MATH 3339
Statistics for the Sciences

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Lecture 2 - 3339

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Outline

Probability Models

$$0 \le P(\text{event}) \le 1$$

- A **probability measure** is a function which assign numbers between 0 and 1 to any event in the sample space Ω.
- If the sample space Ω, the collection of events, and the probability measure are all specified, they constitute a **probability model** of the random experiment.

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Assigning probabilities

- **Classical method** is used when all the experimental outcomes are equally likely. If n experimental outcomes are possible, a probability of $1/n$ is assigned to each experimental outcome. Example: Drawing a card from a standard deck of 52 cards. Each card has a 1/52 probability of being selected.

- **Relative frequency method** is used to assign probabilities when data are available to estimate the proportion of the time the experimental outcome will occur if the experiment is repeated a large number of times. That is for any outcome, E, probability of E is
$$P(E) = \frac{\text{number of times E occurs}}{\text{total number of observations}} = \frac{\#(E)}{N}$$

- $P(E)$ is a probability model for any event E that is a subset of Ω.

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Example of Probabilities

Relative frequency method: An insurance company determined the number of accidents in a year. A sample of 100 people were surveyed to determine the number of accidents they were in a year: 0 accidents 25 people, 1 accident 45 people, 2 accidents 20 people, 3 or more accidents 10 people. The following table shows the relative frequency for the outcomes.

Number of accidents	Frequency (count)	Relative frequency
0	25	$\frac{25}{100} = 0.25$
1	45	$\frac{45}{100} = 0.45$
2	20	$\frac{20}{100} = 0.20$
3 or more	10	$\frac{10}{100} = 0.10$
Total	100	1

Pair of Dice

$P(\text{sum of } 12) = \frac{1}{36}$

$P(\text{sum of } 15) = 0$

	⚀	⚁	⚂	⚃	⚄	⚅
	2	3				
⚀	(1,1)	(2,1)	(3,1)	(4,1)	(5,1)	(6,1)
⚁	(1,2)	(2,2)	(3,2)	(4,2)	(5,2)	(6,2)
⚂	(1,3)	(2,3)	(3,3)	(4,3)	(5,3)	(6,3)
⚃	(1,4)	(2,4)	(3,4)	(4,4)	(5,4)	(6,4)
⚄	(1,5)	(2,5)	(3,5)	(4,5)	(5,5)	(6,5)
⚅	(1,6)	(2,6)	(3,6)	(4,6)	(5,6)	(6,6)
						12

What is the probabiltiy of getting a sum of 5? $\frac{4}{36} = \frac{1}{9}$

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How to get $\#(E)$

- $\#(E)$ denotes the number of elements in the subset E.
- Sometimes it is not as obvious as the previous example. Thus we need to use some counting techniques to determine $\#(E)$.

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Beginning Example

In the city of Milford, applications for zoning changes go through a two-step process:

1. A review by the planning commission.
2. A final decision by the city council.

- At step 1 the planning commission reviews the zoning change request and makes a positive or negative recommendation concerning the change.
- At step 2 the city council reviews the planning commission's recommendation and then votes to approve or to disapprove the zoning change.

How many possible decisions can be made for a zoning change in Milford? $2 \times 2 = 4$

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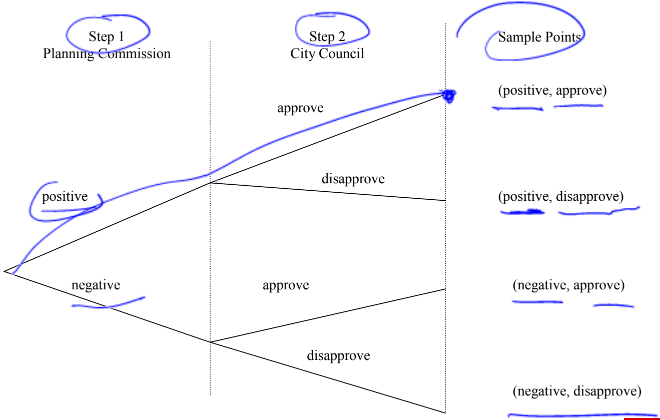
Counting Rules

- If an experiment can be described as a sequence of k steps with n_1 possible outcomes on the first step, n_2 possible outcomes on the second step, and so on, then the total number of experimental outcomes is given by $(n_1)(n_2)\ldots(n_k)$.
- A **tree diagram** can be used as a graphical representation in visualizing a multiple-step experiment.

