MORPHOLOGICAL IMAGE PROCESSING

- 9.1 Preliminaries
- 9.2 Erosion and Dilation
- 9.3 Opening and Closing
- 9.4 The Hit-or-Miss Transformation
- 9.5 Some Basic Morphological Algorithms
- 9.6 Morphological Reconstruction
- 9.7 Summary of Morphological Operations on Binary Images
- 9.8 Grayscale Morphology

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9.1 Preliminaries

9.1.1 Objects and Structuring Elements

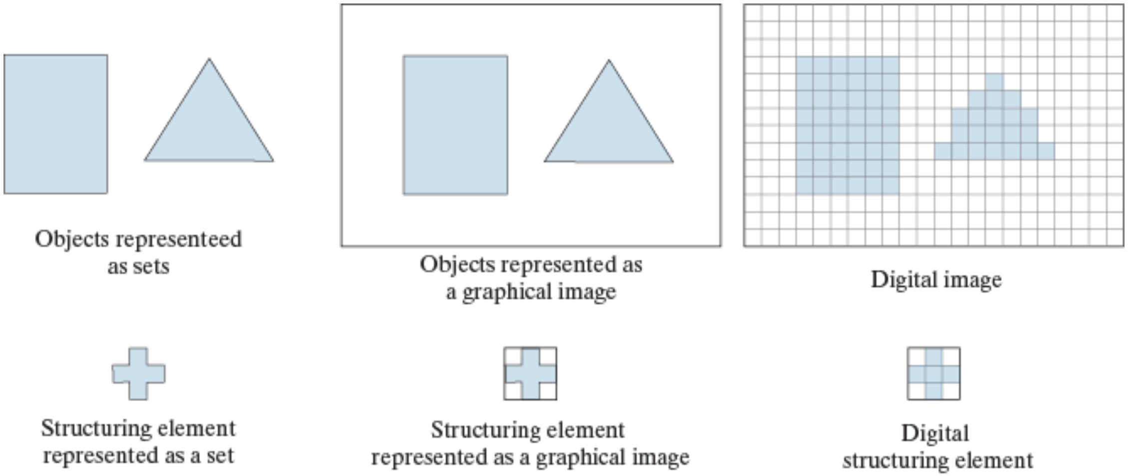


FIGURE 9.1 Top row. *Left:* Objects represented as graphical sets. *Center:* Objects embedded in a background to form a graphical image. *Right:* Object and background are digitized to form a digital image (note the grid). Second row: Example of a structuring element represented as a set, a graphical image, and finally as a digital SE.

9.1 Preliminaries

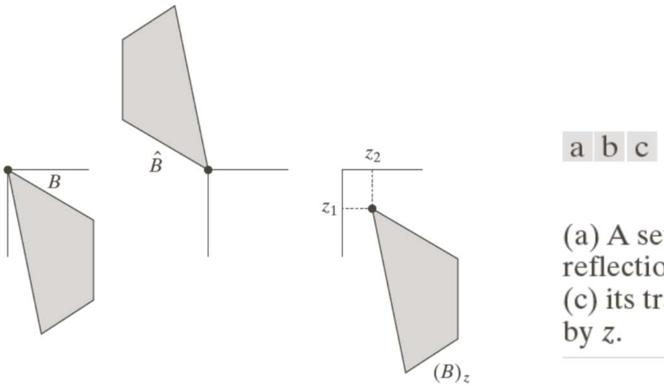
9.1.1 Objects and Structuring Elements

Reflection

$$\hat{B} = \{w \mid w = -b, \ \ for\ b \in B\} \qquad (9\text{-}1)$$

Translation

$$(B)_z = \{c \mid c = b + z,\ for\ b \in B\} \qquad (9\text{-}2)$$



(a) A set, (b) its reflection, and (c) its translation by z.

9.1 Preliminaries

9.1.1 Objects and Structuring Elements

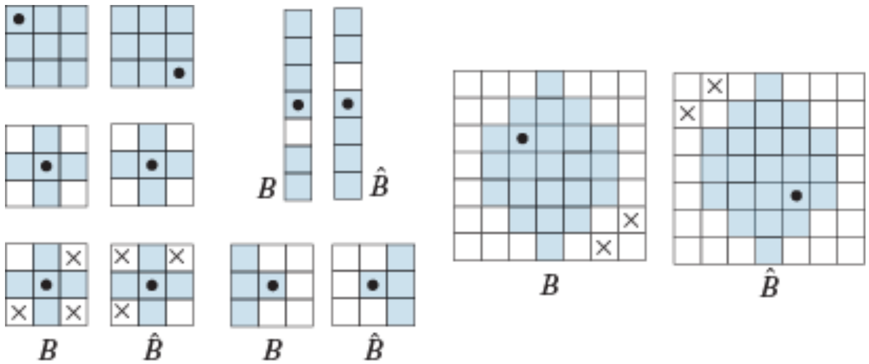
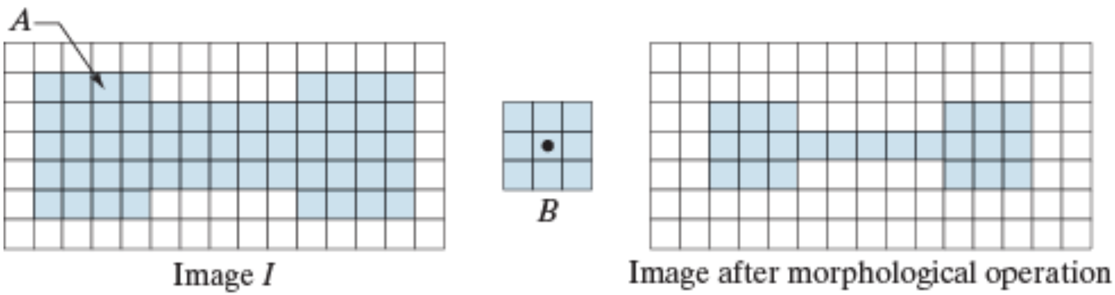


FIGURE 9.2 Structuring elements and their reflections about the origin (the ×'s are don't care elements, and the dots denote the origin). Reflection is rotation by 180° of an SE about its origin.

