ECOS2001 Notes

Lecture 1 - Constraints on Consumption Choice

- Budgetary, time, and other resource limitations constrain consumption choice.

Budget Constraints

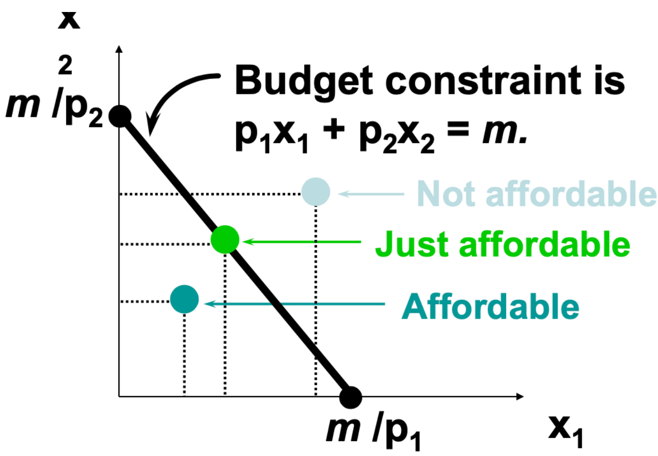
- A **consumption bundle** $\{x_1, x_2, \ldots, x_n\}$ is affordable at given prices $p_1, p_2, \ldots, p_n$ when $p_1x_1 + \ldots + p_nx_n \leq m$.
  - x_n refers to the units of commodity n within the bundle.
  - m is the consumer's disposable income.
- The bundles that are only just affordable form the consumer's **budget constraint**:

$$\{(x_1, \ldots, x_n) | x_1 \geq 0, \ldots, x_n \geq 0 \text{ and } p_1x_1 + \ldots + p_nx_n = m\}$$

- The consumer's **budget set** is the set of all affordable bundles:

$$B(p_1, \ldots, p_n, m) = \{(x_1, \ldots, x_n) | x_1 \geq 0, \ldots, x_n \geq 0 \text{ and } p_1x_1 + \ldots + p_nx_n \leq m\}$$

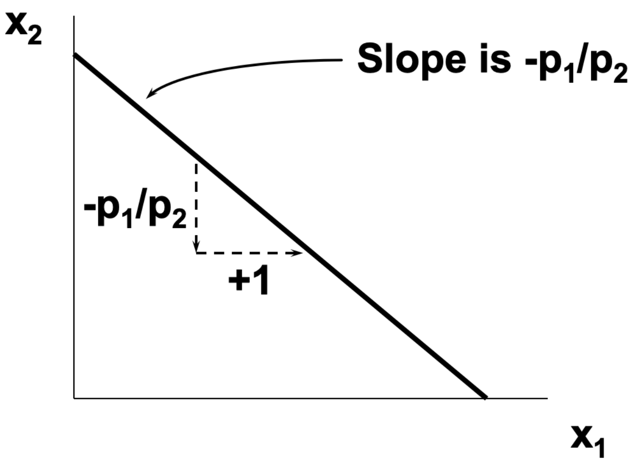
Budget Set and Constraint for Two Commodities



- The budget set makes up the triangular area under the budget constraint line.
- For two commodities, the slope can be rearranged to be $(-\frac{p_1}{p_2})$.

$$p_1x_1 + p_2x_2 = m$$
$$\equiv x_2 = (-\frac{p_1}{p_2})x_1 + \frac{m}{p_2}$$

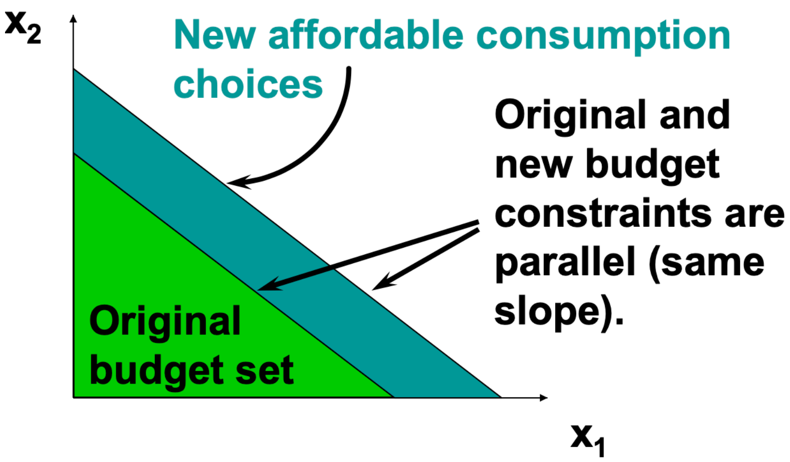
- Therefore, the opportunity cost of an extra unit of commodity 1 is $\frac{p_1}{p_2}$ units foregone of commodity 2. The opportunity cost of an extra unit of commodity 2 is $\frac{p_2}{p_1}$ units foregone of commodity 1.



Income and Price Changes

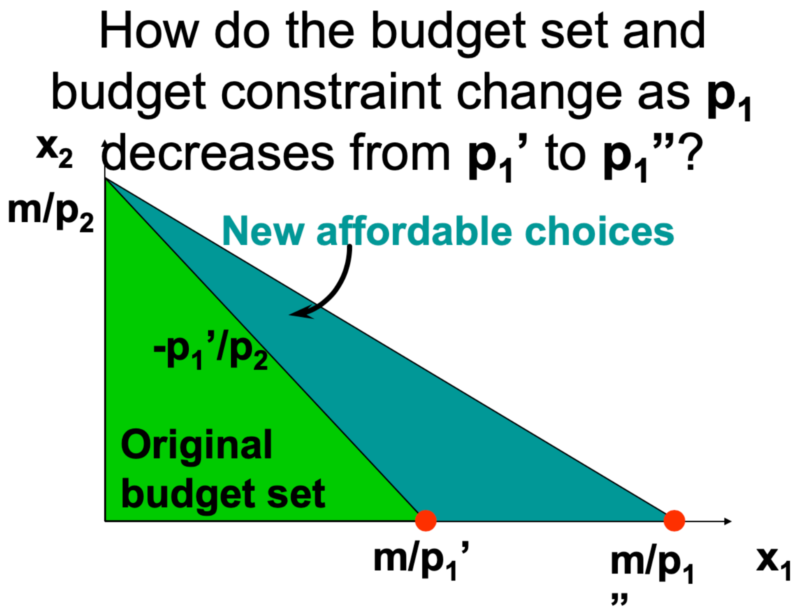
- How do the budget set and budget constraint change as income m changes?

Higher Income Gives More Choice



- Increases/decreases in income m shift the constraint outward/inward in a parallel manner, thereby enlarging/shrinking the budget set and improving/reducing choice.
- No original choice is lost and new choices are added when income increases, so higher income cannot make a consumer worse off. An income decrease may (and typically will) make the consumer worse off.

Lower Prices Gives More Choice

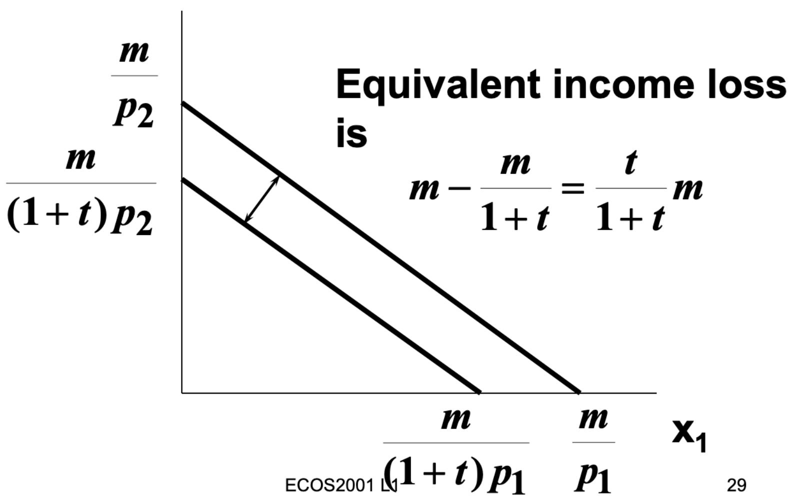


- Reducing the price of one commodity pivots the constraint outward. No old choice is lost and new choices are added, so reducing one price cannot make the consumer worse off and vice versa.

Uniform Ad Valorem Sales Taxes

- An **ad valorem sales tax** levied at a rate of t% increases all prices by t%, from p to $(1+t)p$.
- A uniform sales tax is applied uniformly to all commodities.
- A uniform sales tax levied at rate t changes the constraint from:
  - $p_1x_1 + p_2x_2 = m$
  - to $(1+t)p_1x_1 + (1+t)p_2x_2 = m$
  - i.e. $p_1x_1 + p_2x_2 = m/(1+t)$

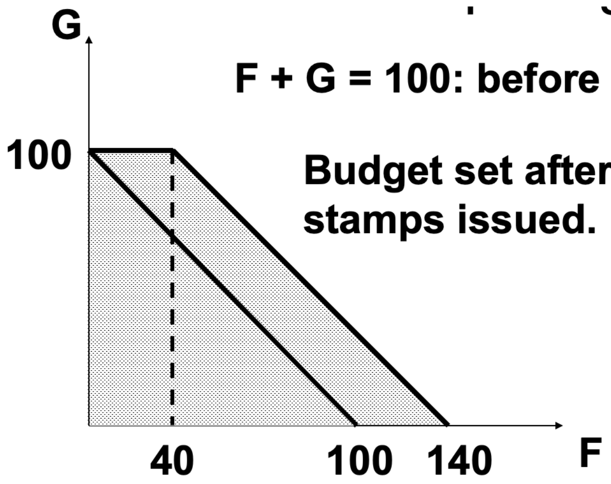
x_2



- This is to say that a uniform ad valorem sales tax levied at rate t is equivalent to an income tax levied at rate $t/(1+t)$.

The Food Stamp Program

- **Food stamps** are coupons that can be legally exchanged only for food.
- How does a commodity-specific gift such as a food stamp alter a family's budget constraint?
  - Suppose $m = \$100$, $p_F = \$1$ and the price of 'other goods' is $p_G = \$1$. The budget constraint is then $F + G = 100$.



Preferences: Rationality in Economics

- Behavioural Postulate: A decisionmaker always chooses its most preferred alternative from its set of available alternatives.
  - To model choice, we must model decisionmakers' preferences.
- Comparing two different consumption bundles, x and y, we get preference relations.
  - **Strict Preference**: x is more preferred than y: $x \succ y$
  - **Weak Preference**: x is at least as preferred than y: $x \succsim y$
  - **Indifference**: x is exactly as preferred as y. $x \sim y$
- Preference relations are ordinal relations - they only state the order in which bundles are preferred.

Note

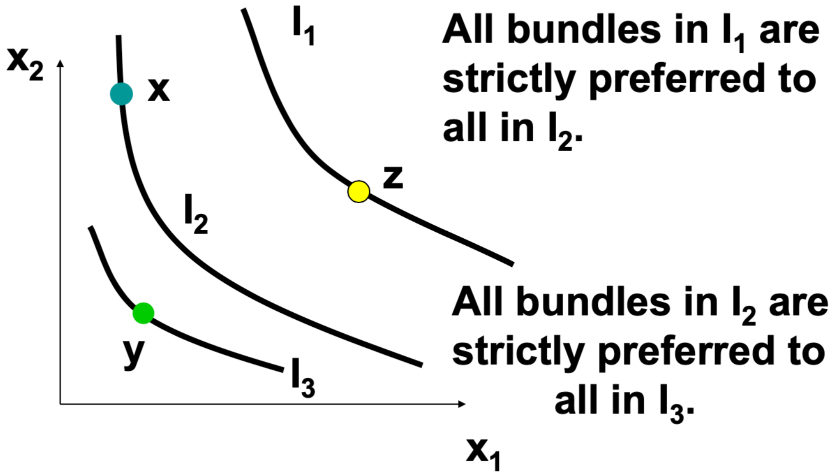
- $x \succsim y$ and $y \succsim x$ implies $x \sim y$.
- $x \succsim y$ and (not y $\succsim$ x) implies $x \succ y$.

Assumptions About Preference Relations

- **Completeness:** For any two bundles x and y, it is always possible to make the statement that either $x \succsim y$ or $y \succsim x$.
- **Reflexivity:** Any bundle x is always at least as preferred as itself: $x \succsim x$.
- **Transitivity:** If $x \succsim y$ and $y \succsim z \to x \succsim z$.

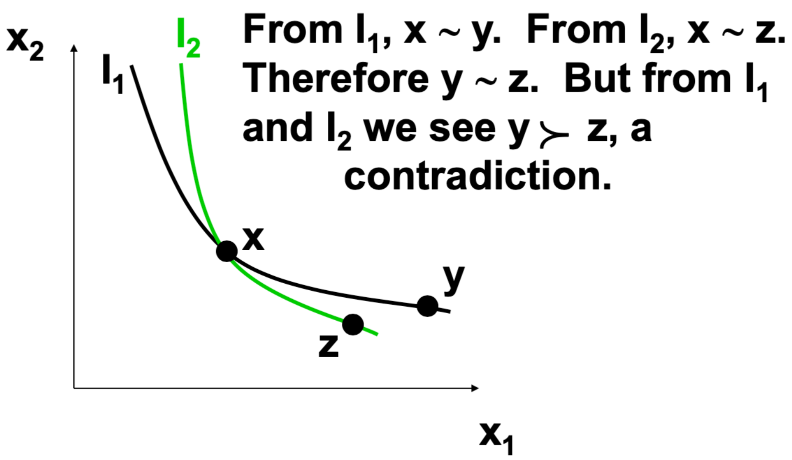
Indifference Curves

- Take a reference bundle x'. The set of all bundles equally preferred to x' is the **indifference curve** containing x' - the set of all bundles $y \sim x'$.
- Since an indifference 'curve' is not always a curve, a better name might be an indifference 'set'.



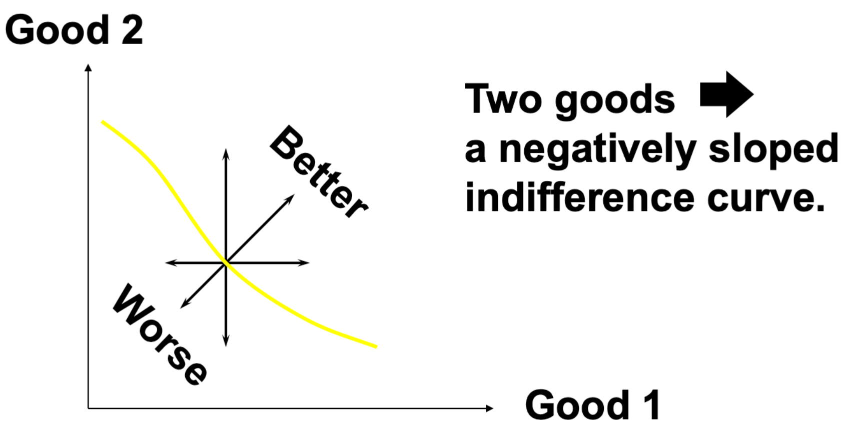
- In this case:
  - $z \succ x \succ y$
  - $WP(x)$ represents the set of bundles that are weakly preferred to x, including I_2. This is the area above the I_2 line.
  - $SP(x)$ represents the set of bundles strictly preferred to x, not including I_2. This is the area above the I_2 line, where I_2 is dotted.

Indifference Curves Cannot Intersect

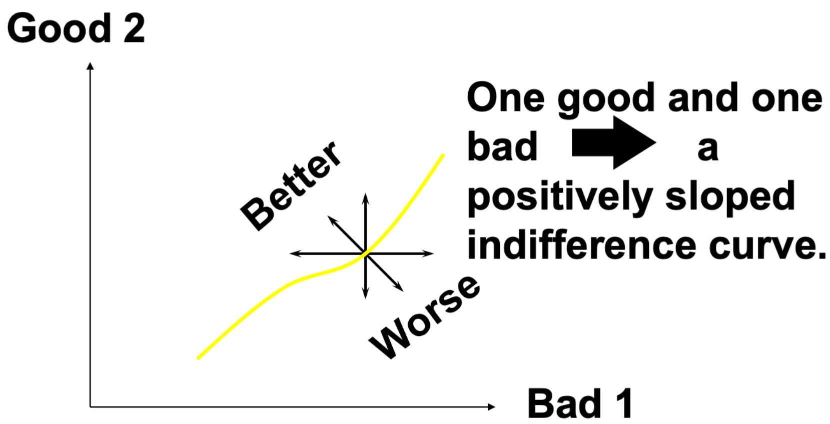


Slopes of Indifference Curves

- When more of a commodity is always preferred, the commodity is a **good**.
  - if every commodity is a good, then indifference curves are negatively sloped.



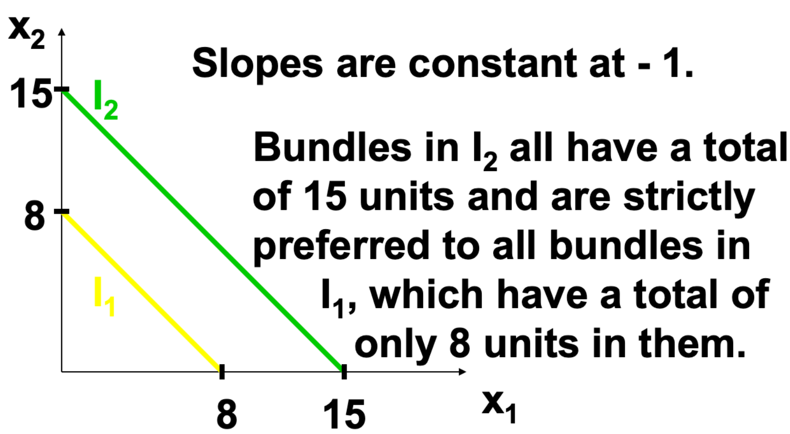
- if less of a commodity is always preferred, then the commodity is a **bad**.



Extreme Cases of Indifference Curves

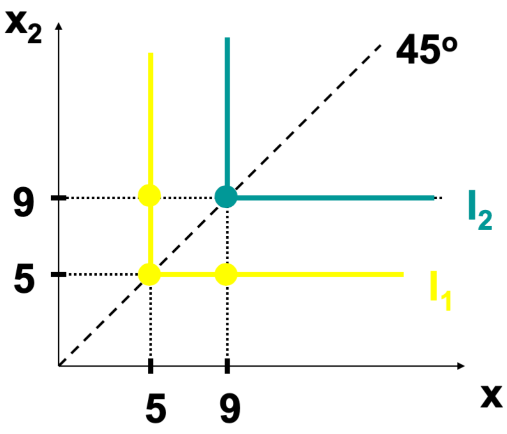
Perfect Substitutes

- If a consumer always regards units of commodities 1 and 2 as equivalent, then the commodities are perfect substitutes and only the total amount of **total** commodities in bundles determines their preference rank-order.
    - Note that the indifference curve has to be a straight line.



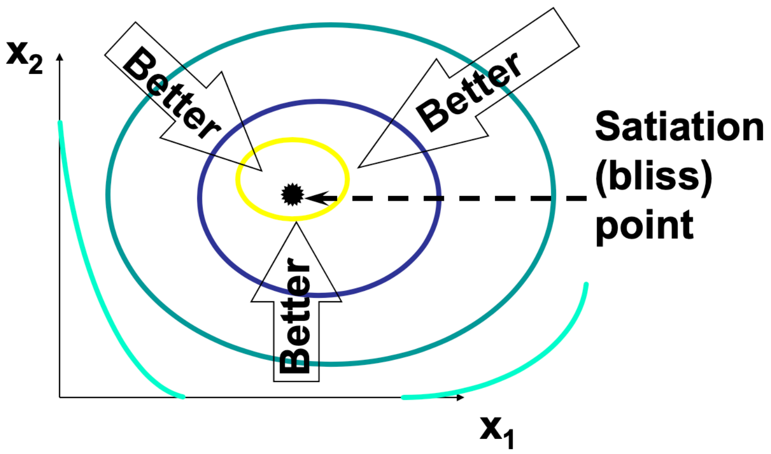
Perfect Complements

- If a consumer always consumes commodities 1 and 2 in fixed proportion (one-to-one), then the commodities are perfect complements and only the number of **pairs** of units of the two commodities determines the preference rank-order of bundles.
  - Since $(5, 5)$, $(5, 9)$ and $(9, 5)$ contain 5 pairs, they are equally preferred. The bundle $(9, 9)$ is strictly preferred since it contains 9 pairs.



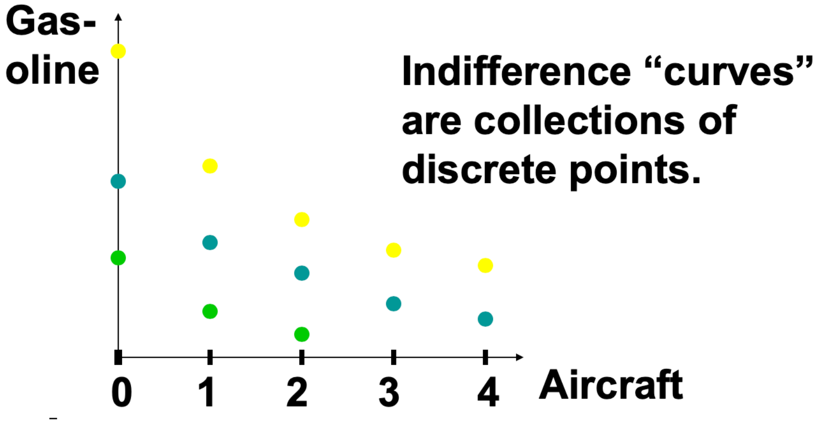
Preferences Exhibiting Satiation

- A bundle strictly preferred to any other is a **satiation point** or a bliss point.
  - Bundles of equal preference form circular indifference curves (or in this case, sets) around the satiation point.



Discrete Commodities

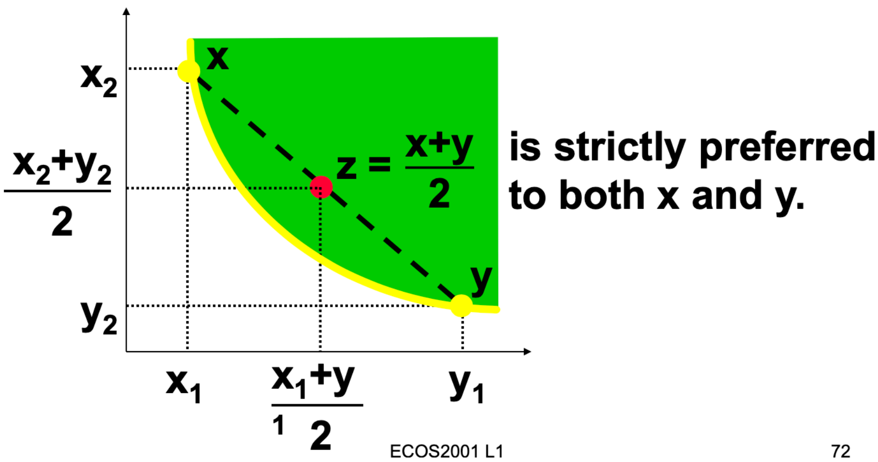
- A commodity is **infinitely divisible** if it can be acquired in any quantity e.g. water or cheese.
- A commodity is **discrete** if it comes in unit sums of 1, 2, 3...e.g. aircraft, ships and refrigerators.



- In this case, gasoline is discrete and aircrafts are discrete.

Well-Behaved Preferences

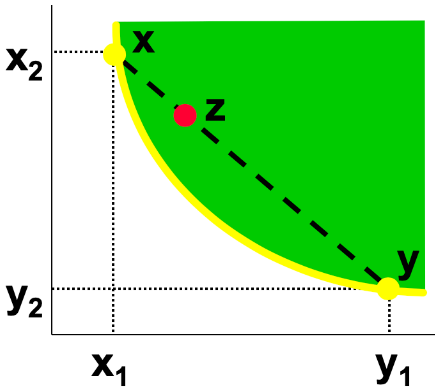
- A preference relation is well-behaved if it is monotonic and convex.
  - **Monotonicity**: more of any commodity is always preferred (i.e. no satiation, and every commodity is a good).
  - **Convexity**: mixtures of bundles are (at least weakly) preferred to the bundles themselves e.g. the 50-50 mixture of the bundles x and y is $z = 0.5x + 0.5y$.
    - z Is at least as preferred as x or y.



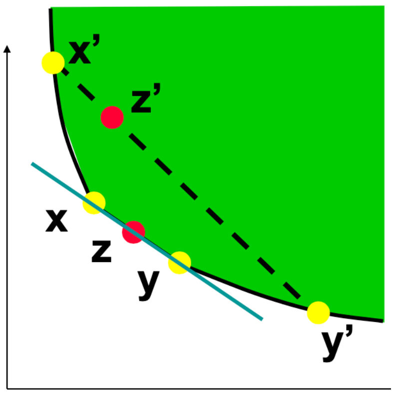
- Or more generally for any point z on that dotted line between x and y:

$$z = (tx_1 + (1-t)y_1, tx_2 + (1-t)y_2) \text{ is preferred to x and y for all } 0 < t < 1$$

- Preferences are **strictly convex** when all mixtures z are strictly preferred to their component bundles x and y.

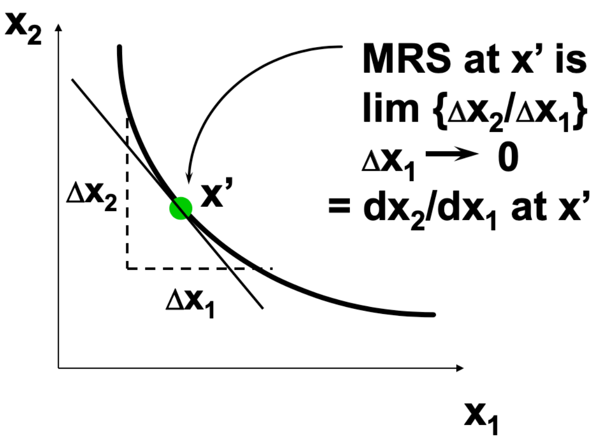


- Preferences are **weakly convex** if at least one mixture z is equally preferred to a component bundle.

