Sketch of a variant Laporta algorithm

We represent an IBP identity as a descending list of terms:

$$0 = \sum_i c_i I_i \Leftrightarrow \texttt{[c1*I1,c2*I2, ..]}$$

where I_1 is the most complicated integral. Among the system of IBP identities, we group the equations with the same highest integral, and place like the following form:

$$\begin{pmatrix}
\bullet & \bullet & \bullet & & & & & \\
\bullet & \bullet & \bullet & \bullet & \bullet & & & \\
\bullet & \bullet & \bullet & \bullet & \bullet & \bullet & & \\
\bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \\
 & \bullet & \bullet & & & & & \\
 & \bullet & \bullet & \bullet & & & & \\
 & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\
 & & \bullet & \bullet & \bullet & & & \\
 & & \bullet & \bullet & \bullet & \bullet & \bullet & \\
 & & & \vdots & & & &
\end{pmatrix}$$

I.e., in each subset of equations, they share the highest integral (the left most $\bullet$) and they line up from shorter to longer.

Within the subset of equations, substituting the top element into the successors, we eventually have the right triangular form:

$$\begin{pmatrix}
\bullet & \bullet & \bullet & & & & & \\
 & \bullet & \bullet & \bullet & \bullet & \bullet & & \\
 & & \bullet & & & & & \\
 & & & \bullet & \bullet & \bullet & \bullet & \bullet \\
 & & & & \bullet & \bullet & & \\
 & & & & & \vdots & &
\end{pmatrix}$$

Once we reach this right triangular form, use back substitution; for the first two lists, using second equation, we substitute it in the first and represent the highest integral via lower integrals:

$$\begin{pmatrix}
\bullet & \bullet & \bullet & & & \\
 & \bullet & \bullet & \bullet & \bullet & \bullet
\end{pmatrix} \mapsto \begin{pmatrix}
\bullet & 0 & \bullet & \bullet & \bullet & \bullet \\
 & \bullet & \bullet & \bullet & \bullet & \bullet
\end{pmatrix}$$

Finally we have the following form of equations:

$$\begin{pmatrix}
\bullet & 0 & 0 & 0 & 0 & \star & \star & \star \\
 & \bullet & 0 & 0 & 0 & \star & \star & \star \\
 & & \bullet & 0 & 0 & \star & \star & \star \\
 & & & \bullet & 0 & \star & \star & \star \\
 & & & & \bullet & \star & \star & \star \\
 & & & & & \star & \star & \star
\end{pmatrix}$$

The $\star$'s are the Master Integrals of this system.