Tree diagram

2 * 2 = 4



Examples

1. How many ways can we select 4 digits and 3 letters?
   - If digits and letters are allowed to repeat?

     $10*10*10*10 \; * \; 26*26*26$

   - If digits are letters are not allowed to repeat?

     $10*9*8*7 \; * \; 26*25*24$

2. In how many ways can 4 people be seated in 6 seats?

   $6*5*4*3 = 360$

Allowing Repeated Values

When we allow repeated values, The number of orderings of n objects taken r at a time, with repetition is n^r.

- Example 3: In how many ways can you write 4 letters on a tag using each of the letters C O U G A R with repetition?

$$6 * 6 * 6 * 6 = 6^4$$

Permutations

when 3 to be selted from 5

$n=5$, $r=3$

It allows one to compute the number of outcomes when r objects are to be selected from a set of n objects where the order of selection is important. The number of permutations is given by

$$P_3^5 = \frac{5!}{(5-3)!} \quad P_r^n = \frac{n!}{(n-r)!}$$

- Where $n! = n(n-1)(n-2)\cdots(2)(1)$
- Rcode for $n!$: factorial(n)

$5! = 5 * 4 * 3 * 2 * 1$

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Combinations

$n = 5$

$r = 3$

Counts the number of experimental outcomes when the experiment involves selecting *r* objects from a (usually larger) set of *n* objects. The number of combinations of *n* objects taken *r* unordered at a time is

$$C_r^n = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

$$C_3^5 = \binom{5}{3} = \frac{5!}{3!(5-3)!}$$

Rcode: choose(n,r)

```
> factorial(5)/(factorial(3)*factorial(5-3))
[1] 10
```

```
> choose(5,3)
[1] 10
```

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Examples

4. In how many ways can a committee of 5 be chosen from a group of 12 people?

P^{12}_{5} or $\boxed{C^{12}_{5}}$

choose (12 , 5)

5. In a manufacturing company they have to choose 5 out of 50 boxes to be sent to a store. How many ways can they choose the 5 boxes?

choose (50 , 5)

```
> choose(50,5)
[1] 2118760
```

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Difference Between Combinations and Permutations

Three Boxes A, B, C, select 2 $n=3$, $r=2$

1. Permutation:

order is important,

AB	BA
AC	CA
BC	CB

6 possible outcomes.

$P^3_2 = \frac{3!}{(3-2)!} = 6$

2. combination

order is NOT important.

AB $\Leftrightarrow$ BA

AC

BC

3 possible outcomes.

$C^3_2 = \frac{3!}{2!(3-2)!} = 3$

Example 6

From a committee of 10 people.

a) In how many ways can we choose a chair person, a vice-chair person, and a secretary, assuming that one person cannot hold more than one position?

Permutation

$$P^{10}_{3} = \frac{10!}{(10-3)!} = \boxed{720}$$

b) In how many ways can we select a subcommittee of 3 people?

Combination

$$\text{choose}(10, 3) = C^{10}_{3} = {}_{10}C_3 = \boxed{120}$$

```
> choose(10,3)
[1] 120
```

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Example

7. A researcher randomly selects 3 fish from a tank of 12 and puts each of the 3 fish into different containers. How many ways can this be done?

$n=12 \quad r=3$

P^{12}_{3}

choose(8, 4) * choose(2, 1)

$=\boxed{140}$

8. Among 10 electrical components 2 are known not to function. If 5 components are randomly selected, how many ways can we have only one of components not functioning?