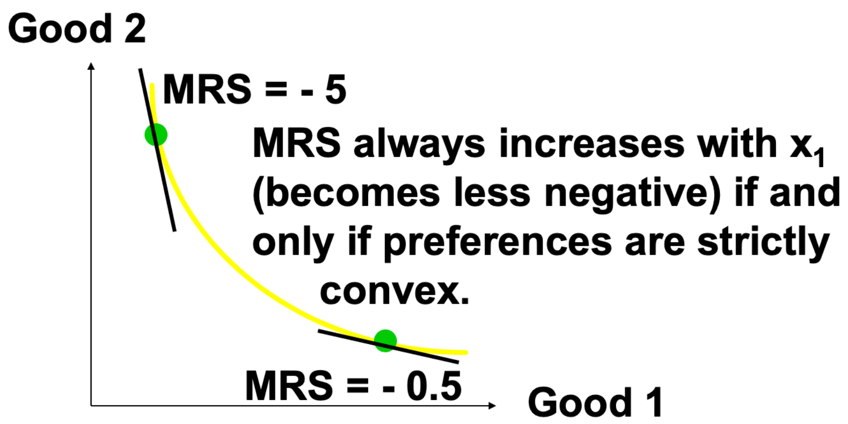


Slopes of Indifference Curves

- The slope of an indifference curve at a point x' is its marginal rate-of-substitution (MRS).
  - The MRS is the rate at which the consumer is only just willing to exchange commodity 2 (dx_2) for a small amount of commodity 1 (dx_1).



- With two goods, there is a negatively sloped indifference curve where MRS < 0.
- With one good and one bad, there is a positively sloped indifference curve where MRS > 0.



Lecture 2 - Utility

- A preference relation that is complete, reflexive, transitive and continuous can be represented by a continuous utility function.
    - Continuity means that small changes to a consumption bundle cause only small changes to the preference level.
- A **utility function** $U(x)$ represents a preference relation if and only if:
    - $x' \succ x'' \rightarrow U(x') > U(x'')$
    - $x'' \succ x \rightarrow U(x'') > U(x)$
    - $x' \sim x'' \rightarrow U(x') = U(x'')$
- Utility is an ordinal concept - magnitude is not important, and it is meaningless to ask how much better something is, or how good it is.

Utility Functions and Indifference Curves

- An indifference curve contains equally preferred bundles, which have the same utility level.
- The collection of all indifference curves for a given preference relation is an indifference map, which is equivalent to a utility function.
- There is no unique utility function representation of a preference relation - in fact, there are infinite utility functions.

Example

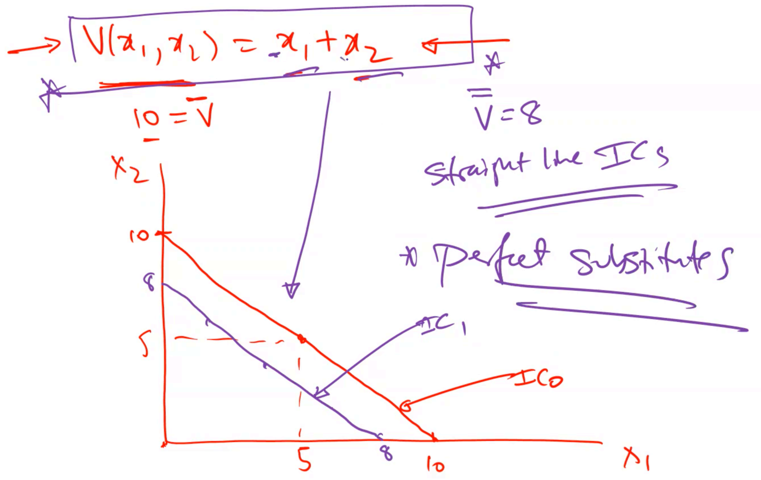
- Suppose $U(x_1, x_2) = x_1 x_2$ represents a preference relation.
- Consider the bundles $(4, 1), (2, 3), (2, 2)$:
    - $U(2, 3) = 6 > U(4, 1) = U(2, 2) = 4$
    - That is, $(2, 3) \succ (4, 1) \sim (2, 2)$
- Similarly, $V(x_1, x_2) = U^2$ yields the same results, since it is a monotonic transformation.

Goods, Bads and Neutrals

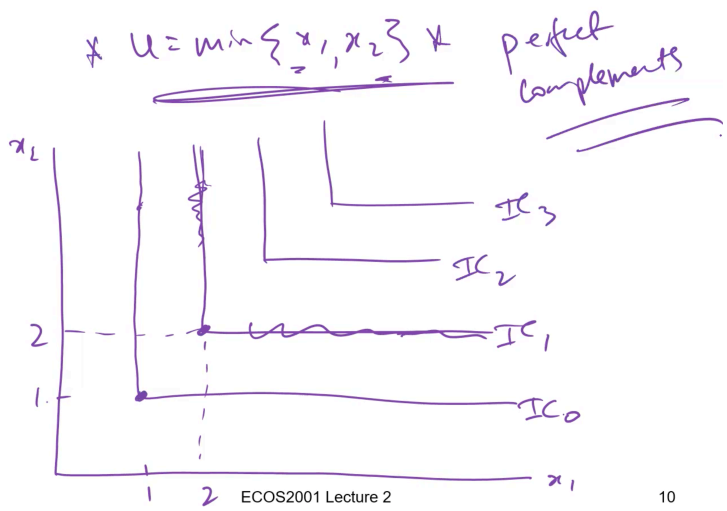
- A **good** is a commodity unit which increases utility (gives a more preferred bundle).
- A **bad** is a commodity unit which decreases utility (gives a less preferred bundle).
- A **neutral** is a commodity unit which does not change utility (gives an equally preferred bundle).

Utility Functions and Their Indifference Curves

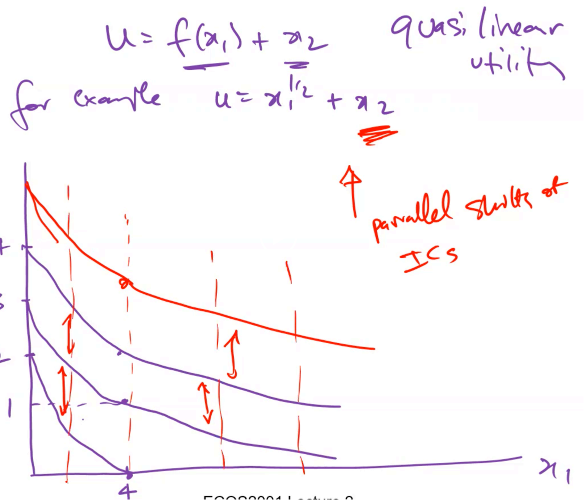
$V(x_1, x_2) = x_1 + x_2$ **- Perfect Substitutes**



$W(x_1, x_2) = \min\{x_1, x_2\}$ **- Perfect Complements**

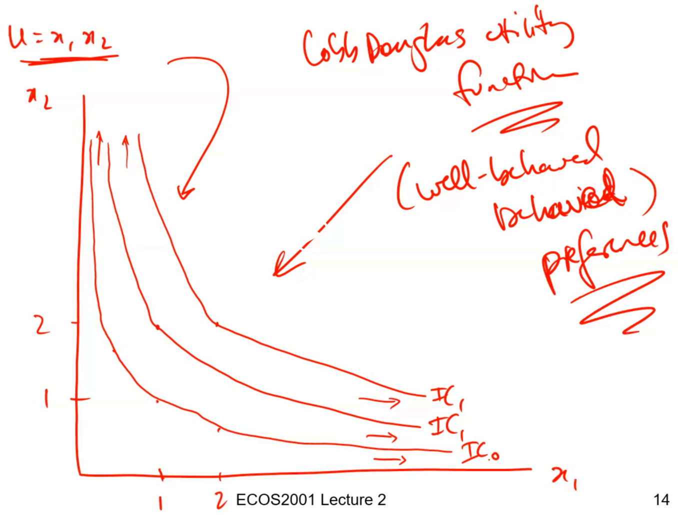


$U(x_1, x_2) = f(x_1) + x_2$ **- Quasi Linear Utility**



Cobb-Douglas Utility Function

- Any function of the form $U(x_1, x_2) = x_1^a x_2^b$ where $a > 0$ and $b < 0$ is called a Cobb-Douglas utility function.
  - $U(x_1, x_2) = x_1^{1/2} x_2^{1/2}$ where $a = b = 1/2$
  - $V(x_1, x_2) = x_1 x_2^3$ where $a = 1, b = 3$
- All curves are hyperbolic, asymptoting to but never touching any axis.



Marginal Utilities

- Marginal means incremental. The **marginal utility** of commodity i is the rate-of-change of total utility as the quantity of commodity i consumed changes.

$$MU_i = \frac{\partial U}{\partial x_i}$$

- If $U(x_1, x_2) = x_1^{1/2} x_2^{1/2}$:

$$MU_1 = \frac{\partial U}{\partial x_1} = \frac{1}{2} x_1^{-1/2} x_2^2$$

$$MU_2 = \frac{\partial U}{\partial x_2} = 2x_1^{1/2} x_2$$

Marginal Rates of Substitution

- The general equation for an indifference curve is $U(x_1, x_2) = k$, a constant. Totally differentiating this identity gives:

$$\frac{\partial U}{\partial x_1}\mathrm{dx}_1 + \frac{\partial U}{\partial x_2}\mathrm{dx}_2 = 0$$

$$\frac{\partial U}{\partial x_1}\mathrm{dx}_1 = -\frac{\partial U}{\partial x_2}\mathrm{dx}_2$$

- Intuitively, this makes sense since a small increase to x_1 ($\frac{\partial U}{\partial x_1}$) must be compensated by a decrease to x_2 to maintain the constant 0.

- Rearranged to find the **marginal rate of substitution**:

$$MRS_{2\to 1} = \frac{\mathrm{dx}_2}{\mathrm{dx}_1} = -\frac{\partial U/\partial x_1}{\partial U/\partial x_2} = -\frac{MU_1}{MU_2}$$

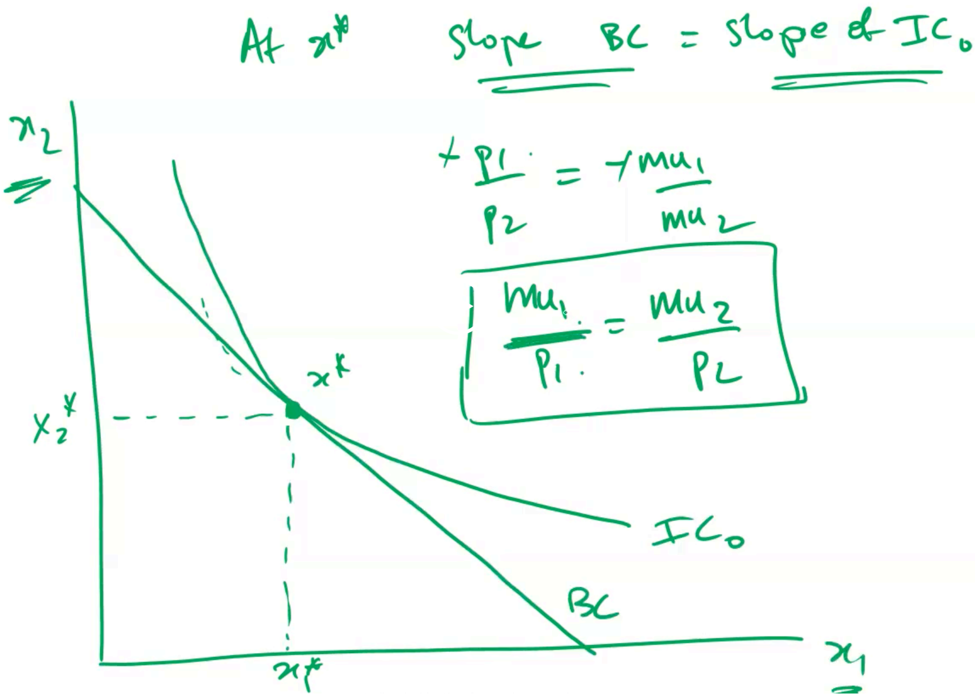
- For $U(x_1, x_2) = x_1 x_2$, $MRS = -x_2/x_1$. The MRS is equal to the slope of the indifference curve.

Monotonic Transmforations and Marginal Rates-of-Substitution

- Applying a monotonic transformation to a utility function representing a preference relation simply creates another utility function representing the same preference relation.
- MRS is unchanged by a positive monotonic transformation.

Rational Constrained Choice

- A rational choice is noted to be utility maximisation subject to the budget constraint. Here, (x_1^*, x_2^*) is the most preferred affordable bundle.



- In essence it is saying that the marginal utility per dollar spend is equal across the two products.
  - If the marginal utility per dollar spend is higher for x_1, then you increase utility by adding more x_1 to your bundle.

Ordinary Demand

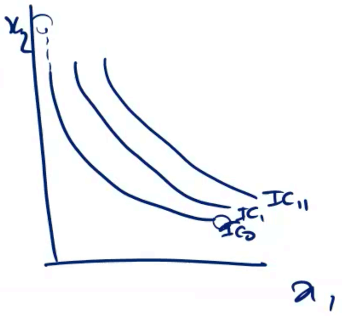
- The most preferred affordable bundle is called the consumer's **ordinary demand** at the given prices and budget.
  - This is denoted by $x_1*(p_1, p_2, m)$ and $x_2*(p_1, p_2, m)$.
- When $x_1^* > 0$ and $x_2^* > 0$, the demanded bundle is an **interior solution**. If buying (x_1^*, x_2^*) costs $m, then the budget is exhausted.
- Again, the slope of the IC at the most preferred affordable bundle is equal to the slope of the budget constraint.

Most Preferred Affordable Bundle

Cobb-Douglas Utility

$$\boxed{U(x_1,x_2)=x_1^a x_2^b}$$

$$p_1x_1+p_2x_2=m$$



$$MRS_{2\to1} = -\frac{mu_1}{mu_2} = -\frac{a x_1^{a-1}\cdot x_2^b}{b x_1^a \cdot x_2^{b-1}} = -\frac{a x_2}{b x_1}$$

$$\frac{a x_2}{b x_1}=\frac{p_1}{p_2}$$

at utility max. bundle:

$$\frac{p_1}{p_2}=MRS_{2\to1}=\frac{a x_2}{b x_1} \quad ; \quad x_2=\frac{p_1}{p_2}\cdot\frac{b x_1}{a}$$

$$\boxed{x_2=\frac{p_1 b x_1}{p_2 a}}$$

B.C

$$p_1x_1+p_2x_2=m$$

$$p_1x_1+\cancel{p_2}\left[\frac{p_1 b x_1}{\cancel{p_2}\, a}\right]=m$$

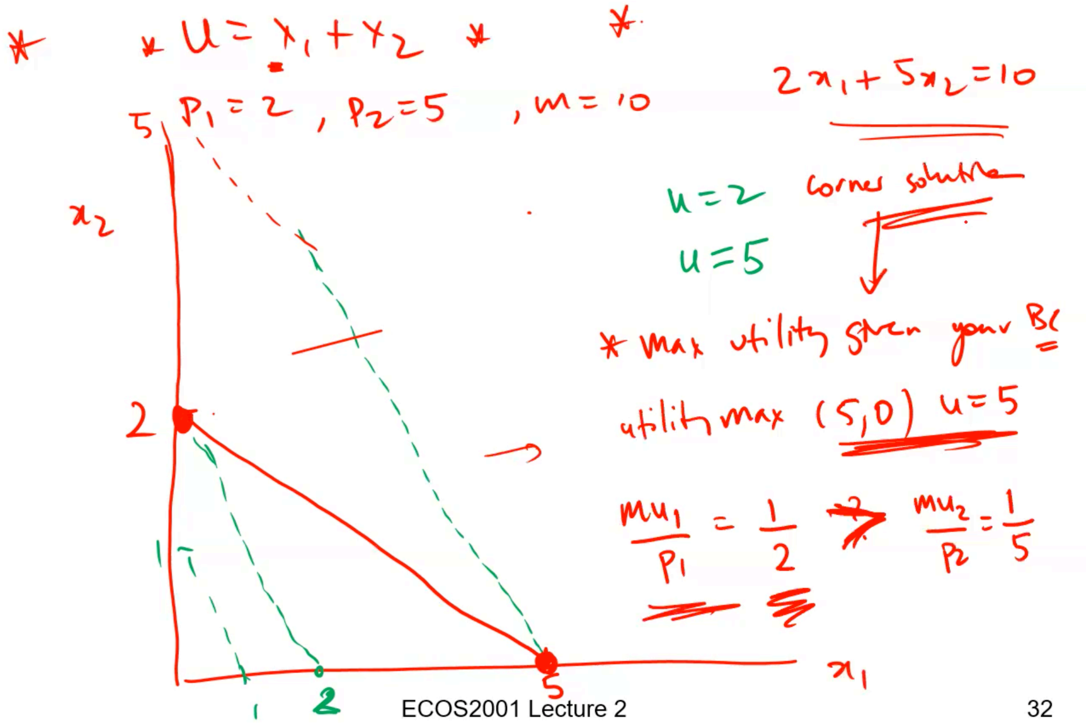
$$x_1\cdot p_1\left[\frac{a+b}{a}\right]=m$$

$$\boxed{x_1^*=\frac{am}{(a+b)p_1}}$$

$$x_2=\frac{p_1 b\cdot am}{p_2\cdot a(a+b)\cancel{p_1}} \qquad \boxed{x_2^*=\frac{bm}{p_2(a+b)}}$$

- Note that solutions should resemble something like the red boxes above ^.

Perfect Substitutes

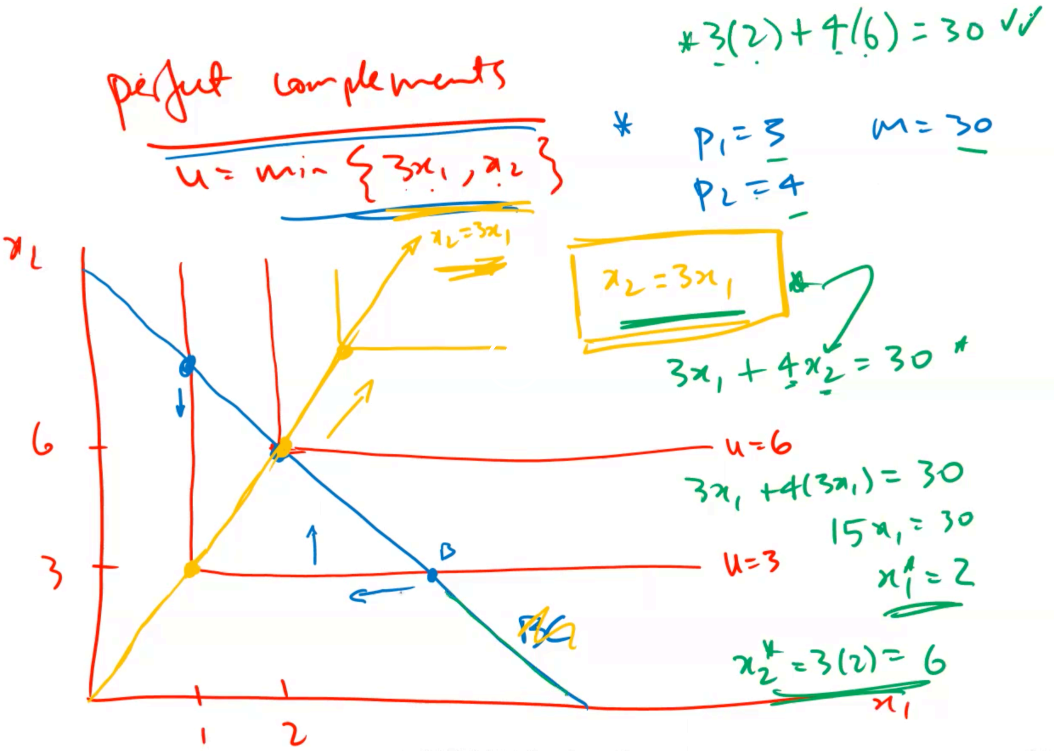


- This is to say that the most preferred affordable bundle will be determined by which good has the greater marginal utility per dollar spend.
  - In this case, x_1 gives better 'bang for your buck', so it is that corner solution that is best.
- Formally:

$$(x_1^*, x_2^*) = (\frac{x}{p_1}, 0) \text{ if } p_1 < p_2$$
$$(x_1^*, x_2^*) = (0, \frac{y}{p_2}) \text{ if } p_1 > p_2$$

Perfect Complements

- In this case, a consumer will attempt to consume on their budget constraint at one of the kinks.

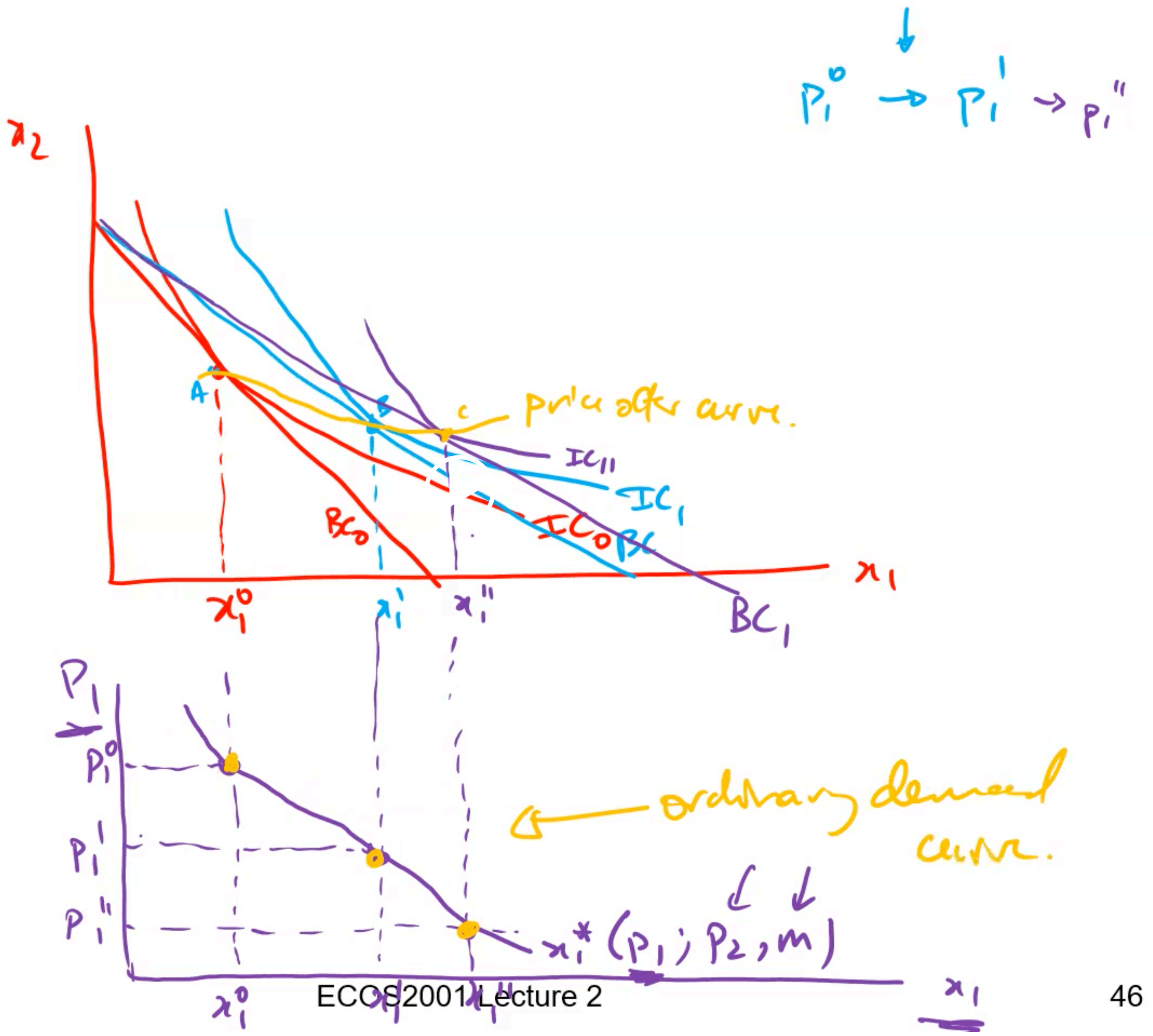


Demand Functions

- **Comparative statics analysis of ordinary demand functions** is the study of how ordinary demands $x_1*(p_1; p_2, y)$ and $x_2*(p_2; p_1, y)$ change as prices p_1, p_2 and y change, holding everything else constant.

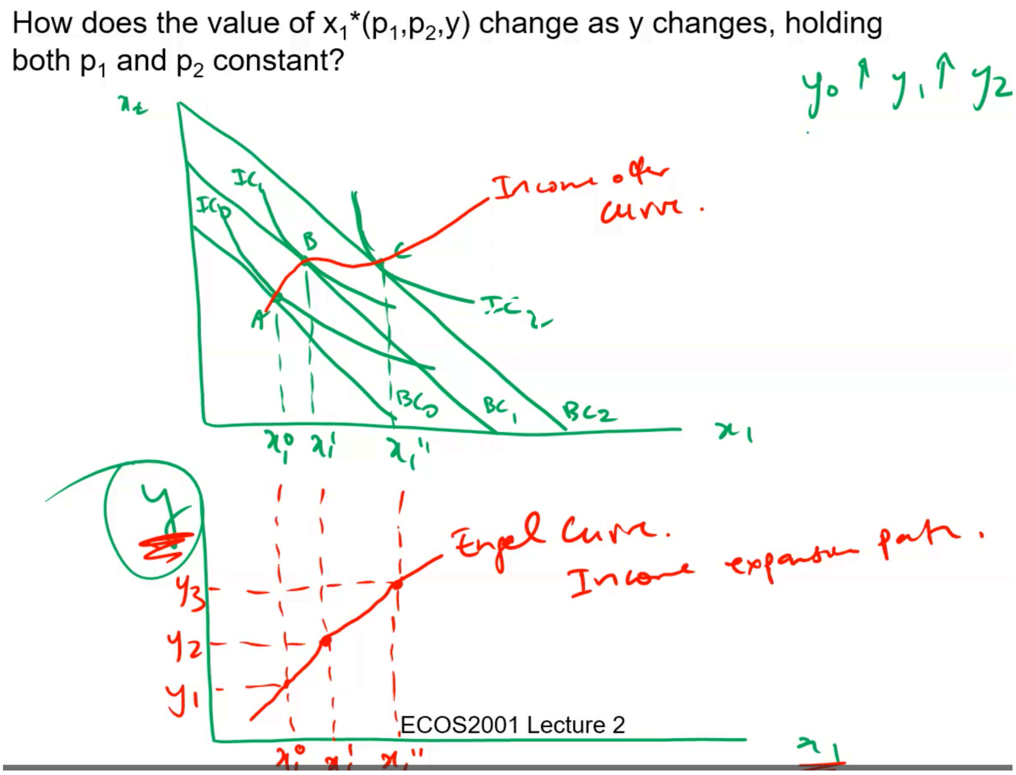
Own-Price Changes

- In the examp3le below, how does $x_1*(p_1; p_2, y)$ change as p_1 changes, holding p_2 and y constant?



