 0$

Suppose line L passes through points C = (3, 4) and B = (5, -2). Choosing R(x,y) as the point on the line L, the point normal form of the line L is:

If vector **a** = (a_x, a_y), then the vector perpendicular to **a** in the *counterclockwise* sense is $a^{\perp} = (-a_y, a_x)$, and in the *clockwise* sense it is $-a^{\perp}$.



Build vector **b** = B – C = (2, -6) thus $\mathbf{b}^{\perp} = (6, 2)$

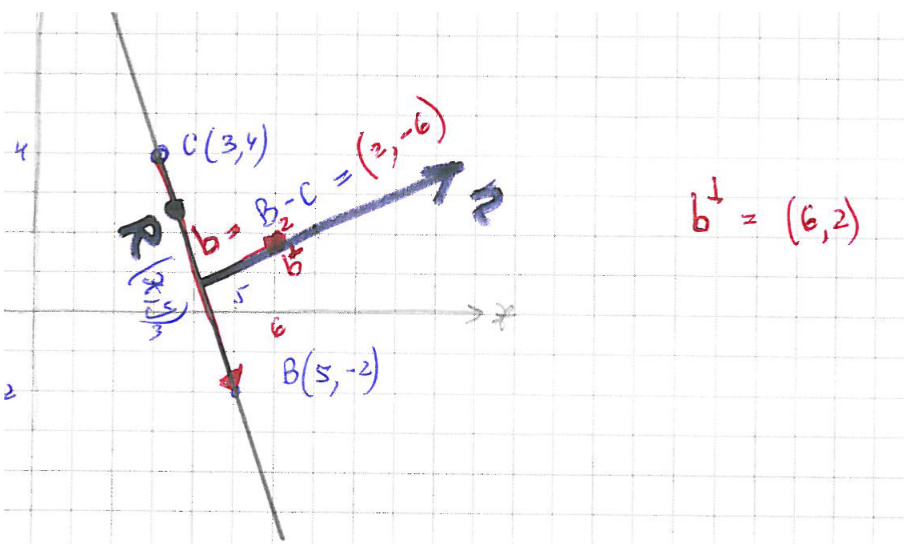
n • (R – C) = 0.