9.1 Preliminaries

9.1.1 Objects and Structuring Elements



a b c

FIGURE 9.3
(a) A binary image containing one object (set), A. (b) A structuring element, B. (c) Image resulting from a morphological operation (see text).

9.2 Erosion and Dilation

9.2.1 Erosion

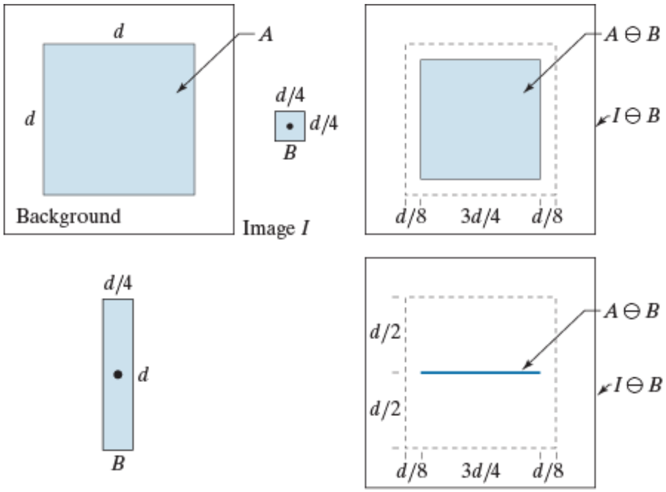
$$A \Theta B = \{z | (B)_z \subseteq A\} \qquad (9.3)$$

$$I \Theta B = \{z | (B)_z \subseteq A \text{ and } A \subseteq I\} \cup \{A^c | A^c \subseteq I\} \qquad (9.4)$$

$$A \Theta B = \{z | (B)_z \cap A^c = \varnothing\} \qquad (9.5)$$

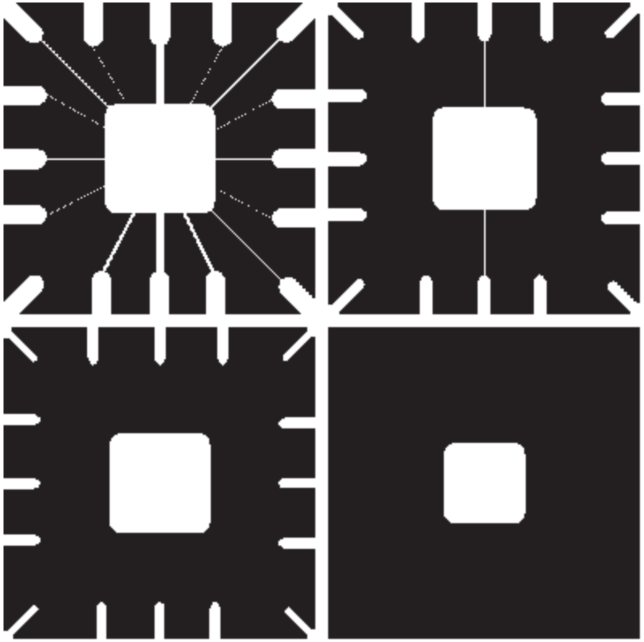
a b c
d e

FIGURE 9.4
(a) Image I, consisting of a set (object) A, and background.
(b) Square SE, B (the dot is the origin).
(c) Erosion of A by B (shown shaded in the resulting image).
(d) Elongated SE.
(e) Erosion of A by B. (The erosion is a line.) The dotted border in (c) and (e) is the boundary of A, shown for reference.



9.2 Erosion and Dilation

9.2.1 Erosion



a b
c d

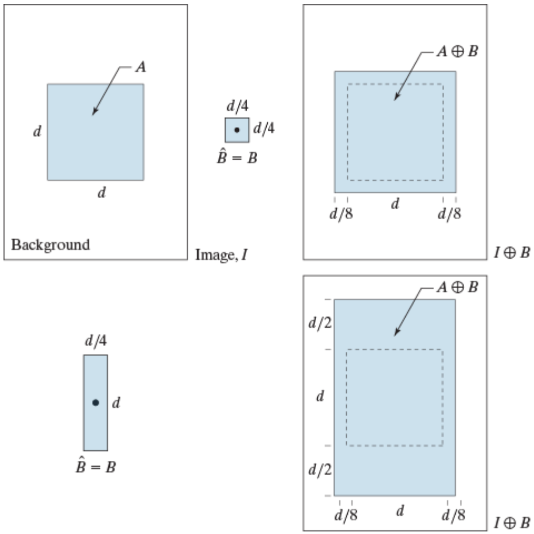
FIGURE 9.5 Using erosion to remove image components. (a) A 486×486 binary image of a wire-bond mask in which foreground pixels are shown in white. (b)–(d) Image eroded using square structuring elements of sizes 11×11, 15×15, and 45×45 elements, respectively, all valued 1.