- The curve containing all the utility maximising bundles traced out as p_1 changes is the p_1 **Price Offer Curve**.
- The plot of the x_1-coordinates of the p_1 price offer curve against p_1 is the **Ordinary Demand Curve** for commodity 1.

Income Changes



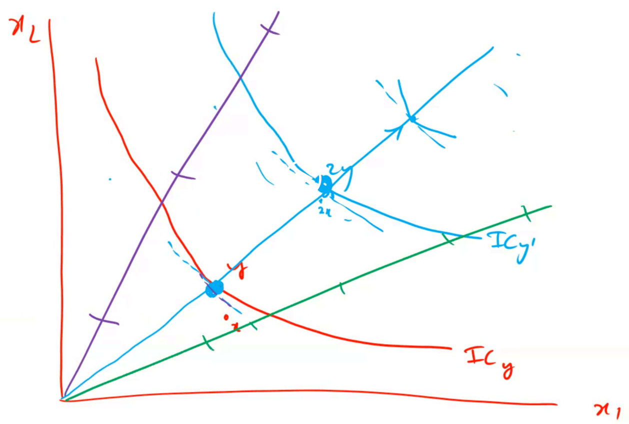
- The **Income Offer Curve** plots changes in x_1 and x_2 given changes in y.
- A plot of quantity demanded against income is called an **Engel Curve** or an **Income Expansion Path**.

Engel Curves

- Engel Curves are straight lines if the consumer's preferences are homothetic, i.e. if and only if

$$(y_1, y_2) \succ (x_1, x_2) \iff (ky_1, ky_2) \succ (kx_1, kx_2) \text{ for every k>0}$$

- That is to say that the consumer's MRS is the same anywhere on a straight line drawn from the origin (regardless of what ray you draw from the origin, the slope of the IC will be the same where they meet).



Types of Goods

Income Effects

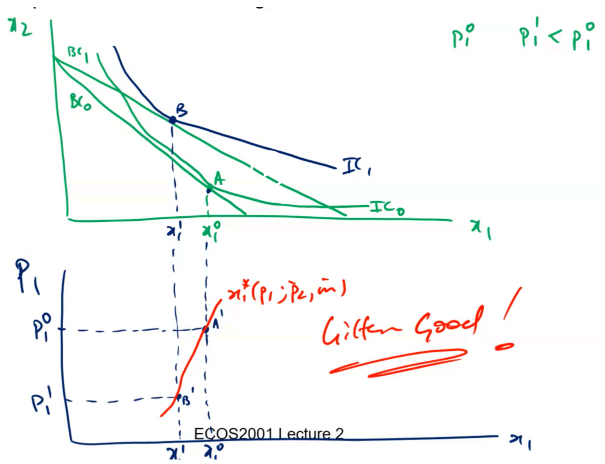
- A good for which quantity demanded (x_1) rises with income y is called **normal**.
  - Therefore, a normal good's engel curve is positively sloped.
- A good for which quantity demanded falls as income increases is called **income inferior**.
  - Therefore, an income inferior good's Engel curve is negatively sloped.

Ordinary Goods

- A good is called **ordinary** if the quantity demanded of it always increases as its own price decreases, keeping everything else constant.

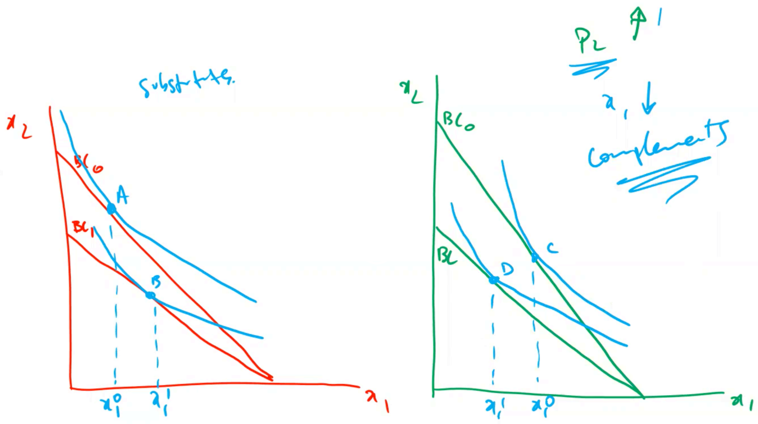
Giffen Goods

- If, for some values of its own price, the quantity demanded of a good rises as its own-price increases, then the good is called a **Giffen good**.



Cross-Price Effects

- If an increase in p_2:
  - increases demand for commodity 1 then commodity 1 is a **gross substitute** for commodity 2.
  - reduces demand for commodity 1 then commodity 1 is a **gross complement** for commodity 2.

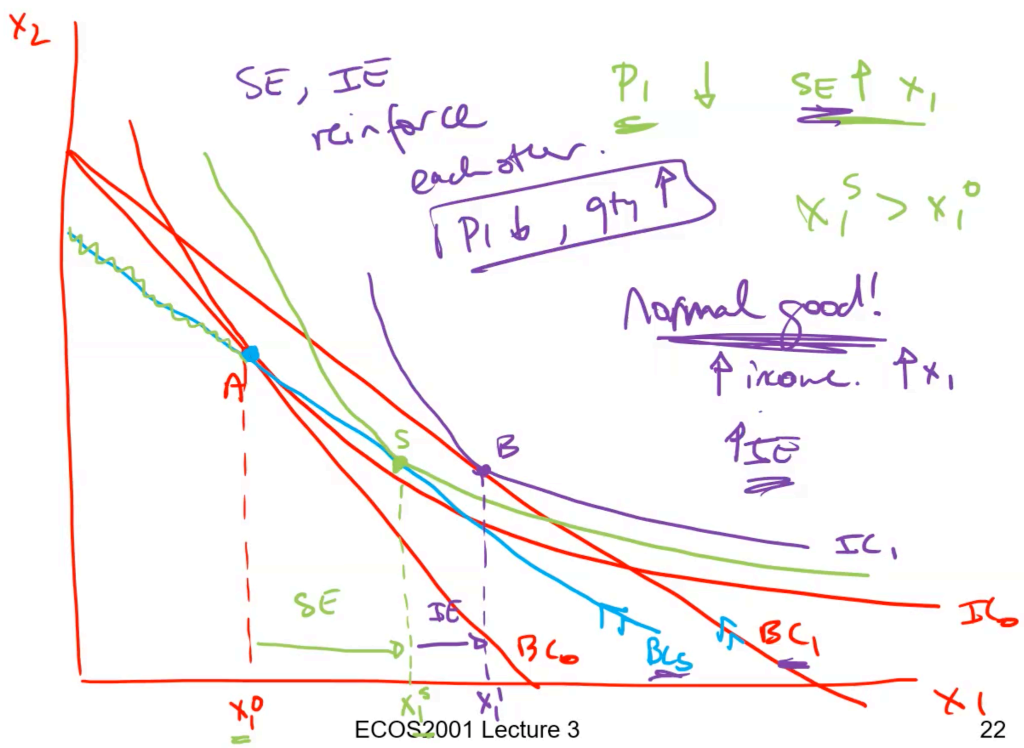


Lecture 3 - Price Change, Inter-Temporal Choice

Effects of a Price Change

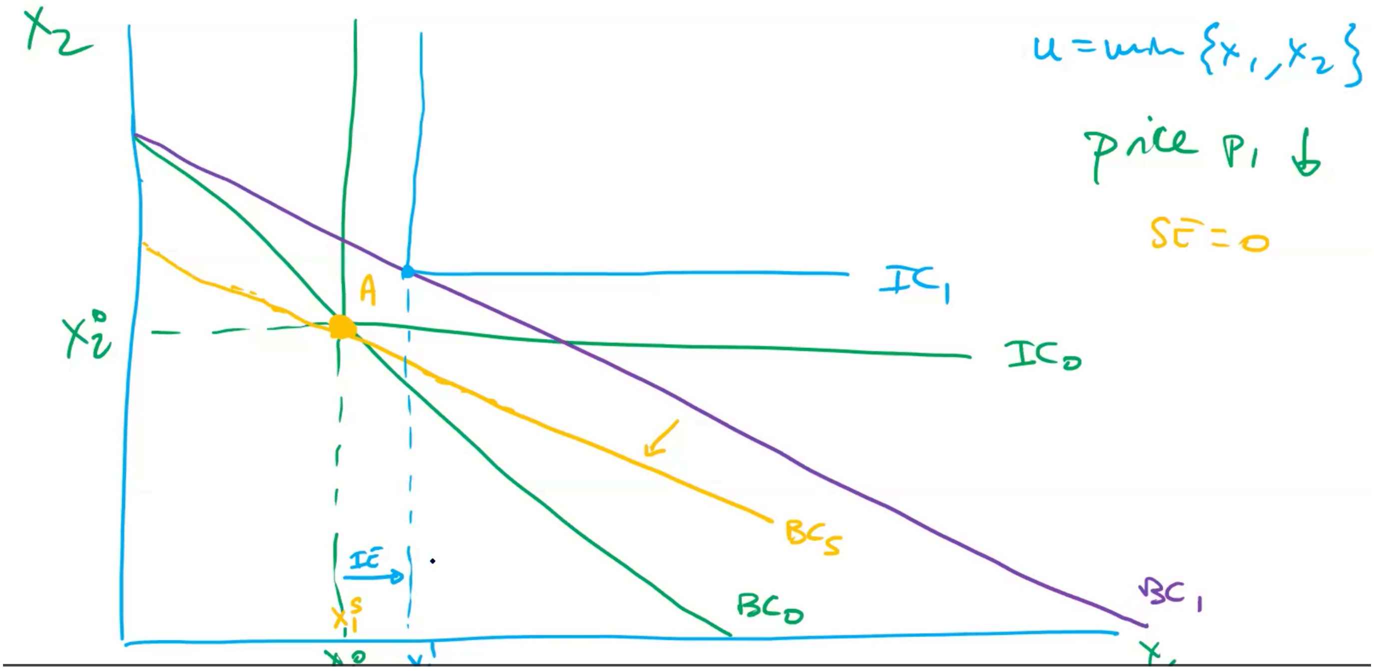
- What happens when a commodity's price decreases?
  - **Substitution Effect**: the commodity is relatively cheaper, so consumers substitute it for now relatively more expensive other commodities.
  - **Income Effect**: the consumer's budget of y can purchase more than before, as if the consumer's income rose, with consequent income effects on quantities demanded.
- Slutsky discovered that changes to demand from a price change are always the sum of a pure substitution effect and an income effect.

Stepping Through Price Change - Slutsky



- Assume you start with BC_0, with maximum utility bundle at point A where IC_0 is tangential.
- A decrease in the price of commodity 1 brings you to BC_1.
- For the sake of measuring the substitution effect, imagine that your income now decreases such that the budget constraint intersects at point A.
  - This is a parallel shift from BC_1 to BC_S.
- Taking the new maximum utility bundle of BC_s brings you to point S where IC_S is tangential.
  - Since commodity 1 is relatively cheaper, the pure **substitution effect** is measured as the increase in commodity 1 bought.
  - Note that BC_0 is preferred to BC_S between 0 and X_1^0. Therefore, any consumer will not decrease their utility by reducing consumption of commodity 1. Instead, they will move onto the new budget cosntraint that they could not reach with the old income.
- Since you cannot decrease an income, the difference between X_1^S and the consumption of commodity 1 at maximum utility bundle B is the **income effect**.
- **Therefore, the change to demand due to lower p_1 is the sum of the income and substitution effects.**

- For perfect complements, you want to consume at one of the kinks, so there is no substitution effect. Any change in quantity is based off income.



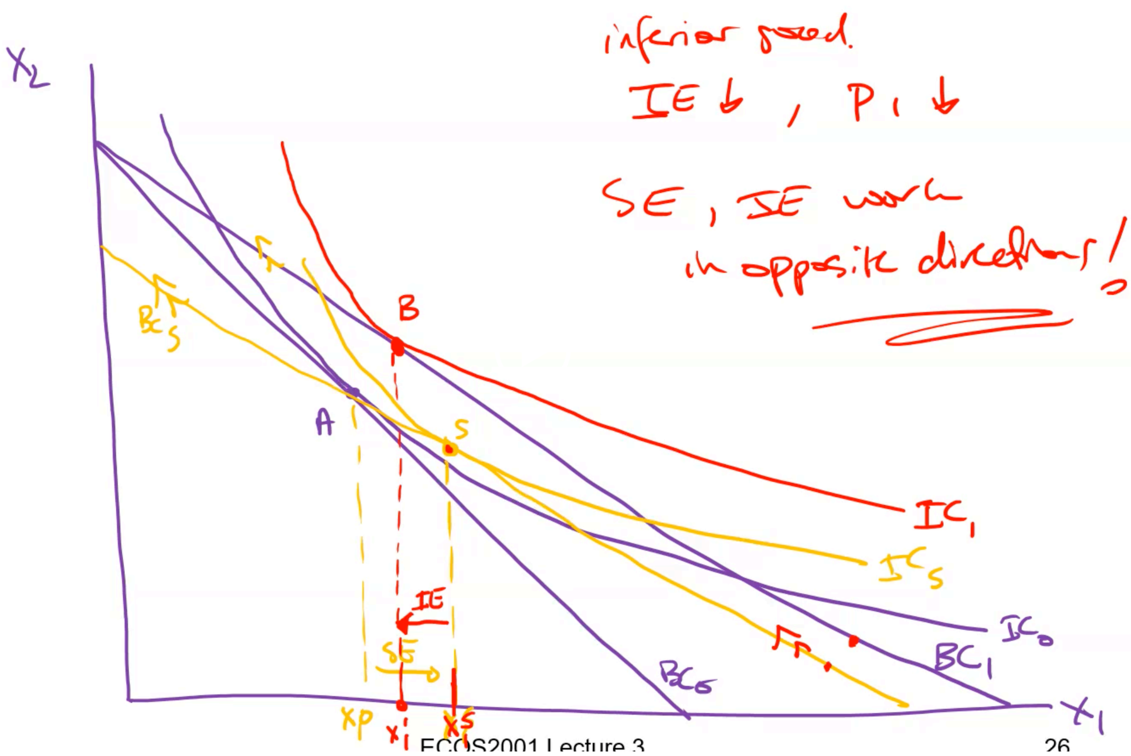
- For perfect substitutes, if you only consume one good, and the price of the other good falls such that you only consume the other good, that is pure substitution effect.

Slutsky's Effects for Normal Goods

- Most goods are normal (i.e. demand increases with income).
    - The substitution and income effects reinforce each other when a normal good's own price changes.
    - A normal good's ordinary demand curve slopes down.
    - The Law of Downward-Sloping Demand therefore always applies to normal goods.

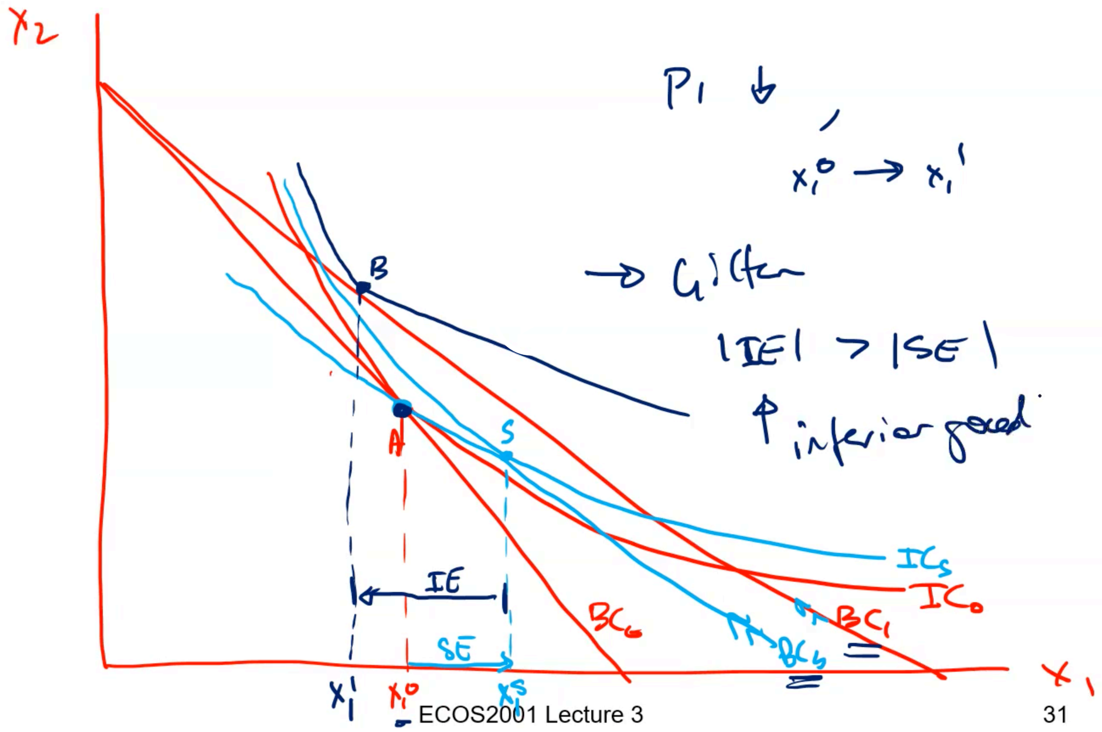
Slutsky's Effects for Income-Inferior Goods

- Some goods are income-inferior (i.e. demand is reduced by higher income).
    - The substitution and income effects oppose each other when an income-inferior good's own price changes.
- The pure substitution effect is as for a normal good. The income effect is in the opposite direction. However, the SE dominates, and the law of demand still holds.



Slutsky's Effects for Giffen Goods

- Cases of extreme income-inferiority, the income effect may be larger in size than the substitution effect, causing quantity demanded to fall as own-price falls.
  - These are called Giffen goods.
  - Law of Downward-Sloping Demand is violated for extremely income-inferior goods.



Inter-temporal Choice

- People often receive income in "lumps" e.g. monthly salary.
- How is a lump of income spread over the following month (saving now for consumption later)?
- Or how is consumption financed by borrowing now against income to be received at the end of the month?

Future Value

- Given the interest rate per period is r, the future value one period from now of m is:

$$FV = m(1 + r)$$

- The present value of m available at the start of the next period is:

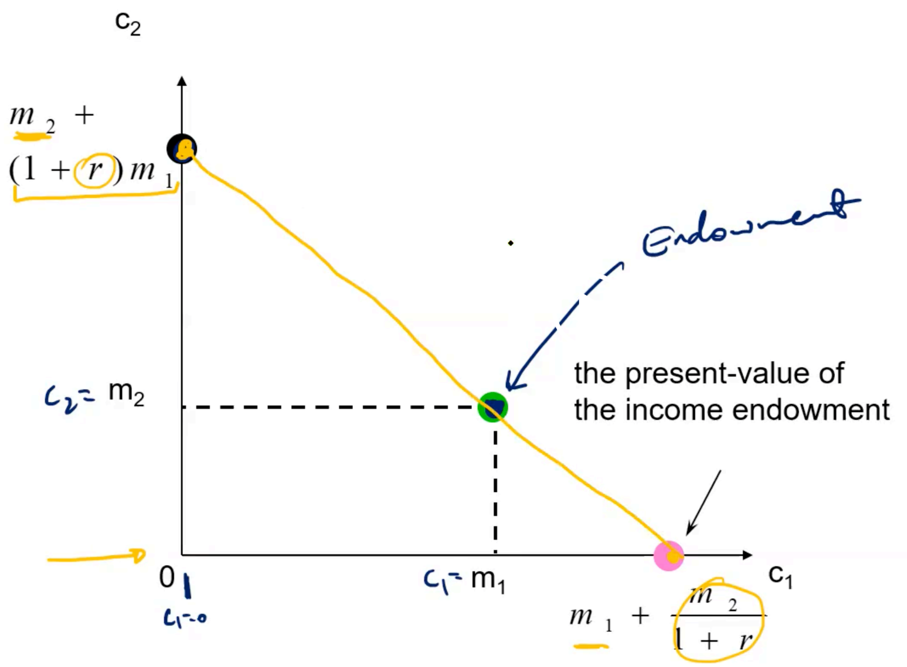
$$PV = m/(1 + r)$$

Inter-temporal Choice Problem

- Let m_1 and m_2 be incomes received in periods 1 and 2.
- Let c_1 and c_2 be consumptions in periods 1 and 2.
- Let p_1 and p_2 be the prices of consumption in periods 1 and 2.
- The inter-temporal choice problem is:
  - Given incomes m_1 and m_2 and given the consumption prices p_1 and p_2, what is the most preferred inter-temporal consumption bundle (c_1, c_2)?
- Depends on:
  - The inter-temporal budget constraint.
  - The inter-temporal consumption preferences.

Inter-temporal Budget Constraint

- **Suppose no savings and no borrowing:**
  - $c_1 = m_1$
  - $c_2 = m_2$
- **Saving:** suppose the consumer spends nothing in period 1, $c_1 = 0$ and saves $s_1 = m_1$. The interest rate is r, so period 2's consumption level is:
  - $c_2 = m_2 + (1 + r)m_1$
- **Spending:** suppose the consumer spends everything possible on consumption in period 1, $c_2 = 0$.
  - They can at most borrow $b_1 = m_2/(1 + r)$ given that they can only use m_2 to pay back.
  - The largest possible period 1 consumption level is thus: $c_1 = m_1 + m_2/(1 + r)$.



- For the **budget constraint equation**, $c_2 = m_2 + (1+r)(m_1 - c_1)$ which can be rearranged to become:
  - Note that the slope is equal to the opportunity cost of a dollar spent today as opposed to tomorrow.

$$c_2 = -(1+r)c_1 + m_2 + (1+r)m_1 \text{ with slope } -(1+r).$$

- The future valued form of the budget constraint where all terms are in period 2 values.

$$(1+r)c_1 + c_2 = (1+r)m_1 + m_2$$

- The present valued form the constraint in period 1 values is:

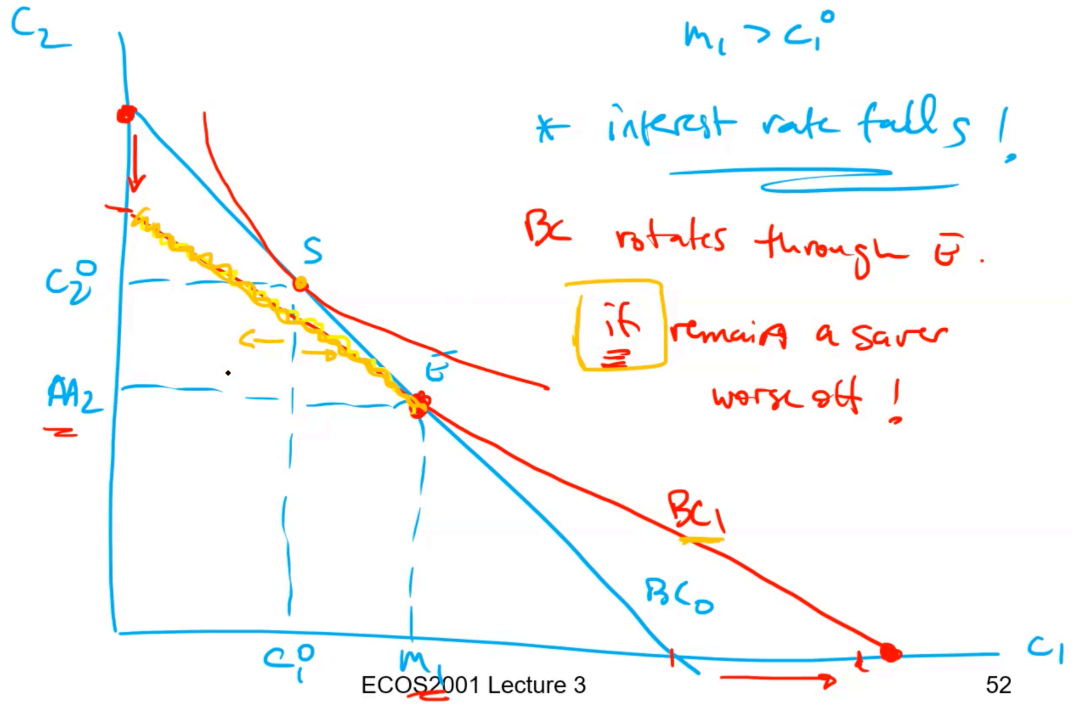
$$c_1 + c_2/(1+r) = m_1 + m_2/(1+r)$$

Comparative Statics

- Regardless of interest rates, you'll always be able to consume $c_1 = m_1$ and $c_2 = m_2$. If interest rates fall, the budget constraint curve will pivot outwards

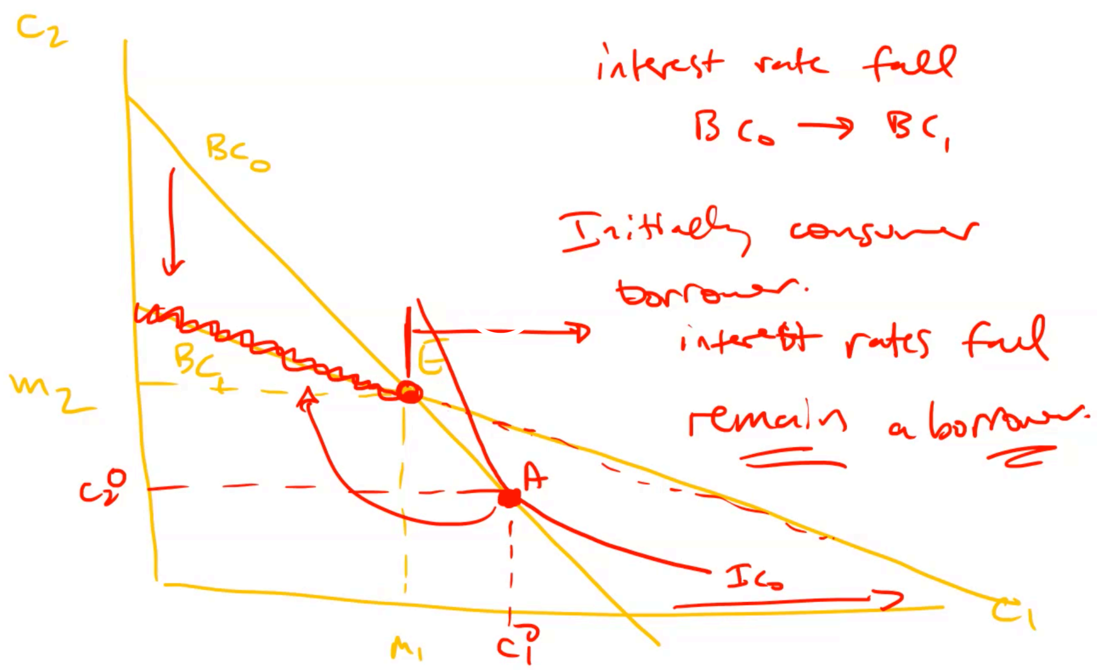
Consumer Saves

- If interest rate falls, if they remain a saver, their welfare is reduced by a lower interest rate.
  - Only if c_2 is a normal good can we say for sure that savings will fall.



