# Experimental Methods: Lecture 3

## Attrition

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# Road Map to Lecture 3: Attrition

- Non-interference re-visited
- Attrition
- RAND case study
- Combating Attrition
- MIPO
- MIPO—X and IPW
- Bounds

# Non-interference

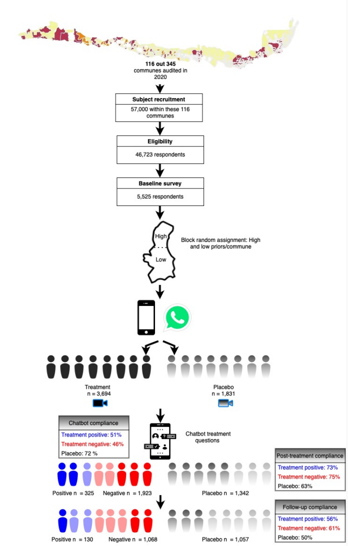
$\text{ATE} = \text{E}\left[Yi(1)_j D_i = 1\right]\ \text{E}\left[Yi(0)_j D_i = 0\right]$

- Thus, average outcomes in the control and treatment groups in the sample are unbiased estimators of E [Yi (1)] and E [Yi (0)]
- But implicit in this description is the assumption that the researcher observes Yi for all subjects assigned
- Attrition occurs when outcome data are missing
- If attrition systematically related to potential outcomes, remaining subjects are no longer random samples of original group of subjects
- Thus, comparison of remaining group averages may no longer be an unbiased estimator of ATE

# How Might Attrition Occur

- Subjects refuse to cooperate with researchers
    - Respondents refuse post-treatment questionnaire
- Researchers lose track of experimental subjects
    - Subjects change address or name
- Firms, organizations, or governments block researchers' access to outcomes
    - Common with experiments on sensitive topics, e.g. corruption
- Outcome variable unavailable for some subjects
    - For a job training program treatment that wants to measure wages six months later, how do you measure this outcome for subjects without jobs?
- Researchers deliberately discard observations
    - Subjects not understanding instructions get discarded

# Duch and Torres



# Duch and Torres

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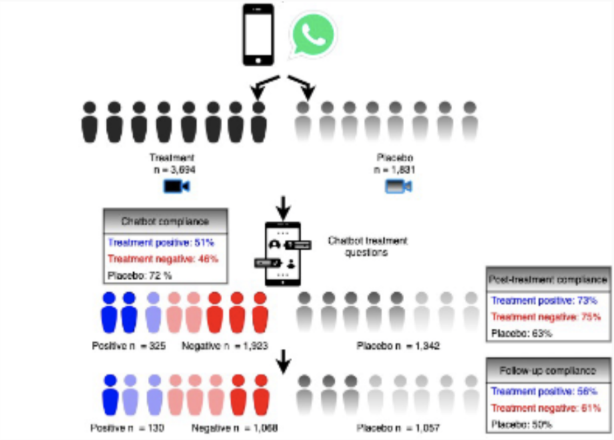

# Rand Health Insurance Experiment: Setup

- Famous study illustrating the dangers of attrition for causal inference Cost \$80 million (1974 USD), or about \$300 million today
- Examines how copayment schemes in health insurance affect health service consumption and outcomes
- Four groups: 5%, 50% 75%, and 100% cost coverage treatments
- Costs over \$1,000 for last three groups fully insured
- Incentives so you "cannot lose financially by participating"
- As a related aside, Finkelstein et al's Oregon Health Insurance experiment

# Rand Health Insurance Experiment: Setup

- Evaluation 3-5 years after treatment
- Those paying larger share made fewer physician visits and had lower rates of hospital admissions
- 100% group consumed 46% more in health services than 5% group
- 100% group no healthier on average than others, based on a wide array of health assessments
- Traditional interpretation of result: Requiring large co-pays does not have an adverse impact on health incomes for the average person
- But what about attrition? (Newhouse 1989)
- Any thoughts on how attrition might happen in a way that is systematically related to potential outcomes?