8 good ← 4

2 not function ← 1

10   5

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Assigning probabilities

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- **Relative frequency method** is used when assigning probabilities is appropriate when data are available to estimate the proportion of the time the experimental outcome will occur if the experiment is repeated a large number of times. That is for any event E, probability of E is

$$P(E) = \frac{\text{number of times E occurs}}{\text{total number of observations}} = \frac{n(E)}{n(S)}$$

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Example 1

If 5 marbles are drawn at random all at once from a bag containing 8 white and 6 black marbles, what is the probability the 2 will be white and 3 will be black?

$$P(2W+3B) = \frac{\#(2W+3B)}{\#(\text{randomly choose 5 from 14})}$$

$$= \frac{\text{choose}(8,2) * \text{choose}(6,3)}{\text{choose}(14,5)}$$

$$= \boxed{0.2797}$$

8W   2W

6B   3B

14

Example 2

$$\frac{7W}{\overset{5M}{12}} \qquad \frac{?W}{\overset{?M}{6}}$$

The qualified applicant pool for six management trainee positions consists of seven women and five men.

1. What is the probability that a randomly selected trainee class will consist entirely of women?

$$P(6W + 0M) = \frac{\text{choose}(7,6) * \text{choose}(5,0)}{\text{choose}(12,6)}$$

$$* \ \text{choose}(n,0) = 1$$

2. What is the probability that a randomly selected trainee class will consist of an equal number of men and women?

$$P(3W + 3M) = \frac{\text{choose}(7,3) * \text{choose}(5,3)}{\text{choose}(12,6)}$$

$$= \boxed{0.3788}$$

Example 3

Suppose a box contains 3 defective light bulbs and 12 good bulbs. Suppose we draw a simple random sample of 4 light bulbs, find the probability that one of the bulbs drawn is defective. Which of the following is the correct result?

a) $\dfrac{\text{choose}(3,1)*\text{choose}(12,3)}{\text{choose}(12,4)}$

b) $\dfrac{\text{choose}(3,1)*\text{choose}(12,3)}{\text{choose}(15,4)}$

c) $\dfrac{\text{choose}(3,1)}{\text{choose}(15,4)}$

d) $\dfrac{3!}{12!}$

3 def. ← 1 def.

12 good ← 3 good

15 . 4.

Example 4

Suppose a box contains 3 defective light bulbs and 12 good bulbs. Suppose we draw a simple random sample of 4 light bulbs,

1. What is the probability that none of bulbs drawn are defective?

$$\frac{\text{choose}(12, 4) * \text{choose}}{\text{choose}(15, 4)} = ?$$

2. What is the probability that at least one of the bulbs drawn is defective?

$$P(\text{at least } 1 \text{ def}) = 1 - P(\text{no def.})$$
$$= 1 - ?$$
$$= \boxed{0.6374}.$$

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Example 5

Suppose we select randomly 4 marbles drawn from a bag containing 8 white and 6 black marbles.

1. What is the probability that half of the marbles drawn are white?

$$\begin{array}{c} 8W \\ 6B \\ \hline 14 \end{array} \qquad \begin{array}{c} 2W \\ 2B \\ \hline 4 \end{array} \qquad \frac{\text{choose}(8,2) * \text{choose}(6,2)}{\text{choose}(14,4)}$$

2. What is the probability that at least 2 of the marbles drawn are white?

$$P(\text{at least } 2W) = P(2W, 3W, 4W)$$

$$= P(2W) + P(3W) + P(4W)$$

$$= \frac{\text{choose}(8,2) * \text{choose}(6,2)}{\text{choose}(14,4)} + \frac{\text{choose}(8,3) * \text{choose}(6,1)}{\text{choose}(14,4)} + \frac{\text{choose}(8,4)}{\text{choose}(14,4)}$$

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$$P(\text{at least } 2W) = 1 - P(0W, 1W)$$

$$= 1 - P(0W) - P(1W)$$

$$= \boxed{0.8252}$$

Basic Probability Rules

1. $0 \leq P(E) \leq 1$ for each event E.
2. $P(\Omega) = 1$
3. If $E_i \cap E_j = \varnothing$ for all $i \neq j$ we say that the events $E_1, E_2, \ldots$ are **pairwise disjoint.**

   If $E_1, E_2, \ldots$ is a finite or infinite sequence of events such that $E_i \cap E_j = \varnothing$ for $i \neq j$, then $P(\bigcup_i E_i) = \sum_i P(E_i)$.

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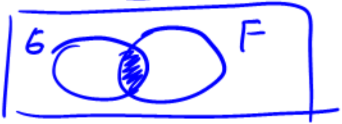
Other Probability Rules

$$P(A^c) = P(\bar{A}) = 1 - P(A)$$

4. **Complement Rule**: $P(E \cap {\sim}F) = P(E) - P(E \cap F)$. In particular, $P({\sim}E) = 1 - P(E)$.