Consumer Borrows

- If interest rates fall and the consumer will remain a borrower, their welfare is increased.
  - Borrowing could increase or decrease as a result of lower interest rates.



Lecture 4 - From Individual to Market Demand Functions

- Think of an economy containing n consumers, denoted by $i = 1, \ldots, n$. Consumer i's ordinary demand function for commodity j is:

$$x_j^{*i}(p_1, p_2, m^i)$$

- When all consumers are price-takers, the market demand function for commodity j is:

$$X_j(p_1, p_2, m^1, \ldots, m^n) = \sum_{i=1}^{n} x_j^{*i}(p_1, p_2, m^i)$$

- If all consumers are identical, where $M = nm$:

$$X_j(p_1, p_2, M) = n \times x_j^*(p_1, p_2, m)$$

- The market demand curve is the 'horizontal sum' of the individual consumers' demand curves.

Elasticities

- Elasticity measures the 'sensitivity' of one variable with respect to another.
  - Used in a wide range of applications: quantity demanded changes with respect to own price, another good's price, income; quantity supplied etc.
- The elasticity of variable X with respect to variable Y is:

$$\epsilon_{x,y} = \frac{\%\Delta x}{\%\Delta y}$$

- Own-price elasticity of demand is a sensitivity measure that is independent of units of measurement since it is a ratio of percentages.
  - It is preferred to the slope of a demand curve, which depends upon arbitrary units of measurement for quantity demanded.
  - It can also be **written as**, where the first part of the second last term is the inverse of slope:

$$\epsilon_{ii} = \frac{\Delta x_i}{x_i} \Big/ \frac{\Delta p_i}{p_i} = \frac{\Delta x_i}{\Delta p_i} \times \frac{p_i}{x_i} = \frac{dx_i}{dp_i} \times \frac{p_i}{x_i}$$

Arc Elasticities

$$\epsilon_{x_i^*, p^i} = \frac{(x_2 - x_1)}{x_m} \Big/ \frac{(p_2 - p_1)}{p_m}$$

- where:

$$x_m = \frac{x_1 + x_2}{2} \text{ and } p_m = \frac{p_1 + p_2}{2}$$

Point Elasticities

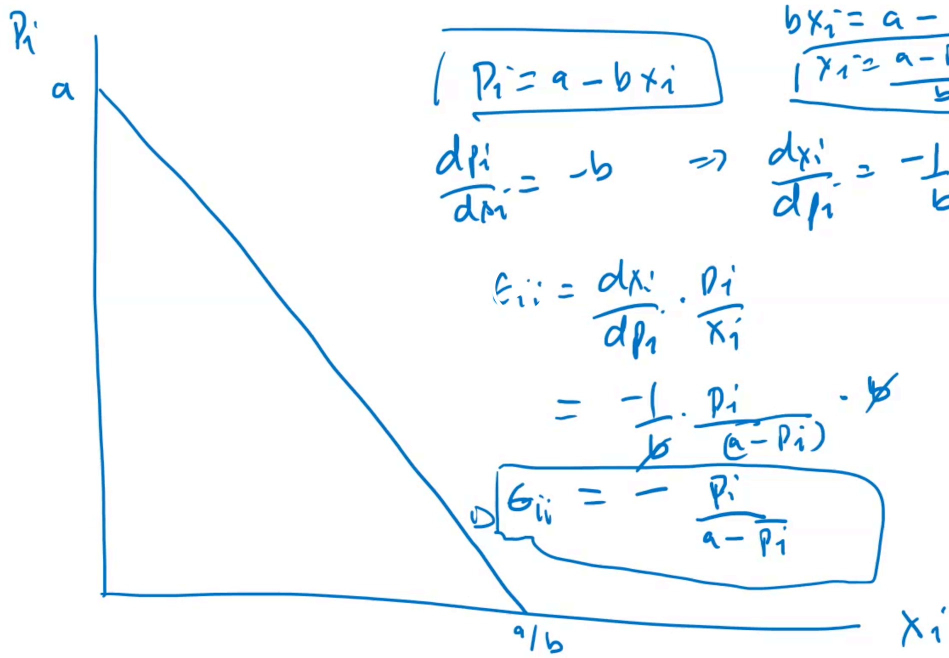
- The **elasticity at the point** is the own-price elasticity of demand in a very small interval of prices centred on p_i'.

$$\epsilon_{X_i^*, p_i} = \frac{p_i'}{X_i'} \times \frac{dX_i^*}{dp_i}$$

- Given a linear demand curve, $p_i = a - bx_i$, the elasticities are as followed:
  - Note that elasticities of demand curves are negative. When written as a positive number, the

author has just taken the magnitude of the elasticity.

- 0 to -1 to negative infinity, inelastic to unit elastic to elastic.



$$b x_i = a - p_i$$

$$\boxed{p_i = a - b x_i} \qquad \boxed{x_i = \frac{a - p_i}{b}}$$

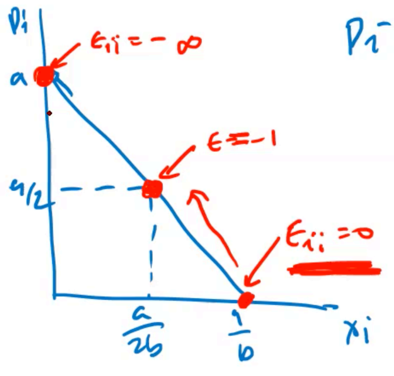
$$\frac{dp_i}{dx_i} = -b \Rightarrow \frac{dx_i}{dp_i} = -\frac{1}{b}$$

$$\epsilon_{ii} = \frac{dx_i}{dp_i} \cdot \frac{p_i}{x_i}$$

$$= -\frac{1}{\cancel{b}} \cdot \frac{p_i}{(a - p_i)} \cdot \cancel{b}$$

$$\boxed{\epsilon_{ii} = -\frac{p_i}{a - p_i}}$$

$$\epsilon_{ii} = -\frac{p_i}{(a - p_i)}$$



$p_i = 0 \qquad \epsilon_{ii} = 0$

$p_i \to a \qquad \epsilon_{ii} = -\frac{a}{a - a} \to -\infty$

$p_i = \frac{a}{2} \qquad \epsilon_{ii} = -\frac{a/2}{a - a/2} = -1$

Constant Price Elasticity

$$\boxed{x_i = k p_i^{-2}}$$

$$\cancel{x_i = \frac{k}{p_i^{2}}}$$

$$\frac{dx_i}{dp_i} = -2k p_i^{-3}$$

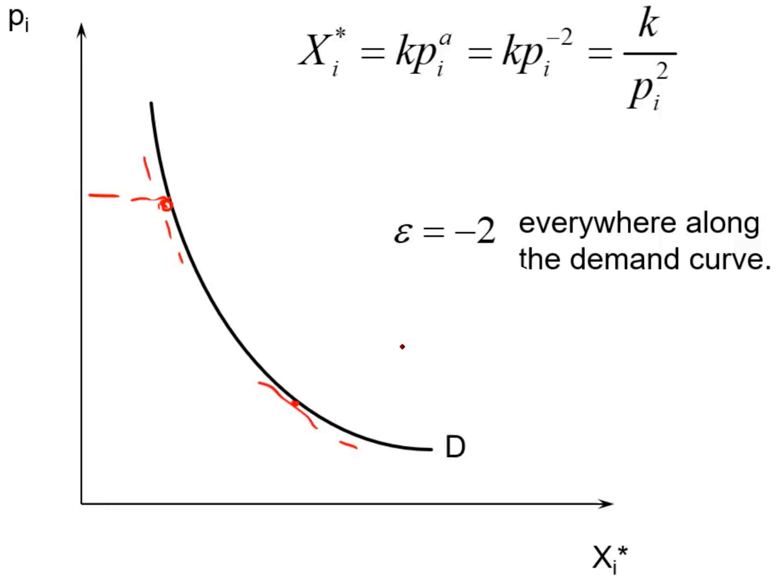
constant price elasticity

$$\epsilon_{ii} = \frac{dx_i}{dp_i} \cdot \frac{p_i}{x_i}$$

$$= \frac{-2k p_i^{-3} \cdot p_i}{k p_i^{-2}}$$

$$= -2 \frac{\cancel{p_i^{-2}}}{\cancel{p_i^{-2}}}$$

$$\epsilon_{ii} = -2$$



Revenue and Own-Price Elasticity of Demand

- If raising a commodity's price causes little decrease in quantity demanded, then sellers' revenues rise.
  - Own-price inelastic demand causes sellers' revenues to rise as price rises.
- If raising a commodity's price causes a large decrease in quantity demanded, then sellers' revenues fall.
  - Own-price elastic demand causes sellers' revenues to fall as price rises.
- Sellers' revenue is the price of a commodity times the amount demanded:

$$R(p) = p \times X^*(p)$$

- So:

$$\frac{dR}{dp} = X^*(p) + p\frac{dX^*}{dp}$$
$$= X*(p)[1 + \frac{p}{X^*(p)}\frac{dX^*}{dp}]$$
$$= X^*(p)[1 + \epsilon]$$

- If $\epsilon = -1$, then $\frac{dR}{dP} = 0$.
- If $-1 < \epsilon \leq 0$, then $\frac{dR}{dP} > 0$. This is in the inelastic range.
- If $\epsilon < -1$, then $\frac{dR}{dP} < 0$. This is in the elastic range.

Marginal Revenue and Own-Price Elasticity of Demand

- A seller's marginal revenue is the rate at which revenue changes with the number of units sold by the seller.

$$MR(q) = \frac{dR(q)}{dq}$$

- Below, $p(q)$ denotes the seller's inverse demand function i.e. the price at which the seller can sell q units. Then:

$$R(q) = p(q) \times q$$

- So:

$$MR(q) = \frac{dR(q)}{dq} = \frac{dp(q)}{dq}q + p(q)$$
$$= p(q)[1 + \frac{q}{p(q)}\frac{dp(q)}{dq}]$$
$$= p(q)[1 + \frac{1}{\epsilon}]$$

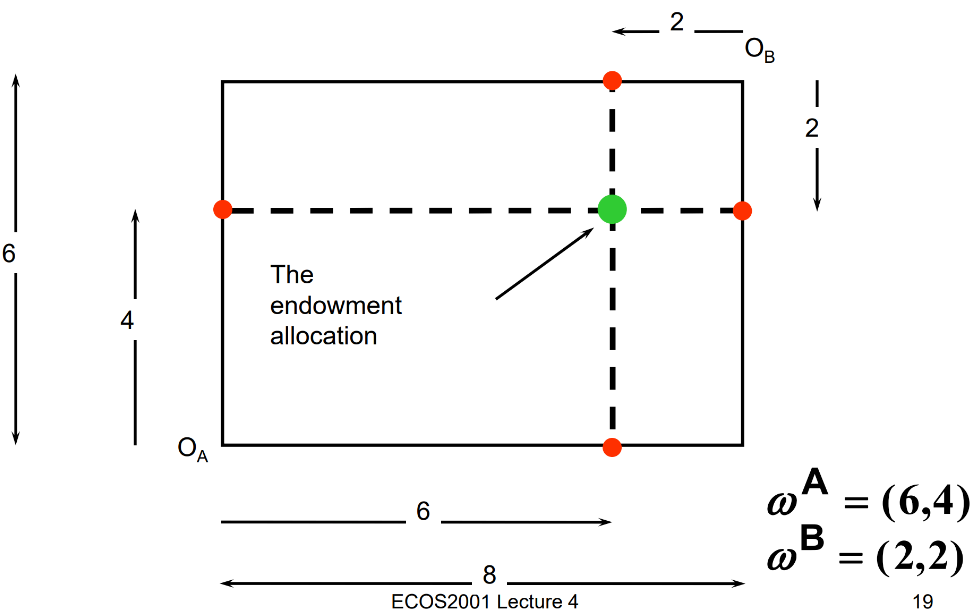
- The way a seller's revenue changes with the number of units it sells depends on the sensitivity of quantity demanded to price i.e. upon its own-price elasticity of demand.
- If $\epsilon = -1$, then $MR(q) = 0$.
- If $-1 < \epsilon \leq 0$, then $MR(q) < 0$. This is in the inelastic range.
- If $\epsilon < -1$, then $MR(q) > 0$. This is in the elastic range.

Exchange

- Two consumers, A and B. Their endowments of goods 1 and 2 are:
  - $\omega^A = (\omega_1^A, \omega_2^B)$ and $\omega^B = (\omega_1^B, \omega_2^B)$
  - E.g. $\omega^A = (6, 4)$ and $\omega^B = (2, 2)$
- The total quantities available are:
  - $\omega_1^A + \omega_1^B = 6 + 2 = 8$
  - $\omega_2^A + \omega_2^B = 4 + 2 = 6$

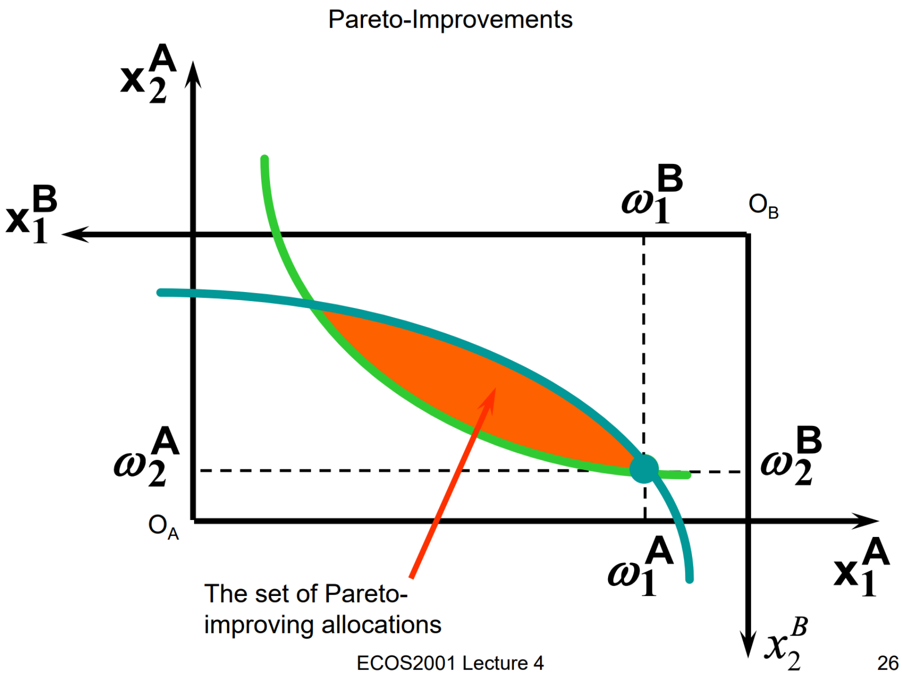
Edgeworth Box

- The dimensions of the box are the quantities available of the good.

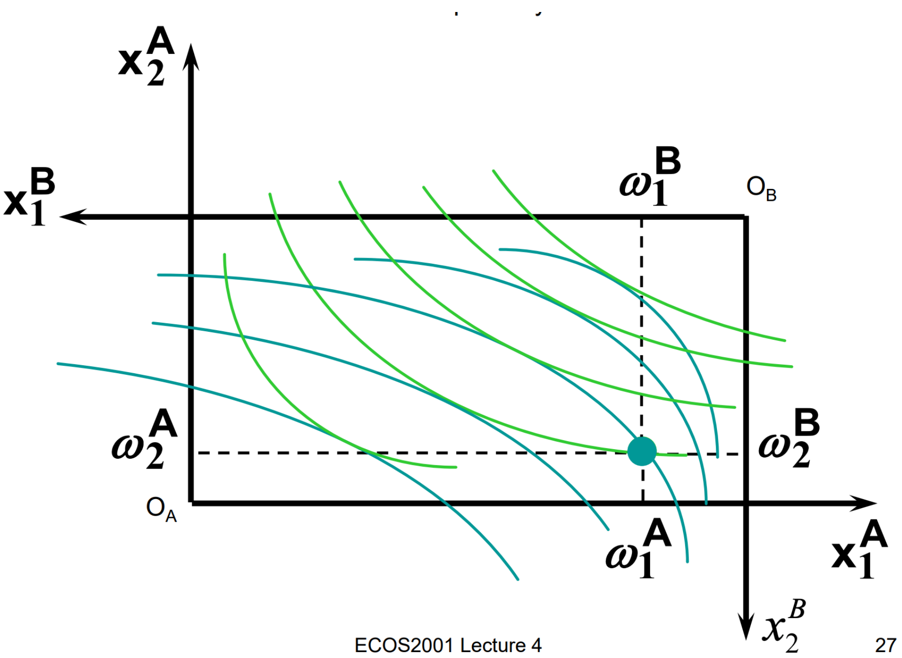


- All points in the box including the boundary represent feasible allocations of the combined endowments.

- This is essentially another way of depicting the budget constraint.
- For consumers A and B, we can add preferences to the edgeworth box.
    - At a given endowment allocation, a different endowment allocation that improves the welfare of a consumer without reducing the welfare of another is a **Pareto-improving allocation**.
- Since consumers can refuse to trade if it makes them worse off, the only possible outcomes from exchange are Pareto-improving allocations.

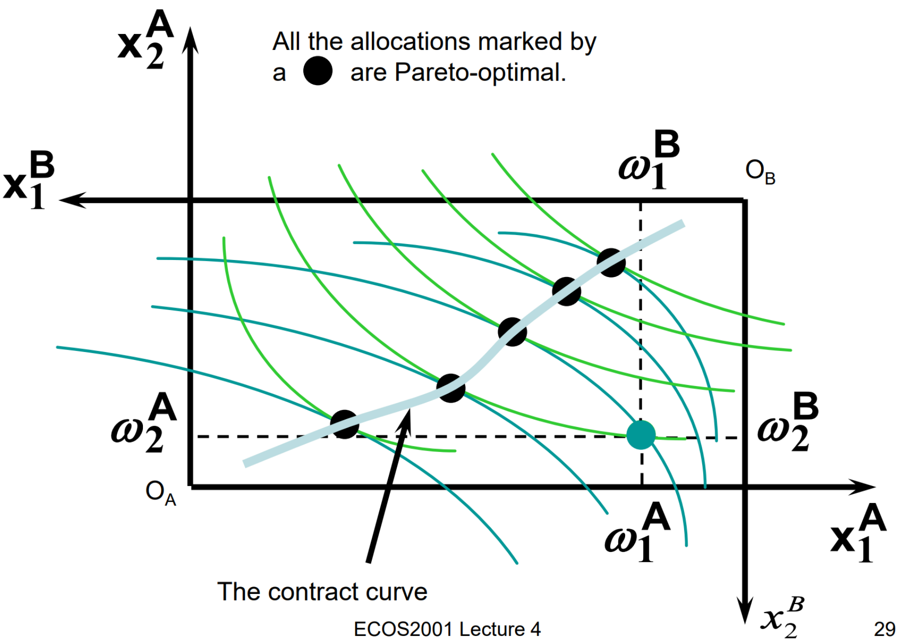


- There are a number of sets of Pareto-improving allocations, depending on where the initial endowment allocation is.



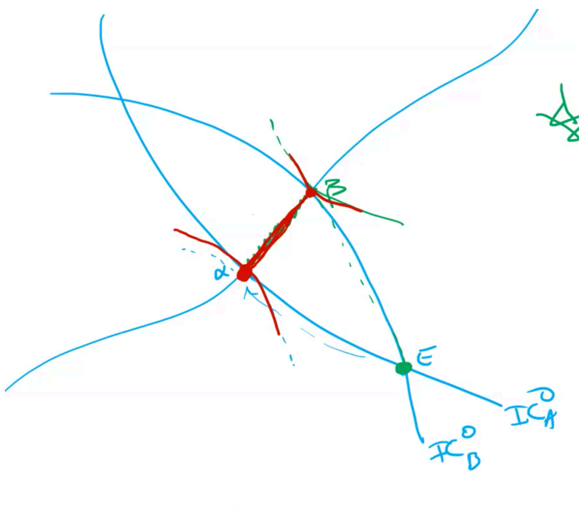
Pareto-Optimality

- The best endowment allocations are at the points where the indifference curves are tangential to one another.
    - This is because A wants to move to the north east, and B wants to move to the south west.
- The **contract curve** is the set of all Pareto-optimal allocations.



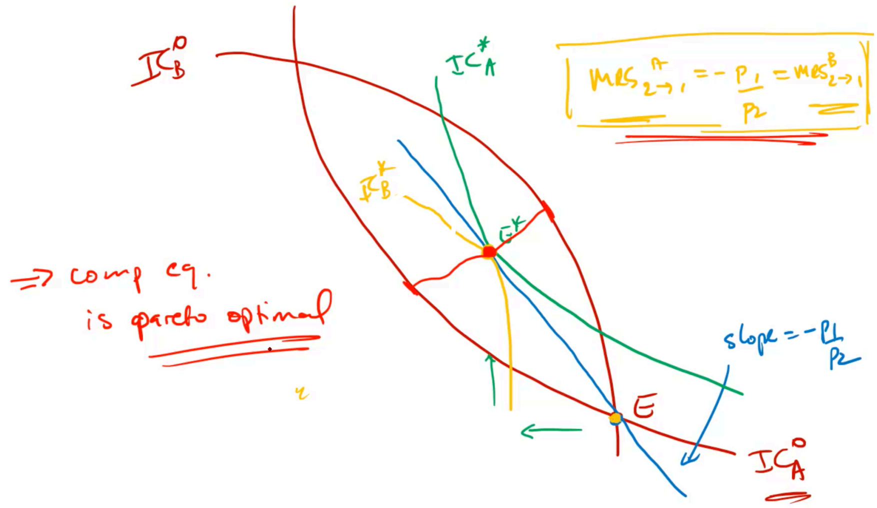
Which Allocation on the Contract Curve Will Consumers Trade?

- It depends upon how trade is conducted e.g. perfectly competitive markets? One-on-one bargaining?
- The **core** is the set of all Pareto-optimal allocations that are welfare-improving for both consumers relative to their own endowments.
    - Rational trade should achieve a core allocation. The core is in the 'lens', given an endowment allocation.

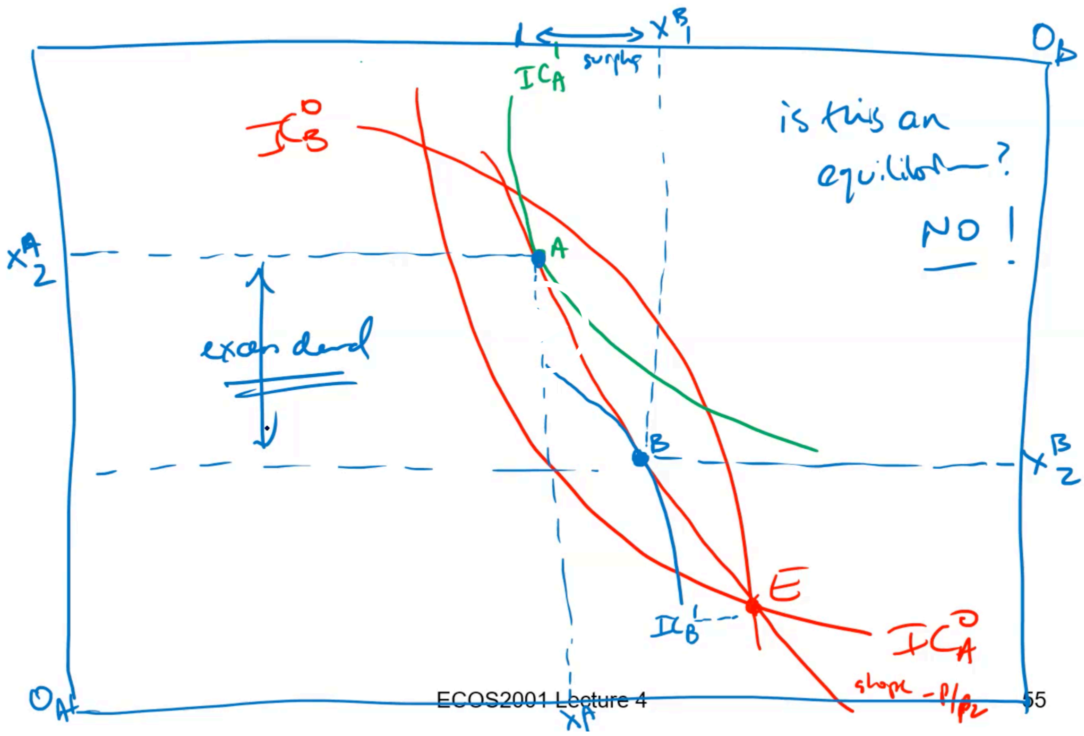


Trade in Competitive Markets

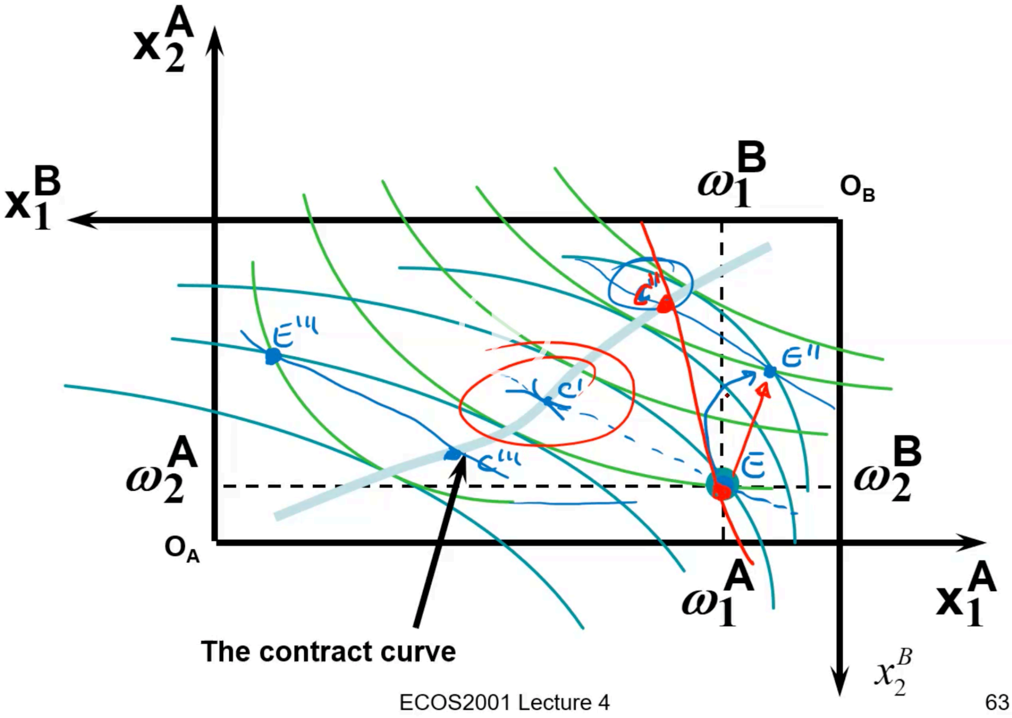
- Consider trade in perfectly competitive markets.
  - Each consumer is a price-taker trying to maximise their own utility given p_1, p_2 and their own endowment.
  - They will aim to trade on their budget constraint/price-ratio line.
- At the new prices, both markets clear and there is a general equilibrium.



