Fermi Estimate

What is Fermi Estimate?

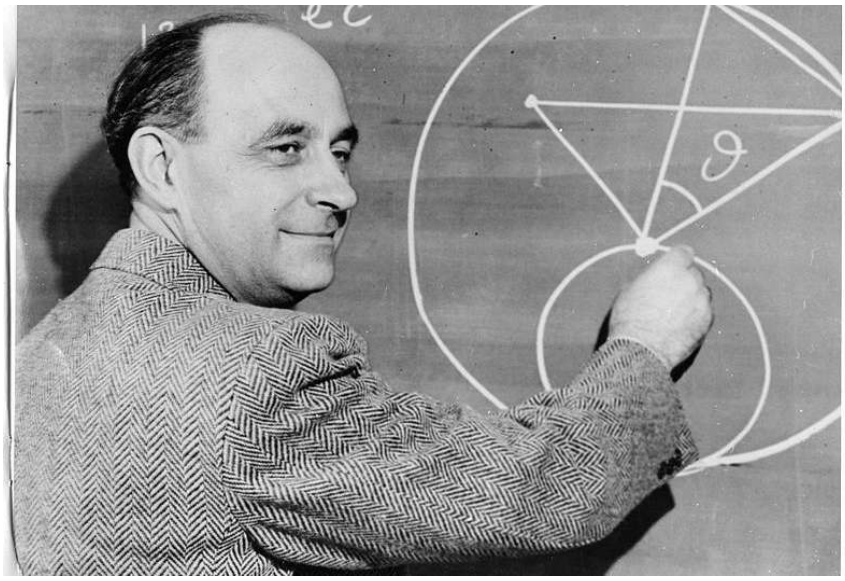
> In physics or engineering education, a Fermi problem, Fermi quiz, Fermi question, Fermi estimate, order-of-magnitude problem, order-of-magnitude estimate, or order estimation is an estimation problem designed to teach dimensional analysis or approximation of extreme scientific calculations, and such a problem is usually a back-of-the-envelope calculation. - Wikipedia

A Fermi estimate is one done using back-of-the-envelope calculations and rough generalizations to estimate values which would require extensive analysis or experimentation to determine exactly.

> back-of-the-envelope calcualtions is a rough calculation, typically jotted down on any scrap of paper such as envelope. It is more than a guess and less than an accurate calculation or mathematical proof. The defining characteristic of back-of-the-envelope is the use of simplified assumptions. -- Wikipedia

Physics is celebrated for its ability to make extremely accurate predictions about tough problems, but accuracy comes with the cost of difficulty in calculations. If the math is too complex to perform, it is usually a good idea to relax demands and accept an approach that is imprecise but can help to forward.

The History of Fermi Estimate



Fermi is celebrated for his fast and excellent calculations with little or no concrete data. In one well-known example, when the first atomic bomb was detonated during Manhattan Project, Fermi dropped a few scraps of paper as the shock wave from the detonation passed. After some coarse calculation, Fermi estimated the power of the blast from the motion of the scapes as they fell. Fermi's guesstimate of 10 kilotons of TNT was remarkbly close to the now-established value of 20 kilotons, even though the estimate was 50% off the actual number.

> Fermi believed that the ability to make fast and less accurate calculation was an essential skill for physicists. A good way to solve physics problems - and complex in any line of work - is by coming up with simple shortcuts to make approximate, but meaningful calcualtions.

Example of Fermi Problems

What is the size of market for razors in China? How mamy trash cans are there in Beijing? How many piano tuners are there in world?

Such non-standard problems are called "Fermi problems", which are very bewildering to many people, at least if we want to guess the answer in one step.

One distintive feature of fermi problem is that precision is impossible to achieve quickly, but it's easier to arrive at fast estimate of the range for the right answer. Before investing a big effort to measure something with precision, problem-solvers can estimate answer approximately - and only then determine if it's essential to do the eextra steps to arrive at the accurate answer.

What a role does Fermi Estimate play in decision making or when should we use effective guesstimation techniques?

In real life, especially business world, many problems are too complicated to give an accurate answer immediately. In analyzing such problems, precision is impossible to achieve quickly, but we can quickly estimate the range of correct answer. Hence before putting much effort into calculating anything with precision, make a rough estimate of the answer, then decide whether it's worth investigating further.

How to solve fermi problem?

