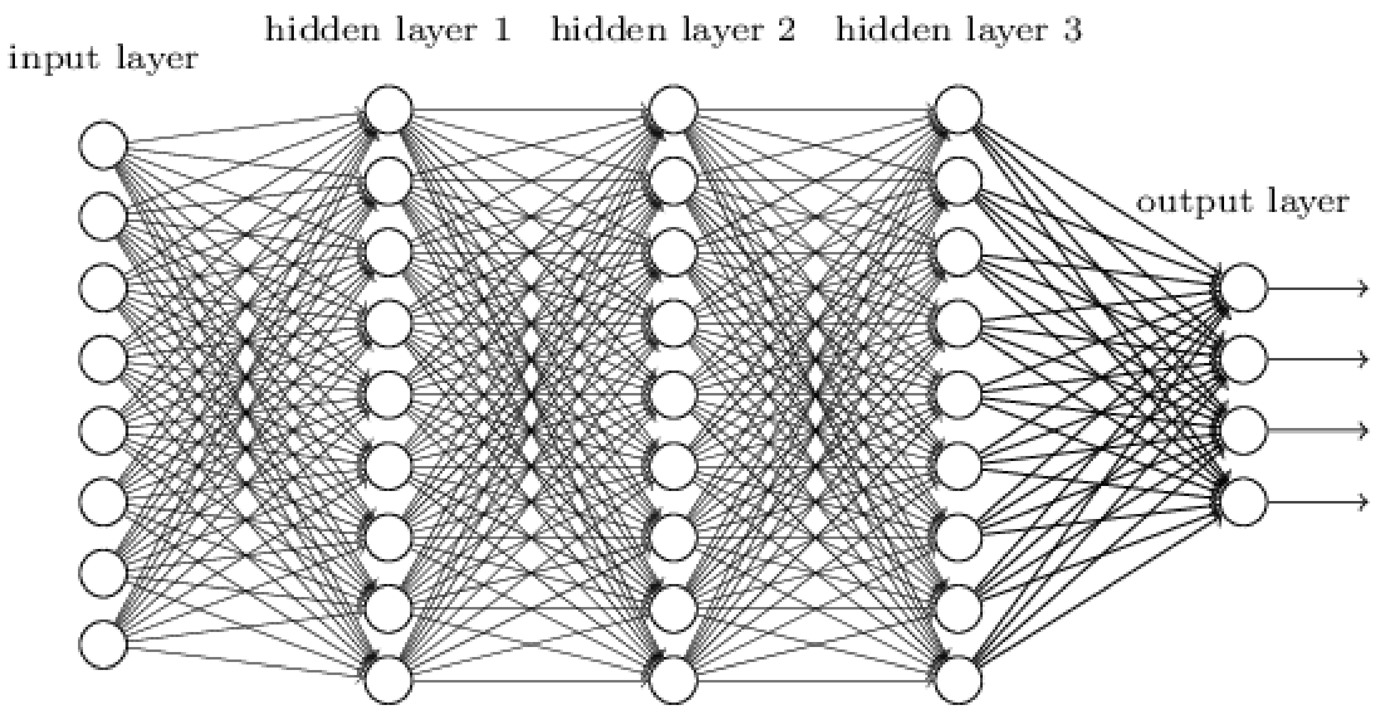
COSC 4368 Fundamentals of Artificial Intelligence

Convolutional Neural Networks
October 23th, 2023
(slides modified from U Waterloo CS 898)

Convolutional Neural Network

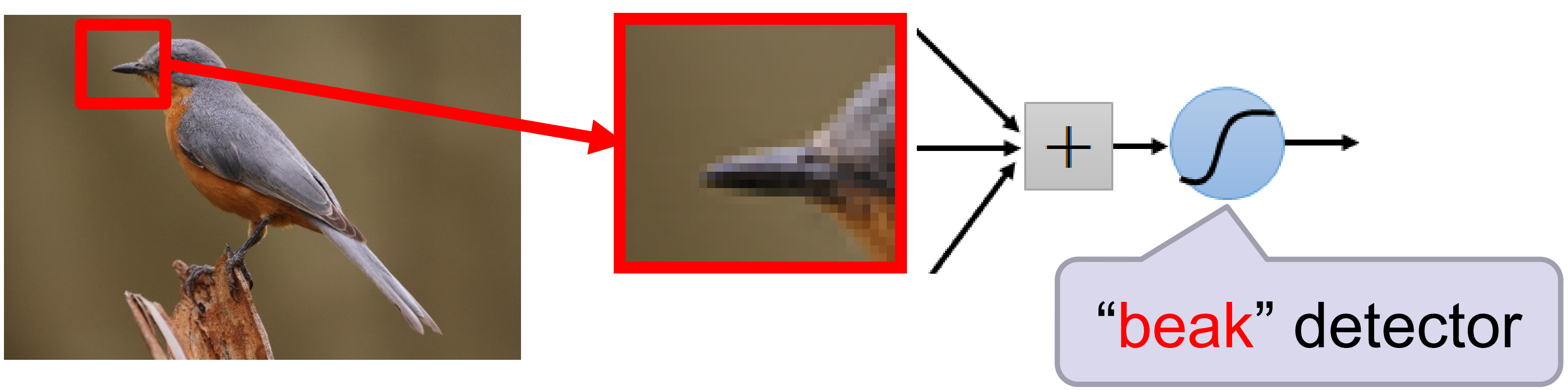
- We know it is good to learn a small model.
- From this fully connected model, do we really need all the edges?
- Can some of these be shared?