- **First Fundamental Theorem of Welfare Economics:** Given that consumers' preferences are well-behaved, trading in perfectly competitive markets implements a Pareto-optimal allocation of the economy's endowment.
  - In a market not in equilibrium, excess demand/supply will be fixed by changing prices (the budget constraint line will become flatter).



- **Second Fundamental Theorem of Welfare Economics:** Given that consumers' preferences are well-behaved, for any Pareto-optimal allocation, there are prices and an allocation of the total endowment that makes the Pareto-optimal allocation implementable by trading in competitive markets.
  - Say that we are not happy with endowment point E and want to move to E''. The perfectly competitive market will only move us towards c', and a change in prices will cause disquilibrium.
    - In this case, policy to change endowments is the way to reach a 'better' endowment allocation c'' on the contract curve. This can be done using lump sum transfers.



Walras' Law

- Every consumer's preferences are well-behaved so, for any positive prices (p_1, p_2), each consumer spends all of their budget.
  - For consumer A:

$$p_1 x_1^{*A} + p_2 x_2^{*A} = p_1 \omega_1^A + p_2 \omega_2^A$$

  - For consumer B:

$$p_1 x_1^{*B} + p_2 x_2^{*B} = p_1 \omega_1^B + p_2 \omega_2^B$$

- Summed and rearranged to become:
  - This says that the summed market value of excess demands is zero for any positive prices p_1 and p_2. This is **Walras' Law**.

$$p_1(x_1^{*A} + x_1^{*B} - \omega_1^A - \omega_1^B) + p_2(x_2^{*A} + x_2^{*B} - \omega_2^B - \omega_2^B) = 0$$

Implications of Walras' Law

- For a two-commodity exchange economy:
  - If one market is in equilibrium, then the other market must also be in equilibrium.
  - If there is excess supply in one market, this implies an excess demand in the other market.

Lecture 5 - Technology, Production and Costs

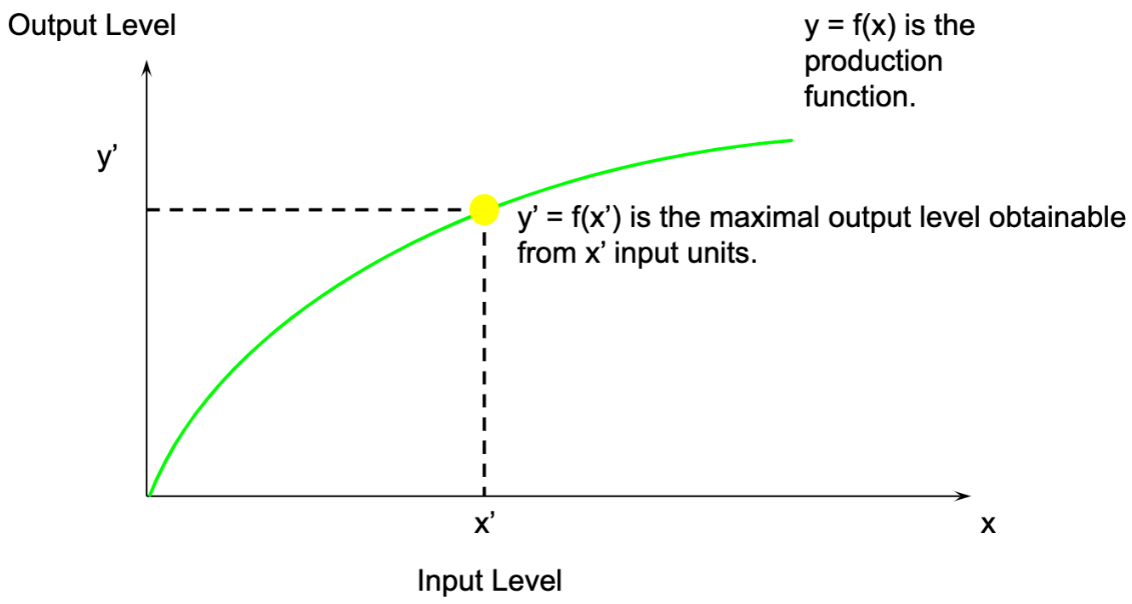
- A **technology is a process by which inputs are converted to an output**.
  - Usually several technologies will produce the same product. Which technology is the best and how do we compare technologies?

Input Bundles

- x_i denotes the amount used of input i i.e. the level of input i.
- An **input bundle** is a vector of the input levels $(x_1, x_2, \ldots, x_n)$.
- y denotes the **output level**.
- The technology's **production function** thus states the maximum amount of output possible from an input bundle:

$$y = f(x_1, \ldots, x_n)$$

- Given one input and one output:



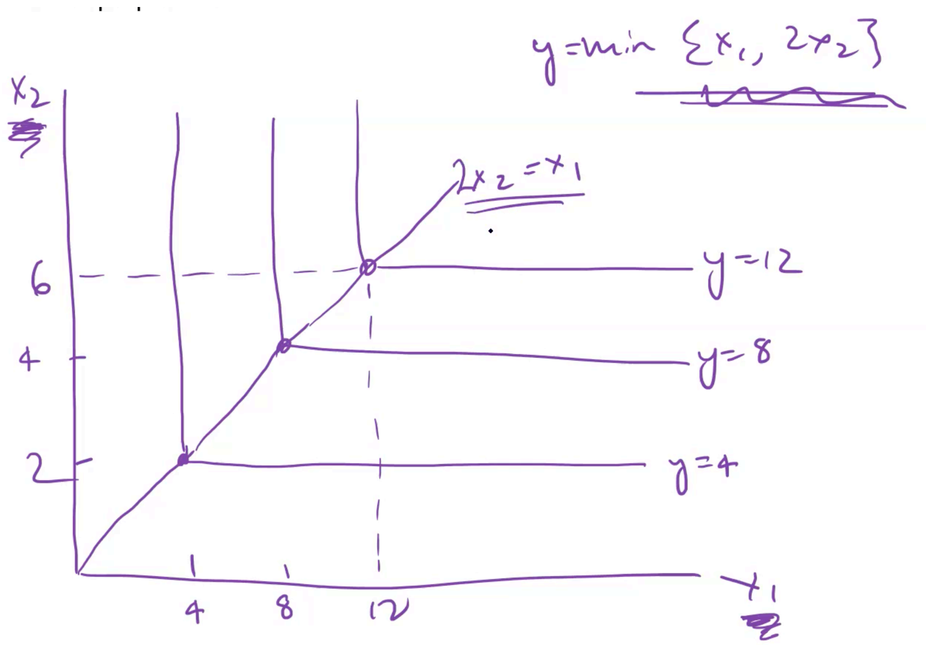
Technology Sets

- A **production plan** is an input bundle and an output level $\rightarrow (x_1, \ldots, x_n, y)$.
- A production plan is **feasible** if $y \leq f(x_1, \ldots, x_2)$.
- The collection of all feasible production plans is the **technology set**.

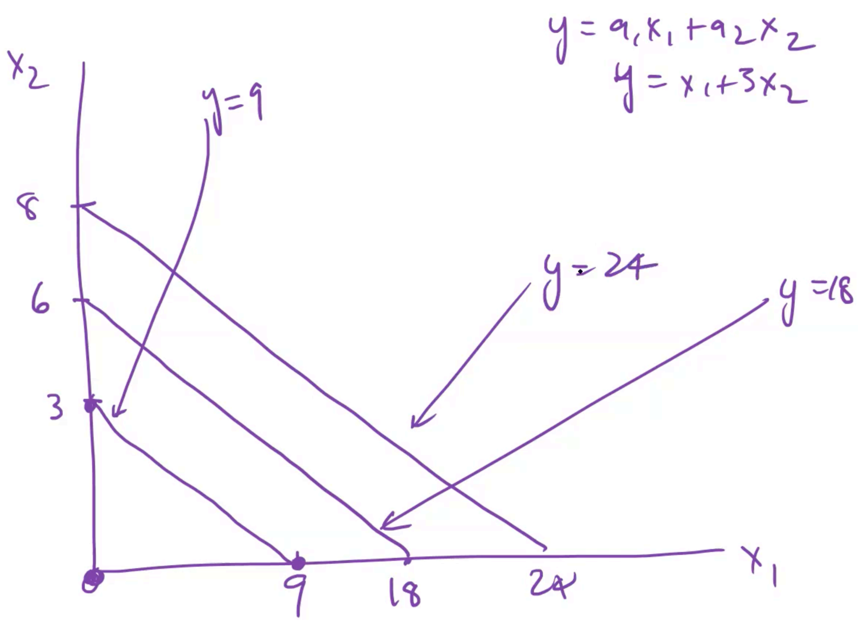
Technologies with Multiple Inputs

- We consider the two input case with levels x_1, x_2. Output level is y.
- The y output unit **isoquant** is the set of all input bundles that yield at most the same output level y.
  - They are similar to indifference curves, although isoquants represent real, existing output.

Fixed Proportions

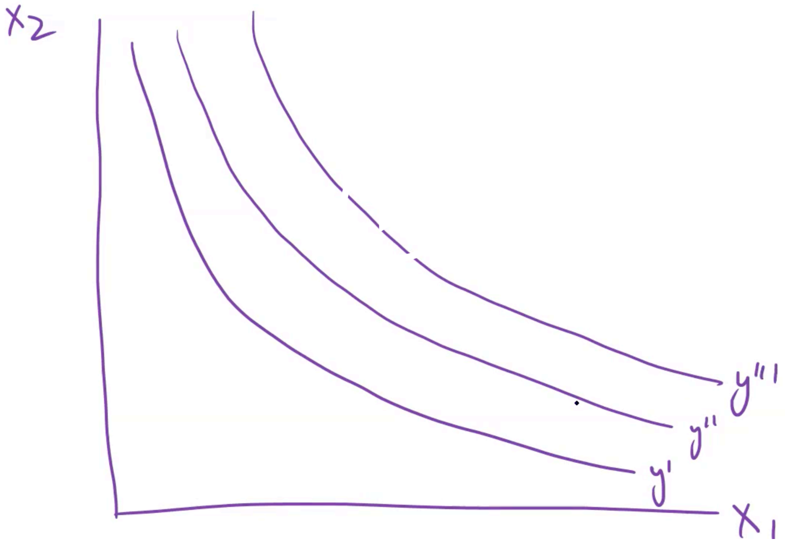


Perfect Substitutes



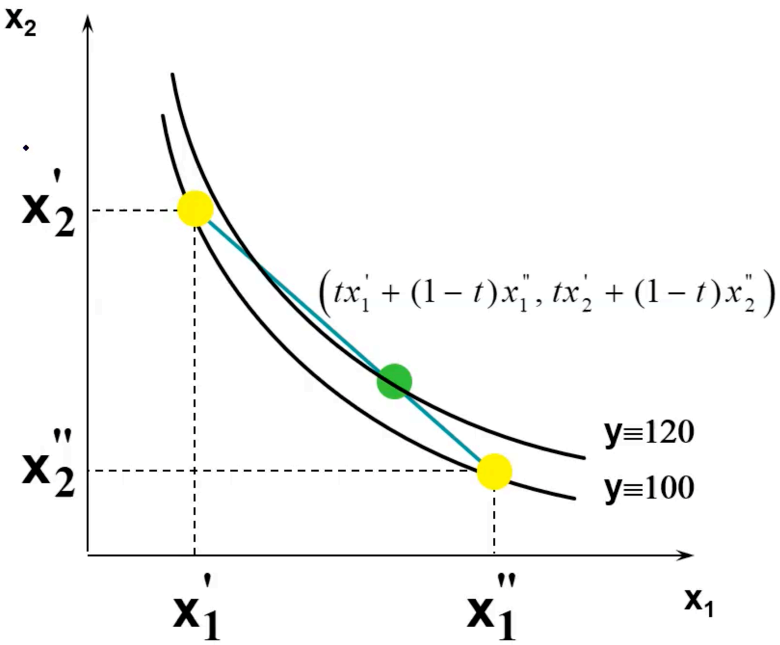
Cobb-Douglas Production Function

$$y = A x_1^{\alpha} x_2^{\beta} = x_1^{1/3} x_2^{1/3}$$



Properties of Technology

- **Monotonic**: if you increase at least one input, you should produce as much output as originally.
  - This is to say that any increase in inputs will never hurt your output e.g. hiring an extra chef won't cause traffic in the kitchen, you can always sit them out $\rightarrow$ free disposal.
- **Convex:** if the input bundles x' and x'' both provide y units of output, then the mixture $tx' + (1-t)x''$ provides at least y units of outputs for any $0 < t < 1$.



Marginal (Physical) Products

$$y = f(x_1, \ldots, x_n)$$

- The **marginal product** of input i is the rate-of-change of the output level as the level of input i changes, holding all other output levels fixed.

$$MP_i = \frac{\Delta y}{\Delta x_i} = \frac{\partial y}{\partial x_i}$$

- Typically, the marginal product of one input depends upon the amount used of other inputs.
- **Diminishing MP**: MP of input i is diminishing if it becomes smaller as the level of input i increases.

$$\frac{\partial MP_i}{\partial x_i} = \frac{\partial}{\partial x_i}(\frac{\partial y}{\partial x_i}) = \frac{\partial^2 y}{\partial x_i^2} < 0$$

Technical Rate of Substitution

- Measures the rate at which the firm will have to substitute one input for another to keep the output constant i.e. **slope of the isoquant**.

$$y = f(x_1, \ldots, x_n)$$

$$dy = \frac{\partial y}{\partial x_1}dx_1 + \frac{\partial y}{\partial x_2} = 0 \text{ NOTE: dy=0 since output does NOT change}$$

$$TRS(x_1, x_2) = \frac{dx_2}{dx_1} = -\frac{\partial y/\partial x_1}{\partial y/\partial x_2} = -\frac{MP_1(x_1, x_2)}{MP_2(x_1, x_2)}$$

- **Diminishing TRS**: as you incresae the amount of factor 1 and reduce factor 2 to keep y the same, the TRS declines
  - The slope of the isoquant must decrease in absolute value as you move along the isoquant increasing x_1, and must increase as you move in the direction of increasing x_2.
  - Isoquants have 'well-behaved' convex shapes.

Profit Maximisation

- **Profits** are revenues minus costs (including opportunity costs). The **competitive** firm takes all output prices and all input prices as given constants.
  - Note: we need to ensure time scales for all factors are the same.

$$\Pi = p_1 y_1 + \ldots + p_n y_n - w_1 x_1 - \ldots w_m x_m$$

- **Fixed and Variable Costs:** due to fixed costs, it is possible to make **negative profits in the short run** (fixed factors need to be paid for even in the case of zero output).

Long and Short Run

- **Short Run**: at least one factor of production is fixed (often capital, workers).
- **Long Run**: all factors are variable.
- **Return to Scale**: returns to scale describes how the output level changes as all input levels change in direct proportion (e.g. all input levels doubled, or halves). This is typically a long-run concept.
  - Constant Returns to Scale: $f(kx_1, kx_2, \ldots, kx_n) = kf(x_1, x_2, \ldots, x_n)$
  - Decreasing Returns to Scale: $f(kx_1, kx_2, \ldots, kx_n) < kf(x_1, x_2, \ldots, x_n)$
  - Increasing Returns to Scale: $f(kx_1, kx_2, \ldots, kx_n) > kf(x_1, x_2, \ldots, x_n)$
- Can a technology exihibit increasing returns to scale even if all of its marginal products are diminishing?

    - Yes, for example, Cobb-Douglas return functions (powers increase by more than the factor increase).

Short-Run Profit Maximisation

- Suppose the firm is in a short-run circumstance in which $x_2 \equiv \tilde{x_2}$ i.e. x_2 is fixed.
- Its short-run production function is $y = f(x_1, \tilde{x_2})$.
- The firm's fixed cost is $FC = w_2\tilde{x_2}$, with it's variable costs as w_1x_1.
- The profit function is $\Pi = py - w_1x_1 - w_2\tilde{x_2}$
- The **profit-maximisation problem** for the firm is to:

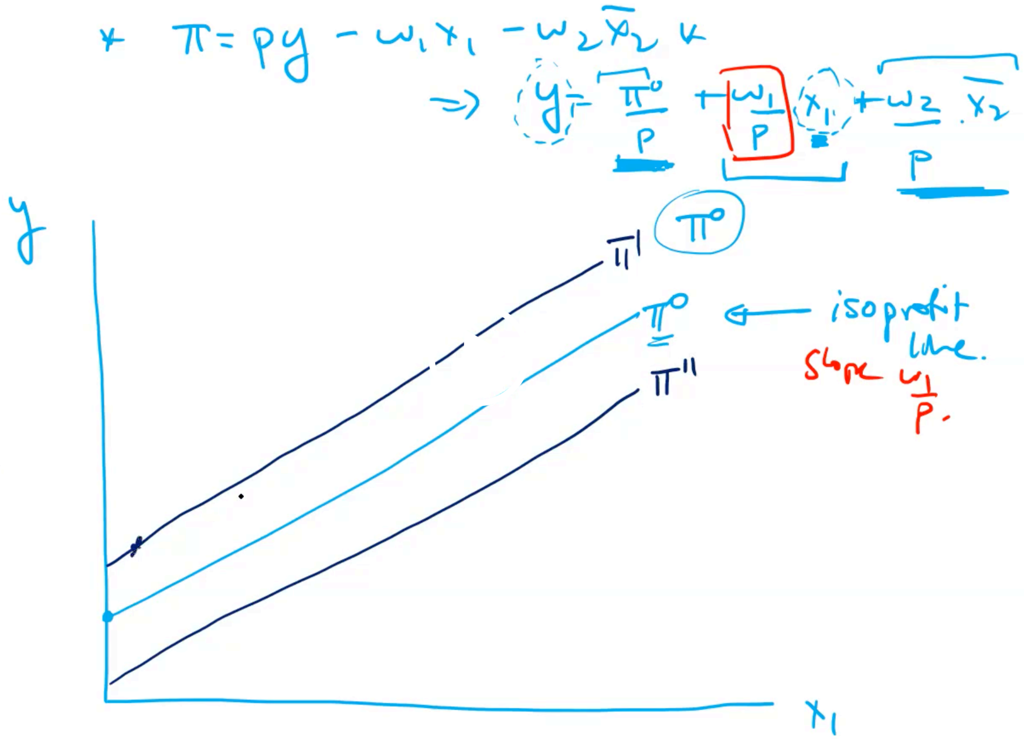
$$\max_{x_1} pf(x_1, \tilde{x_2}) - w_1x_1 - w_2\tilde{x_2}$$

- Hence, the value of the **marginal revenue product** of a factor should equal its price.
    - This is found by deriving the equation above.
    - Essentially, **an input's cost should equal the value of the marginal product that they produce**.

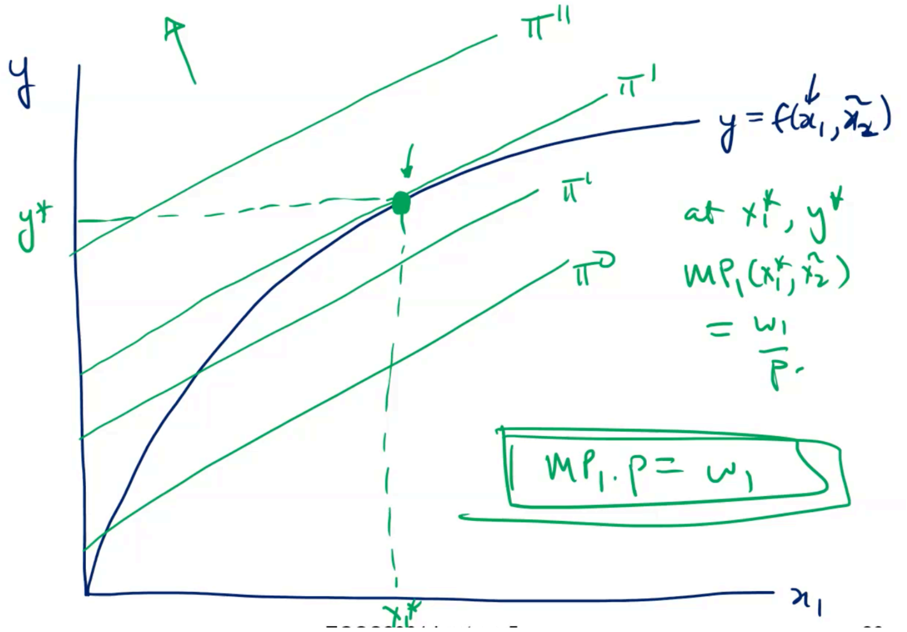
$$pMP_1(x_1^*, \tilde{x_2}) = w_1$$
$$-MRP_1 = w_1$$

Isoprofit Curves

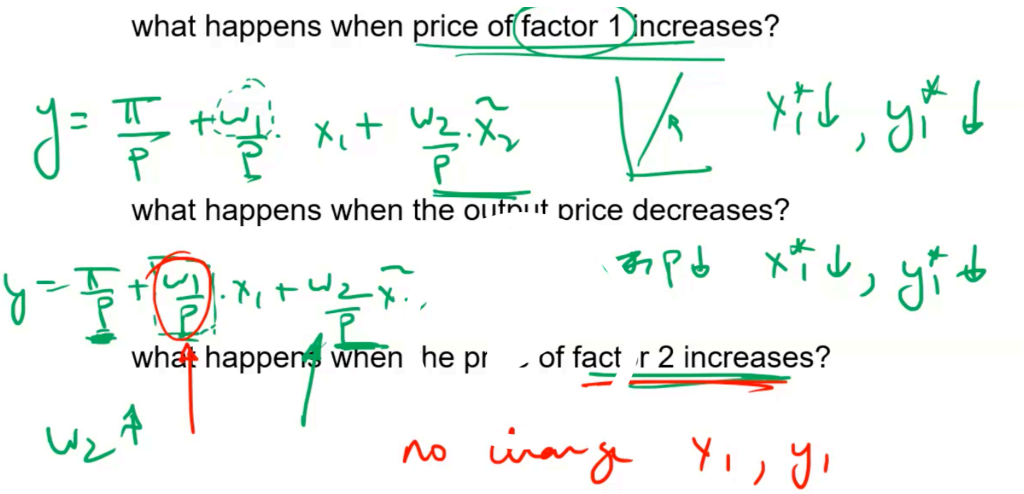
- Given a profit function, isolating an output y gives us the following equation. We fix profits at Π^0, making the second term the only one that is variable.
    - Graphing this gives us various isoprofit curves, where costs and outputs are balanced to give a constant level of profit.
    - Ideally, a firm would like to increase output and profit with less inputs (go northwest).



- The profit maximising point is the point where the production function is tangent to the highest possible isoprofit line.
  - This is the point where $\mathrm{MP}_1 = w_1/p$.



Comparative Statics



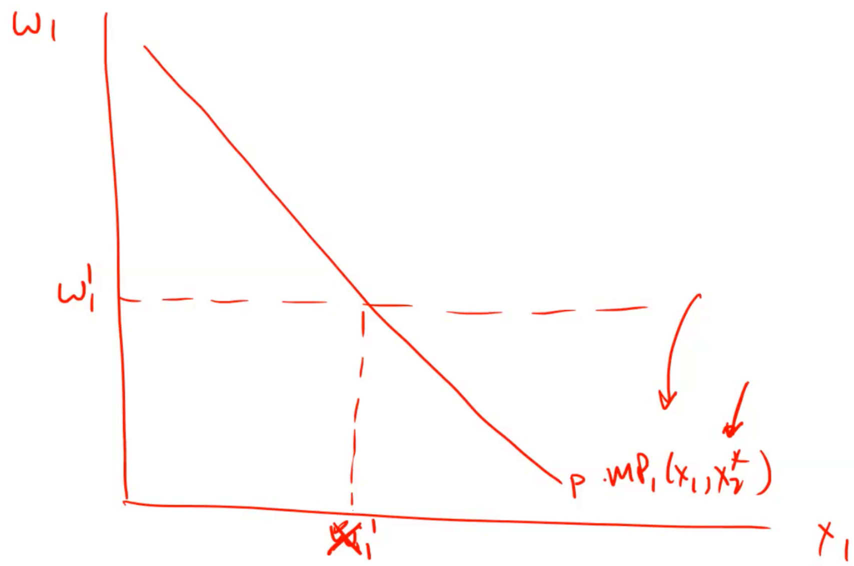
Long-Run Profit Maximisation

$$\max_{x_1, x_2} pf(x_1, x_2) - w_1 x_1 - w_2 x_2$$

- Given that both factors can be changed:

$$p\mathrm{MP}_1(x_1^*, x_2^*) = w_1$$
$$p\mathrm{MP}_2(x_1^*, x_2^*) = w_2$$

- The optimal choice of each factor as a function of the prices can be shown using **factor demand curves**.
  - These show the relationship between a factor and a profit-maximising choice of that factor.