- Based on our common sense, divide the big problem into a number of sub-problems using demographic or geographic features.
- Make simplified assumptions.
- Address the divided small problem one by one.

Look at the typical example: 中国人一年消费多少猪肉?

Most of people are really bad with huge numbers, and thus we need to first decompose the target metric into small components. When decomposing a huge metric, we have two approaches: decompose it by formula or b y some common features, such as gender, age, education, country, etc.

在这，很明显，我们可以先根据计算公式拆解中国一年猪肉的总消费量， 如下：

$$\text{中国猪肉年总消费量} = \text{中国平均每人每天猪肉消耗量} \times \text{一年的天数}(365) \times \text{中国总人口}$$

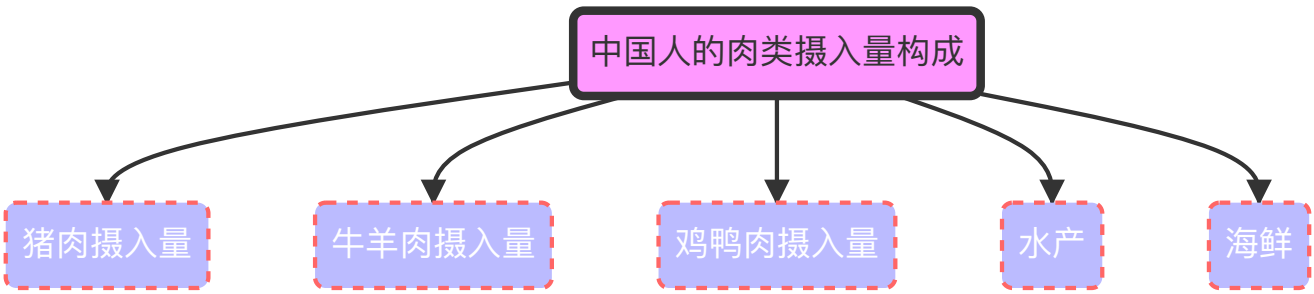
而`中国平均每人猪肉消耗量`又可以根据中国人每日的食肉摄入类别进行拆解，因为中国人的蛋白质来源不仅限于猪肉，还有其他的蛋白质类别。

$$\text{中国平均每人每天猪肉消耗量} = \text{猪肉消耗占比} \times \text{中国平均每天肉消耗量}$$

目前，未知的两个参数为`percentage of consumption of pork per person per day`和`total consumption of meat per person per day`. 迅速获得一个精确的数值肯定不可能，所以我们需要根据经验来估计这两个参数。

首先，一个标准的牛排重量为120g-150g，所以午餐和晚餐都摄入一个牛排重量的肉类的情况下，每人每天消耗的肉量大约为240g。当然，我们无法确定是否也有早餐含有肉类的情况，但是根据生活经验，中国人的早餐大部分由鸡蛋和面粉类食物构成。To move forward, 我们这里做一个简单的假设：*中国人基本上在中午和晚上会摄入肉类食品。*

其次，我们可以将中国每天的肉类摄入量进行拆解，从而大致估计猪肉的消耗占比。



这里为们可以make a wild guess about the percentage of pork consumption. 鉴于中国人的主要摄入肉类为猪肉，我们不妨猜测猪肉的消耗占总肉类消耗的50%。因此，可以得出中国平均每人每天的猪肉消耗量大约为：

$$\text{consumption of pork per person perday} = 240g \times 50\% = 120g$$

Using the estimate, we can give an approximate answer of the yearly consumption of pork in China:

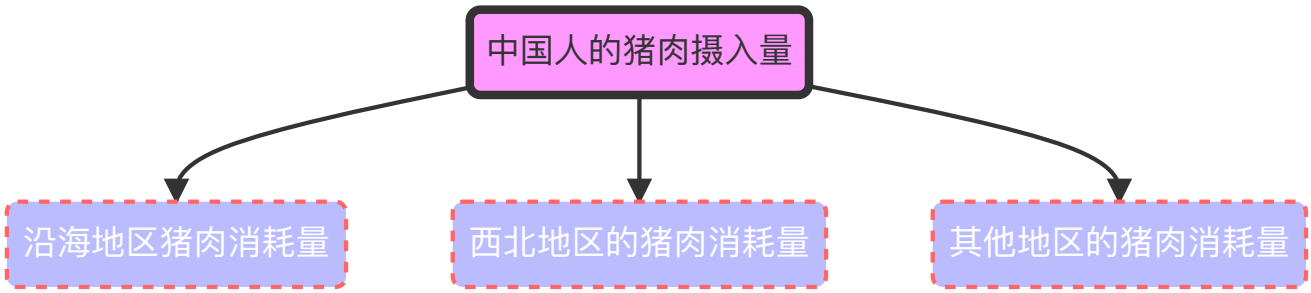
$$120g \times 365 \times 1.4\text{billion} = 61320 \text{ billion grams}$$
$$\because 1 \text{ ton} = 907 \text{ kilograms} = 1000 \times 1000 \text{ grams} = 1000000 \text{ grams}$$
$$\therefore 61320 \div 1000000 \times 10^{10} = 61.32 \text{ million tons}$$

所以，最后估计给出的数值是6132万吨猪肉。网上搜索，新闻里给出的数值为5600多万吨猪肉。我们估计出的数值虽然相差好几百万吨，但是其实已经是一个比较准确的数值。

对于此题，我们还可以从多个不同角度出发，将中国人的猪肉的消耗占比进行进一步的拆解，例如地区和年龄。上述的计算暗含的假设是猪肉的消耗在全中国各地和不同的年龄阶层都一致。所以我们可以练习，如何从这两个角度出发，将指标进行进一步的拆解。

地区

如果从省份的角度划分的话，计算会变得复杂要估计的数字也很多，因为中国总共有34个省级行政区。因此，不妨直接把中国划分成*沿海地区，西北地区，和其他*，这里的其他就是指非西北和沿海地区。这么划分的理由是，西北地区以牛羊肉为主要蛋白质来源，而沿海地区依靠海鲜。



沿海地区主要依赖海鲜为生，我们可以猜测猪肉的消耗量占比为20%；同理，西北地区主要食用牛羊，依然可以猜测猪肉的消耗占到20%。而其他地区，以猪肉消耗同时消耗淡水产品，所以我猜测60%。此时，中国的猪肉年消耗量为：

$$\text{中国年猪肉消耗量} = \text{西北地区年猪肉消耗量} + \text{沿海地区猪肉消耗量} + \text{其他地区猪肉消耗量}$$

$$\text{西北地区年猪肉消耗量} = \text{西北地区每人每天猪肉消耗量} \times 365\text{天} \times \text{西北地区总人口}$$

$$\text{沿海地区年猪肉消耗量} = \text{沿海地区每人每天猪肉消耗量} \times 365\text{天} \times \text{沿海地区总人口}$$

$$\text{其他地区年猪肉消耗量} = \text{其他地区每人每天猪肉消耗量} \times 365\text{天} \times \text{其他地区总人口}$$

很容易发现，未知的参数为**西北地区人口，沿海地区人口，和其他地区人口**。鉴于西北地区地广人稀，我们肯定猜测较小的数值，而沿海地区和非沿海地区是中国的人口密集区，所以我会猜词一个很大的数值。西北地区大约有1亿人，沿海省份主要包括了：广西，广东，澳门，香港，福建，浙江，江苏，山东和辽宁。鉴于省份不是所有省市靠海，这里我保守猜测沿海地区的人有3亿，所以其余地区为10亿人口。所以，最终结果为：

$$\text{西北地区年猪肉消耗量} = \text{西北地区每人每天猪肉消耗量} \times 365\text{天} \times \text{西北地区总人口} =$$
$$20\% \times 240g \times 1\text{亿} \times 365 \div 1000000 = 175.2\text{万吨}$$

$$\text{沿海地区年猪肉消耗量} = \text{沿海地区每人每天猪肉消耗量} \times 365\text{天} \times \text{沿海地区总人口} =$$
$$20\% \times 240g \times 3\text{亿} \times 365 \div 1000000 = 525.6\text{万吨}$$

$$\text{其他地区年猪肉消耗量} = \text{其他地区每人每天猪肉消耗量} \times 365\text{天} \times \text{其他地区总人口} =$$
$$60\% \times 240g \times 10\text{亿} \times 365 \div 1000000 = 5256\text{万吨}$$

$$\text{中国年猪肉消耗量} = 175.2 + 525.6 + 5256 = 5956.8\text{万吨}$$