9.2 Erosion and Dilation

9.2.2 Dilation

$$A \oplus B = \{z | (\hat{B})_z \cap A \neq \varnothing\} \qquad (9\text{-}6)$$

$$A \oplus B = \{z | \; [(\hat{B})_z \cap A] \subset A\} \qquad (9\text{-}7)$$

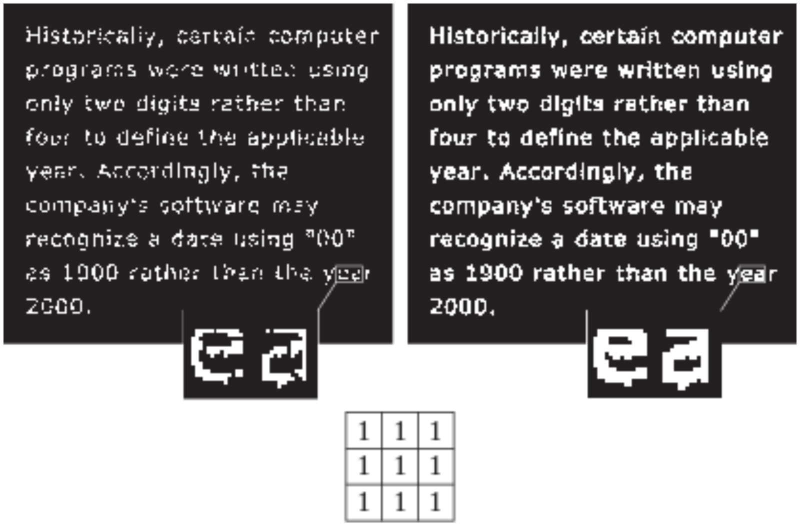


a b c
d e

FIGURE 9.6 (a) Image I, composed of set (object) A and background. (b) Square SE (the dot is the origin). (c) Dilation of A by B (shown shaded). (d) Elongated SE. (e) Dilation of A by this element. The dotted line in (c) and (e) is the boundary of A, shown for reference.

9.2 Erosion and Dilation

9.2.2 Dilation



a c
b

FIGURE 9.7
(a) Low-resolution text showing broken characters (see magnified view).
(b) Structuring element.
(c) Dilation of (a) by (b). Broken segments were joined.

9.2 Erosion and Dilation

9.2.3 Duality

$$(A \ominus B)^c = A^c \oplus \hat{B} \qquad \text{(9-8)}$$

$$(A \oplus B)^c = A^c \ominus \hat{B} \qquad \text{(9-9)}$$

$$(A \ominus B)^c = \{z \mid (B)_z \subseteq A\}^c$$

$$(A \ominus B)^c = \{z \mid (B)_z \cap A^c = \varnothing\}^c$$

$$(A \ominus B)^c = \{z \mid (B)_z \cap A^c \neq \varnothing\}$$
$$= A^c \oplus \hat{B}$$

9.3 Opening and Closing

9.3.1 Opening

$$A \circ B = (A \Theta B) \oplus B \tag{9-10}$$

$$A \circ B = \cup\{(B)_z \mid (B)_z \subseteq A\} \tag{9-12}$$

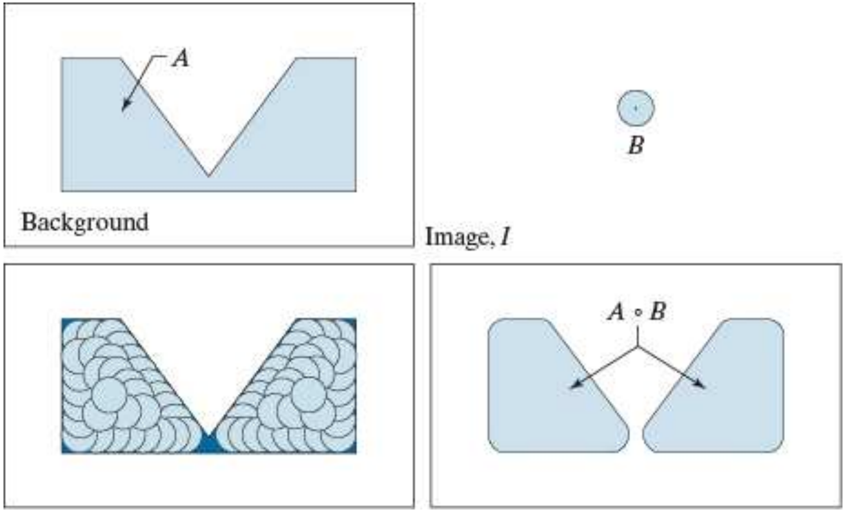


FIGURE 9.8 (a) Image I, composed of set (object) A and background. (b) Structuring element, B. (c) Translations of B while being contained in A. (A is shown dark for clarity.) (d) Opening of A by B.

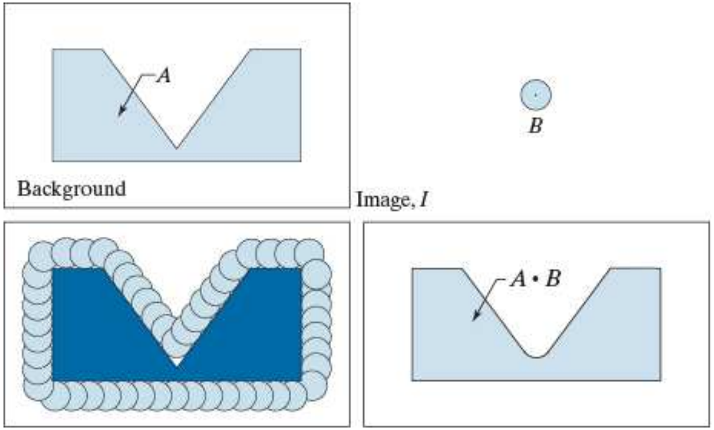
9.3 Opening and Closing

9.3.2 Closing

$$A \bullet B = (A \oplus B) \ominus B \qquad (9\text{-}11)$$

$$(A \bullet B)^c = (A^c \circ \hat{B}) \qquad (9\text{-}15)$$

$$(A \circ B)^c = (A^c \bullet \hat{B}) \qquad (9\text{-}14)$$



a b
c d

FIGURE 9.9 (a) Image I, composed of set (object) A, and background. (b) Structuring element B. (c) Translations of B such that B does not overlap any part of A. (A is shown dark for clarity.) (d) Closing of A by B.