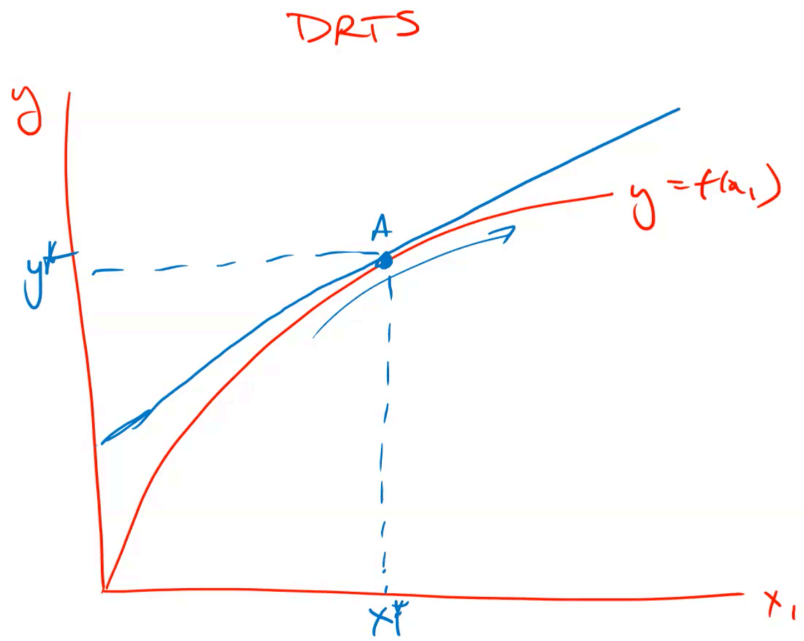


- Given the optimal choice of factor 2, draw the optimal choice of factor 1:
  - The slope is downward curving by assumption of dimishing MP.

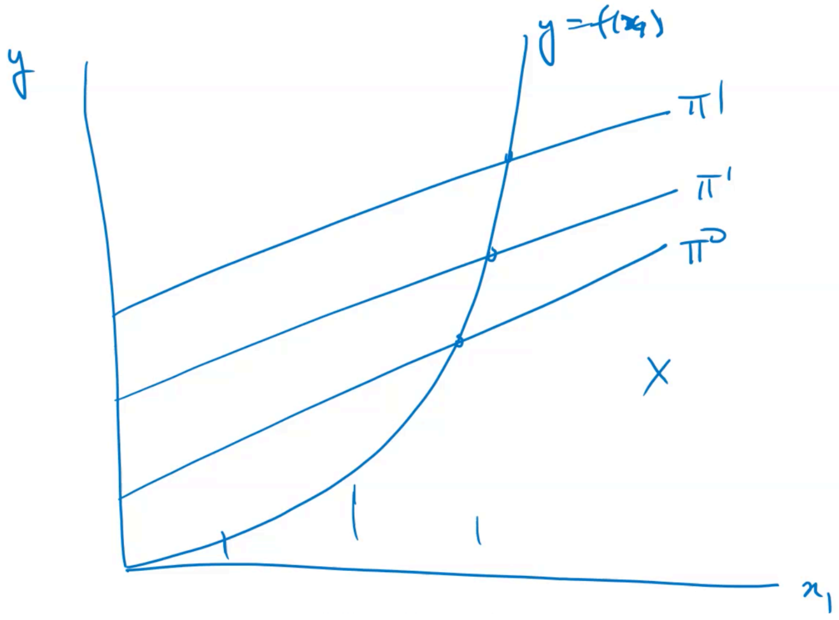
$$p\mathrm{MP}_1(x_1, x_2^*) = w_1$$

Profit Maximisation and Returns to Scale

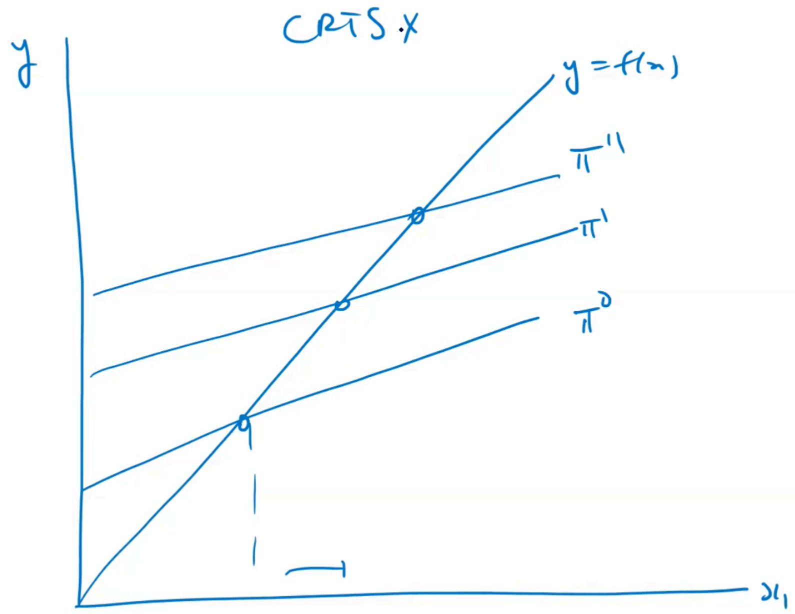
- For industries with decreasing returns to scale:



- For industries with increasing returns to scale:



- For industries with constant returns to scale:



- Given an industry with increasing or constant returns to scale, a firm would want to grow infinitely large. Most modelling thus assumes an industry with decreasing returns to scale.

Cost Minimisation (Alternate Way of Looking at It)

- The cheapest way to produce a given level of output y.

$$\min_{x_1, x_2 \geq 0} w_1 x_1 + w_2 x_2 \text{ subject to } f(x_1, x_2) = y$$

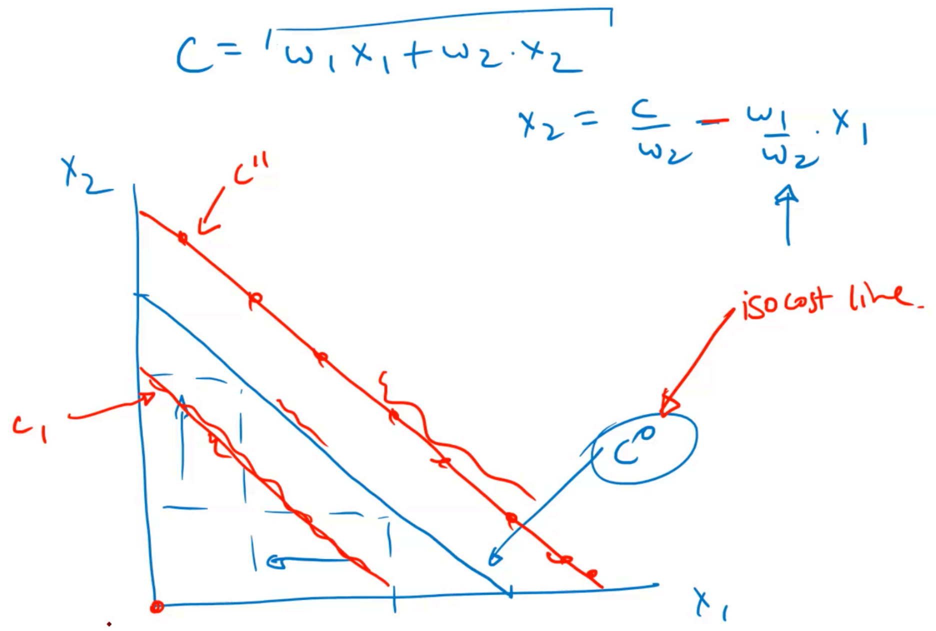
- The solution (min cost necessary to produce the desired level of output) depends on w_1, w_2, y so the **cost function** is:

$$c(w_1, w_2, y)$$

- For a given level of cost C:

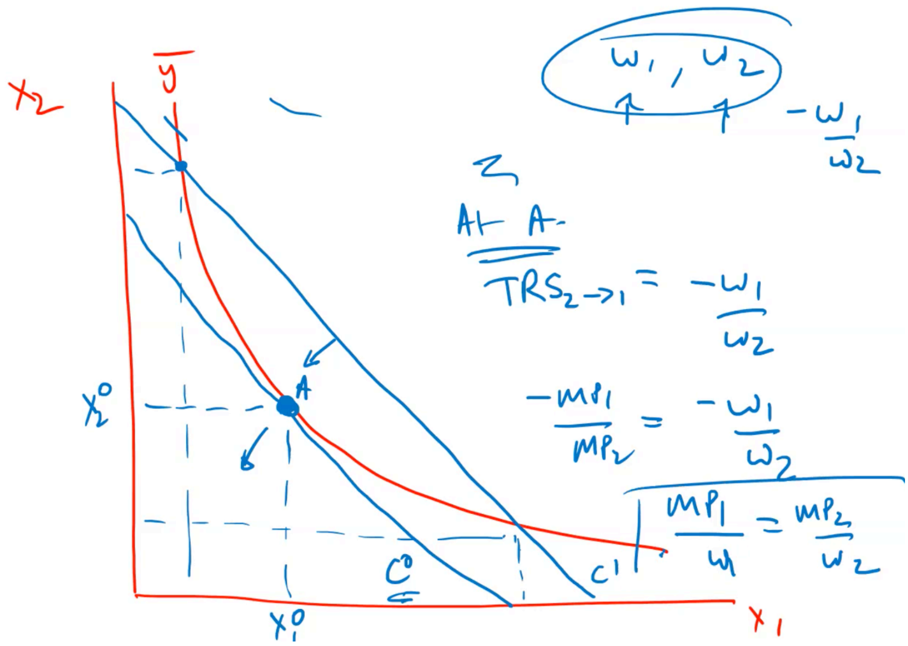
$$w_1 x_1 + w_2 x_2 = C \text{ or that}$$
$$x_2 = -\frac{w_1}{w_2} x_1 + \frac{c}{w_2}$$

- This is a straight line with slope $-w_1/w_2$. Every point on an **isocost line** has the same cost and higher isocost lines have higher costs.



- **Cost Minimisation means:**
    - Find a point on the isoquant that has the lowest possible isocost line associated with it.
    - With an interior solution and if the isoquant is a smooth curve (Cobb-Douglas), the cost minimisation point will be tangency
    - The technical rate of substitution must equal the factor price ratio:

$$-\frac{w_1}{w_2} = \mathrm{TRS} = -\frac{\mathrm{MP}_1}{\mathrm{MP}_2} \text{ at } (x_1^*, x_2^*)$$



- You should use an input mix such that the marginal cost per dollar spend is equal.

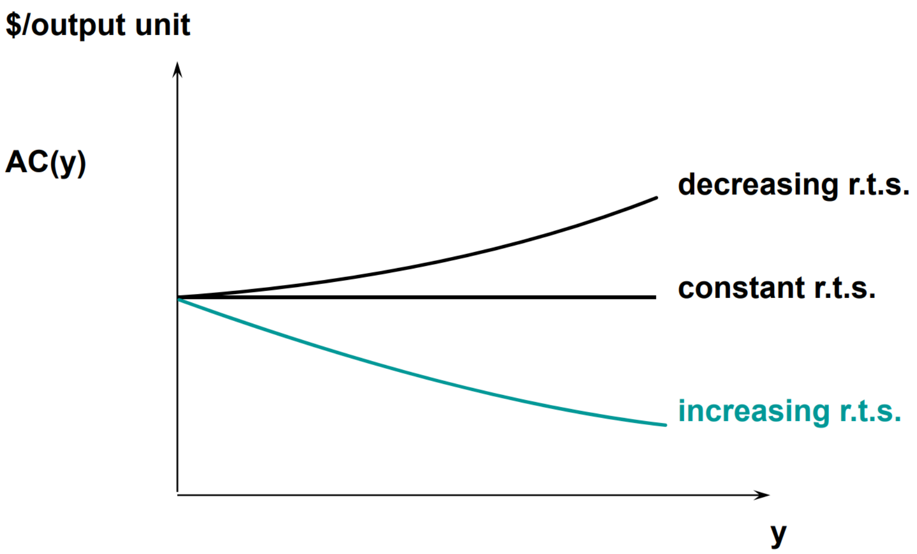
Returns to Scale and Costs

- Given a unit cost function that produces 1 unit of output.

$$c(w_1, w_2, 1)$$

- If the minimum cost way of producing y units is $yc(w_1, w_2, 1), then there is constant returns to scale.
- Increasing returns to scale: costs increase less than linearly in output.
- Decreasing returns to scale: costs increase more than linearly in output.
- Implications for Average Cost:

$$AC(w_1, w_2, y) = \frac{c(w_1, w_2, y)}{y}$$



Short and Long Run Total Costs

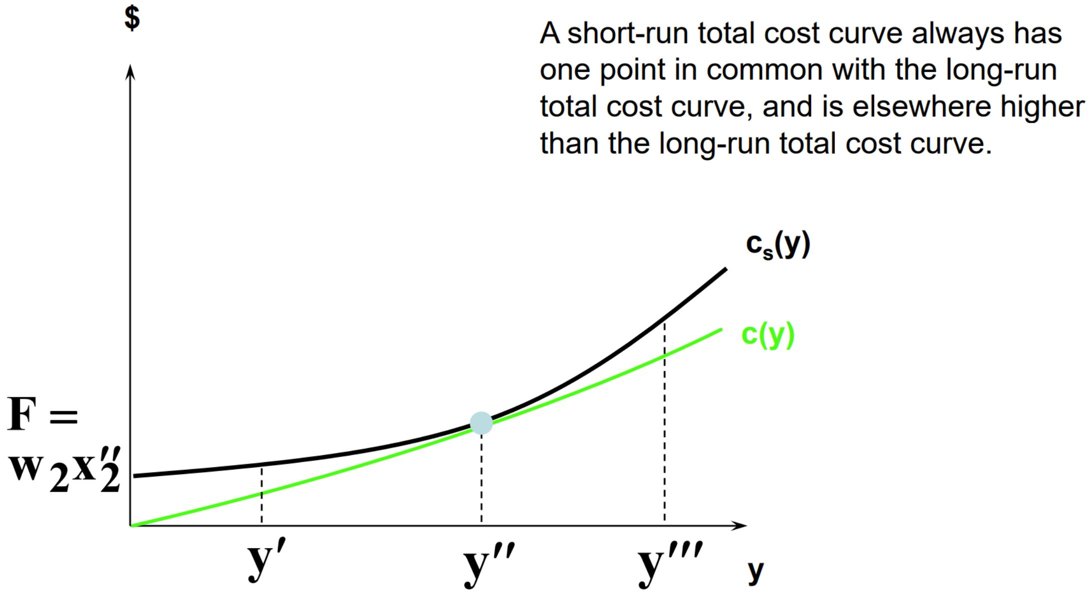
- In the long-run, a firm can vary all of its input levels.
    - The long run-cost minimisation problem is:

$$\min_{x_1, x_2 \geq 0} w_1x_1 + w_2x_2 \text{ subject to } f(x_1, x_2) = y$$

- Consider a firm that cannot change its input 2 level from x_2' units.
    - The short run cost minimisation problem is:

$$\min_{x_1, x_2 \geq 0} w_1x_1 + w_2x_2' \text{ subject to } f(x_1, x_2') = y$$

- The short run cost-minimisation problem is the long-run problem subject to an extra constraint.
    - Short-run total cost exceeds long-run total cost except for the output level where the short-run level restriction is the long-run input level choice.
    - In every other case, the long-run total cost must be lower than the short-run total cost (since you have the ability to change your input mix to one where costs are lower).

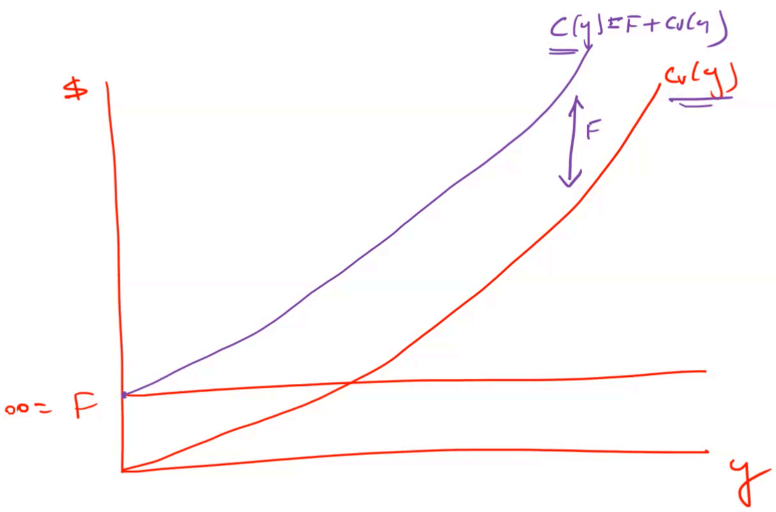


Lecture 6 - Cost Curves and Perfect Competition

Fixed, Variable and Total Cost Functions

- **Fixed Costs**: F is the total cost to a firm of its short-run fixed inputs, and do not vary with the firm's output level.
- **Variable Cost Function**: $c_v(y)$ is the total cost to a firm of its variable inputs when producing y output units $\rightarrow$ depends upon the levels of the fixed inputs.
- **Total Cost Function**: $c(y)$ is the total cost of all inputs, fixed and variable, when producing y output units.

$$c(y) = F + c_v(y)$$

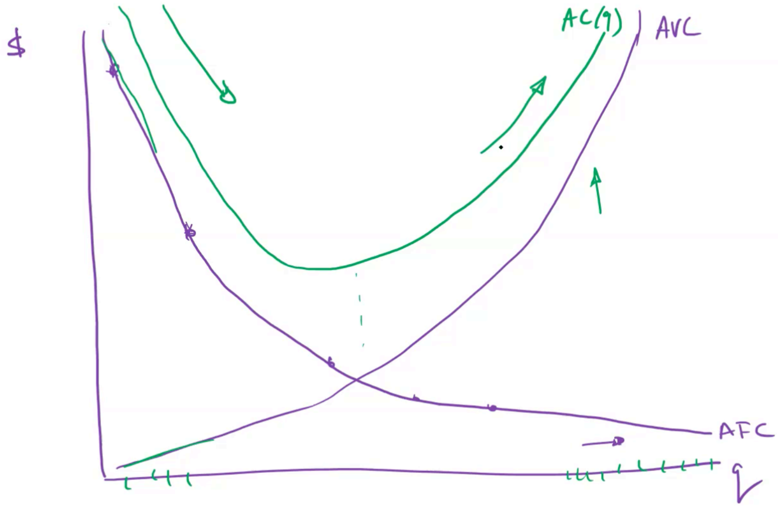


Average Fixed, Variable and Total Cost Curves

Average Total Cost Function:

$$\begin{aligned} \mathrm{AC}(y) &= \frac{F}{y} + \frac{c_v(y)}{y} \\ &= \mathrm{AFC(y)} + \mathrm{AVC}(y) \end{aligned}$$

- Average Variable Costs tends to rise quickly due to dimishing returns.



Marginal Cost Function

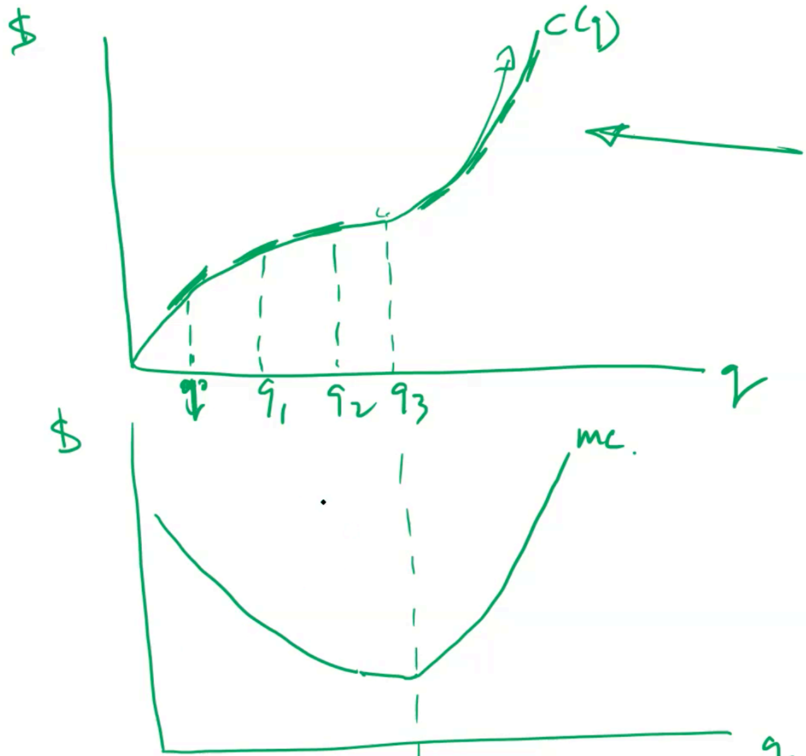
- **Marginal cost** is the rate-of-change of variable production cost as the output level changes. That is:

$$\mathrm{MC}(y) = \frac{\partial c_v(y)}{\partial y}$$

also:

$$MC(y) = \frac{\partial\, c_v(y)}{\partial\, y} \quad \Rightarrow \quad c_v(y) = \int_0^y MC(z)dz.$$

- Marginal Cost is also the **slope** of the total cost function.



Marginal and Average Cost Function

- Since:

$$AVC(y) = \frac{c_v(y)}{y},$$

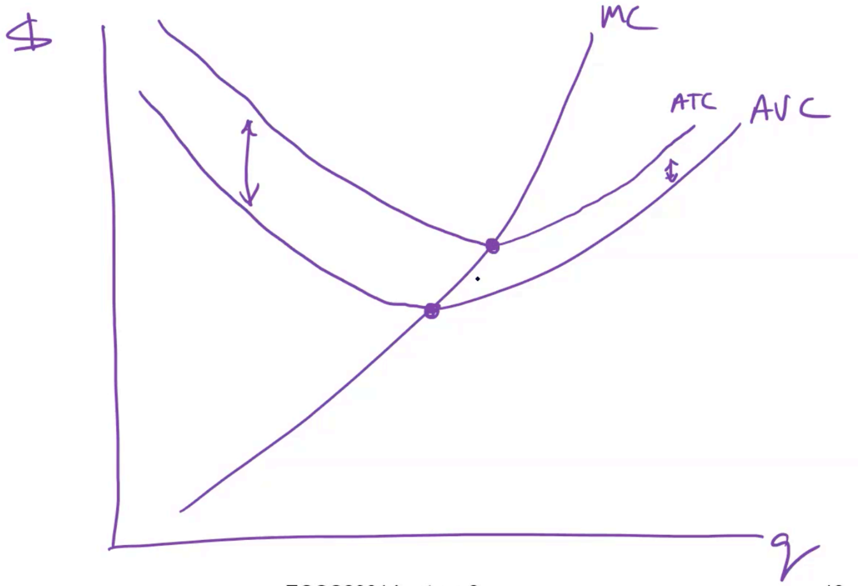
$$\frac{\partial\ AVC(y)}{\partial\ y} = \frac{y \times MC(y) - 1 \times c_v(y)}{y^2}$$

- Hence:

$$\frac{\partial\ AVC(y)}{\partial\ y} \gtreqless 0 \quad \text{if} \quad y \times MC(y) \gtreqless c_v(y)$$

$$\frac{\partial\ AVC(y)}{\partial\ y} \gtreqless 0 \quad \text{as} \quad MC(y) \gtreqless AVC(y)$$

- The short-run MC curve intersects the short-run AVC curve from below at the AVC curves' minimum.



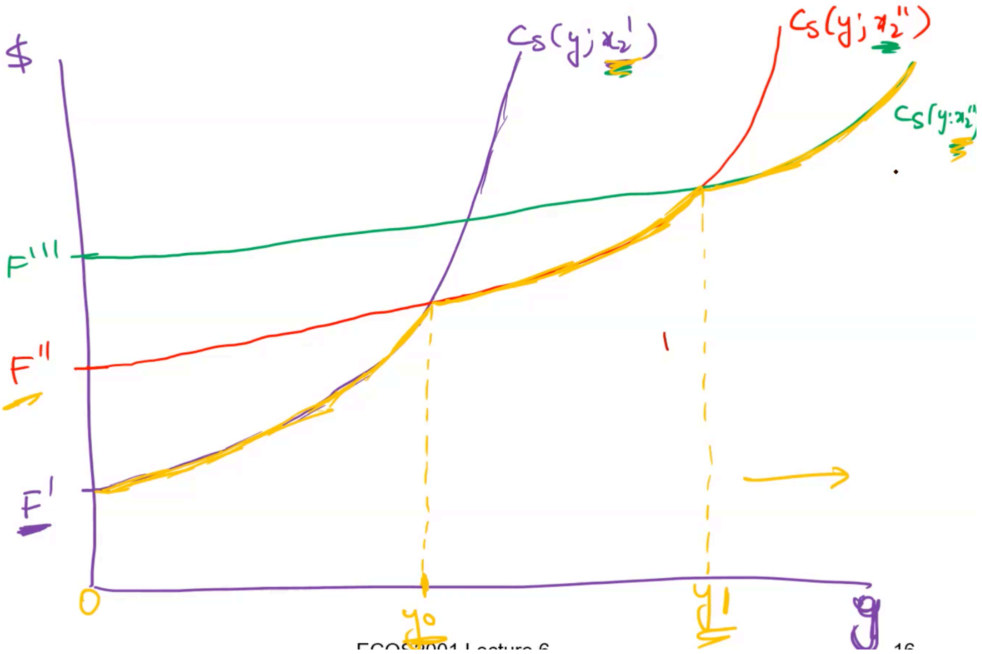
- There is a similar relationship between ATC and MC.
  - The short-run MC curve intersects the short-run AVC from below at the AVC curve's minimum. This occurs with the short-run ATC curve as well.