(6, 2) • ((x, y) – (3, 4)) = 0

(6, 2) • (x - 3, y - 4) = 0

6 · (x – 3) + **2** · (y - 4) = 0

6 · x + **2** · y - 26 = 0



C = (3,4)   The point normal

$n \cdot (R - C) = 0$

$(6,2) \cdot ((x,y) - (3,4)) = 0$

$(6,2) \cdot (x-3, y-4) = 0$

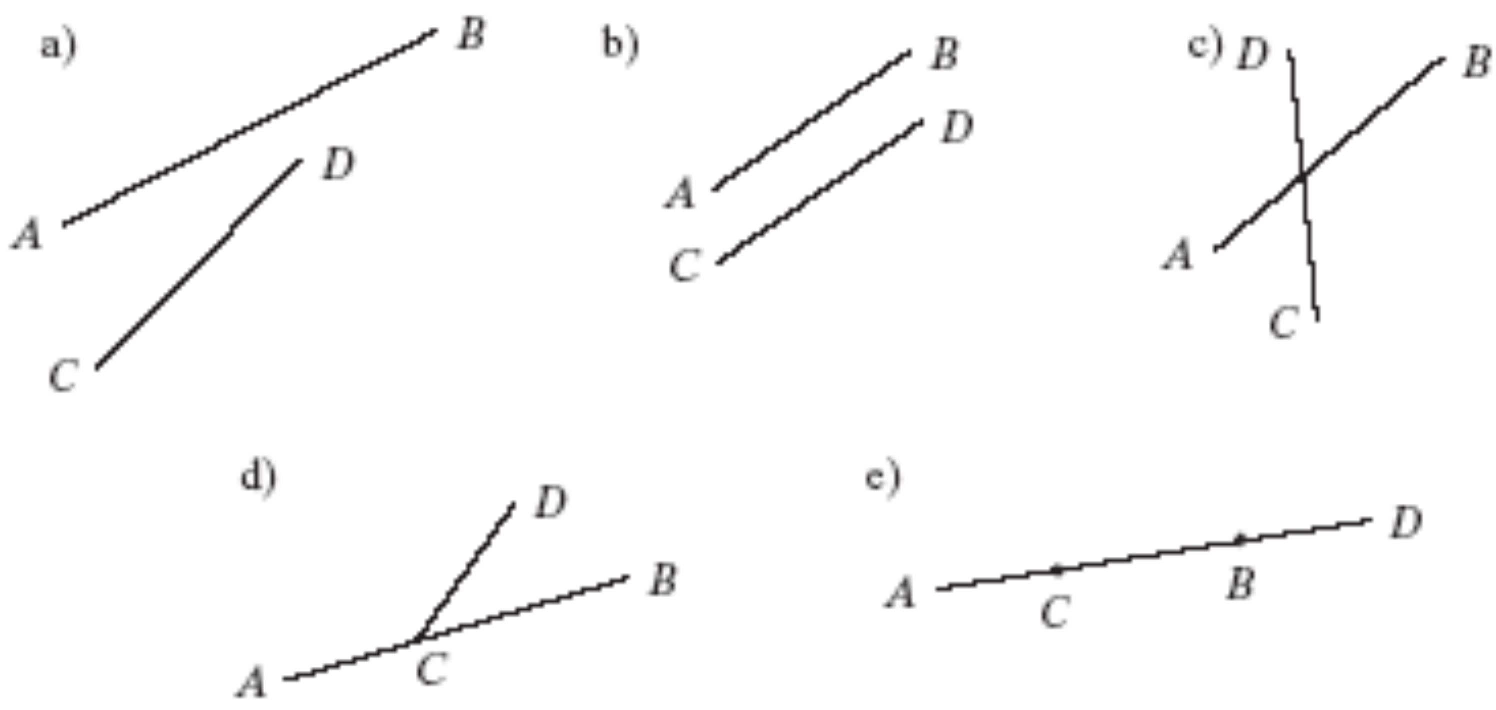
$6(x-3) + 2(y-4) = 0$

$6x + 2y = 26$

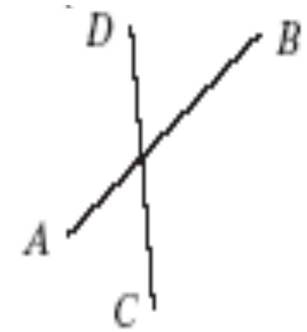
4.6 FINDING THE INTERSECTION OF TWO LINE SEGMENTS

> If the facts don't fit the theory, change the facts.
>
> *Albert Einstein (1879–1955)*

- They can miss each other (a and b), overlap in one point (c and d), or even overlap over some region (e). They may or may not be parallel.



Intersection of 2 Line Segments

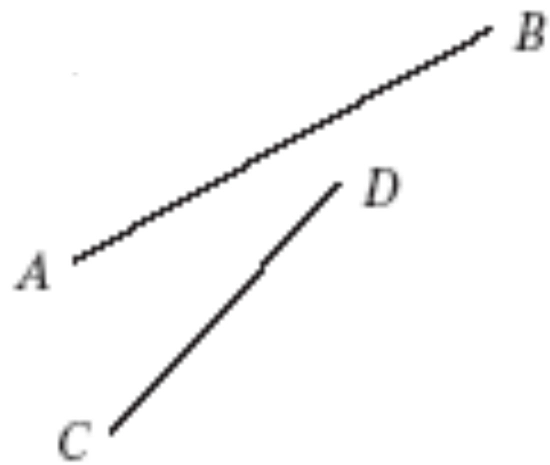


Every line segment has a **parent line**, the infinite line of which it is part. Unless two parent lines are parallel, they will intersect at some point in 2D. **We locate this point.**

Using parametric representations for each of the line segments in question, call ***AB*** the segment from *A* to *B*. Then $AB(t) = A + \mathbf{b}\, t$, where for convenience we define $\mathbf{b} = B - A$.

As t varies from 0 to 1 each point on the finite line segment is crossed exactly once.