9.3 Opening and Closing

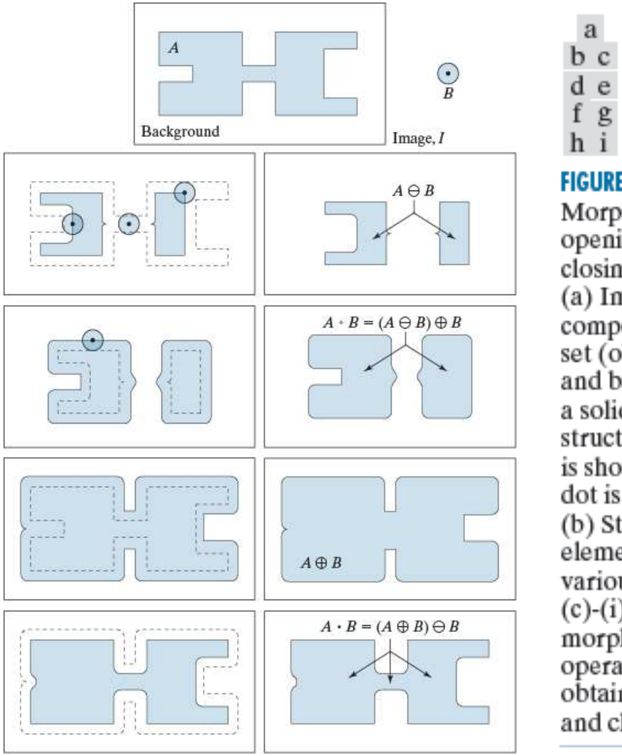


FIGURE 9.10 Morphological opening and closing. (a) Image I, composed of a set (object) A and background; a solid, circular structuring element is shown also. (The dot is the origin.) (b) Structuring element in various positions. (c)-(i) The morphological operations used to obtain the opening and closing.

9.3 Opening and Closing

9.3.3 Properties of Opening and Closing

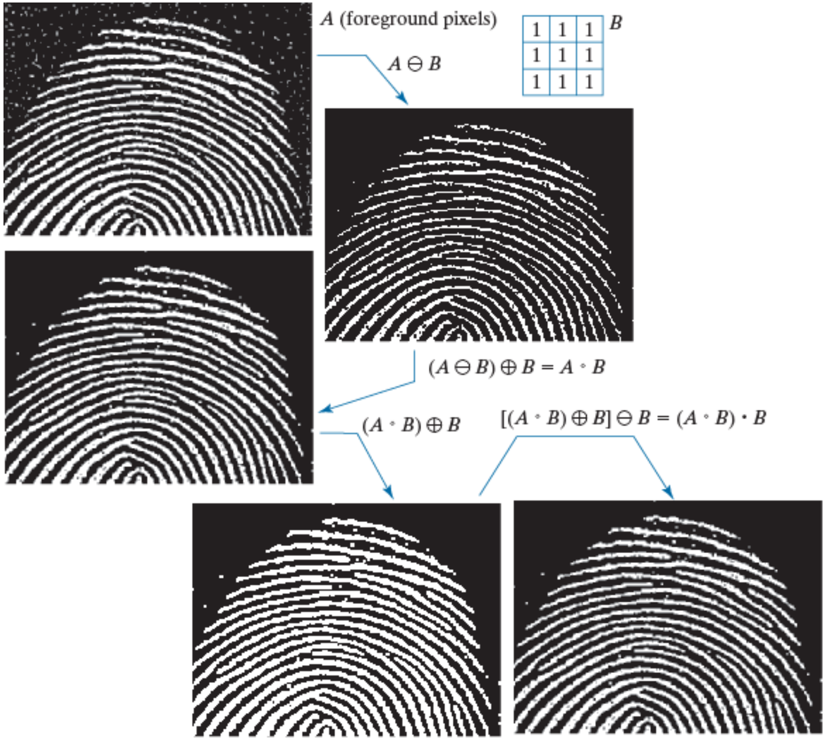
Property of Opening

1. $A \circ B$ is a subset(subimage) of A
2. If C is a subset of D, then $C \circ B$ is a subset of $D \circ B$
3. $(A \circ B) \circ B = A \circ B$

Property of Closing

1. A is a subset(subimage) of $A \bullet B$
2. If C is a subset of D, then $C \bullet B$ is a subset of $D \bullet B$
3. $(A \bullet B) \bullet B = A \bullet B$

9.3 Opening and Closing



a b
c d
e f

FIGURE 9.11 (a) Noisy image. (b) Structuring element. (c) Eroded image. (d) Dilation of the erosion (opening of A). (e) Dilation of the opening. (f) Closing of the opening. (Original image courtesy of the National Institute of Standards and Technology.)

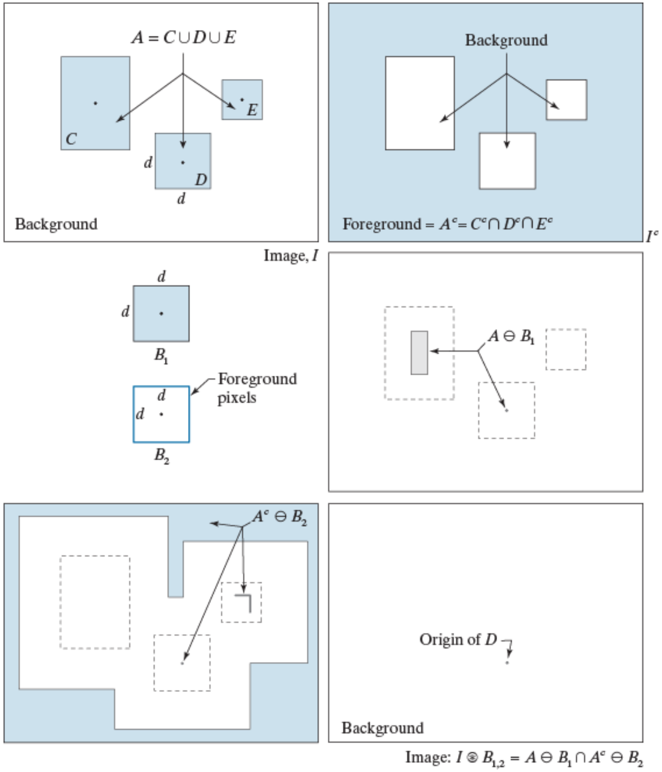
9.4 The Hit-or-Miss Transformation

- The morphological ***hit-or-miss transform (HMT)*** is a basic tool for shape detection
  - The HMT utilizes two structuring elements: B_1, for detecting shapes in the foreground, and B_2, for detecting shapes in the background