5. $P(\varnothing) = 0$

6. **Addition Rule**: $P(E \cup F) = P(E) + P(F) - P(E \cap F)$.

7. If $E_1 \subseteq E_2 \subseteq \ldots$ is an infinite sequence, then $P(\bigcup_i E_i) = lim_{i \to \infty} P(E_i)$.

8. IF $E_1 \supseteq E_2 \supseteq \ldots$ is an infinite sequence, then $P(\bigcap_i E_i) = lim_{i \to \infty} P(E_i)$.

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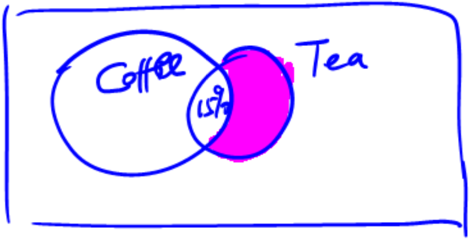
Example of Probability Rules

Suppose that 55% of adults drink coffee, 25% of adults drink tea and 15% of adults drink both coffee and tea.

1. What is the probability that an adult drinks tea but not coffee?



$P(\text{coffee}) = 0.55$

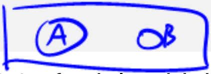
$P(\text{Tea}) = 0.25$

$P(\text{Coffee} \cap \text{Tea}) = 0.15$

$P(\text{Tea} \cap \sim\text{coffee}) = P(\text{Tea}) - P(\text{Tea \& coffee})$

$= 0.25 - 0.15 = \boxed{0.10}$

Example of Probability Rules



$P(A \cap B) = 0$

Suppose that 55% of adults drink coffee, 25% of adults drink tea and 15% of adults drink both coffee and tea.

2. What is the probability that an adult does not drink either beverages?

$P(\sim(\text{coffee} \cup \text{tea}))$

$= 1 - 0.65$

$= \boxed{0.35}$

$P(\text{coffee} \cup \text{Tea}) = P(\text{coffee}) + P(\text{tea})$

$- P(\text{coffee} \cap \text{tea})$

$= 0.55 + 0.25 - 0.15$

$= 0.65$

Example of Probability Rules

Suppose that 55% of adults drink coffee, 25% of adults drink tea and 15% of adults drink both coffee and tea.

3. What is the probability that an adult drinks tea, given that they drink coffee?

Later.

Hospital Patients

$P(E \cup F) = P(E) + P(F) - P(E \cap F)$

Hospital records show that 12% of all patients are admitted for heart disease, 28% are admitted for cancer (oncology) treatment, and 6% receive both coronary and oncology care. What is the probability that a randomly selected patient is admitted for coronary care, oncology or both? (Note that heart disease is a coronary care issue.)

$\boxed{0.34}$

$0.12 + 0.28 - 0.06$

General Multiplication Rule

For any two events E and F

$$P(E \cap F) = P(E) \times P(F|E)$$

or

$$P(E \cap F) = P(F) \times P(E|F)$$

Where $P(F|E)$ is the probability of F given that the event E has occurred. Similarly $P(E|F)$ is the probability of E given that F has occurred. These types of probabilities are called **conditional probability.** An easy way to determine this calculation is through a tree diagram.

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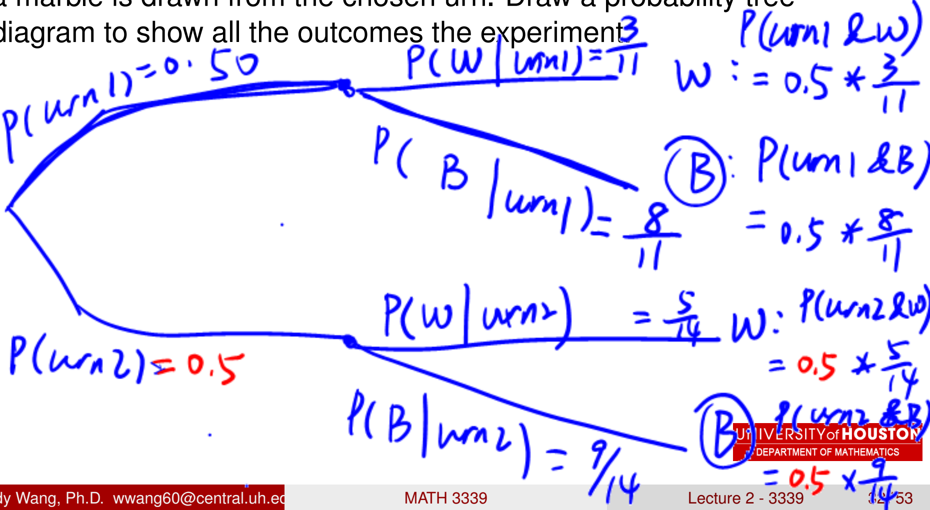
Example of Tree Diagram

