CS310 – AI Foundations
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Week 4: A* Search

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Welcome!

- Welcome to CS310 week 4!
- This week we will be introducing more costs and heuristics
  - Best first Search
  - A Algorithm
  - A* Algorithm
- Adversarial Search
  - Game Trees

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Last Week

- Iterative Deepening Search
- Visited list
- Introduction to Heuristics
- Hill Climbing

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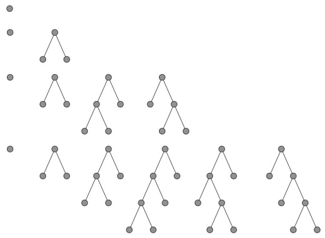
Iterative Deepening

- Algorithm: do (repeated) depth-limited search, but with increasing depth.
- Expands nodes multiple times, but time complexity is of the same order of magnitude



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Breadth-First Search with visited list

- let Agenda = [S_0]
- let Visited $= []
- while Agenda ≠ [] do
  - let Current = remove-first (Agenda)
  - let Agenda = rest(Agenda)
  - if Goal (Current) then return ("Found it!")
  - let Next = NextStates (Current)
  - let Agenda = Agenda + Next – Visited)
  - Visited.append(Current)

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Making Search cost-aware: Uniform-cost search

- Goal: find the ``cheapest'' path from the initial state to a goal state.
- Idea: same algorithm as BFS but we use a priority queue as agenda
  - Previously, we added states to the agenda arbitrarily
    - Simply gone Left to Right in our examples
    - Because it doesn't matter!
  - Now we consider costs when deciding order
- Nodes are added to agenda in increasing order of path so far
  - Agenda is now ordered in terms of priority
  - Or we find cheapest solution
- Guaranteed to find an optimal solution!

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Uniform-cost search

- let Agenda = [(S_0,0)]
- let Visited $= []
- while Agenda ≠ = [ ]:
  - let (Current,i) = lowest-cost (Agenda)
  - let Agenda = remainder(Agenda)
  - if Goal (Current): return ("Found it!")
  - if Current not in Visited:
    - let Next = NextStatesWithCosts(Current)
    - let Agenda = Agenda + Next
    - Visited.append(Current)

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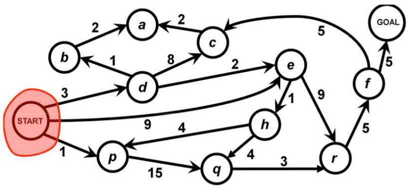
Example of algorithm variant

- PQ = Priority Queue
- Want to reach goal state



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Heuristic Search & Hill Climbing

- DFS and BFS are both searching blind " they search all possibilities in an order dictated by NextStates(S_i)
  - Its still uninformed search
- When people search, we look in the most promising places first
  - Solving maze with logical rules
  - Rule of thumb (looking for lost items)
- e.g. in the block world at state {[a],[b],[c]}, the most promising move is to {[a],[b,c]}
  - Obvious to (nearly) any human, right?
- The most promising states are often those which are closest to the goal state
- But how can we know how close we are to the goal state?

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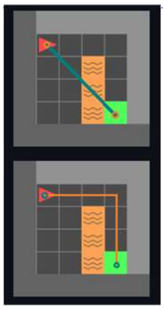
Heuristic Functions

- We had 2 possible metrics
- Euclidean Distance and Manhattan Distance
- Slightly different values
- Both ignore the magma, they assume that there are no constraints on actor movement.
- They relax the problem
- This means that we consider a much simpler and potentially easier to calculate case



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Heuristic Functions

- Consider the sliding blocks puzzle
- We can move 1 block at a time into the blank space
- Can we design a good heuristic function?

7	2	4
5		6
8	3	1

→

1	2	3
4	5	6
7	8	

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Consider a Relaxed Function

- When we relax the problem, we remove some of the restrictions
- Possible examples
  - $h_1(s)$ = number of misplaced tiles on board
  - $h_2(s)$ = sum of all Manhattan distances of putting tiles in correct position
  - $h_{2(s)} = \max(h_1(s), h_2(s))$
    - We can combine admissible heuristics
- We can ignore the rule about having to slide into a blank space
- We relax the problem

7	2	4
5		6
8	3	1

→

1	2	3
4	5	6
7	8	

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Heuristic Search

Arad	366	**Mehadia**	241
Bucharest	0	**Neamt**	234
Craiova	160	**Oradea**	380
Drobeta	242	**Pitesti**	100
Eforie	161	**Rimnicu Vilcea**	193
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Giurgiu	77	**Timisoara**	329
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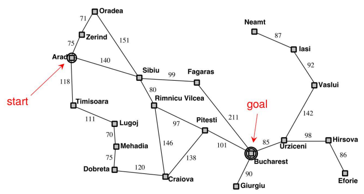
Figure 3.22 Values of h_{SLD}—straight-line distances to Bucharest.