$$
\begin{aligned}
I \circledast B_{1,2} &= \{z | (B_1)_z \subseteq A \; and (B_2)_z \subseteq A^c\} \\
&= (A \ominus B_1) \cap (A^c \ominus B_2) \qquad (9\text{-}16)
\end{aligned}
$$

$$I \circledast B = \{z | (B)_z \subseteq I\} \qquad (9\text{-}17)$$

9.4 The Hit-or-Miss Transformation



a b
c d
e f

FIGURE 9.12 (a) Image consisting of a foreground (1's) equal to the union, A, of set of objects, and a background of 0's. (b) Image with its foreground defined as A^c. (c) Structuring elements designed to detect object D. (d) Erosion of A by B_1. (e) Erosion of A^c by B_2. (f) Intersection of (d) and (e), showing the location of the origin of D, as desired. The dots indicate the origin of their respective components. Each dot is a single pixel.

9.4 The Hit-or-Miss Transformation

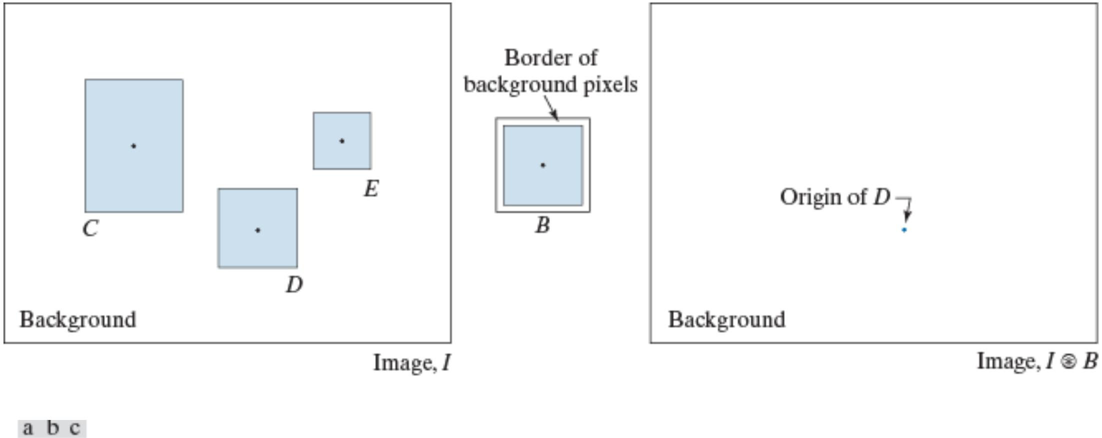
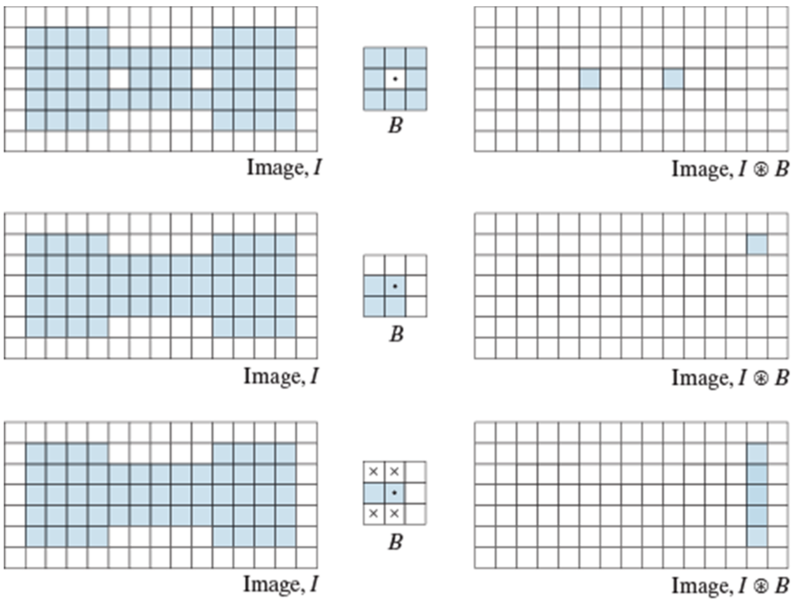


FIGURE 9.13 Same solution as in Fig. 9.12, but using Eq. (9-17) with a single structuring element.

9.4 The Hit-or-Miss Transformation



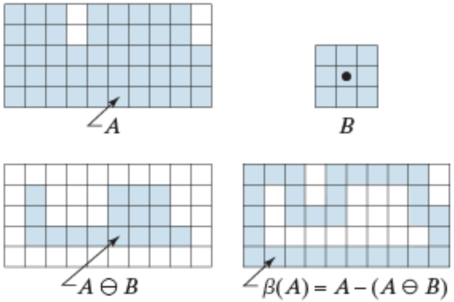
a b c
d e f
g h i

FIGURE 9.14 Three examples of using a single structuring element and Eq. (9-17) to detect specific features. First row: detection of single-pixel holes. Second row: detection of an upper-right corner. Third row: detection of multiple features.