Short-Run and Long-Run Total Cost Curves

- A firm has a different short-run total cost curve for each possible short-run circumstance.
  - The extra amount of input 1 needed for 1 extra output unit is: $1/MP_1$.
  - Each unit of input 1 costs w_1, so the firm's extra cost from producing one extra unit of output is:

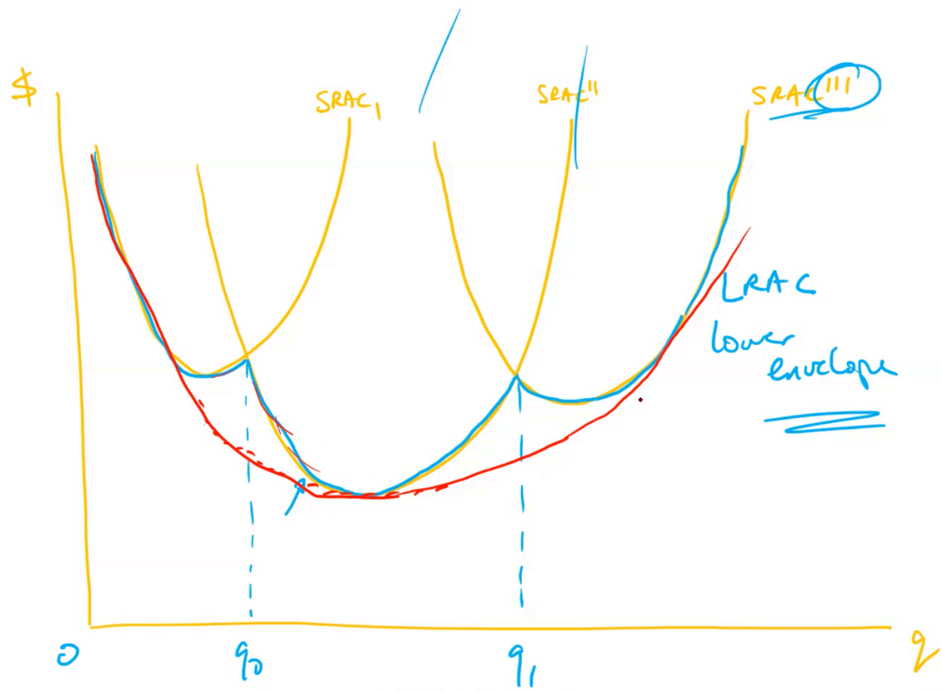
$$MC = \frac{w_1}{MP_1}$$

- The firm's long-run total cost curve consists of the lowest parts of the short-run total cost curves.
  - The long-run total cost curve is the lower envelope of the short-run total cost curves.



Short-Run and Long-Run Average Cost Curves

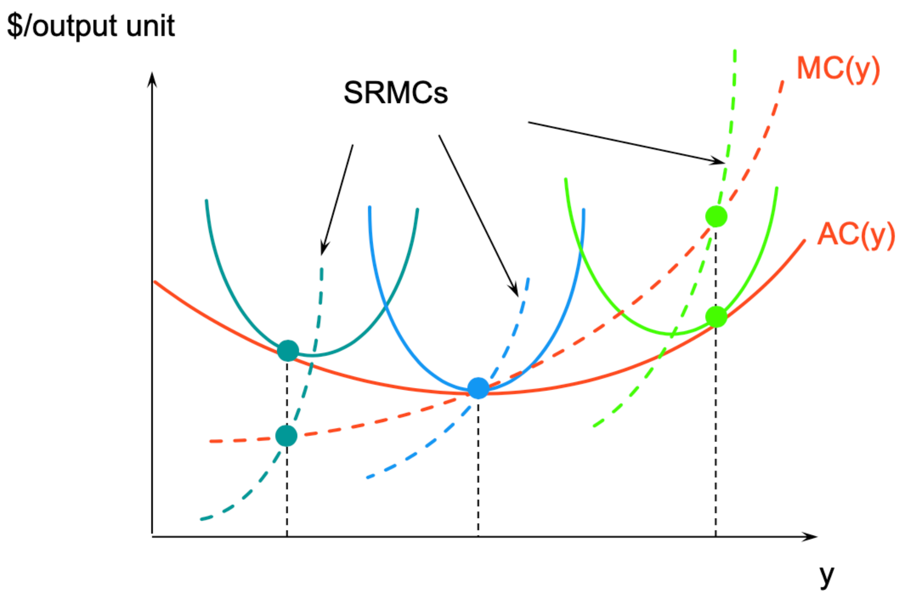
- For any output level y, the long-run total cost curve always gives the lowest possible total production cost.
- Therefore, the long-run av. total cost curve must always give the lowest possible av. total production cost.
- The long-run av. total cost curve must be the **lower envelope** of all of the firm's short-run av. total cost curves



- Ends up smoothing out as the red line shows.

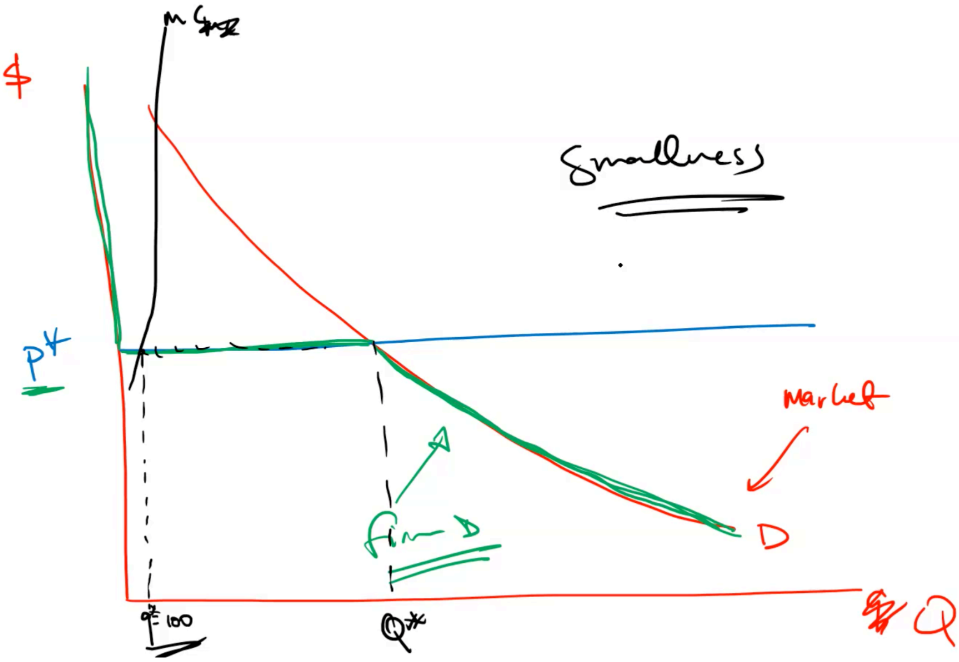
Short-Run and Long-Run Marginal Cost Curves

- For any output level $y > 0$, the long-run marginal cost of production is the marginal cost of production for the short-run chosen by the firm.



Pure Competition

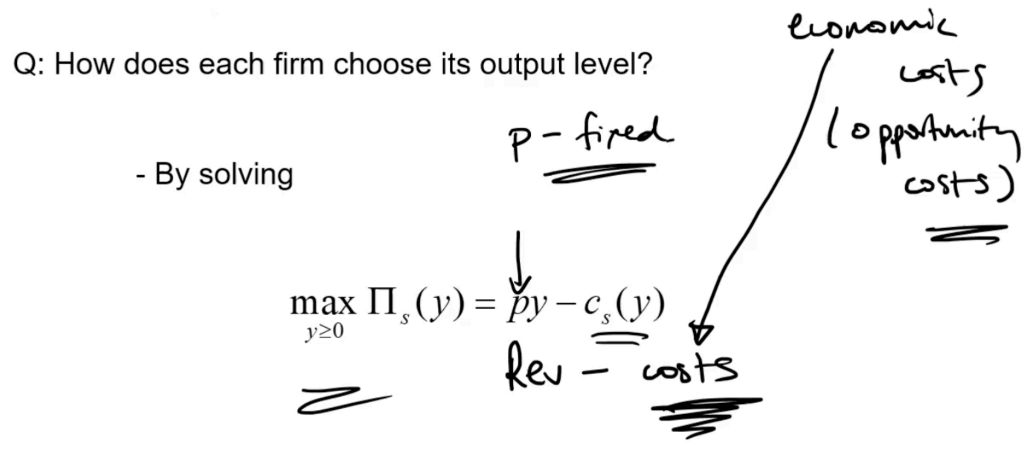
- A firm in a perfectly competitive market knows it has no influence over the market price for its product. The firm is a **market price-taker**. The firm is free to vary its own price.
  - If the firm sets its own price above the market price then the quantity demanded from the firm is zero.
  - If the firm sets its own price below the market price then the quantity demanded from the firm is the entire market quantity-demanded.



- **Smallness**: when someone says that an individual firm is "small relative to the industry", the individual firm's technology causes it always to supply only a small part of the total quantity demanded at the market price.
  - Although they are able to produce anywhere along that horizontal line, they choose to produce less to maximise profits.
  - This also means that for a firm, the only relevant part is the horizontal area $\longrightarrow$ market demand is a **HORIZONTAL LINE**.

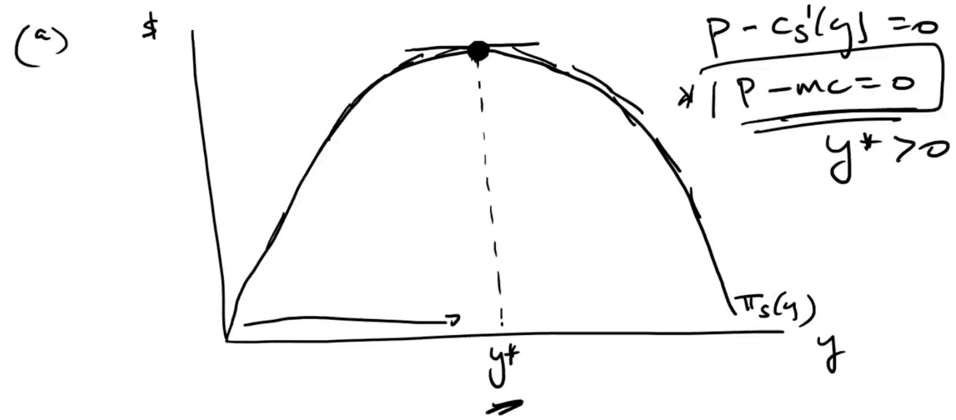
Firm's Short-Run Supply Decision

- We assume that each firm is a profit-maximiser and in the short-run.



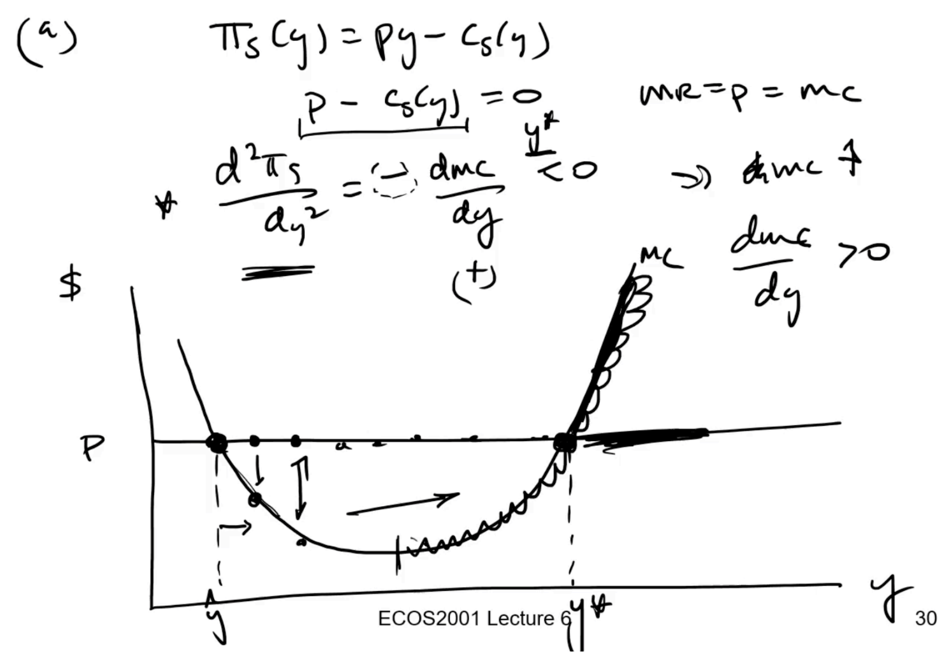
Produces More Than 0: $y_s^* > 0$

- We find that:
  - $(i)\ \dfrac{d\Pi_s(y)}{dy} = p - MC_s(y) = 0$ We produce when $p = MC$.
  - $(ii)\ \dfrac{d^2\Pi_s(y)}{dy^2} < 0\ at\ y = y_s^*.$ We produce on the upwards sloping part of the firm's MC curve.



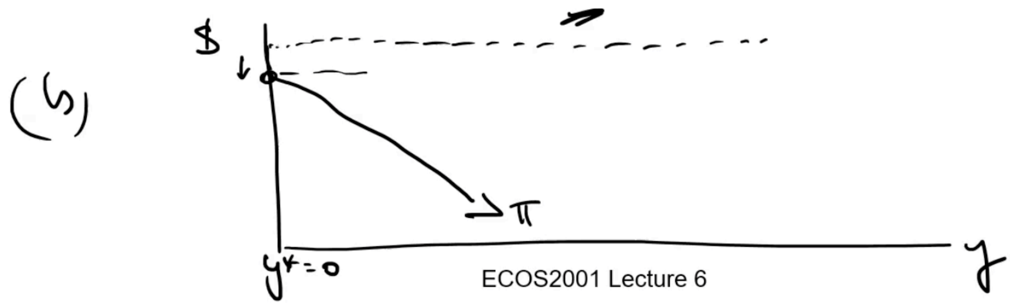
- In order for the second order condition to be true, the firm's MC curve must be upward-sloping/a profit-maximising supply level can lie only on the **upwards sloping part of the firm's MC curve**.

$$\frac{dMC(y_s^*)}{dy} > 0$$



Does Not Produce: $y_s^* = 0$

- We find that:
  - The first order condition is **LESS THAN 0**.
  - The second order condition is the same.



Short-Run Supply Decision Rule

- **PRODUCE WHEN $p \geq AVC_s(y)$** min
  - **PRODUCE AT** $p = MC$.

But not every point on the upward-sloping part of the firm's MC curve represents a profit-maximum.

- the firm will choose an output ~~level~~ y > 0 only if

$$\Pi_s(y) = py - \cancel{F} - c_v(y) \geq \cancel{-F}$$

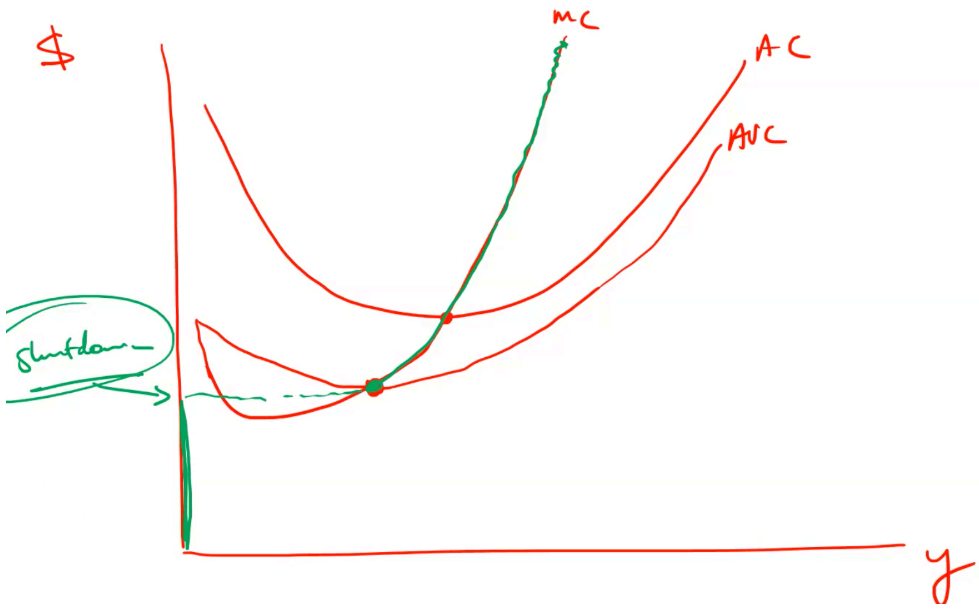
$$py \geq c_v(y)$$

This is equivalent to

$$\boxed{p \geq AVC_s(y)}$$

$$p \geq \frac{c_v(y)}{y} = AVC_s(y)$$

- **Shut-down**: means produce no output (but the firm is still in the industry and suffers its fixed cost). It is a **short-run** concept.
- **Exiting**: means leaving the industry, which can only be done in **long-run**.



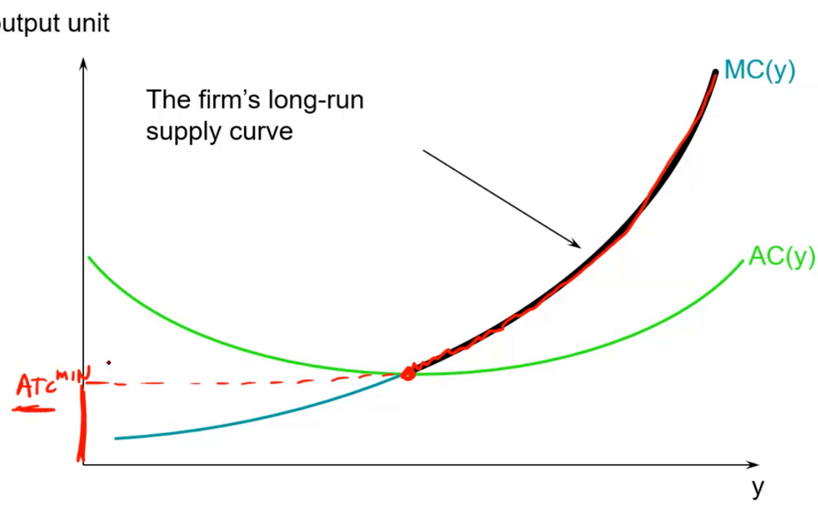
Firm's Long-Run Supply Decision

- The long-run is the circumstance in which the firm can choose amongst all of its short-run circumstances.
- The firm's long-run supply decision is:

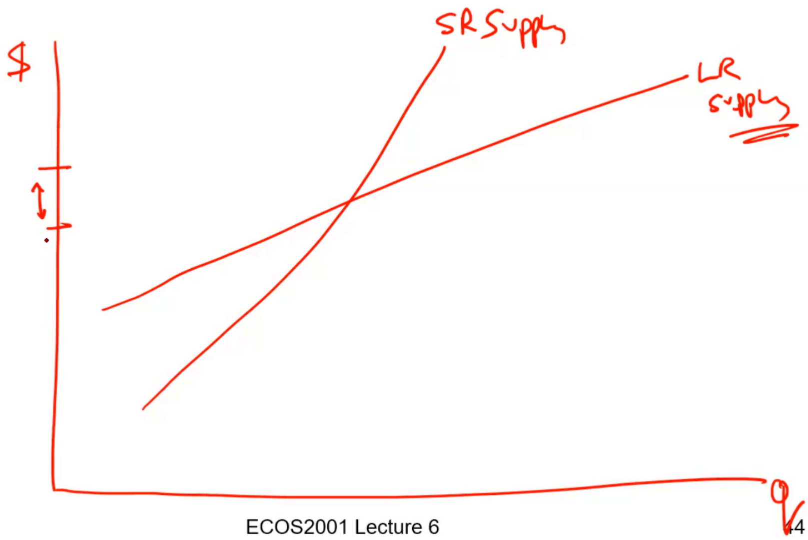
$$p = MC(y) \text{ and}$$
$$\frac{dMC(y)}{dy} > 0$$

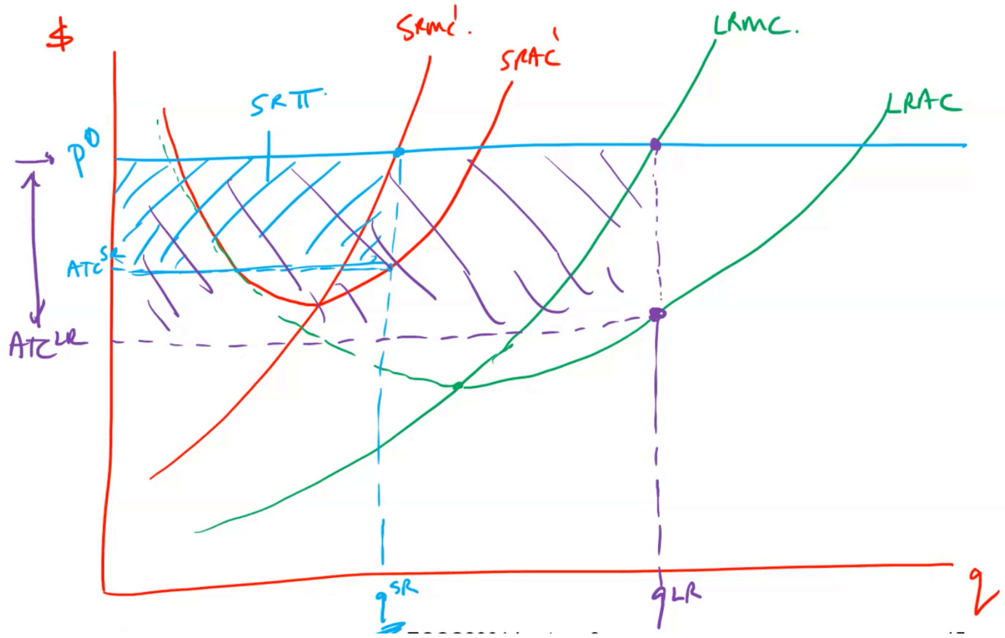
- The firm's economic profit level must not be negative since then the firm would exit the industry.

- **A firm will leave if $\Pi(y) = py - c(y) \geq 0$ or $p < ATC$.**
  - The supply curve then becomes the MC curve.



- How is the firm's long-run supply curve related to all of its short-run supply curves?
  - A long-run supply curve is more **elastic** than a short run average cost curve $\rightarrow$ they are more capable of taking advantage/changing based on a change in price.



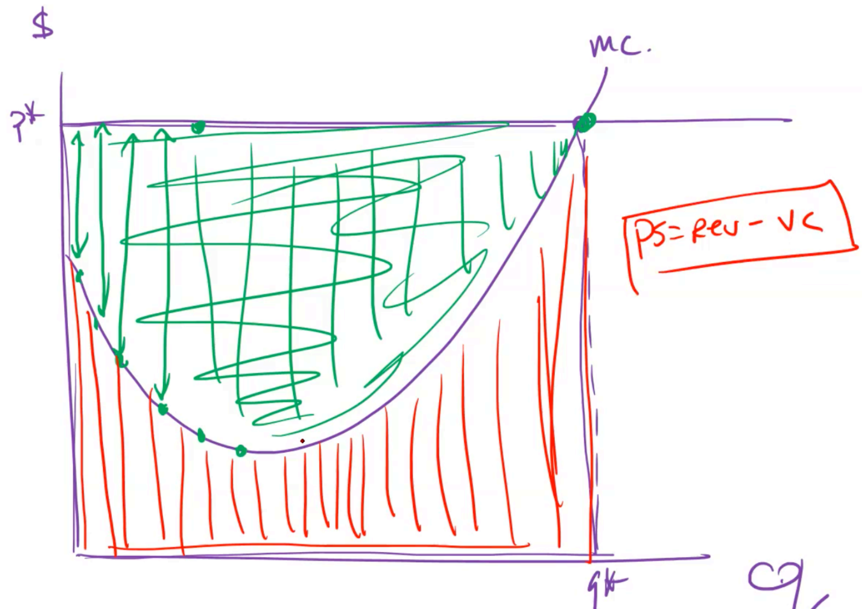


Producer's Surplus

- The firm's producer's surplus is the accumulation, unit by extra unit of output, of extra revenue less extra production cost.

$$
\begin{aligned}
PS(p) &= \int_0^{y^*(p)} \left[p - MC_s(z)\right] d(z) \\
&= py^*(p) - \int_0^{y^*(p)} MC_s(z)d(z) \\
&= py^*(p) - c_v\left(y^*(p)\right).
\end{aligned}
$$

- This essentially says the PS = Revenue - Variable Cost.
  - PS = Profit + Fixed Cost as well, and PS = Profit when FC = 0 in the long run.