Urn 1 contains 3 white and 8 blue marbles. Urn 2 contains 5 white and 9 blue marbles. One of the two urns is chosen at random with one as likely to be chosen as the other. An urn is selected at random and then a marble is drawn from the chosen urn. Draw a probability tree diagram to show all the outcomes the experiment.



a. What is the probability that Urn 2 was chosen?

$$0.5 = P(\text{urn}2)$$

b. What is the probability that a white marble was chosen, given that Urn 2 was chosen?

$$P(\text{White} \mid \text{urn } 2) = \frac{5}{14}$$

c. What is the probability that Urn 1 was chosen and that a blue marble was chosen?

$$P(\text{urn}1 \text{ and Blue}) = 0.5 * \frac{8}{11}$$

d. What is the probability that a blue marble was chosen?

$$P(\text{Blue}) = P(\text{Blue} \ \& \ \text{urn}1) + P(\text{Blue} \ \& \ \text{urn}2)$$
$$= 0.5 * 8/11 + 0.5 * 9/14$$

e. What is the probability that the marble drawn was white?

$$P(\text{white}) = 1 - P(\text{Blue})$$

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Example General Multiplication Rule

A person must select one of three boxes, each filled with toy cars. The probability of box A being selected is 0.19, of box B being selected is 0.18, and of box C being selected is 0.63. The probability of finding a red car in box A is 0.2, in box B is 0.4, and in box C is 0.9. We are selecting one of the toy cars.

$P(\sim red \mid A) = 1 - 0.2 = 0.8$

1. What is the probability that the toy car is red and in box A?

$P(A) = 0.19$ $\qquad$ $P(red \mid A) = 0.2$

$P(B) = 0.18$ $\qquad$ $P(red \mid B) = 0.4$

$P(C) = 0.63$ $\qquad$ $P(red \mid C) = 0.9$

? $P(Red \text{ and } A) = P(red \mid A) * P(A)$

$= 0.2 * 0.19 = 0.038$

Example General Multiplication Rule

A person must select one of three boxes, each filled with toy cars. The probability of box A being selected is 0.19, of box B being selected is 0.18, and of box C being selected is 0.63. The probability of finding a red car in box A is 0.2, in box B is 0.4, and in box C is 0.9. We are selecting one of the toy cars.

2. What is the probability that the toy car is red and in box B?

$$P(\text{red and } B) = 0.4 * 0.18$$

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Example General Multiplication Rule

A person must select one of three boxes, each filled with toy cars. The probability of box A being selected is 0.19, of box B being selected is 0.18, and of box C being selected is 0.63. The probability of finding a red car in box A is 0.2, in box B is 0.4, and in box C is 0.9. We are selecting one of the toy cars.

3. What is the probability that the toy car is red and in box C?

$$0.9 * 0.63$$

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Example 14

$P(F) = 0.30$ $P(\sim F) = 0.70$

Thirty percent of the students at a local high school face a disciplinary action of some kind before they graduate. Of those "felony" students, 40% go on to college. Of the ones who do not face disciplinary action 60% go on to college.

F: felony
C: college

$P(\sim C \mid F) = 0.60$

$P(C \mid F) = 0.40$

$P(C \mid \sim F) = 0.60$

1. What is the probability that a randomly selected student both faced a disciplinary action and went on to college? $P(F \cap C)$.