9.5 Basic Morphological Algorithms

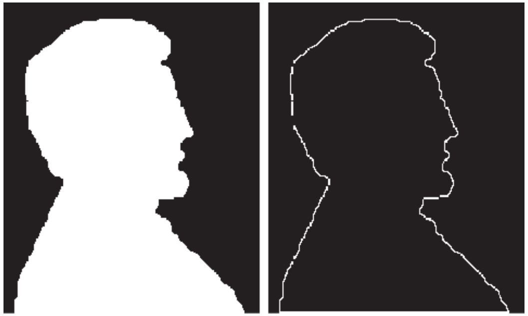
9.5.1 Boundary Extraction

$$\beta(A) = A - (A \Theta B) \qquad (9\text{-}18)$$



a b
c d

FIGURE 9.15
(a) Set, A, of foreground pixels.
(b) Structuring element.
(c) A eroded by B.
(d) Boundary of A.



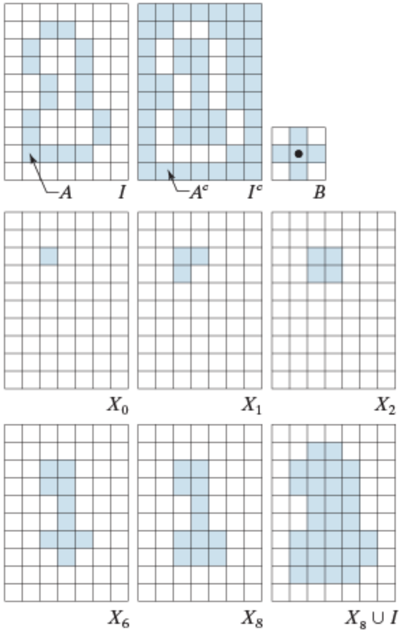
a b

FIGURE 9.16
(a) A binary image.
(b) Result of using Eq. (9-18) with the structuring element in Fig. 9.15(b).

9.5 Basic Morphological Algorithms

9.5.2 Hole Filling

$$X_k = (X_{k-1} \oplus B) \cap A^c \quad k = 1,2,3\ldots \qquad (9\text{-}19)$$

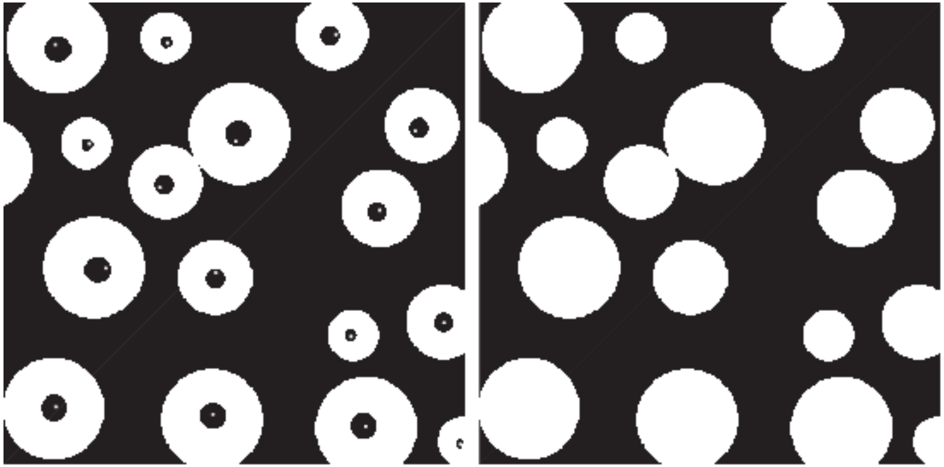


a b c
d e f
g h i

FIGURE 9.17 Hole filling. (a) Set A (shown shaded) contained in image I. (b) Complement of I. (c) Structuring element B. *Only* the foreground elements are used in computations (d) Initial point inside hole, set to 1. (e)–(h) Various steps of Eq. (9-19). (i) Final result [union of (a) and (h)].

9.5 Basic Morphological Algorithms

9.5.2 Hole Filling



a b

FIGURE 9.18 (a) Binary image. The white dots inside the regions (shown enlarged for clarity) are the starting points for the hole-filling algorithm. (b) Result of filling all holes.