Intersection of 2 Line Segments



$AB(t) = A + \mathbf{b}t, \ \mathbf{b} = B - A$

$CD(u) = C + \mathbf{d}u, \ \mathbf{d} = C - D$

At the intersection, $A + \mathbf{b}t = C + \mathbf{d}u$,

or $\mathbf{b}t = \mathbf{c} + \mathbf{d}u$, with $\mathbf{c} = C - A$

Taking dot product $\bullet$ with $\mathbf{d}^{\perp}$ gives

$a^{\perp} \bullet a = a \bullet a^{\perp} = 0$

$$\mathbf{b} \bullet \mathbf{d}^{\perp} t = \mathbf{c} \bullet \mathbf{d}^{\perp} + \mathbf{d} \bullet \mathbf{d}^{\perp} u$$

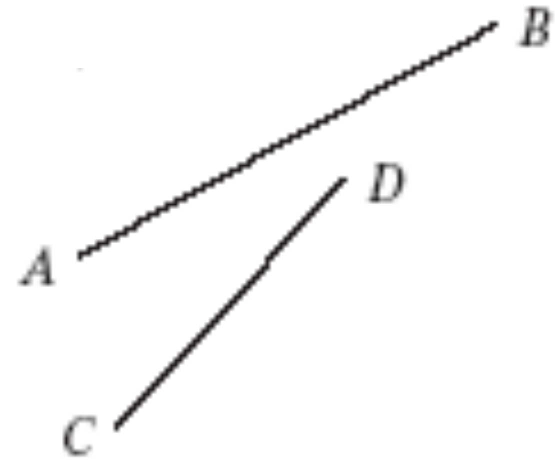
Taking dot product $\bullet$ with $\mathbf{b}^{\perp}$ gives

$$- \mathbf{c} \bullet \mathbf{b}^{\perp} = \mathbf{d} \bullet \mathbf{b}^{\perp} u$$

Intersection of 2 Line Segments

$AB(t) = A + \mathbf{b}t$, $\mathbf{b} = B - A$

$CD(u) = C + \mathbf{d}u$, $\mathbf{d} = C - D$



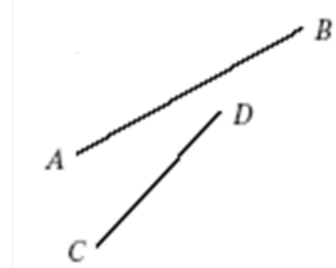
$$\mathbf{b} \bullet \mathbf{d}^{\perp} t = \mathbf{c} \bullet \mathbf{d}^{\perp}$$

$$- \mathbf{c} \bullet \mathbf{b}^{\perp} = \mathbf{d} \bullet \mathbf{b}^{\perp} u$$

Intersection of 2 Line Segments

$\mathbf{b} \bullet \mathbf{d}^{\perp} t = \mathbf{c} \bullet \mathbf{d}^{\perp}$

$-\mathbf{c} \bullet \mathbf{b}^{\perp} = \mathbf{d} \bullet \mathbf{b}^{\perp} u$



- **Case1:** $\mathbf{b} \bullet \mathbf{d}^{\perp} = 0$ means $\mathbf{d} \bullet \mathbf{b}^{\perp} = 0$ and the lines are either the same line or parallel lines.

  There is no intersection.

- **Case 2:** $\mathbf{b} \bullet \mathbf{d}^{\perp} \neq 0$ gives $t = \mathbf{c} \bullet \mathbf{d}^{\perp} / \mathbf{b} \bullet \mathbf{d}^{\perp}$

  and $u = -\mathbf{c} \bullet \mathbf{b}^{\perp} / \mathbf{d} \bullet \mathbf{b}^{\perp}$

- In this case, the line segments intersect if and only if $0 \le t \le 1$ and $0 \le u \le 1$, at

$$P = A + \mathbf{b}(\mathbf{c} \bullet \mathbf{d}^{\perp} / \mathbf{b} \bullet \mathbf{d}^{\perp}).$$

$AB(t) = A + \mathbf{b}t$

Example

For the two segments AB and CD, determine whether the segments intersect, and if so, where.

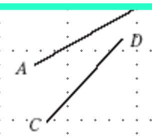
A = (1, 4), and B = (7,1/2),

C = (5, 0), and D = (0, 7).

Draw these segments.