Hill Climbing

- The easiest way to use a heuristic estimate to search is to require that every single move we make takes us closer to the goal
- The form of search doesn't even require an agenda, since at each decision point, we take the action that looks best to us and repeat until we are done
- Very low memory footprint, as no agenda or visited list needed
- Problems: dead ends, plateaus, solution quality (i.e. the number of steps can be very poor)
- Used to good effect in planning software

Hill Climbing

```
HCS (Graph, state):
      current ← state
      while true do
            if goal(current) then
                  return current
            E = next_states(current) in Graph
            current ← element s' of E with min h(s')
```


- Problem: Navigate from Iasi to Oradea
- Option, Neamt or Vaslui
- Heuristic says Neamt is closer!
- But then, dead end, so go back. Use lowest cost
- That is also Neamt! (distance 87)

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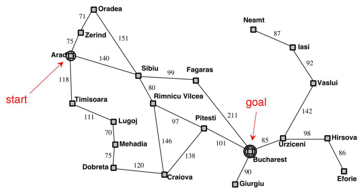

Enforced Hill Climbing

- Problems: dead ends, plateaus, solution quality (i.e. the number of steps can be very poor)
- Hill climbing can get stuck
- Enforced hill climbing uses Breadth first search
  - goal condition that h(s') < h(s)
- Doesn't look for the minimum, but used breadth first search to identify a better successor
- Can identify a better route than just hill climb (may avoid plateaus)

EnforcedHill Climbing

```
HCS (Graph, state):
      current ← state
      heuristic_val ←h(state)
      while true do
            if goal(current) then
                  return current
            current ← BreadthFirstSearch(Graph, state)
```

- Does not look for the minimum h, looks for the first h better than the heuristic_val
- Can depend how you order the search

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Best First Search

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Greedy Best First Search

- Enforced hill climbing is great when it works, but for some problems its better to keep track of the nodes we havent yet expanded, using the agenda
  - Keep our agenda and visited list
- We can then use the heuristic function to determine which node to expand next
- As new states are discovered, we add them to the agenda and record the value of the heuristic function
- When we pick the next node to explore, we choose the one which has the lowest value for the heuristic function (i.e. the one that looks nearest to the goal)
- We do this by picking the best heuristic value from the agenda
  - We do not consider actual values, purely the heuristic
- Very similar to Uniform Cost Search, but using no cost information

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- let Agenda = [(S_0,0)]
- let Visited $= []
- while Agenda ≠ = [ ]:
  - let (Current,i) = lowest-heuristic (Agenda)
  - let Agenda = remainder(Agenda)
  - if Goal (Current): return ("Found it!")
  - if Current not in Visited:
    - let Next = NextStatesWithCosts(Current)
    - let Agenda = Agenda + Next
    - Visited.append(Current)

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Analysing Greedy Best-First Search

- The good:
  - Complete in finite search spaces – it will find a solution
  - Best case complexity of O(bm)
  - Easy to implement
  - With a good heuristic, very efficient
- The bad:
  - Very dependent on heuristic
  - Not complete in infinite space
  - Can be slow
- The ugly:
  - **Its greedy**
  - Its not always the best thing to try to get to the goal as quickly as possible

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Best First Search and Algorithm A

- Best-first search can speed up the search by a very large factor, but can it isn't guaranteed to return the shortest solution
- When deciding to expand a node, we need to take account of how long the path is so far, and add that on to the heuristic value:
- $f(S_i, G) = g(S_0, Si) + h(Si, G)$
- Here, g represents the actual cost value (i.e. the distance covered from the initial state to the current location, and h is the heuristic value for the remaining distance
- This will give a search which has elements of both breadth-first search (uniform cost search) and best-first search
- This type of search is called "Algorithm A"

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Algorithm A*

- If h(Si ,G) never over-estimates the distance from S_i to the goal G, it is called an admissible heuristic
- If h(Si ;G) is admissible, then Algorithm A will always return the shortest path (like breadth-first search) but will omit much of the work if the heuristic function is informative
- The use of an admissible heuristic turns Algorithm A into Algorithm A^*
- Uses: problem solving, route finding, path planning in robotics, computer games, etc.
- Complete as long as heuristic is safe.
- Optimal as long as the heuristic is admissible.

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Why is A* Optimal?

- Suppose a suboptimal goal node, S_k, appears in the agenda -we have not selected it yet, so we do not yet know that it is a goal node
- Also on the agenda, there must be a node, S_i which is on the optimal path from S_0 to the goal state
- Since the heuristic function, h(Si , G), is admissible, this means:
  - $g(S_0; S_k) > g(S_0; S_i) + h(S_i; G)$
  - (as the right hand side will be smaller or equal to the actual costs of reaching the goal G)
- Therefore S_k will never be selected over Si for expansion

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A* And visited List

- To optimise the algorithm one can maintain a visited list (similar to UCS)
- in this case you need to record the expanded state together with the value at which it was expanded
- if we meet a node that has been expanded already at a higher cost value, we expand again!

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- A* Can use the best of both worlds

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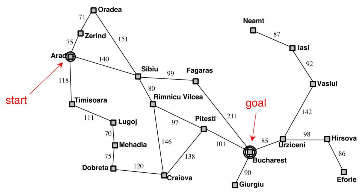

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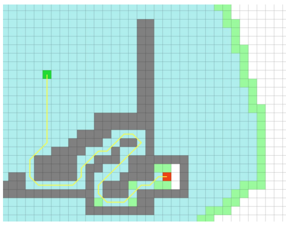
Search Algorithm Examples

- http://qiao.github.io/PathFinding.js/visual/
- You can test different Heuristics and Algorithms



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Summary

- Try these algorithms out!
  - Not just for maps, but for states too
- Greedy Best First Search
- A* Algorithm

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