9.5 Basic Morphological Algorithms

9.5.3 Extraction of Connected Components

$$X_k = (X_{k-1} \oplus B) \cap A \qquad k = 1,2,3\ldots \qquad (9\text{-}20)$$

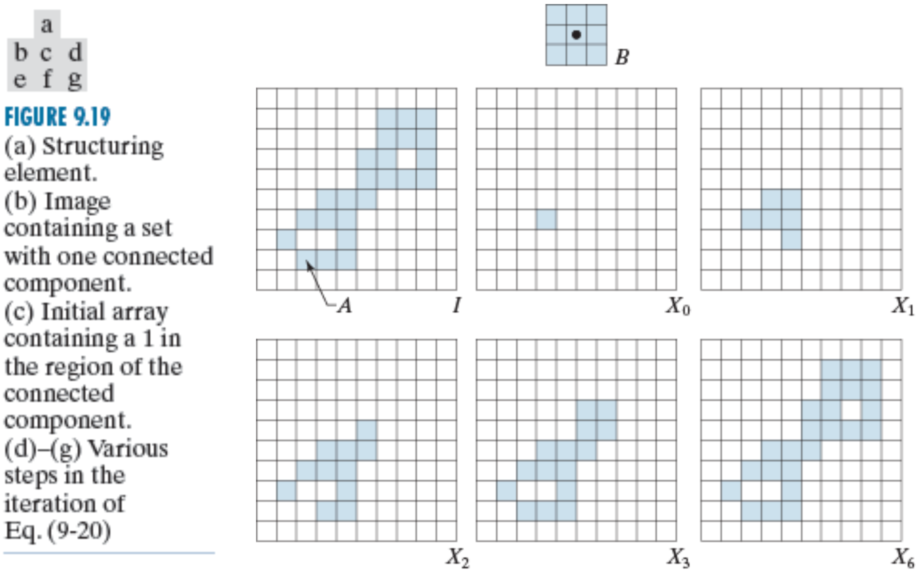
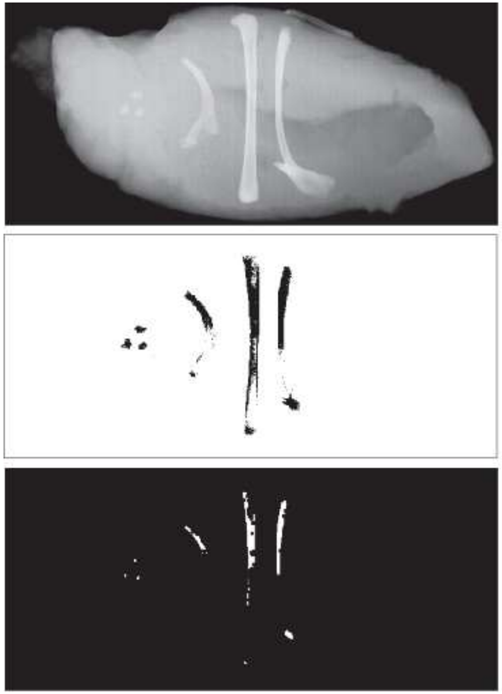


FIGURE 9.19 (a) Structuring element. (b) Image containing a set with one connected component. (c) Initial array containing a 1 in the region of the connected component. (d)–(g) Various steps in the iteration of Eq. (9-20)

9.5 Basic Morphological Algorithms

9.5.3 Extraction of Connected Components



Connected component	No. of pixels in connected comp
01	11
02	9
03	9
04	39
05	133
06	1
07	1
08	743
09	7
10	11
11	11
12	9
13	9
14	674
15	85

a
b
c d

FIGURE 9.20 (a) X-ray image of a chicken filet with bone fragments. (b) Thresholded image (shown as the negative for clarity). (c) Image eroded with a 5×5 SE of 1's. (d) Number of pixels in the connected components of (c). (Image (a) courtesy of NTB Elektronische Geraete GmbH, Diepholz, Germany, www.ntbxray.com.)

9.5 Basic Morphological Algorithms

9.5.4 Convex Hull

$$X_k^i = (X_{k-1} \circledast B^i) \cup A \quad i = 1,2,3,4 \quad and \quad k = 1,2,3,\ldots \tag{9-21}$$

$$C(A) = \bigcup_{i=1}^{4} D^i \tag{9-22}$$

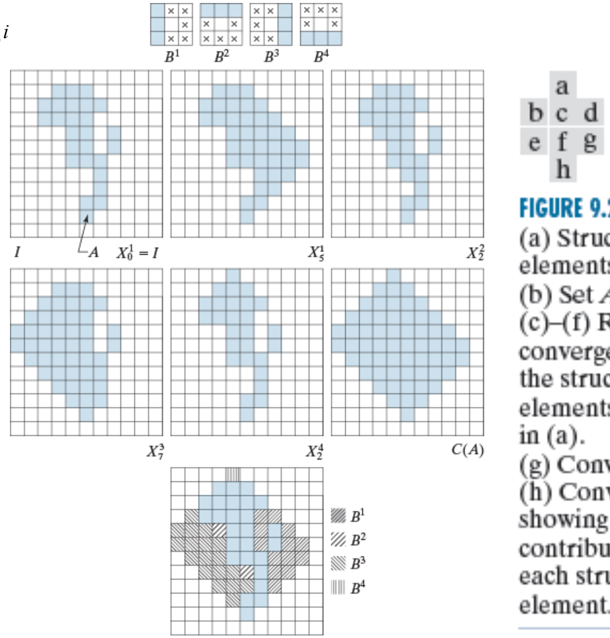
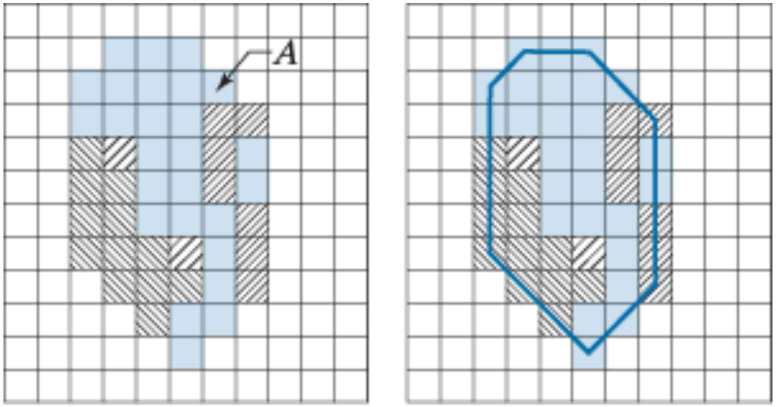


FIGURE 9.21 (a) Structuring elements. (b) Set A. (c)–(f) Results of convergence with the structuring elements shown in (a). (g) Convex hull. (h) Convex hull showing the contribution of each structuring element.