$AB(t) = A + bt$, $b = B – A$

$CD(u) = C + du$, $d = C – D$

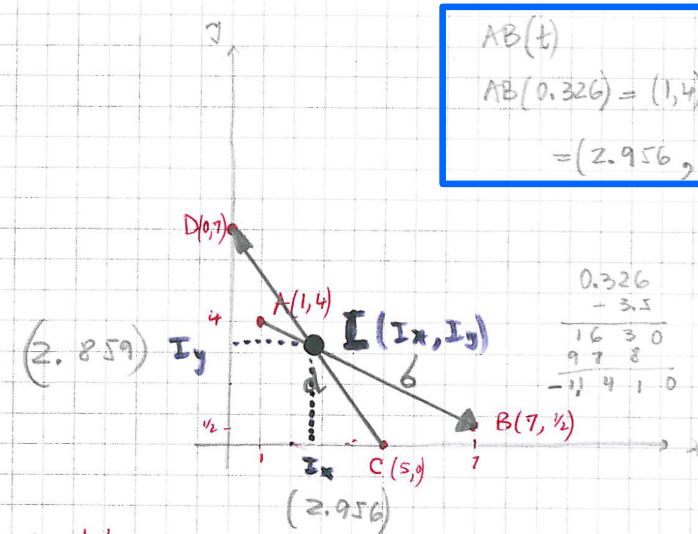


At the intersection, $A + bt = C + du$,

or $\boxed{bt = c + du}$ with $c = C – A$

Example

A	(1, 4)
B	(7, 0.5)
C	(5, 0)
D	(0, 7)
b	(6, -3.5)
c	(4, -4)
d	(-5, 7)
b $^{\perp}$	(3.5 , 6)
d $^{\perp}$	(-7, -5)
d · **b**$^{\perp}$	-24.5
t	0.326531
u	0.408163
I	(2.959, 2.857)



$AB(t)$

$AB(0.326) = (1,4) + (6,-3.5)\cdot 0.326$

$= (2.956,\ 2.859)$

$AB(t) = A + bt$

$AB(t) = (1,4) + (6,-3.5)\,t$

$b = (7, ½) - (1,4) = (6, -3.5)$

$CD(u) = C + du$

$CD(u) = (5,0) + (-5,7)\,u$

$d = D - C = (0,7) - (5,0) = (-5, 7)$

$A + bt = C + du$

$bt = (C - A) + du$

$c = (5,0) - (1,4) = (4,-4)$

$(6,-3.5)\,t = (4,-4) + (-5,7)\,u$

$6t = 4 - 5u$

$-3.5t = -4 + 7u$

$t = \frac{4-5u}{6}$

$-3.5 \cdot \frac{4-5u}{6} = -4 + 7u$

$-\frac{14}{6} + \frac{18.5u}{6} - 7u = -4$

$18.5u - 42u = -24 + 14 = -10$

$-23.5u = -10 \Rightarrow \boxed{u = 0.408}$

$\boxed{t = 0.326}$

⊥

Example

Suppose we have a line segment defined by the 2 endpoints $p_1 = [x_1, y_1]^T$ and $p_2 = [x_2, y_2]^T$.

The parametric form of the line segment can be defined in a **matrix form** as:

$$p(\alpha) = p_1 + \alpha\,(p_2 - p_1)$$

or as **scalar equations**:

$$x(\alpha) = x_1 + \alpha\,(x_2 - x_1)$$

$$y(\alpha) = y_1 + \alpha\,(y_2 - y_1)$$

Example

Suppose we have a **line segment** defined by the 2 endpoints $P_1 = [x_1, y_1]^T$ and $P_2 = [x_2, y_2]^T$.

As the parameter α varies from 0 to 1, we move along the segment from P_1 to P_2.

Negative values of α yield points on the line on the other side of P_1 from P_2.

Similarly, values of $\alpha > 1$ give points on the line past P_2 going off to infinity.

Example

Suppose we have a **line segment** defined by the 2 endpoints $p_1 = [x_1, y_1]^T$ and $p_2 = [x_2, y_2]^T$.

Find α for the intersection of **this line segment** with

line $\mathbf{y} = \mathbf{y}_{\mathbf{max}}$

$y_{max} = y(\alpha) = y_1 + \alpha\,(y_2 - y_1)$

To avoid division $\boldsymbol{\alpha} = (\mathbf{y_2 - y_1}) / (\mathbf{y_{max} - y_1})$

$\alpha = (y_2 - y_1) / (y_{max} - y_1)$

Keep $\alpha\,\Delta Y = \Delta Y_{max}$

Knowing α allows us to get x coordinate of the intersection point:

$\boldsymbol{\Delta Y} = (\mathbf{y_2 - y_1})$

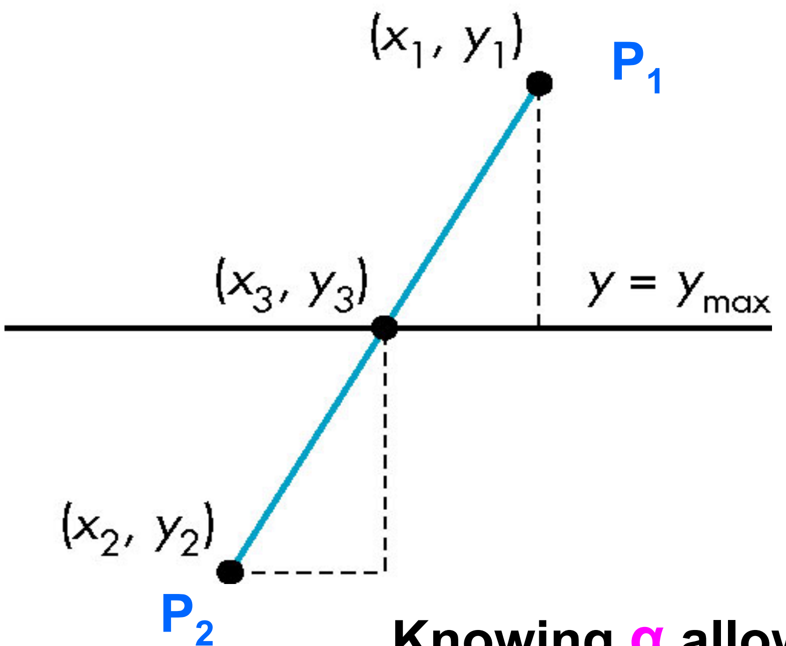
$x = x(\alpha) = x1 + \alpha\,(x2 - x1)$

$\boldsymbol{\Delta Y_{max}} = (\mathbf{y_{max} - y_1})$

Same Example

Suppose we have a **line segment** defined by the 2 endpoints **P**$_1$ = $[x_1, y_1]^T$ and **P**$_2$ = $[x_2, y_2]^T$.

Find intersection point (x_3, y_3) for the intersection of **this line segment from P$_1$ to P$_2$** with **line y = y$_{max}$**



$x_3 = x(\alpha) = x1 + \alpha\,(x2 - x1)$

$y_3 = y_{max} = y(\alpha) = y_1 + \alpha\,(y_2 - y_1)$

$\alpha = (y_2 - y_1) / (y_{max} - y_1)$

Knowing α allows us to get x_3 coordinate of the intersection point:

$x_3 = x1 + \alpha\,(x2 - x1) =$

$= x1 + (y2 - y1) / (ymax - y1)\,(x2 - x1)$

4.4 THE CROSS PRODUCT OF TWO VECTORS x

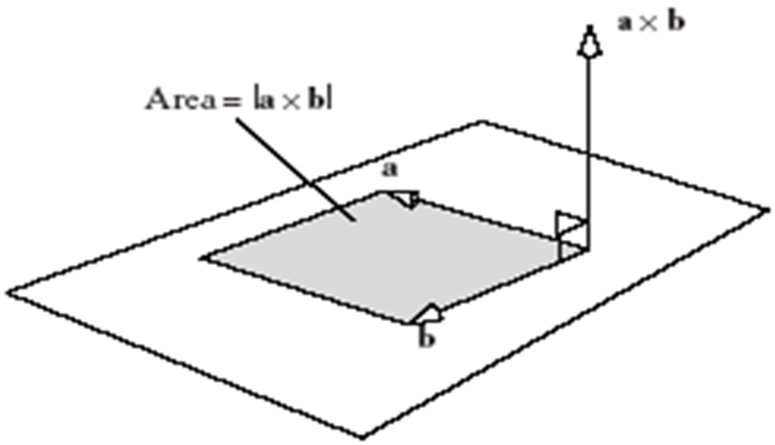
Let us grant that the pursuit of mathematics is a divine madness of the human spirit

Alfred North Whitehead
(1861–1947)

$$\mathbf{a} \times \mathbf{b} = (a_y b_z - a_z b_y)\mathbf{i} + (a_z b_x - a_x b_z)\mathbf{j} + (a_x b_y - a_y b_x)\mathbf{k}$$

The determinant below also gives the result:

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$



Properties of the Cross-Product X

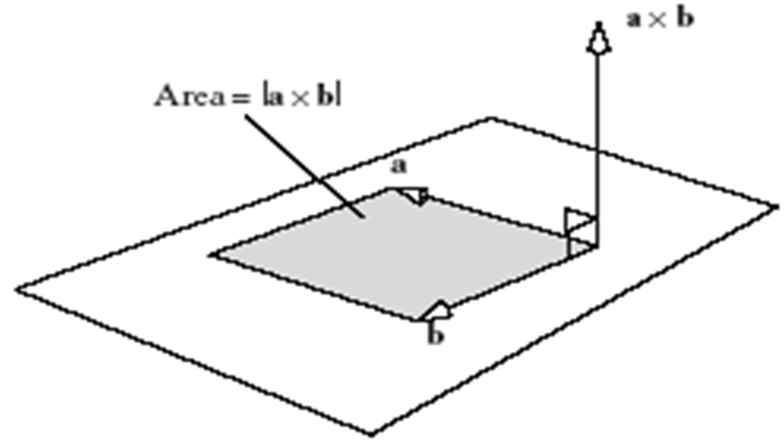
i x j = **k**; **j x k** = **i**; **k x i** = **j**

a x b = - **b x a**;

a x (**b** ± **c**) = **a x b** ± **a x c**;

(s**a**) **x b** = s(**a x b**)

a x (**b x c**) ≠ (**a x b**) **x c**



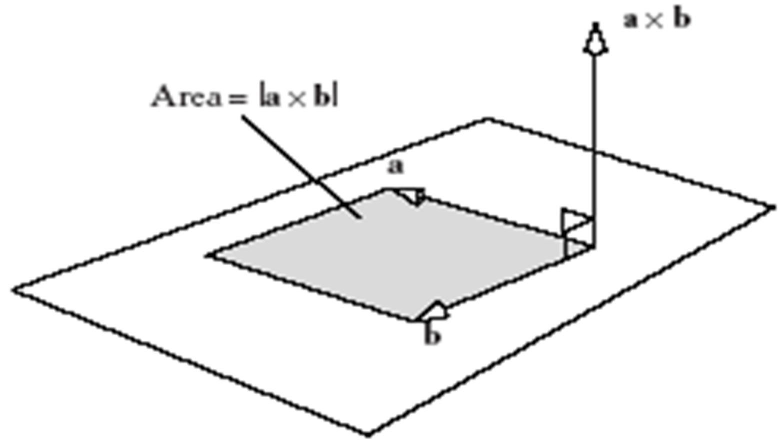
– for example,

a = $(a_x, a_y, 0)$, **b** = $(b_x, b_y, 0)$, **c** = $(0, 0, c_z)$

c = **a x b** is perpendicular to **a** and to **b**.

The direction of **c** is given by a **right/left hand rule in a right/left-handed coordinate system.**