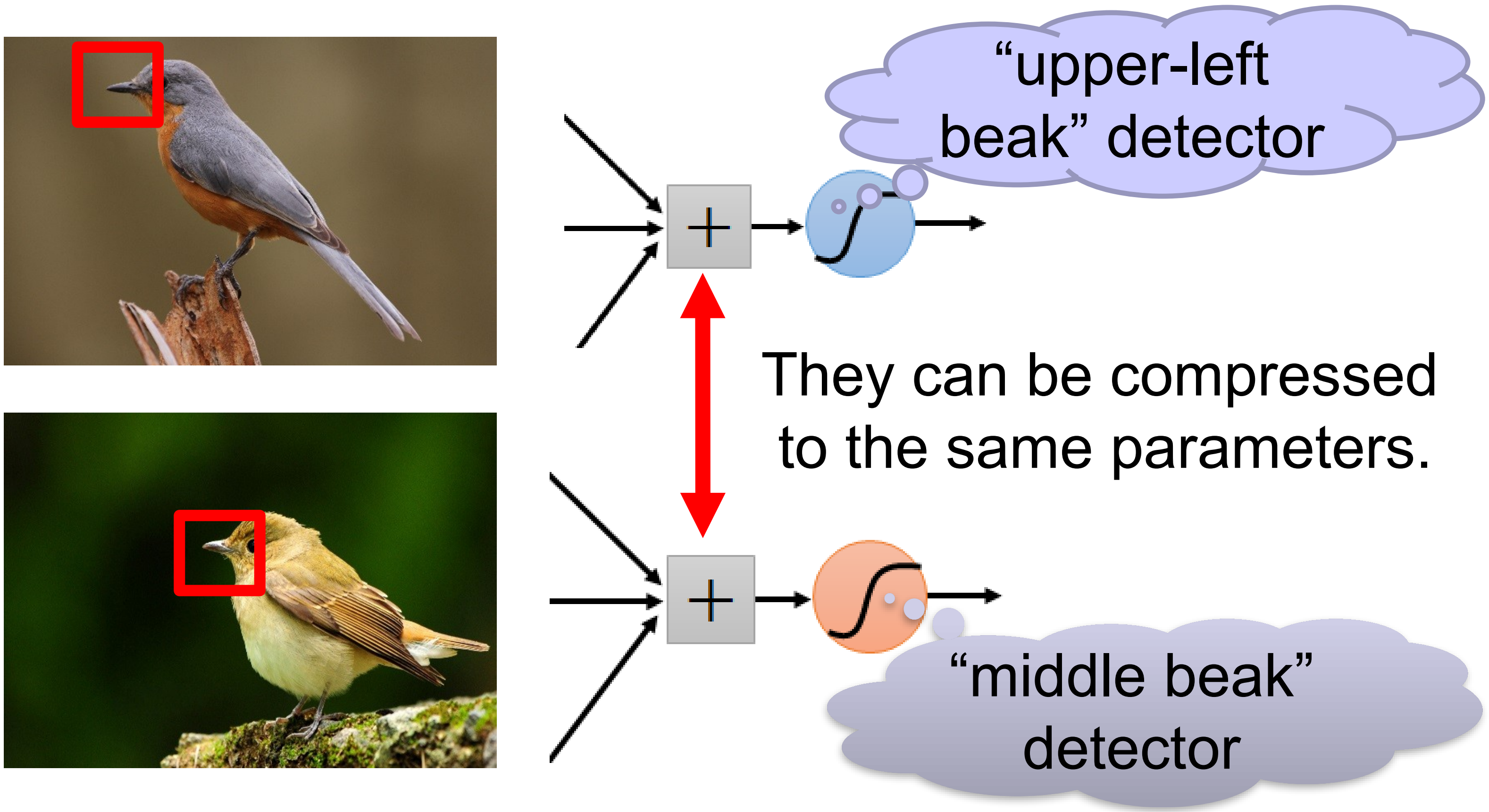
Consider learning an image:

- Some patterns are much smaller than the whole image

Can represent a small region with fewer parameters

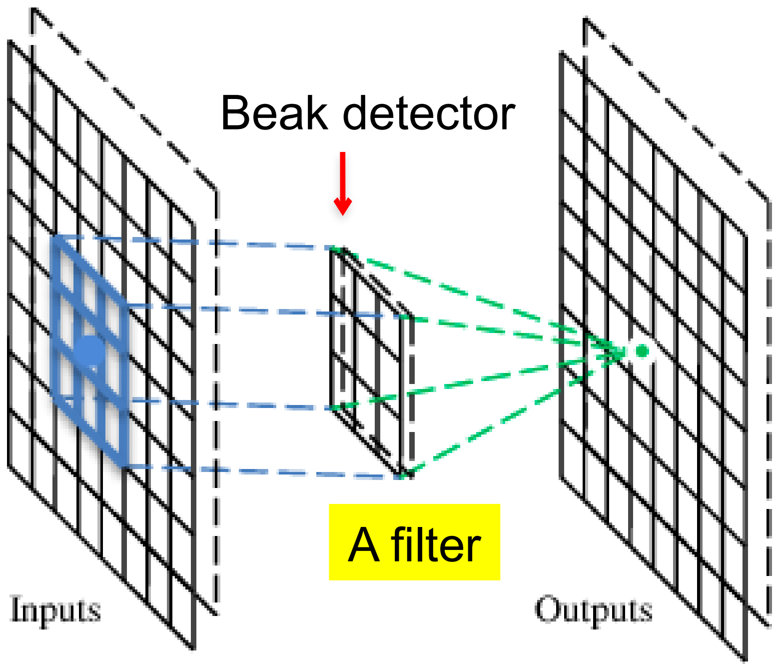


Same pattern appears in different places:
They can be compressed!
What about training a lot of such "small" detectors and each detector must "move around".



A convolutional layer

A CNN is a neural network with some convolutional layers (and some other layers). A convolutional layer has a number of filters that does convolutional operation.



Convolution

These are the network parameters to be learned.

1	0	0	0	0	1
0	1	0	0	1	0
0	0	1	1	0	0
1	0	0	0	1	0
0	1	0	0	1	0
0	0	1	0	1	0

6 x 6 image

Filter 1

1	-1	-1
-1	1	-1
-1	-1	1

Filter 2

-1	1	-1
-1	1	-1
-1	1	-1

⋮ ⋮

Each filter detects a small pattern (3 x 3).