$P(F \text{ and } C) = P(F \cap C) = P(F \,\&\, C)$

$= P(C \mid F) * P(F)$

$= 0.40 * 0.30 = \boxed{0.12}$

$P(\sim F \cap C)$

$= P(C \mid \sim F) * P(\sim F)$

$= 0.60 * 0.70$

$= 0.42$

Example

Thirty percent of the students at a local high school face a disciplinary action of some kind before they graduate. Of those "felony" students, 40% go on to college. Of the ones who do not face disciplinary action 60% go on to college.

2. What percent of the students from the high school go on to college?

$$P(C) = P(C \cap F) + P(C \cap \sim F)$$

$$= 0.12 + 0.42$$

$$= \boxed{0.54}$$

Example of General Multiplication Rule

Suppose we draw two cards from a deck of 52 fair playing cards, what is the probability of getting an ace on the first draw and a king on the second draw?

"A" "k"

- Without replacement.

$$P(\text{"A" and "k"}) = \frac{4}{52} * \frac{4}{51}$$

- With replacement.

$$\frac{4}{52} * \frac{4}{52}$$

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Conditional Probability

Let A and B be events with $P(B) > 0$. The **conditional probability** of A, given B is:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

General rule for multiplication: For any two events E and F, $P(E \cap F) = P(E) \times P(F|E)$ or $P(E \cap F) = P(F) \times P(E|F)$.

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Two Frequently Asked Questions

1. When do I add and when do I multiply?
   - Add when finding the chance of events A **or** B or both happening.

$$P(A \text{ or } B) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

   - Multiply when finding the chance that both events A **and** B happen.

$$P(A \text{ and } B) = P(A \cap B) = P(A) \times P(B, \text{ given } A) = P(A)P(B|A)$$

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Two Frequently Asked Questions

2. What's the difference between disjoint (mutually exclusive) and independent?
   - Two events are disjoint if the occurrence of one prevents the other from happening.
   $$P(A \cap B) = 0$$
   - Two events are independent if the occurrence of one does not change the *probability* of the other.
   $$P(A|B) = P(A)$$

A : male

B : raining

A and B are independent but not disjoint.

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Example

Thirty percent of the students at a local high school face a disciplinary action of some kind before they graduate. Of those "felony" students, 40% go on to college. Of the ones who do not face disciplinary action 60% go on to college.

1. Show if events {faced disciplinary action} and {went to college} are independent or not.

because $P(C|F) \neq P(C)$,

C and F are NOT independent.

if $P(A \text{ and } B) = P(A) * P(B)$,

A and B are independent.

$P(\text{cat and dog}) = 0.24$;

$P(\text{cat}) = 0.4$ $\qquad$ $P(\text{dog}) = 0.6$

$0.24 = 0.4 * 0.6 \Longrightarrow$ cat and dog independent.

$P(\text{cat or dog}) = P(\text{cat}) + P(\text{dog}) - P(\text{cat \& dog})$

$= 0.4 + 0.6 - 0.4 * 0.6$

Dogs and Cats

$P(\text{dog}) = 0.6$ $\quad P(\text{dog and cat}) = 0.24$

$P(\text{cat}) = 0.4$

The probability of owning a dog is 0.6, the probability of owning a cat is 0.4. The probability of owning a dog and a cat is 0.24.

1. What is the probability that out of cat owners, they also own a dog?

$$P(\text{dog} \mid \text{cat}) = \frac{P(\text{dog and cat})}{P(\text{cat})} = \frac{0.24}{0.4} = \boxed{0.6}$$

2. What is the probability that out of dog owners, they also own a cat?

$$P(\text{cat} \mid \text{dog}) = \frac{P(\text{dog and cat})}{P(\text{dog})} = \frac{0.24}{0.6} = \boxed{0.4}$$

3. Are "owning a dog" and "owning a cat" independent events?

$\checkmark P(A \mid B) = P(A)$?