# Rand Health Insurance Experiment: Critique

- 8% in free group refused to enroll
  - Military service, institutionalization, death, incomplete data collection
- 25% refusal rate in 5
  - 6.7% in free plan left voluntarily after enrolling
  - 0.4% in 5% treatment group left after enrolling
- One interpretation: Those who chose to remain in copay plans anticipated lower health costs
  - Those anticipating illness drop out to pick up cheaper private insurance
- Alternative: Those who dropped out have same unmeasured health outcomes and health service consumption than those remaining
- How would we know? How would we deal with it?

# Four Strategies to Combat Attrition

- Assume missingness is independent of potential outcomes
  - Unbiased inferences but lose sample size
  - Cannot directly assess this
  - Newhouse points out average pre-treatment health outcomes and health service utilization are similar across experimental groups among subjects who do not drop out of the study
  - If one assumes prior health outcomes are indicative of subsequent unmeasured potential outcomes, one could interpret this evidence to mean missingness is independent of potential outcomes

# Four Strategies to Combat Attrition

- Assume missingness independent of potential outcomes within subgroups defined by background attributes
  - Newhouse reports refusal to enroll is correlated with age/education
  - We could recover unbiased ATE by re-weighting the data to "fill in" age/education cells depleted by attrition
- Place bounds by guessing missing values (Manski 1995, 2007)
  - Assume those who disappear are extremely healthy or extremely ill
  - Alternatively, "trim" healthiest observations in free care group until attrition rate as high as cost-sharing groups
- Gather more data from missing subjects (Cochran 1977)
  - RAND eventually gathered health outcomes from 77

# Special Forms of Attrition: MIPO

- If $R_i(z) \perp\!\!\!\perp Y_i(z)$, attrition is (relatively) innocuous
- Missing Independent of Potential Outcomes or MIPO
- Can only gather circumstantial evidence about plausibility
  - If result of random procedure, no systematic relationship between $r_i$ and subject background attributes or experimental assignment should exist (regr F-test)
  - But do covariates at one's disposal include the systematic sources of missingness?
- MIPO assumption more convincing if subjects have little discretion over whether their outcomes will be reported
  - Tutoring: Students miss for midyear exam
  - Voter mobilization: Town clerk fails to record data in timely fashion
  - Arguments for/against MIPO in these cases?

# Special Forms of Attrition: MIPO

- Alternative assumption: Attrition is unrelated to potential outcomes conditional on pre-treatment covariates $X_i$
- Not surprisingly, this is referred to as Missing Independent of Potential Outcomes given X, or MIPO | X
- Example: In student test example, we condition on attendance record prior to the treatment intervention as $X_i$
- Intuition: Assume $N_1 = 30$ Oxford, $N_2 = 10$ Cambridge students treated with tutoring vouchers
  - Let's say 15 Oxford students were missing at random
  - How might you recover an unbiased ATE in this example?

# Weighting under MIPO | X

- Define $\pi_i$ (z = 1; x) as the share of non-missing subjects among those who are treated and have covariate profile $X_i = x$
- When MIPO | X holds,

$$E[Y_i(1)] = \frac{1}{N}\sum_{i=1}^{N}\frac{Y_i(1)r_i(1)}{\pi_i(z=1,x)} \tag{1}$$

- Subjects with missing outcomes do not appear in formula
- Produces accurate estimates of ATE when non-missing observations are good substitutes for missing outcomes
- Also known as inverse-probability weighting because subjects weighted by $\frac{1}{\pi_i(z=1,x)}$
- Typically, logistic regression is used to estimate $\hat{\pi}(z,x)$ weighted regression is then run

# Weighting under MIPO | X

- Primary drawback is the possibility that missingness remains related to potential outcomes
- As before, this is not directly testable or observable
    - Maybe those missing Oxford students weren't randomly missing!
- If attrition biases $A\hat{T}E$ for a subgroup, it may be given more weight in estimates and actually worsen bias
- Also increases sampling variability because more weight placed on subsamples with large share of missing observations