Properties



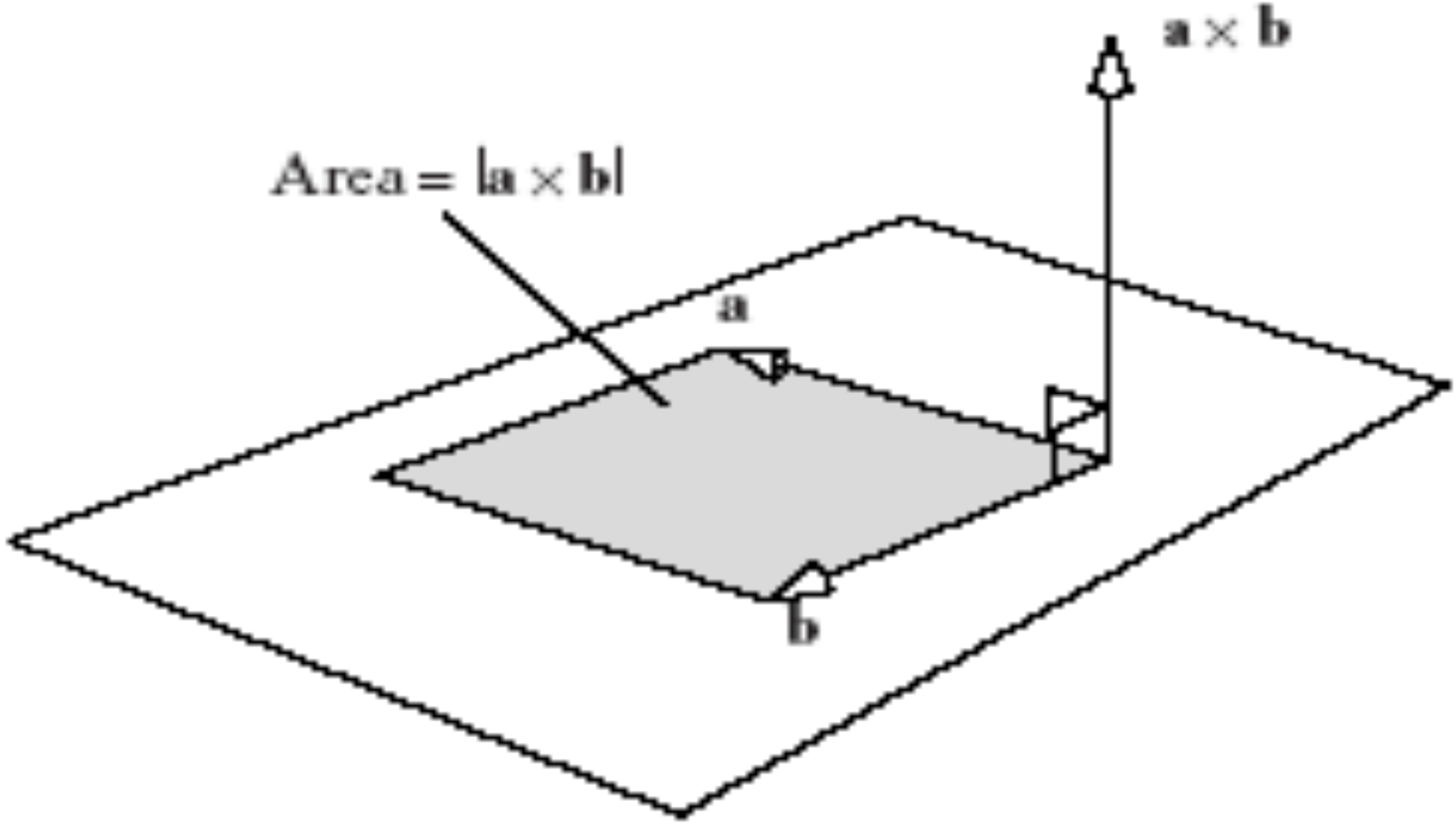
- a • (a x b) = 0

- a x b = |a||b| sin θ, where θ is the smaller angle between a and b

- a x b is also the area of the parallelogram formed by a and b

- a x b = 0 if a and b point in the same or opposite directions, or if one or both has length 0

