**2.12*** Suppose that a flat area with center at $(x_0, y_0)$ is illuminated by a light source with intensity distribution

$$i(x,y) = Ke^{-[(x-x_0)^2 + (y-y_0)^2]}$$

Assume for simplicity that the reflectance of the area is constant and equal to 1.0, and let $K = 255$. If the intensity of the resulting image is quantized using $k$ bits, and the eye can detect an abrupt change of eight intensity levels between adjacent pixels, what is the highest value of $k$ that will cause visible false contouring?

**2.16** Develop an algorithm for converting a one-pixel-thick $m$-path to a 4-path.

**2.18** Consider the image segment shown in the figure that follows.

**(a)*** As in Section 2.5, let $V = \{0,1\}$ be the set of intensity values used to define adjacency. Compute the lengths of the shortest 4-, 8-, and $m$-path between $p$ and $q$ in the following image. If a particular path does not exist between these two points, explain why.

| | | | |
|---|---|---|---|
| 3 | 1 | 2 | 1 $(q)$ |
| 2 | 2 | 0 | 2 |
| 1 | 2 | 1 | 1 |
| $(p)$ 1 | 0 | 1 | 2 |

**(b)** Repeat (a) but using $V = \{1,2\}$.

**2.37** We know from Eq. (2-45) that an affine transformation of coordinates is given by

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \mathbf{A} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

where $(x', y')$ are the transformed coordinates, $(x, y)$ are the original coordinates, and the elements of $\mathbf{A}$ are given in Table 2.3 for various types of transformations. The inverse transformation, $\mathbf{A}^{-1}$, to go from the transformed back to the original coordinates is just as important for performing inverse mappings.

**(a)*** Find the inverse scaling transformation.

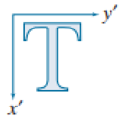
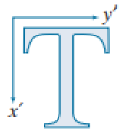
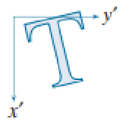
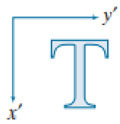
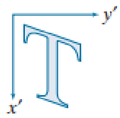
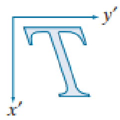
**(b)** Find the inverse translation transformation.

**(c)** Find the inverse vertical and horizontal shearing transformations.

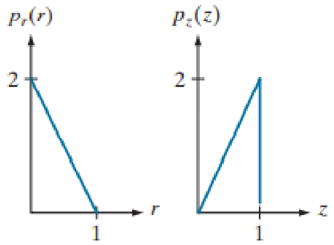
**(d)*** Find the inverse rotation transformation.

**(e)*** Show a composite inverse translation/rotation transformation.

**TABLE 2.3**
Affine transformations based on Eq. (2-45).

| **Transformation Name** | **Affine Matrix, A** | **Coordinate Equations** | **Example** |
|---|---|---|---|
| Identity | $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ | $x' = x$<br>$y' = y$ |  |
| Scaling/Reflection (For reflection, set one scaling factor to −1 and the other to 0) | $\begin{bmatrix} c_x & 0 & 0 \\ 0 & c_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$ | $x' = c_x x$<br>$y' = c_y y$ |  |
| Rotation (about the origin) | $\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$ | $x' = x\cos\theta - y\sin\theta$<br>$y' = x\sin\theta + y\cos\theta$ |  |
| Translation | $\begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}$ | $x' = x + t_x$<br>$y' = y + t_y$ |  |
| Shear (vertical) | $\begin{bmatrix} 1 & s_v & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ | $x' = x + s_v y$<br>$y' = y$ |  |
| Shear (horizontal) | $\begin{bmatrix} 1 & 0 & 0 \\ s_h & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ | $x' = x$<br>$y' = s_h x + y$ |  |

**3.12** An image with intensities in the range [0,1] has the PDF, $p_r(r)$, shown in the following figure. It is desired to transform the intensity levels of this image so that they will have the specified $p_z(z)$ shown in the figure. Assume continuous quantities, and find the transformation (expressed in terms of $r$ and $z$) that will accomplish this.



**3.18** You are given the following kernel and image:

$$w = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} \qquad f = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

**(a)*** Give a sketch of the area encircled by the large ellipse in Fig. 3.28 when the kernel is centered at point (2,3) (2nd row, 3rd col) of the image shown above. Show specific values of $w$ and $f$.

**(b)*** Compute the convolution $w \star f$ using the *minimum* zero padding needed. Show the details of your computations when the kernel is centered on point (2,3) of $f$; and then show the final full convolution result.

**(c)** Repeat (b), but for correlation, $w \,☆\, f$.

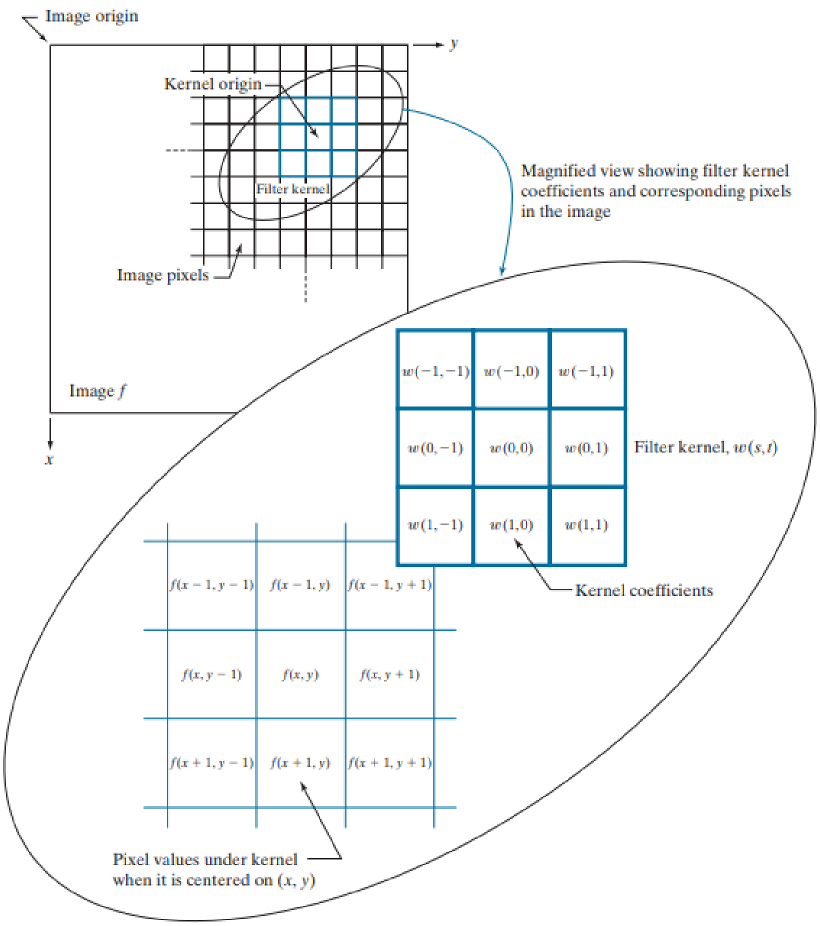



**FIGURE 3.28** The mechanics of linear spatial filtering using a $3 \times 3$ kernel. The pixels are shown as squares to simplify the graphics. Note that the origin of the image is at the top left, but the origin of the kernel is at its center. Placing the origin at the center of spatially symmetric kernels simplifies writing expressions for linear filtering.