Convolution

1	-1	-1
-1	1	-1
-1	-1	1

Filter 1

stride=1

1	0	0	0	0	1
0	1	0	0	1	0
0	0	1	1	0	0
1	0	0	0	1	0
0	1	0	0	1	0
0	0	1	0	1	0

Dot product

3 -1

6 x 6 image

Convolution

1	-1	-1
-1	1	-1
-1	-1	1

Filter 1

If stride=2

1	0	0	0	0	1
0	1	0	0	1	0
0	0	1	1	0	0
1	0	0	0	1	0
0	1	0	0	1	0
0	0	1	0	1	0

6 x 6 image

3 -3

Convolution

1	-1	-1
-1	1	-1
-1	-1	1

Filter 1

stride=1

1	0	0	0	0	1
0	1	0	0	1	0
0	0	1	1	0	0
1	0	0	0	1	0
0	1	0	0	1	0
0	0	1	0	1	0

6 x 6 image

$\boxed{3}$	-1	-3	-1
-3	1	0	-3
-3	-3	0	1
$\boxed{3}$	-2	-2	-1

Convolution

-1	1	-1
-1	1	-1
-1	1	-1

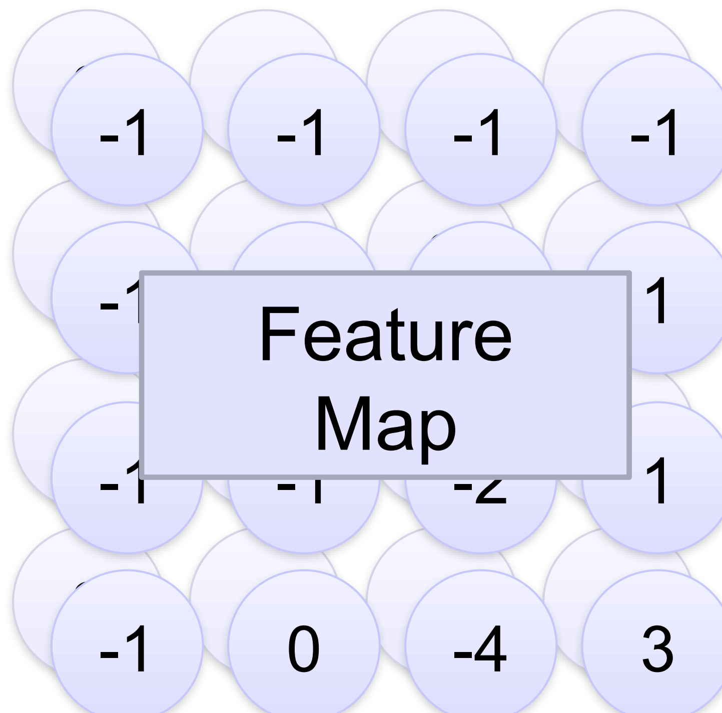
Filter 2

stride=1

1	0	0	0	0	1
0	1	0	0	1	0
0	0	1	1	0	0
1	0	0	0	1	0
0	1	0	0	1	0
0	0	1	0	1	0

6 x 6 image

Repeat this for each filter



Two 4 x 4 images
Forming 2 x 4 x 4 matrix

Color image: RGB 3 channels

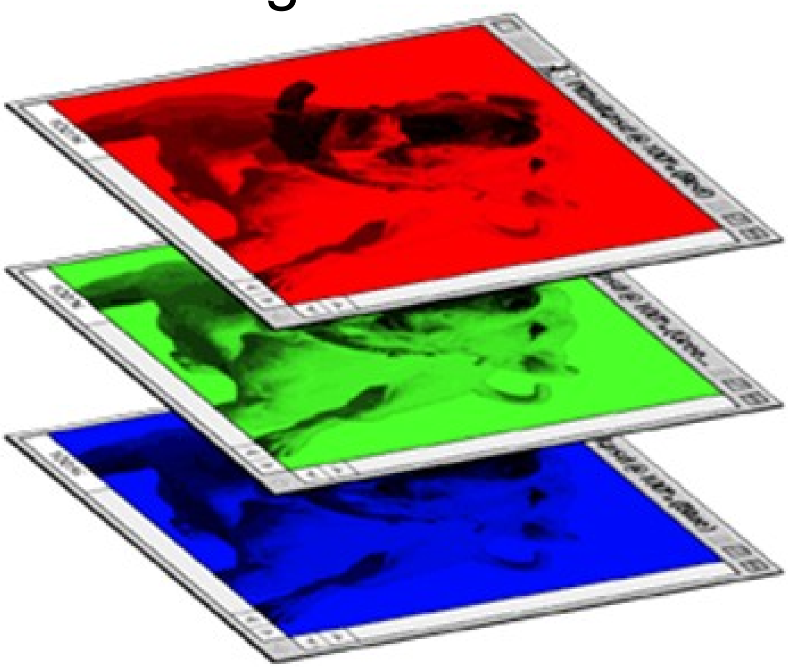
1	-1	-1
-1	1	-1
-1	-1	1

Filter 1

-1	1	-1
-1	1	-1
-1	1	-1

Filter 2

Color image



1	0	0	0	0	1
0	1	0	0	1	0
0	0	1	1	0	0
1	0	0	0	1	0
0	1	0	0	1	0
0	0	1	0	1	0

Convolution v.s. Fully Connected

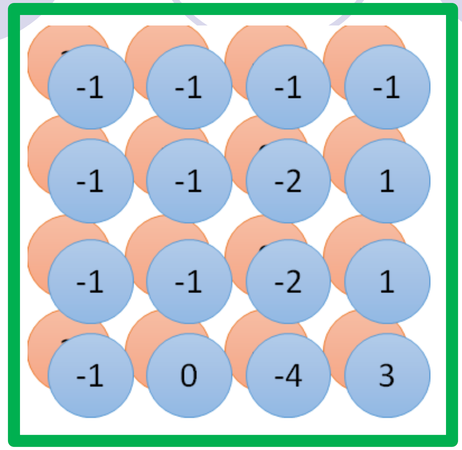
1	0	0	0	0	1
0	1	0	0	1	0
0	0	1	1	0	0
1	0	0	0	1	0
0	1	0	0	1	0
0	0	1	0	1	0