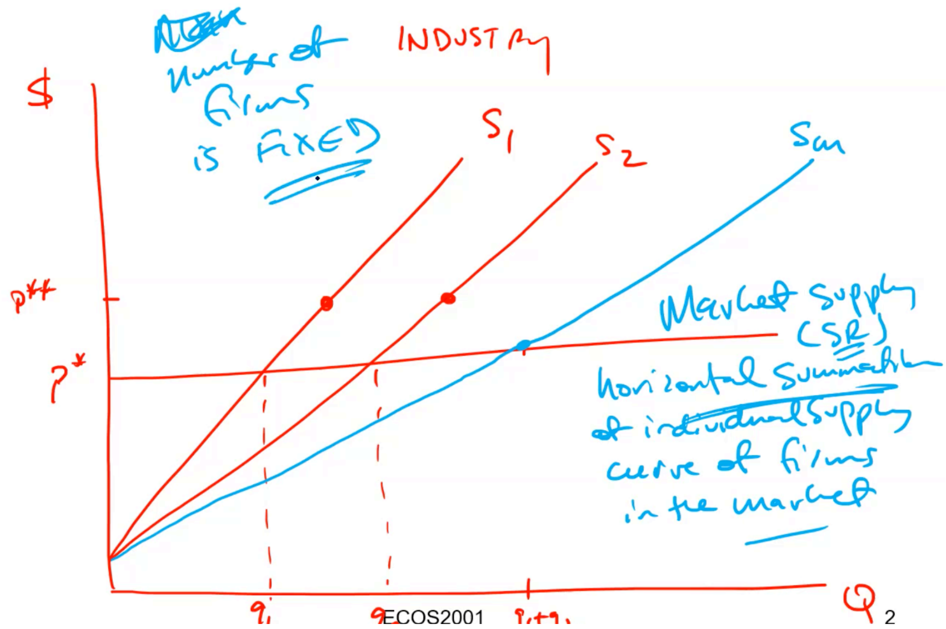
Lecture 7 - Competitive Markets and Monopoly

Industry Supply in Competitive Markets

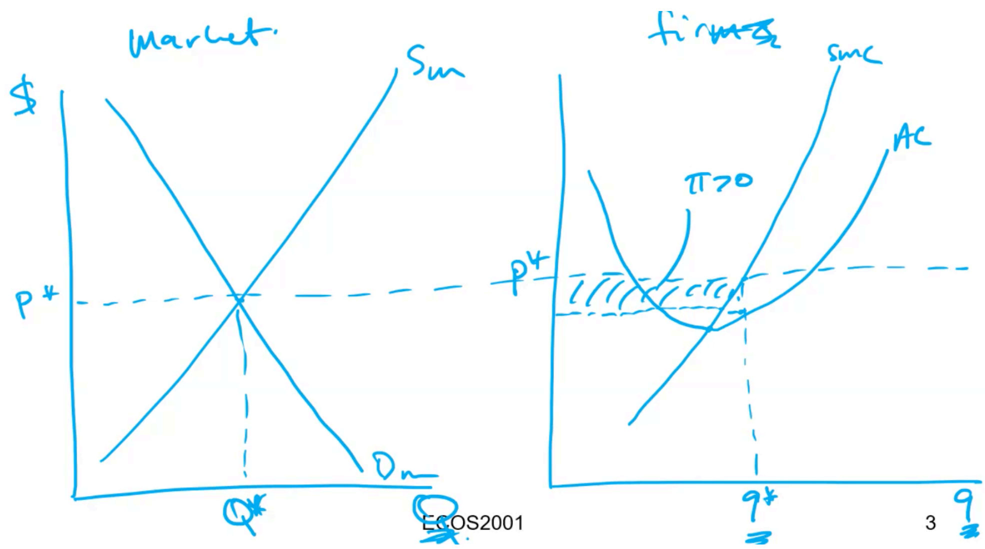
Short-Run Supply

- In a short-run the number of firms in the industry is, temporarily, fixed. Let n be the number of firms; $i = 1, \ldots, n$.
- $S_i(p)$ is firm i's supply function.
- The industry's **short-run supply function** is:

$$S(p) = \sum_{i=1}^{n} S_i(p).$$



- In a short-run, neither entry nor exit can occur.
- Consequently, in a short-run equilibrium, some firms may earn positive economics profits, others may suffer economic losses, and still others may earn zero economic profit.

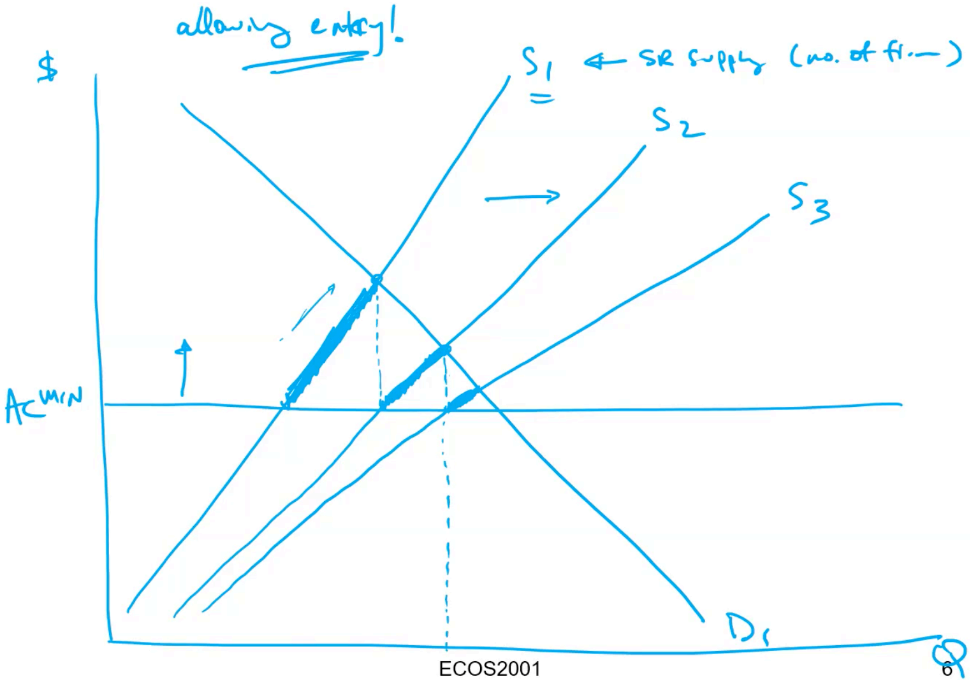


Long-Run Industry Equilibrium

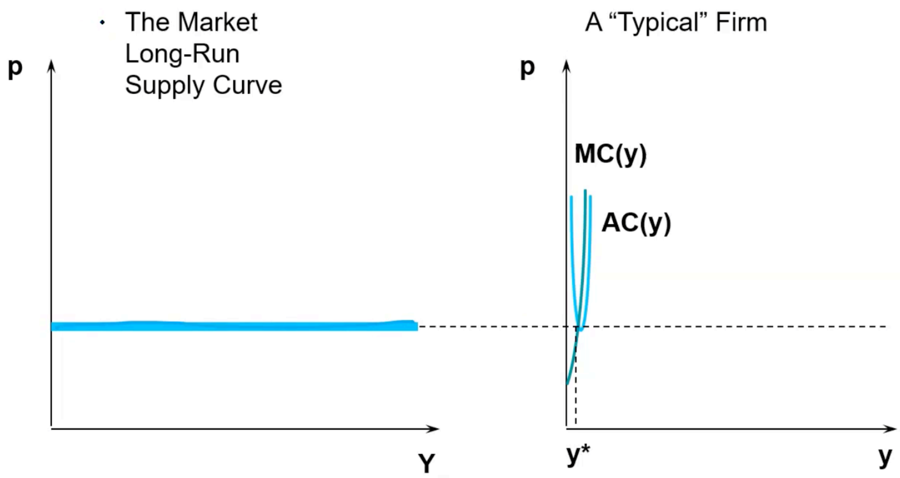
- No fixed factors of production for firms In the long run entry and exit is possible
  - Industry's long-run supply function must account for entry and exit as well as for the supply choices of firms that choose to be in the industry
  - Positive economic profit induces entry.
- Economic profit is positive when the market price p_s^e is higher than a firm's minimum av. total cost:

$$p_s^e > \min AC(y)$$

- Entry increases industry supply, causing p_s^e to fall.



- The **long-run number of firms in the industry** is the largest number for which the market price is at least as large as min AC(y). Now we can construct the industry's long-run supply curve.

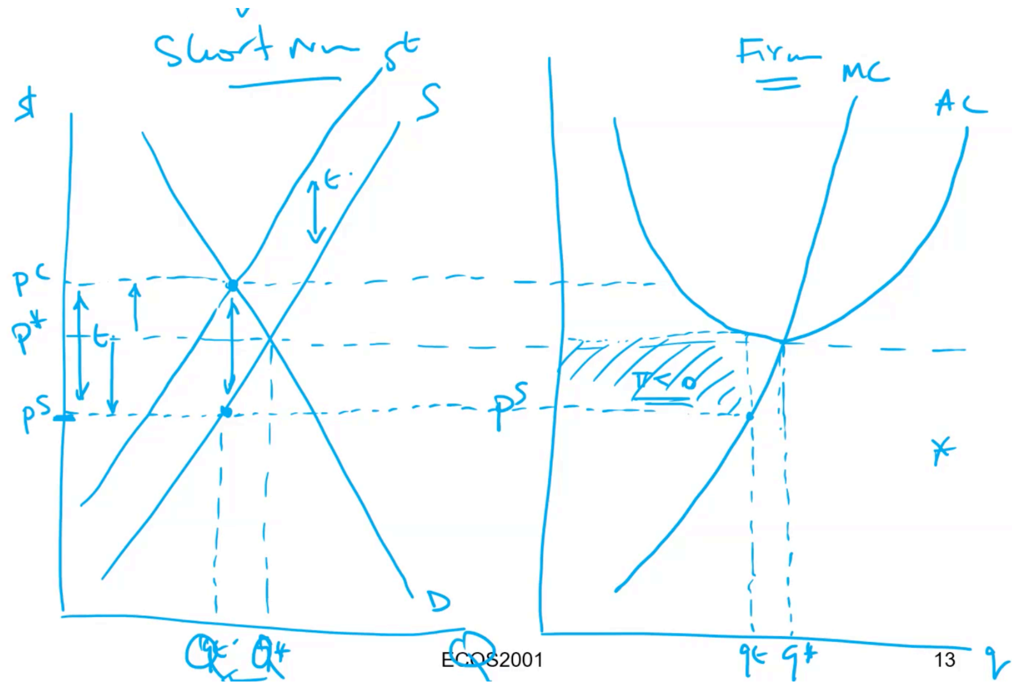


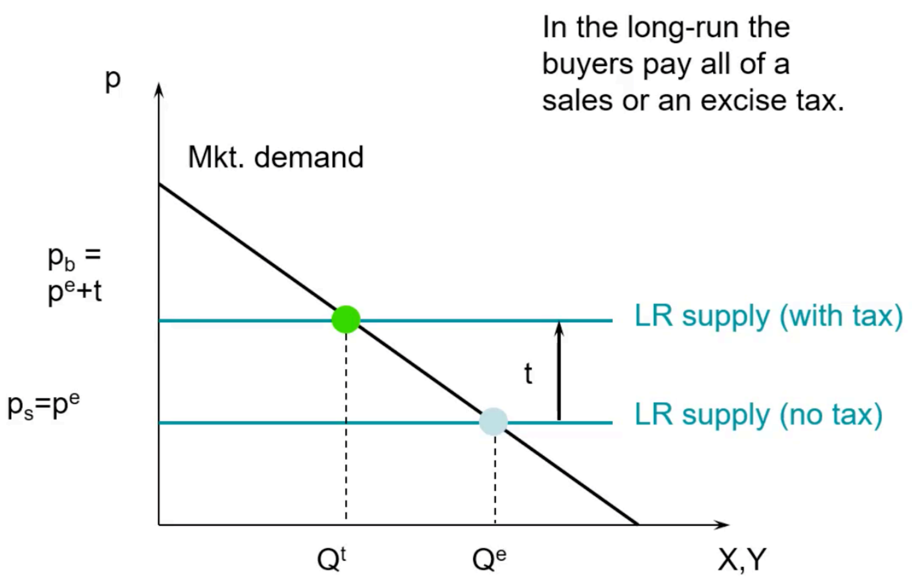
- In the limit, as firms become infinitesimally small, the industry's long-run supply curve is **horizontal** at min AC(y).
- In the long-run market equilibrium, the market price is determined solely by the long-run minimum average production cost. Long-run market price is:

$$p^e = \min_{y>0} AC(y)$$

Long-Run Implications for Taxation

- In a short-run equilibrium, the burden of a sales or an excise tax is typically shared by both buyers and sellers, tax incidence of the tax depending upon the own-price elasticities of demand and supply.

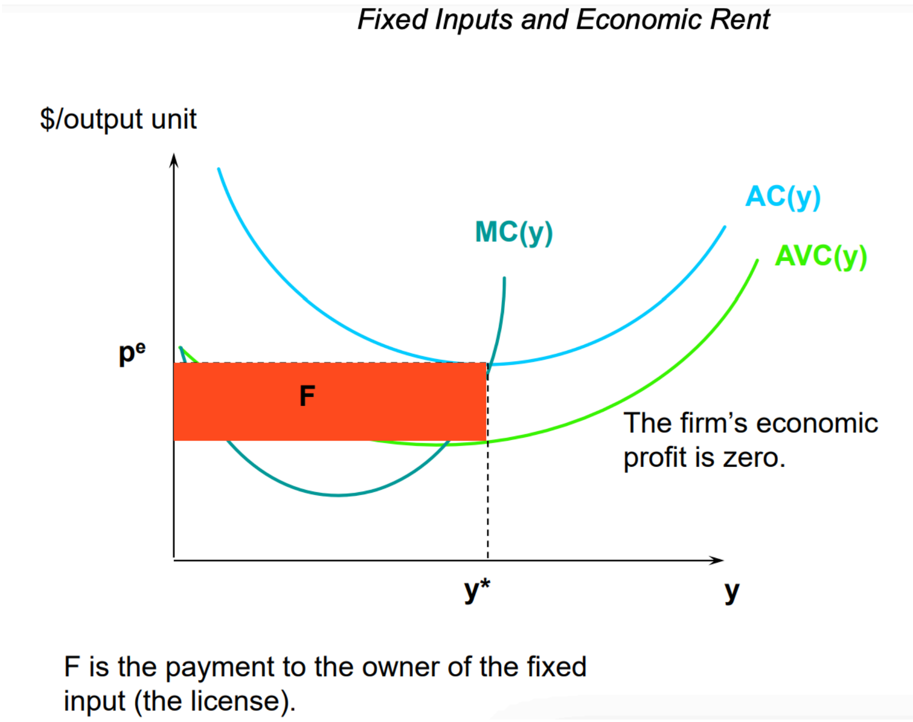




Fixed Inputs and Economic Rent

- What if there is a barriers to entry or exit?
    - The taxi-cab industry has a barrier to entry even though there are lots of cabs competing with each other.
    - Liquor licensing is a barrier to entry into a competitive industry.
- An input (e.g. an operating license) that is fixed in the long-run causes a long-run fixed cost, F.

- Long-run total cost, $c(y) = F + c_v(y)$. And long-run average total cost, $AC(y) = AFC(y) + AVC(y)$. In the long-run equilibrium, what will be the value of F?
- Think of a firm that needs an operating license - the license is a fixed input that is rented but not owned by the firm.
    - If the firm makes a positive economic profit then another firm can offer the license owner a higher price for it. In this way, **all firms' economic profits are competed away, to zero**.
- So in the long-run equilibrium, each firm makes a zero economic profit and each firm's fixed cost is its payment for its operating license



- **Economic rent** is the payment for an input that is in excess of the minimum payment required to have that input supplied.
    - Each license essentially costs zero to supply, so the long-run economic rent paid to the license owner is the firm's long-run fixed cost.
    - F is the payment to the owner of the fixed input (the license); F = economic rent

Pure Monopoly

- Monopoly: single seller
    - Demand curve is the (downward sloping) market demand curve
    - Monopolist can alter the market price by adjusting its output level - price maker.

monopolist's problem

$$\pi(y) = \overbrace{p(y)y}^{\text{Rev}} - \overbrace{c(y)}^{\text{cost.}}$$

$$\frac{d\pi(y)}{dy} = \frac{d}{dy}\left(p(y)(y)\right) - \frac{dc(y)}{dy} = 0$$

~~competitive P = MR~~

$$* \quad \boxed{MR = MC} \quad *$$

- Therefore:

Marginal revenue is the rate-of-change of revenue as the output level y increases;

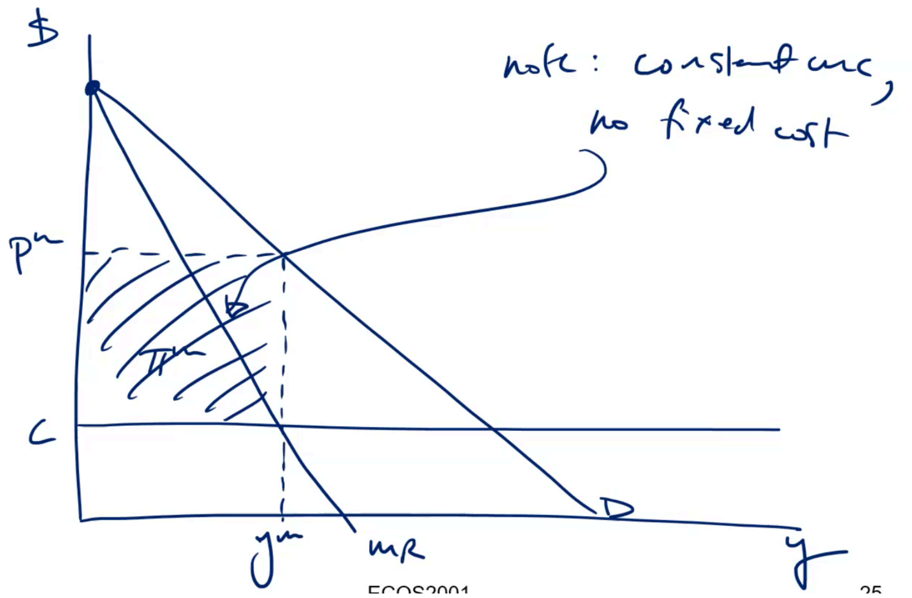
$$MR(y) = \frac{d}{dy}\big(p(y)y\big) = p(y) + y\frac{dp(y)}{dy} \quad *$$

dp(y)/dy is the slope of the market inverse demand function so dp(y)/dy < 0. Therefore for y* > 0

$$MR(y) = p(y) + y\frac{dp(y)}{dy} \quad < \quad p(y)$$

$$MR(y) = \frac{d\,[p(y)y]}{dy} = p(y) + y \cdot p'(y) < p(y) \qquad \frac{dp}{dy} < 0$$

except for very first unit sold.



- Quick calculations, not needed:

At the profit-maximizing output level y*, MR(y*) = MC(y*). So if p(y) = a - by and c(y) = F + ay + by^2 then

$$MR(y^*) = a - 2by^* = \alpha + 2\beta y^* = MC(y^*)$$

and the profit-maximizing output level is

$$y^* = \frac{a-\alpha}{2(b+\beta)}$$

market price

$$p(y^*) = a - by^* = a - b\frac{a-\alpha}{2(b+\beta)}$$

- MARKUP OVER MC AND ELASTICITY OF DEMAND:

Mark-up and elasticity of demand

$$MR(y) = \frac{d}{dy}(p(y)y) = p(y) + y\frac{dp(y)}{dy}$$
$$= p(y)\left[1 + \frac{y}{p(y)}\frac{dp(y)}{dy}\right].$$

Own-price elasticity of demand is

$$\varepsilon = \frac{p(y)}{y}\frac{dy}{dp(y)}$$

So that $$MR(y) = p(y)\left[1 + \frac{1}{\varepsilon}\right]$$

Also sometimes expressed as:

$$\frac{p(y) - MC}{p(y)} = -\frac{1}{\varepsilon}$$

- The more inelastic the customers are, the higher the markup can be.

Quantity Tax Levied on a Monopolist

$P = a - by \qquad \underline{\underline{MC = c}} \; ; \; MC^t = c + t$

$\pi(y^t) = (a - by)y - y(c+t)$

$a - 2by - (c+t) = 0$

$$\boxed{y^t = \frac{a-(c+t)}{2b}}$$

$$\boxed{p^t = \frac{a+c+t}{2}}$$

$\frac{dp^m}{dt} = \frac{1}{2} > 0$

"testing" market power.

\$1 tax ↑ consumer price \$0.50

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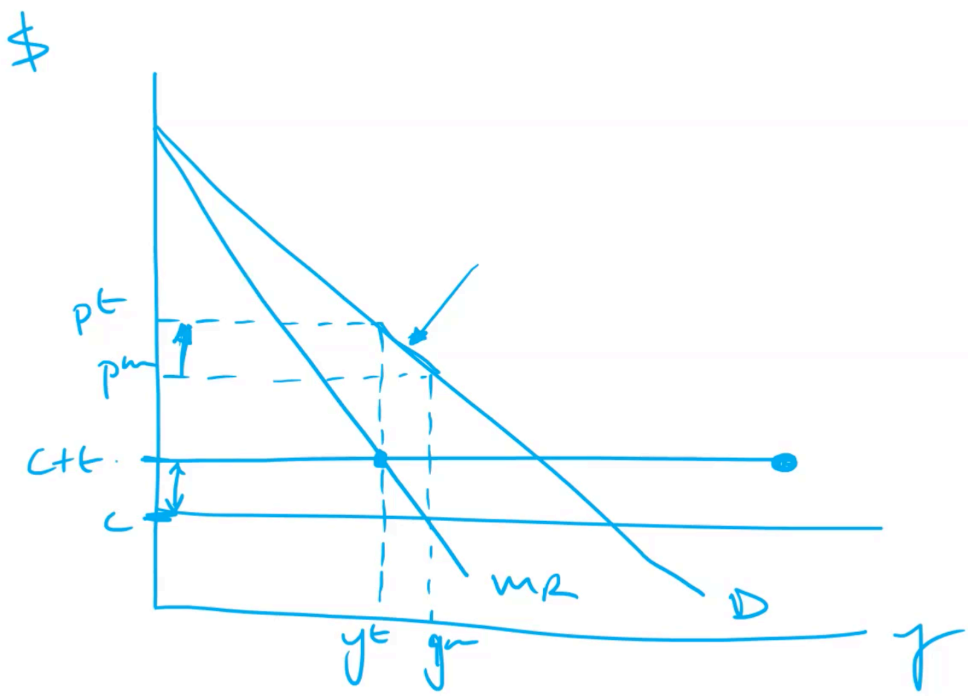
32

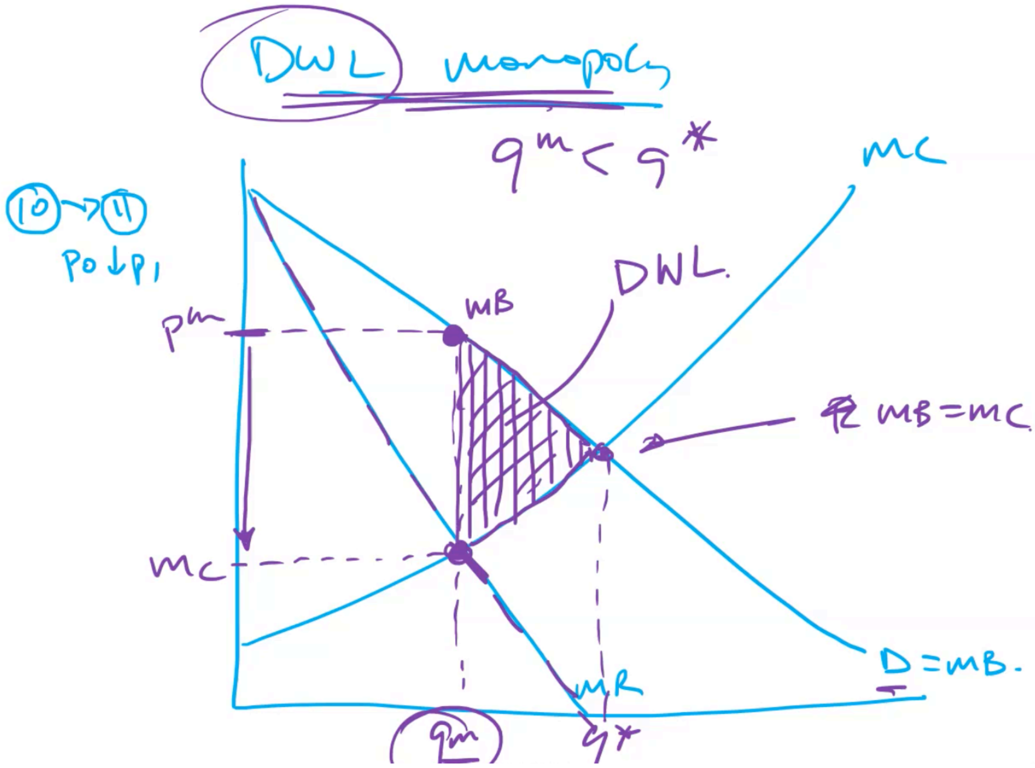
- The monopolist ends up wearing some of the cost of the tax (since demand decreases with their taxation).



Inefficiency of Monopoly

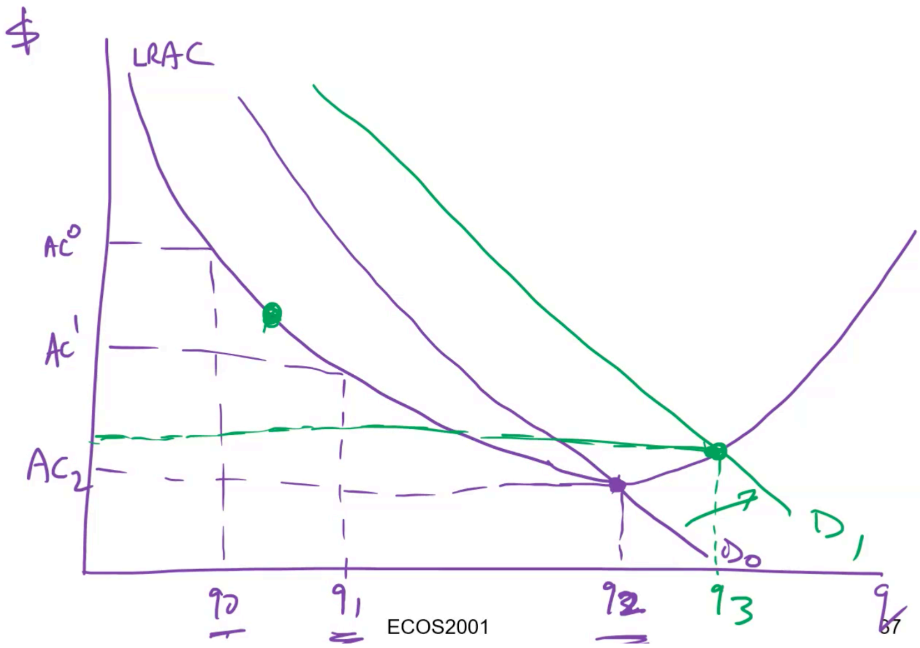
- A market is **Pareto efficient** if it achieves the maximum possible total gains-to-trade.
- Otherwise a market is Pareto inefficient.
- As a monopolist restricts output to below the level at which $MC = MB$, the monopolist prevents all the gains from trade being realise.

- That is $MC(y^*+1) < p(y^*+1)$ so both seller and buyer could gain if the (y^*+1)th unit of output was produced. Hence the market is **Pareto inefficient.**



Natural Monopoly

- It cost less to supply the total output required using one firm, rather than two or more firms
  - Increasing returns to scale imply natural monopoly.
  - A natural monopoly does not necessarily imply increasing returns



- May be better to have a single producer at q_2.