# Simple Example

| Obs | $Y_i(0)$ | $Y_i(1)$ | $R_i(0)$ | $R_i(1)$ | $Y_i(0)\|R_i(0)$ | $Y_i(1)R_i(1)$ | $X_i$ |
|---|---|---|---|---|---|---|---|
| 1 | 3 | 4 | 1 | 1 | 3 | 4 | 1 |
| 2 | 4 | 7 | 1 | 1 | 4 | 7 | 1 |
| 3 | 3 | 4 | 1 | 1 | 3 | 4 | 1 |
| 4 | 4 | 7 | 1 | 1 | 4 | 7 | 1 |
| 5 | 10 | 14 | 0 | 0 | Missing | Missing | 0 |
| 6 | 12 | 18 | 0 | 0 | Missing | Missing | 0 |
| 7 | 10 | 14 | 1 | 1 | 10 | 14 | 0 |
| 8 | 12 | 18 | 1 | 1 | 12 | 18 | 0 |

# Simple Example

- Does MIPO hold? Implications?
- Does MIPO $|$ X hold? Implications?
- What weights?
- Is unweighted expectation measuring something useful?

# Placing Bounds on ATE

- Extreme value bounds gauge consequences of attrition
    - Bounds sample ATE by filling in all missing outcomes
- Substitute extreme values to generate upper/lower bounds
    - Upper bound: Maximum value for missing treatment outcomes, minimum value for missing control outcomes
    - Lower bound: Minimum value for missing treatment outcomes, maximum value for missing control outcomes
- Regression analogy is Leamer's Specification Search

# Placing Bounds on ATE

- Can often bracket true ATE well, but wide bounds given weak assumptions
- As rate of attrition increases, bounds become less informative. Why?
- Can form bounds for transformations for $Y_i$
  - Example: $Y_i' = 1$ if $Y_i > 5$ and $Y_i' = 0$ otherwise)
  - Why might we want to do this?

# Lee 2009 Lee Bounds

$$
\begin{aligned}
q_T &= \frac{\sum_i 1\left(T_i = 1, S_i = 1\right)}{\sum_i 1\left(T_i = 1\right)} \\
q_C &= \frac{\sum_i 1\left(T_i = 0, S_i = 1\right)}{\sum_i 1\left(T_i = 0\right)}
\end{aligned}
$$

$$
q = \frac{q_T - q_C}{q_T}
$$

$$
\begin{aligned}
y_q^T &= G^{-1}_{Y|T=1,S=1}(q) \\
y_{1-q}^T &= G^{-1}_{Y|T=1,S=1}(1-q)
\end{aligned}
$$

# Lee 2009 Lee Bounds

$$\widehat{\theta}^{\text{upper}} = \frac{\sum_i 1\left(T_i = 1, S_i = 1, Y_i \geqslant y_q^T\right) Y_i}{\sum_i 1\left(T_i = 1, S_i = 1, Y_i \geqslant y_q^T\right)} - \frac{\sum_i 1\left(T_i = 0, S_i = 1\right) Y_i}{\sum_i 1\left(T_i = 0, S_i = 1\right)}$$

$$\widehat{\theta}^{\text{lower}} = \frac{\sum_i 1\left(T_i = 1, S_i = 1, Y_i \leqslant y_{1-q}^T\right) Y_i}{\sum_i 1\left(T_i = 1, S_i = 1, Y_i \leqslant y_{1-q}^T\right)} - \frac{\sum_i 1\left(T_i = 0, S_i = 1\right) Y_i}{\sum_i 1\left(T_i = 0, S_i = 1\right)}$$

# VSemenova Leebounds

```r
install.packages("devtools")
library(devtools)
install_github("vsemenova/leebounds")
library(leebounds)

# compute basic Lee (2009) bounds for ATE in week
    208
leedata=data.frame(treat=JobCorps_data_baseline$
    TREATMNT.y, selection=JobCorps_data_employment$
    week_208,outcome=JobCorps_data_wages$week_208)
GetBounds(leebounds(leedata))
```