image

1	-1	-1
-1	1	-1
-1	-1	1

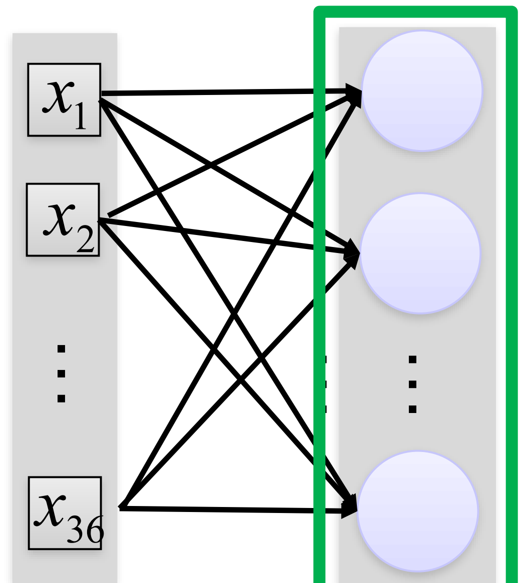
-1	1	-1
-1	1	-1
-1	1	-1

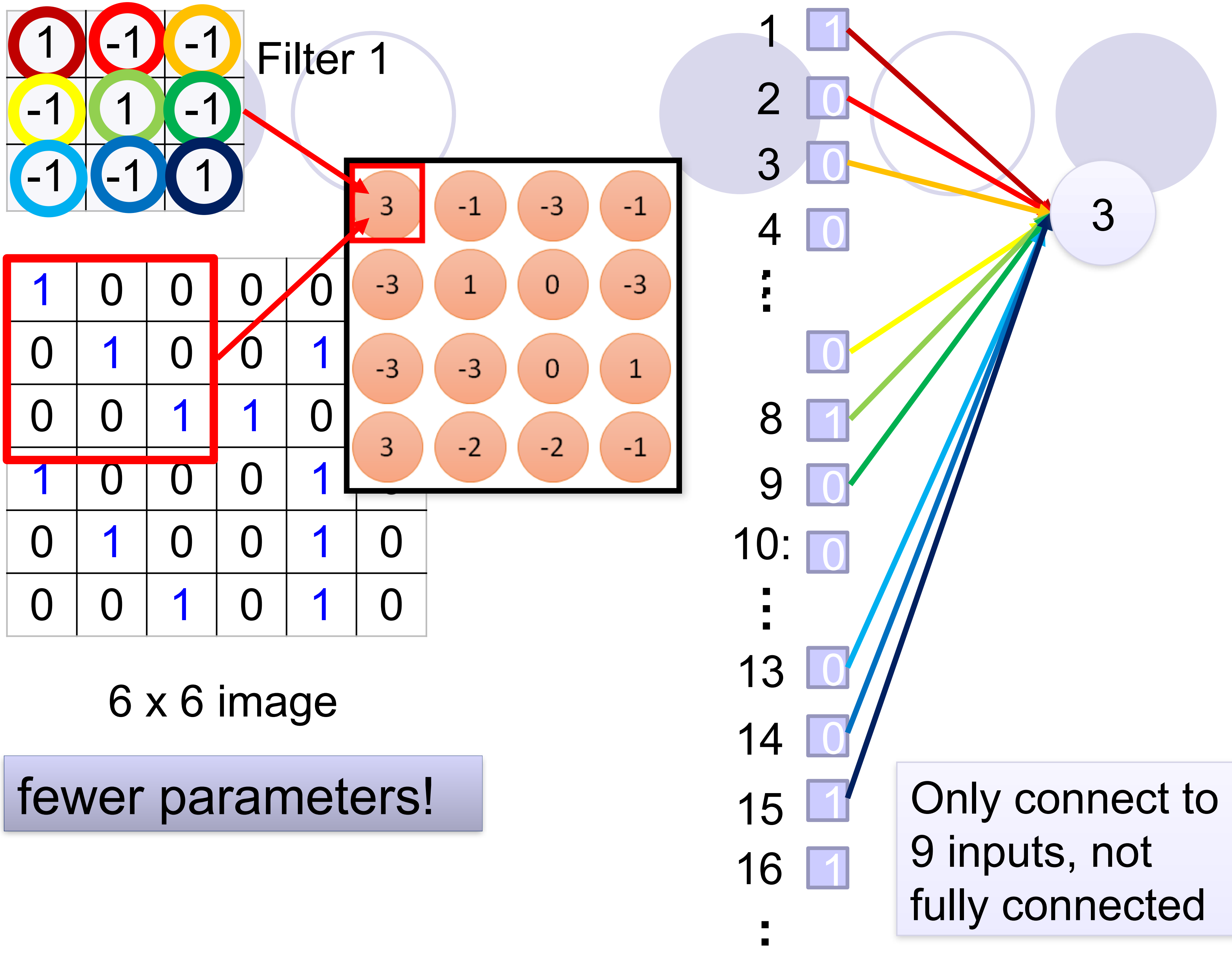
convolution



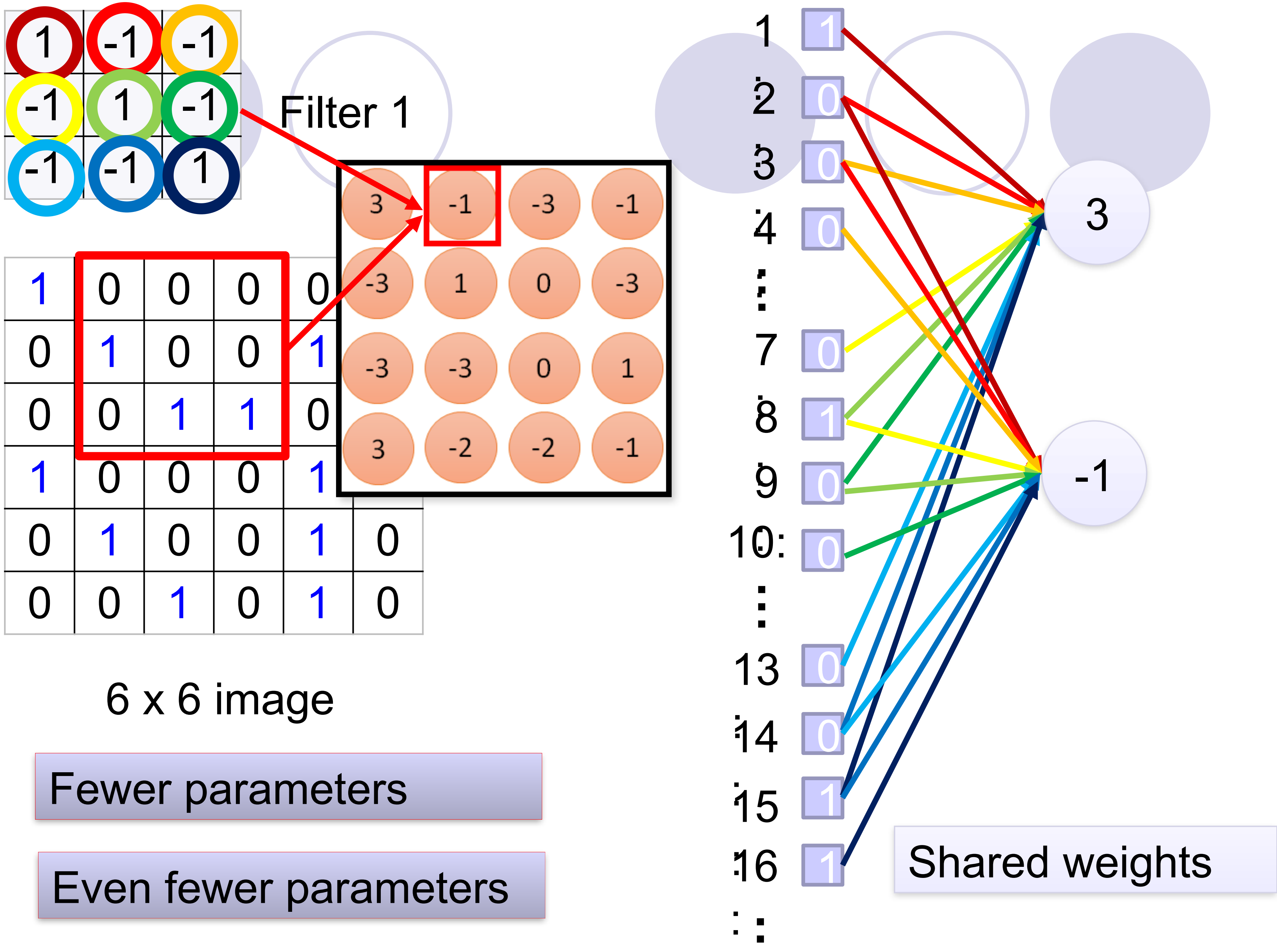
Fully-connected

1	0	0	0	0	1
0	1	0	0	1	0
0	0	1	1	0	0
1	0	0	0	1	0
0	1	0	0	1	0
0	0	1	0	1	0



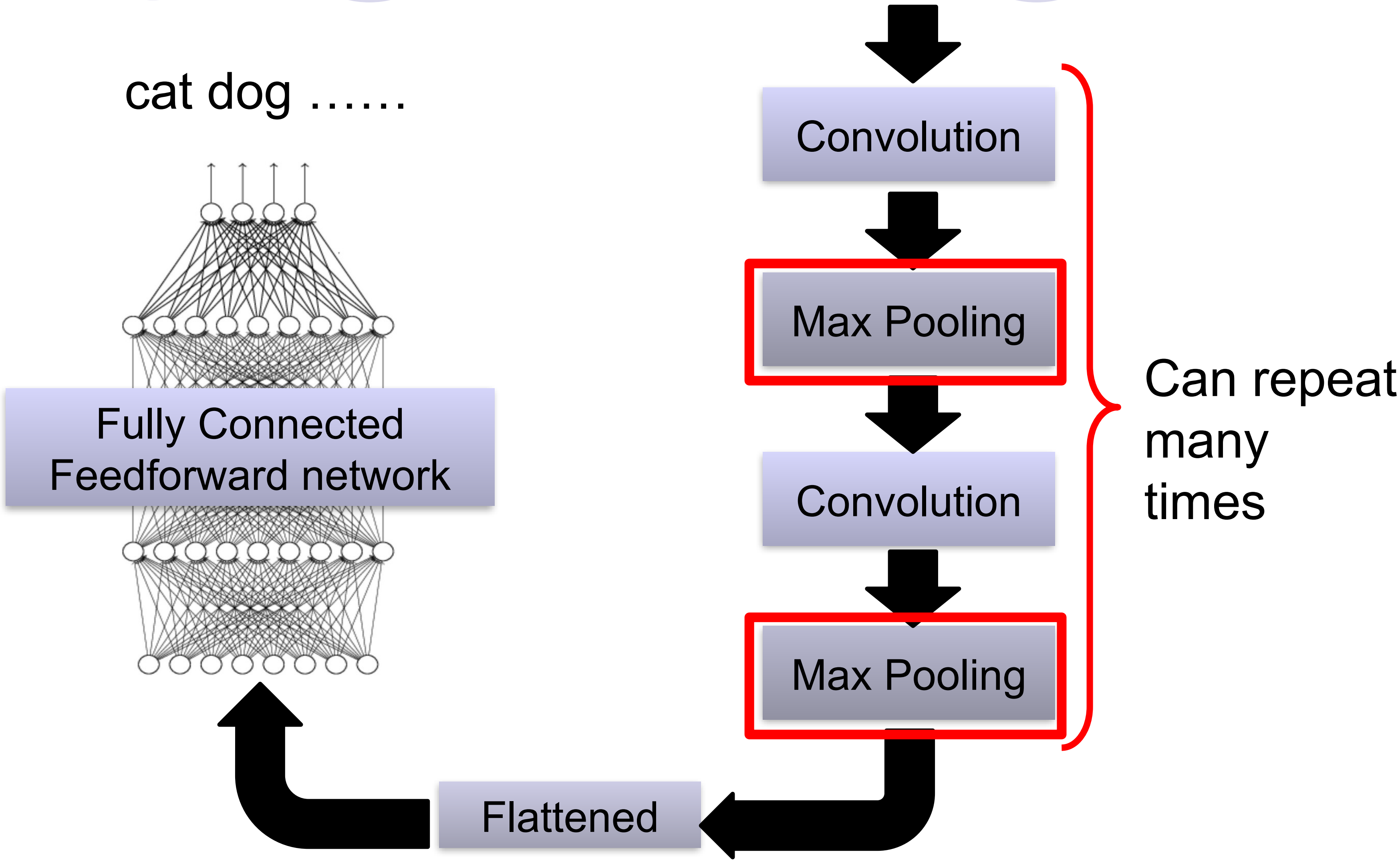


1 -1 -1
-1 1 -1
-1 -1 1
Filter 1
1 0 0 0 0
0 1 0 0 1
0 0 1 1 0
1 0 0 0 1
0 1 0 0 1 0
0 0 1 0 1 0
3 -1 -3 -1
-3 1 0 -3
-3 -3 0 1
3 -2 -2 -1
6 x 6 image
fewer parameters!
1: 1
2: 0
3: 0
4: 0
0
8: 1
9: 0
10: 0
13: 0
14: 0
15: 1
16: 1
3
Only connect to 9 inputs, not fully connected



Filter 1
6 x 6 image