Geometric Interpretation of the Cross Product x

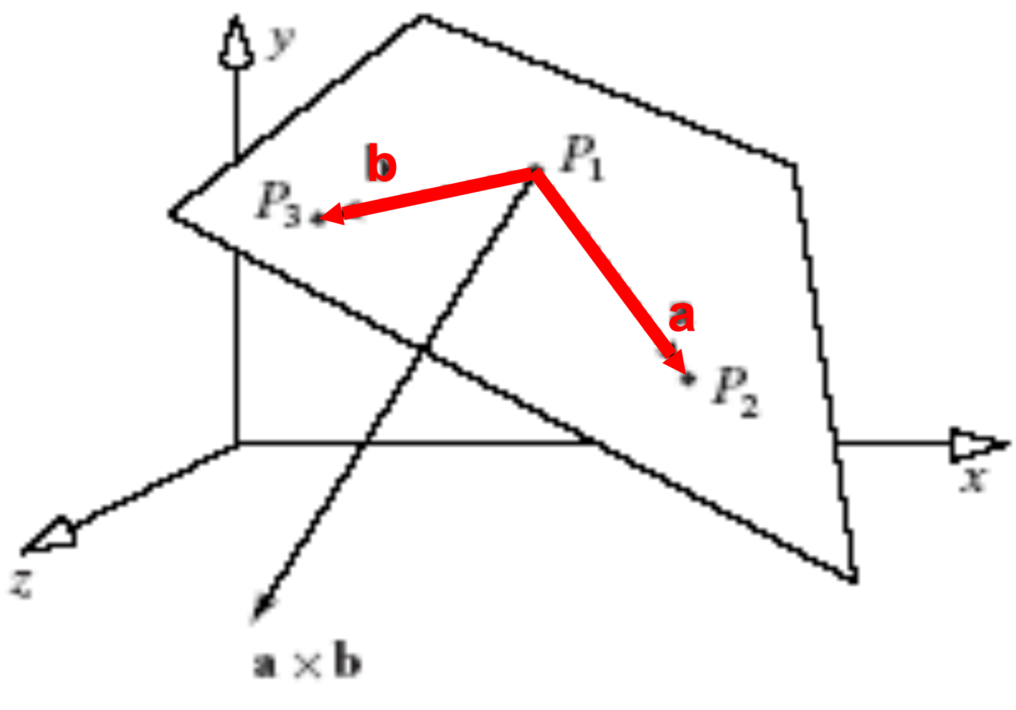


Application: Finding the Normal to a Plane

Given any 3 non-collinear points P_1, P_2, and P_3 in a plane, we can find a **normal n to the plane**:

$$\mathbf{a} = P_2 - P_1, \quad \mathbf{b} = P_3 - P_1, \qquad \mathbf{n} = \mathbf{a} \times \mathbf{b}$$

The normal on the other side of the plane is $-\mathbf{n}$



Plane-Line Intersections

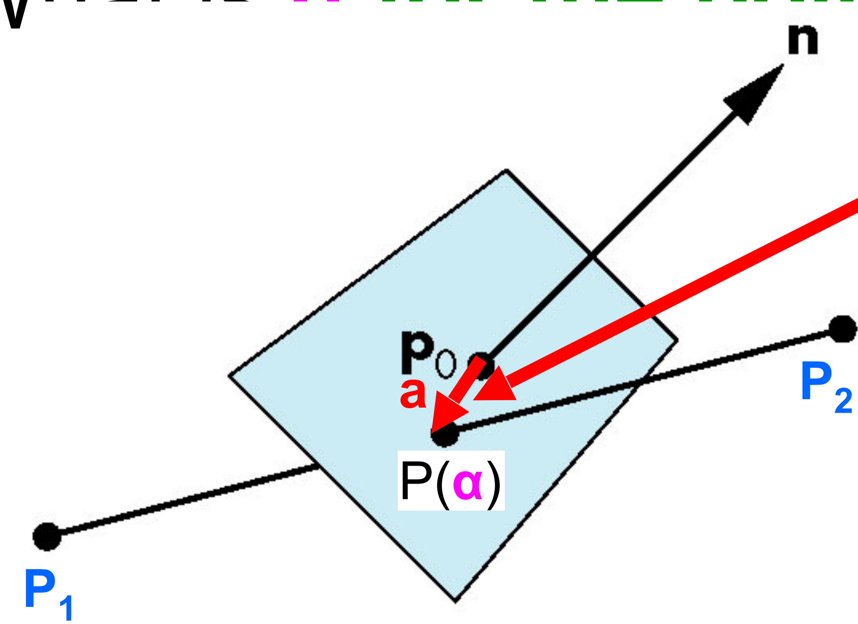
Line segment between P_1 and P_2 parametric α **form**

$$P(\alpha) = P_1 + \alpha\,(P_2 - P_1)$$

Plane equation

$$n \bullet a = 0 \qquad\qquad n \bullet \underline{(P(\alpha) - P_0)} = 0$$

What is α for the point of intersection?



Plane-Line Intersections

$\underline{P(\alpha) = P_1 + \alpha (P_2 - P_1)}$

$n \bullet (\underline{P(\alpha)} - P_0) = 0$

What is α **for the point of intersection?**

$n \bullet (\underline{P_1 + \alpha (P_2 - P_1)} - P_0) = 0$

$n \bullet ((P_1 - P_0) + \alpha (P_2 - P_1)) = 0$

$n \bullet (P_1 - P_0) + \alpha\, n \bullet (P_2 - P_1)) = 0$

$n \bullet (P_0 - P_1) = \alpha\, n \bullet (P_2 - P_1))$

$$\alpha = \frac{n \bullet (p_o - p_1)}{n \bullet (p_2 - p_1)}$$

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The University of New Mexico

09.13.2023 (W 5:30 to 7) (7)	Homework 3	Lecture 4

At 6:45 PM.

End Class 6

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