# Duch and Torres IPW

Table A5—Intention-to-treat: Beliefs about malfeasance - Unweighted and IPW

| | Qualitative | | Resources | | Distribution | |
|---|---|---|---|---|---|---|
| | Unweighted | Unstable | Unweighted | Unstable | Unweighted | Unstable |
| Intercept | 2.446*** | 1.344*** | 2.004*** | 2.168*** | 1.126*** | 1.152*** |
| | (0.085) | (0.024) | (0.086) | (0.010) | (0.038) | (0.015) |
| Prior | 0.478*** | 0.827*** | 0.463*** | 0.828*** | 0.426*** | 0.846*** |
| | (0.014) | (0.012) | (0.016) | (0.009) | (0.016) | (0.015) |
| Treat | 0.780*** | 1.859*** | 0.779*** | 0.373*** | 0.362*** | 0.307*** |
| | (0.071) | (0.060) | (0.078) | (0.073) | (0.027) | (0.030) |
| Covariates | Yes | Yes | Yes | Yes | Yes | Yes |
| Num.Obs. | 3592 | 3592 | 3592 | 3592 | 3592 | 3592 |
| R2 | 0.258 | 0.997 | 0.213 | 0.998 | 0.193 | 0.995 |
| R2 Adj. | 0.257 | 0.997 | 0.212 | 0.998 | 0.192 | 0.995 |

*Note:* This table reports the unweighted and weighted....

*Source:* All waves data

# Duch and Torres Lee Bounds

TABLE A6—INTENTION-TO-TREAT: BELIEFS ABOUT MALFEASANCE - BOUNDS

| | Baseline | | | Upper bound | | | Lower bound | | |
|---|---|---|---|---|---|---|---|---|---|
| | Baseline | Negative | Malfeasance levels | Baseline | Negative | Malfeasance levels | Baseline | Negative | Malfeasance levels |
| Intercept | 2.734*** | 2.471*** | 1.820*** | 1.159** | 0.875* | -0.215 | 4.872*** | 4.695*** | 5.019*** |
| | (0.401) | (0.446) | (0.596) | (0.474) | (0.489) | (0.695) | (0.434) | (0.490) | (0.708) |
| Prior | 0.467*** | 0.466*** | 0.465*** | 0.416*** | 0.416*** | 0.415*** | 0.388*** | 0.388*** | 0.388*** |
| | (0.020) | (0.020) | (0.020) | (0.020) | (0.020) | (0.019) | (0.021) | (0.021) | (0.020) |
| Treat | 0.749*** | 0.575** | 0.292 | 1.981*** | 1.830*** | 1.964*** | -0.729*** | -0.991*** | -2.212*** |
| | (0.079) | (0.208) | (0.458) | (0.091) | (0.258) | (0.619) | (0.095) | (0.208) | (0.533) |
| Negative | | 0.311 | | | 0.339* | | | 0.213 | |
| | | (0.212) | | | (0.179) | | | (0.232) | |
| Log malfeasance | | | 0.062** | | | 0.094*** | | | -0.011 |
| | | | (0.029) | | | (0.034) | | | (0.035) |
| Treat x Negative | | 0.202 | | | 0.174 | | | 0.304 | |
| | | (0.227) | | | (0.276) | | | (0.236) | |
| Treat x Log malfeasance | | | 0.032 | | | 0.001 | | | 0.104*** |
| Covariates | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes | Yes |
| Num.Obs. | 3439 | 3439 | 3439 | 3915 | 3915 | 3915 | 3915 | 3915 | 3915 |
| R2 | 0.263 | 0.267 | 0.268 | 0.268 | 0.271 | 0.273 | 0.151 | 0.154 | 0.154 |
| R2 Adj. | 0.257 | 0.261 | 0.262 | 0.263 | 0.266 | 0.268 | 0.145 | 0.147 | 0.148 |

*Note:* Update notes

*Source:* All waves data