Fewer parameters
Even fewer parameters
Shared weights
3
-1

The whole CNN



Max Pooling

1	-1	-1
-1	1	-1
-1	-1	1

Filter 1

-1	1	-1
-1	1	-1
-1	1	-1

Filter 2

3	-1	-3	-1
-3	1	0	-3
-3	-3	0	1
3	-2	-2	-1

-1	-1	-1	-1
-1	-1	-2	1
-1	-1	-2	1
-1	0	-4	3

Why Pooling

- Subsampling pixels will not change the object

bird

Subsampling

bird

We can subsample the pixels to make image smaller

→ fewer parameters to characterize the image

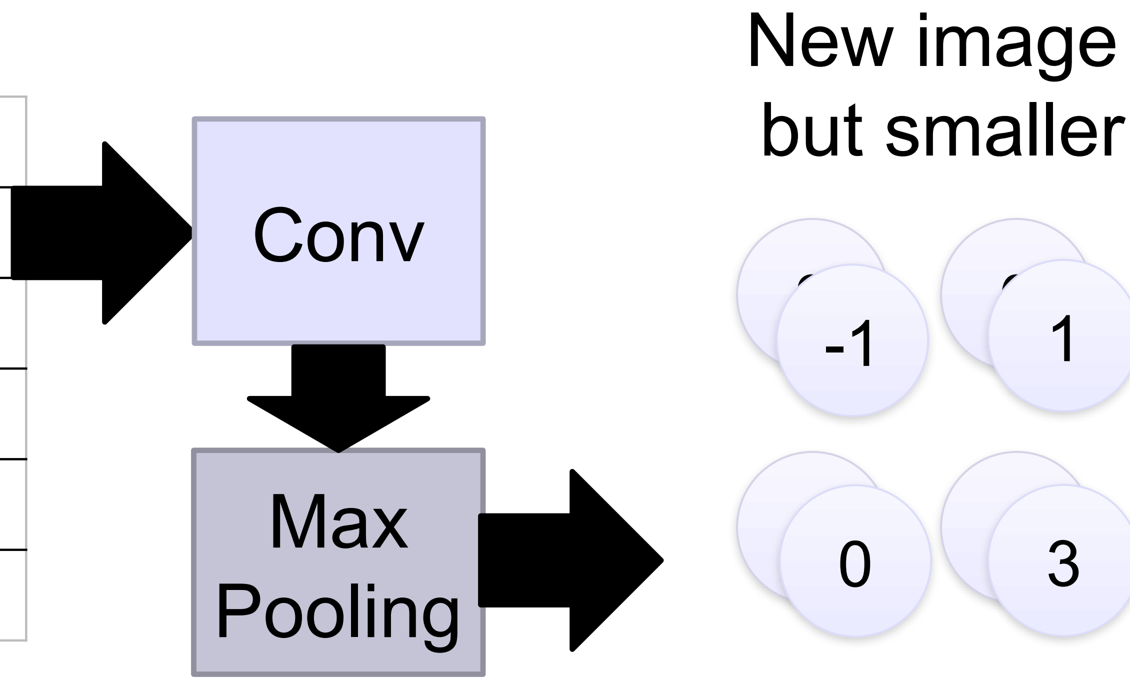
A CNN compresses a fully connected network in two ways:

- Reducing number of connections
- Shared weights on the edges
- Max pooling further reduces the complexity