$\checkmark P(B \mid A) = P(B)$?

since $P(\text{dog} \mid \text{cat}) = P(\text{dog})$

$P(\text{cat} \mid \text{dog}) = P(\text{cat})$

$P(A \cap B) = P(B \cap A)$

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Buyers of Computers $\to P(B \mid \sim A) = 0.40$

Approximately 5 months after the introduction of the iMac, Apple reported that 32% of iMac buyers were first-time computer buyers. At the same time, approximately 5% of all computer sales were of iMacs. Of buyers who did not purchase an iMac, approximately 40% were first-time computer buyers. Let A = the event bought an iMac and B = the event of first-time computer buyer

$P(B) = ?$

1. What is the probability of a person buying an iMac, $P(A)$?

   $P(A) = 0.05 \longrightarrow P(\sim A) = 1 - P(A) = 0.95$

2. What is the probability that a person is a first-time computer buyer, given they bought an iMac, $P(B|A)$?

   $P(B \mid A) = 0.32.$

3. What is the probability that a person bought an iMac *and* is a first-time computer buyer, $P(A \cap B)$?

   $P(A \cap B) = P(B \mid A) * P(A) = 0.32 * 0.05 = 0.016$

4. What is the probability of a person buying a iMac, given they are first-time buyers?

A (iMac), B (first-time buyers)

$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)} = \frac{0.016}{?}$$

$$P(B) = P(\underbrace{B \text{ and } A}) + P(\underbrace{B \text{ and } \sim A})$$

$$= 0.016 + P(B \mid \sim A) * P(\sim A)$$

$$= 0.016 + 0.40 * 0.95$$

$$= \boxed{0.396}$$

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Bayes' Rule

- The probability of a person buying an iMac, given they are first-time buyers is an example of using **Bayes' rule**.
- Given a prior (initial) probability then from sources we obtain additional information about the events.
- From these events we revise the probabilities and get a posterior probability.
- This is an application of the General Multiplication Rule.
- It might be easier to use the tree diagram to calculate this probability.

Let A and $B_1, B_2, \ldots, B_k$ be pairwise disjoint events such that each $P(B_i) > 0$ and $\Omega = B_1 \cup B_2 \cup \ldots \cup B_k$ and assume $P(A) > 0$. Then for each i,

$$P(B_i|A) = \frac{P(A|B_i)P(B_i)}{P(A)} = \frac{P(A \cap B_i)}{P(A)}$$

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Example

A rare disease exists in which only 1 in 500 are affected. A test for the disease exists but of course it is not infallible. A correct positive result (patient actually has the disease) occurs 95% of the time while a false positive result (patient does not have the disease) occurs 1% of the time. If a randomly selected individual is tested and the result is positive, what is the probability that the individual has the disease?

$P(D) = 0.002$

$P(\text{"+"} \mid D) = 0.95$

$P(\text{"−"} \mid D) = 0.05$

$P(\sim D) = 0.998$

$P(\text{"+"} \mid \sim D) = 0.01$

$P(\text{"−"} \mid \sim D) = 0.99$

$$P(D \mid \text{"+"}) = \frac{P(D \text{ and } \text{"+"})}{P(\text{"+"})}$$

$$= \frac{0.002 * 0.95}{?}$$

$$P(\text{"+"}) = P(\text{"+"} \cap D) + P(\text{"+"} \cap \sim D)$$

$$= 0.002 * 0.95 + 0.998 * 0.01$$

Example Two-Way Table

A clothing store targets young customers (ages 18 through 22) wishes to determine whether the size of the purchases related to the method payment. Suppose a customer is picked at random. The following is 300 customers the amount of the purchase and method payment.

	Cash	Credit	Layaway	Total
Under $40	60	30	10	100
$40 or more	40	100	60	200
Total	100	130	70	300

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Example

1. What is the probability that the customer paid with a credit card?

$$P(\text{credit}) = \frac{130}{300}$$

2. What is the probability that the customer purchased under $40?

$$P(< \$40) = \frac{100}{300}$$

3. What is the probability that the customer paid with credit card given that the purchase was under $40?

$30/300$

$$P(\text{Credit} \mid < \$40) = \frac{P(\text{credit} \cap < \$40)}{P(< \$40)} \quad \frac{100}{300}$$

4. What is the probability that the customer paid with credit card and that the purchase was under $40?

$$P(\text{credit} \;\&\; < \$40) = \frac{30}{300}$$

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Example

5. Are type of payment and amount of purchase independent?

$$P(\text{credit}) = \frac{130}{300}$$

$$P(\text{credit} \mid < \$40) = \frac{30}{100}$$

$\Rightarrow$ NOT independent.

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