Max Pooling

1	0	0	0	0	1
0	1	0	0	1	0
0	0	1	1	0	0
1	0	0	0	1	0
0	1	0	0	1	0
0	0	1	0	1	0

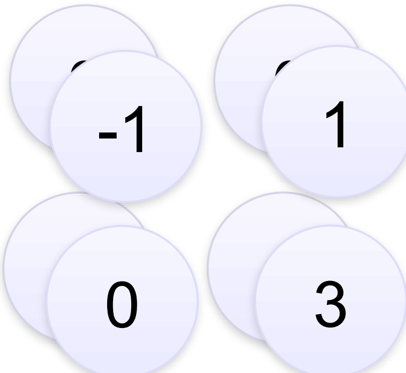
6 x 6 image



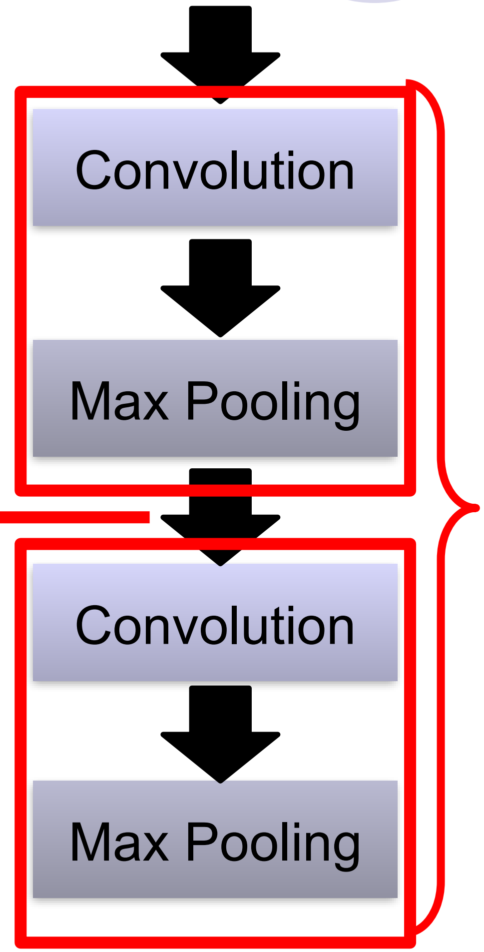
2 x 2 image

Each filter
is a channel

The whole CNN



A new image

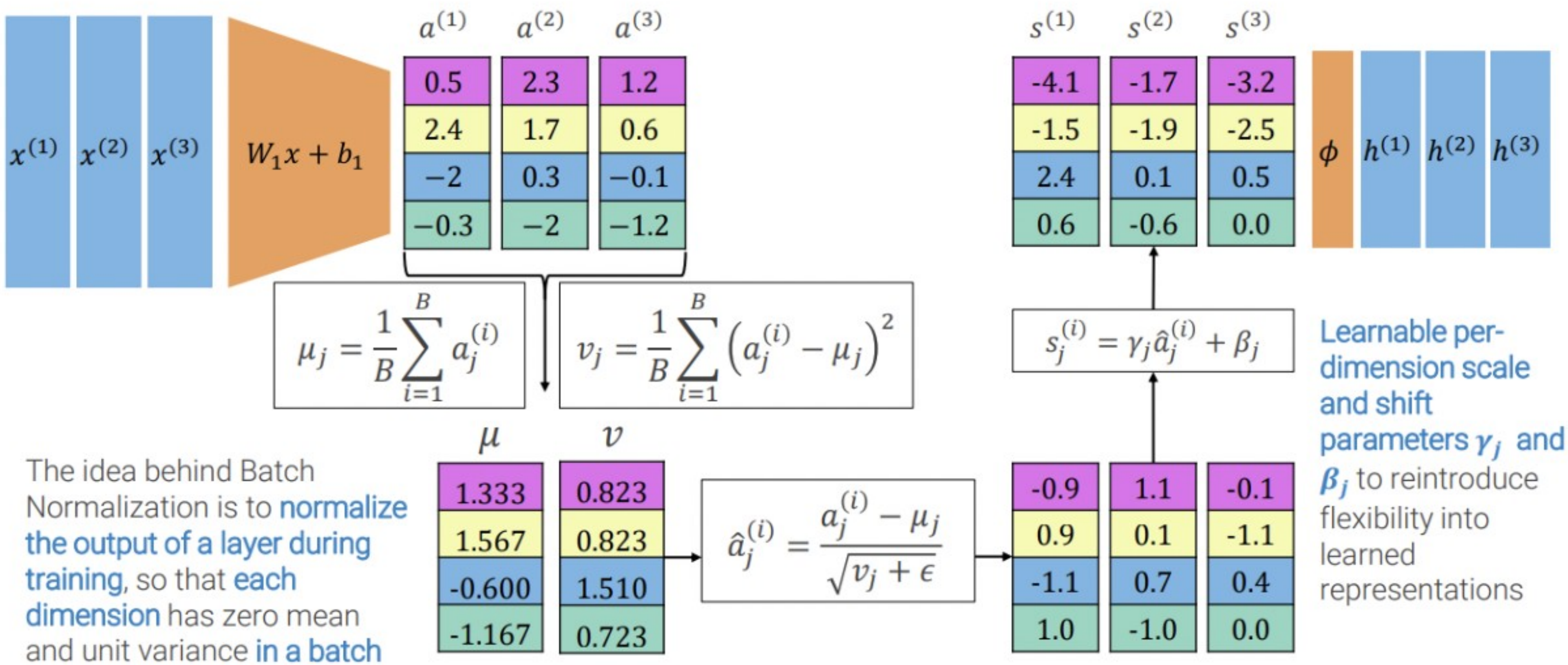


Can repeat many times

Smaller than the original image

The number of channels is the number of filters

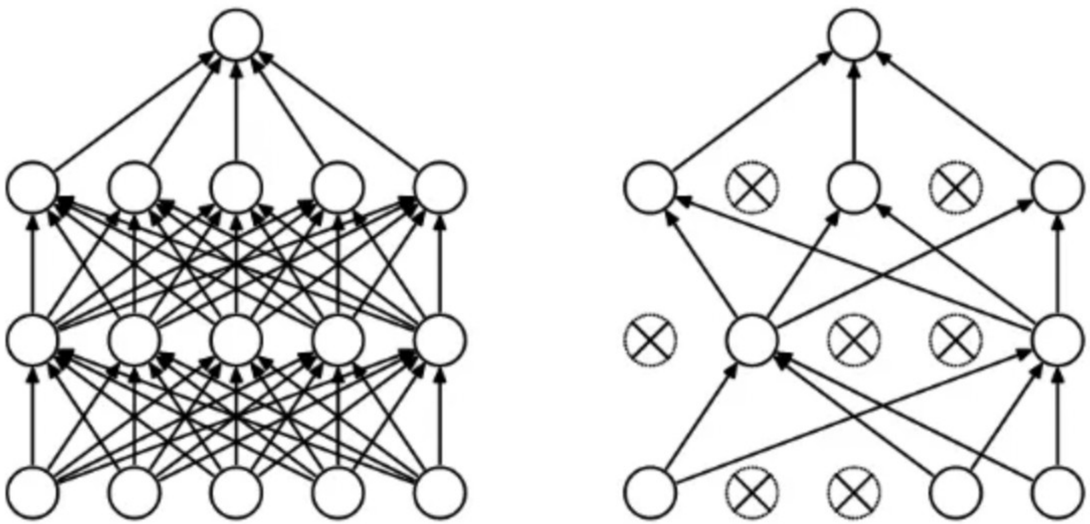
Batch Normalization



A diagram illustrating the Batch Normalization process

Regularize the model, accelerate training, improve generalization, less sensitive to model initialization

Dropout



(a) Standard Neural Net (b) After applying dropout.

$$
\begin{aligned}
z_i^{(l+1)} &= \mathbf{w}_i^{(l+1)} \mathbf{y}^l + b_i^{(l+1)}, \\
y_i^{(l+1)} &= f(z_i^{(l+1)}),
\end{aligned}
$$

$$
\begin{aligned}
r_j^{(l)} &\sim \text{Bernoulli}(p), \\
\widetilde{\mathbf{y}}^{(l)} &= \mathbf{r}^{(l)} * \mathbf{y}^{(l)}, \\
z_i^{(l+1)} &= \mathbf{w}_i^{(l+1)} \widetilde{\mathbf{y}}^l + b_i^{(l+1)}, \\
y_i^{(l+1)} &= f(z_i^{(l+1)}).
\end{aligned}
$$

improve generalization, but need a larger model and more training iterations

The whole CNN

Convolution

Max Pooling

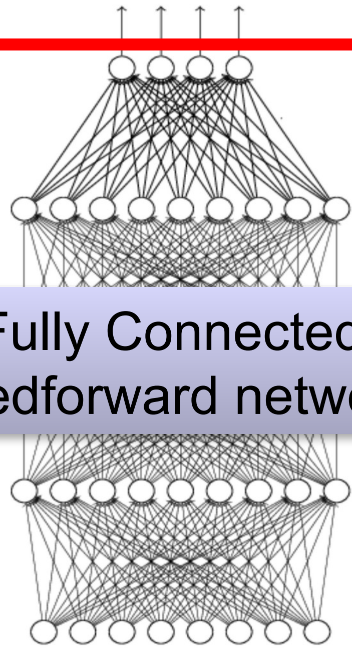
A new image

Convolution

Max Pooling

A new image

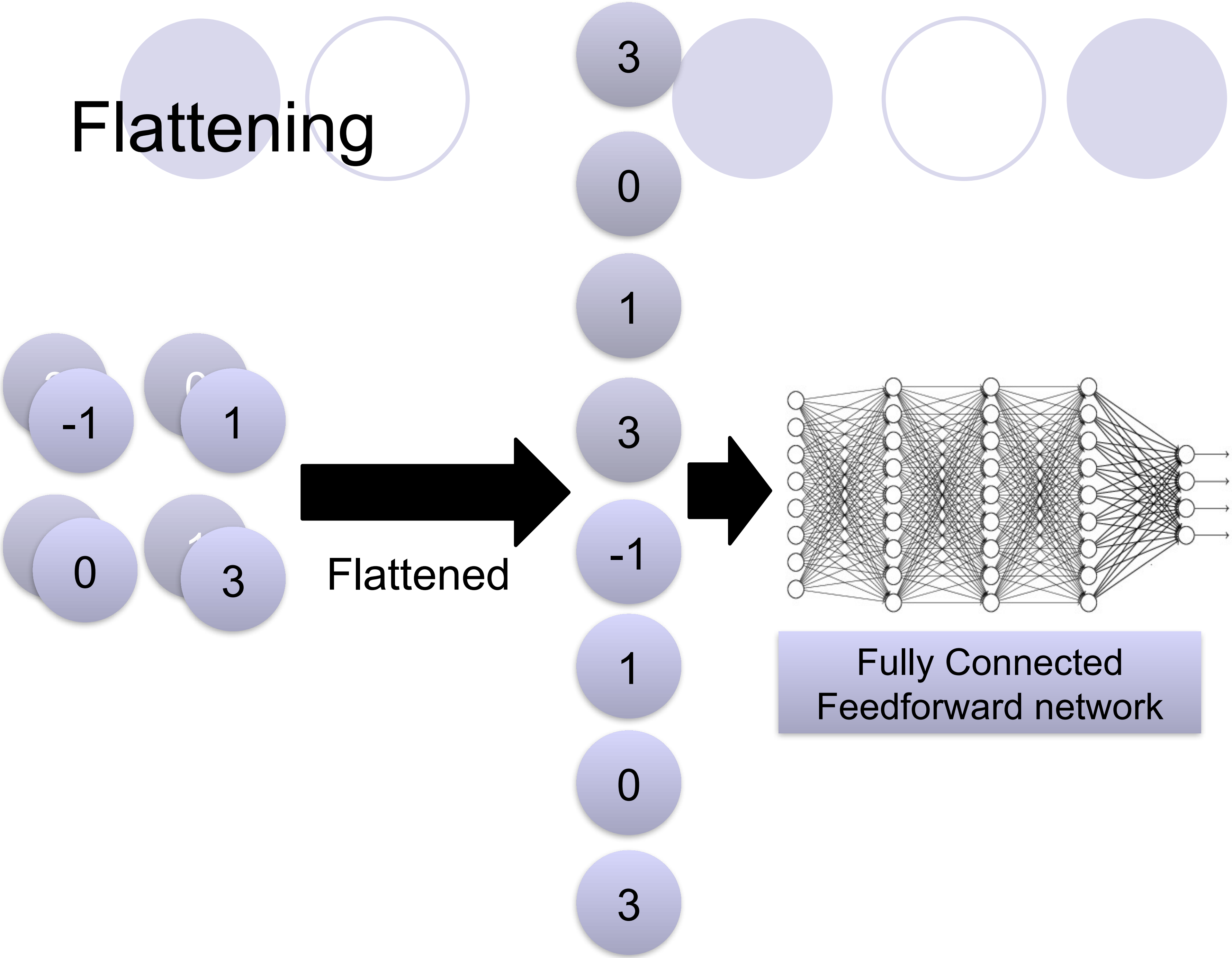
Flattened



Fully Connected Feedforward network

cat dog ……

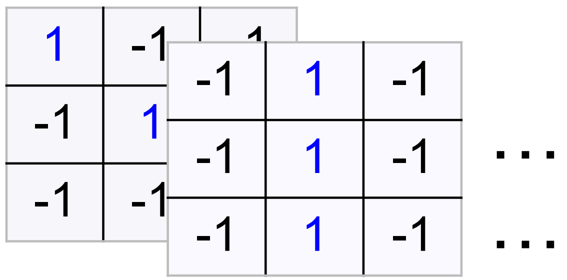
Flattening


-1
1
0
3
Flattened
3
0
1
3
-1
1
0
3
Fully Connected
Feedforward network

CNN in Keras

Only modified the ***network structure*** and ***input format (vector -> 3-D tensor)***

```
model2.add( Convolution2D( 25,3,3,
            input_shape=(28,28,1)) )
```



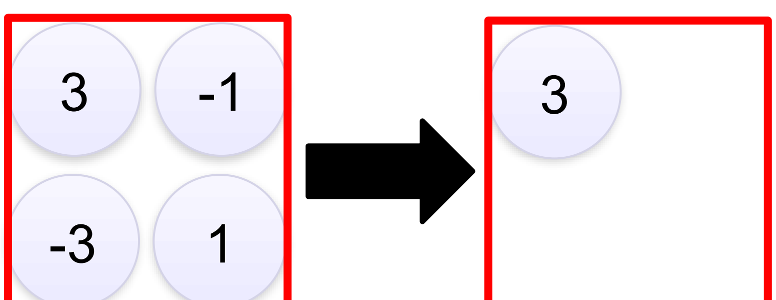
There are 25 3x3 filters.

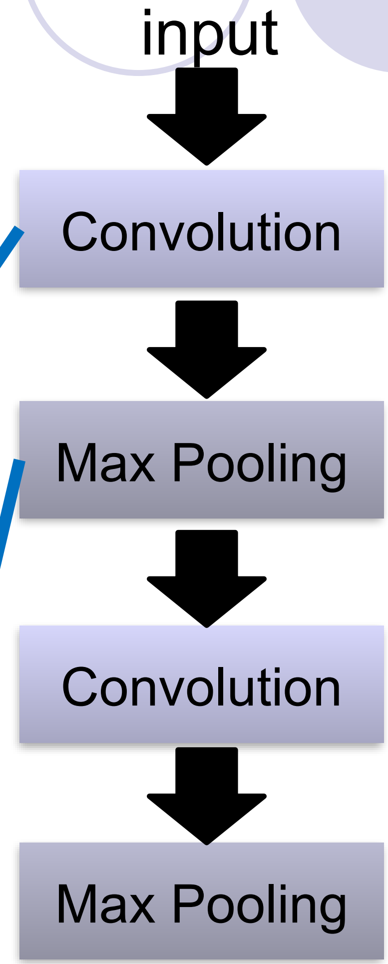
Input_shape = (28 , 28 , 1)

28 x 28 pixels

1: black/white, 3: RGB

```
model2.add(MaxPooling2D((2,2)))
```





CNN in Keras

Only modified the ***network structure*** and ***input format (vector -> 3-D array)***

Input

1 x 28 x 28

Convolution

```
model2.add( Convolution2D( 25,3,3,
            input_shape=(28,28,1)) )
```

25 x 26 x 26

Max Pooling

```
model2.add(MaxPooling2D((2,2)))
```

25 x 13 x 13

Convolution

```
model2.add(Convolution2D(50,3,3))
```

50 x 11 x 11

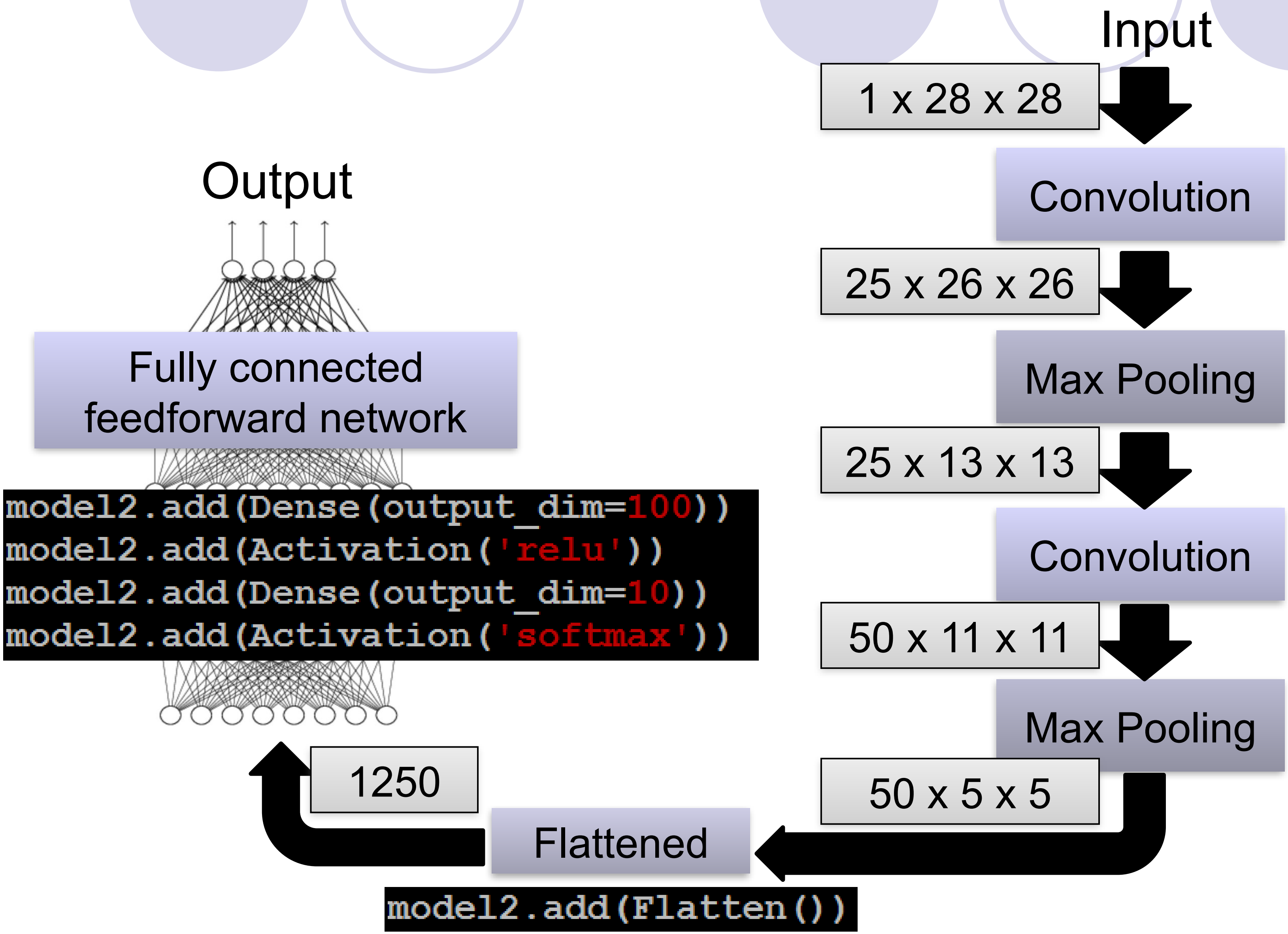
Max Pooling

```
model2.add(MaxPooling2D((2,2)))
```

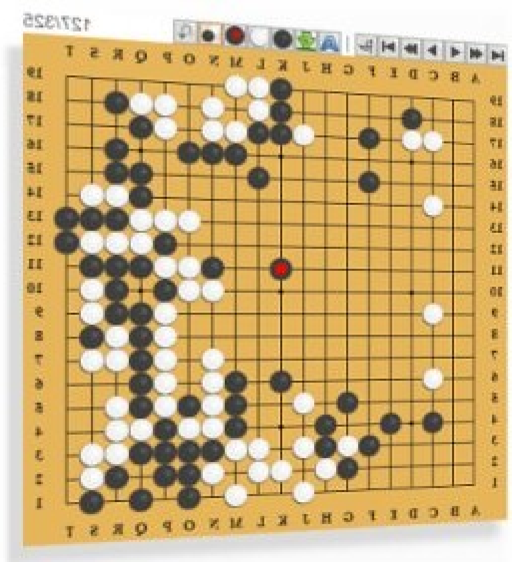
50 x 5 x 5

CNN in Keras

Only modified the ***network structure*** and ***input format (vector -> 3-D array)***



AlphaGo



19 x 19 matrix

Neural Network

Next move

(19 x 19 positions)

Fully-connected feedforward network can be used

But CNN performs much better

AlphaGo's policy network

The following is quotation from their Nature article:

Note: AlphaGo does not use Max Pooling.

Neural network architecture. The input to the policy network is a $19 \times 19 \times 48$ image stack consisting of 48 feature planes. The first hidden layer zero pads the input into a 23×23 image, then convolves k filters of kernel size 5×5 with stride 1 with the input image and applies a rectifier nonlinearity. Each of the subsequent hidden layers 2 to 12 zero pads the respective previous hidden layer into a 21×21 image, then convolves k filters of kernel size 3×3 with stride 1, again followed by a rectifier nonlinearity. The final layer convolves 1 filter of kernel size 1×1 with stride 1, with a different bias for each position, and applies a softmax function. The match version of AlphaGo used $k = 192$ filters; Fig. 2b and Extended Data Table 3 additionally show the results of training with $k = 128$, 256 and 384 filters.