9.5 Basic Morphological Algorithms

9.5.4 Convex Hull

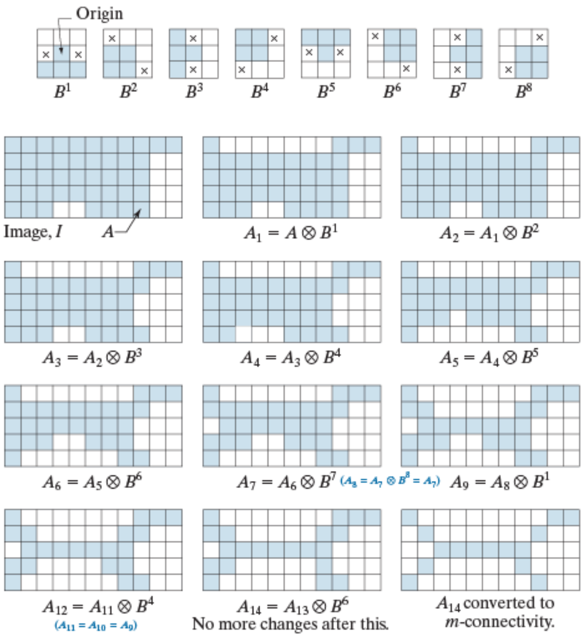


a b

FIGURE 9.22
(a) Result of limiting growth of the convex hull algorithm.
(b) Straight lines connecting the boundary points show that the new set is convex also.

9.5 Basic Morphological Algorithms

9.5.5 Thinning



$$A \otimes B = A - (A \circledast B)$$

$$= A \cap (A \circledast B)^c \qquad \text{(9-23)}$$

$$\{B\} = \{B^1, B^2, B^3, \ldots, B^n\} \qquad \text{(9-24)}$$

$$A \otimes \{B\} = ((\ldots((A \otimes B^1) \otimes B^2)\ldots) \otimes B^n) \qquad \text{(9-25)}$$

a
b c d
e f g
h i j
k l m

FIGURE 9.23
(a) Structuring elements.
(b) Set A.
(c) Result of thinning A with B^1 (shaded).
(d) Result of thinning A_1 with B_2.
(e)–(i) Results of thinning with the next six SEs. (There was no change between A_7 and A_8.)
(j)–(k) Result of using the first four elements again.
(l) Result after convergence.
(m) Result converted to m-connectivity.

9.5 Basic Morphological Algorithms

9.5.6 Thickening

$$A \odot B = A \cup (A \circledast B) \qquad (9\text{-}26)$$

$$A \odot \{B\} = ((\ldots((A \odot B^1) \odot B^2)\ldots) \odot B^n) \qquad (9\text{-}27)$$

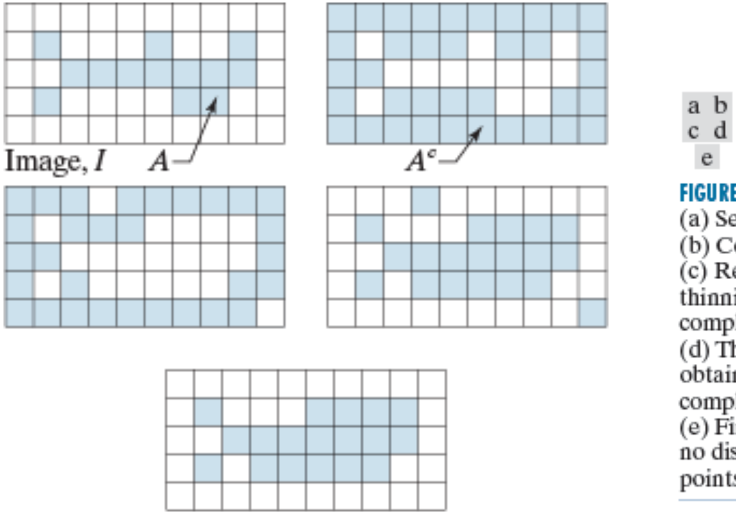
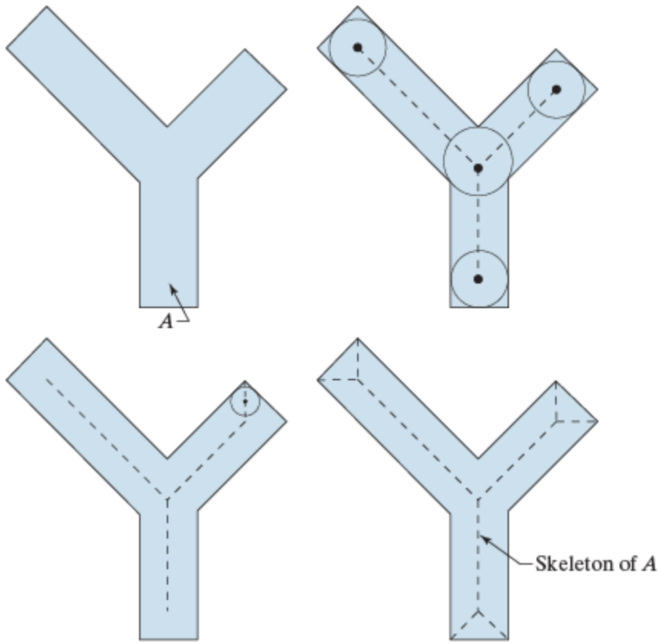


FIGURE 9.24
(a) Set A.
(b) Complement of A.
(c) Result of thinning the complement.
(d) Thickened set obtained by complementing (c).
(e) Final result, with no disconnected points.

9.5 Basic Morphological Algorithms

9.5.7 Skeleton



a b
c d

FIGURE 9.25
(a) Set A.
(b) Various positions of maximum disks whose centers partially define the skeleton of A.
(c) Another maximum disk, whose center defines a different segment of the skeleton of A.
(d) Complete skeleton (dashed).

9.5 Basic Morphological Algorithms

9.5.7 Skeleton

$$S(A) = \bigcup_{k=0}^{K} S_k(A) \tag{9-28}$$

$$S_k(A) = (A \Theta kB) - (A \Theta kB) \circ B \tag{9-29}$$

$$(A \Theta kB) = (\ldots((A \Theta B) \Theta B) \Theta \ldots) \Theta B \tag{9-30}$$

$$K = \max\{k \mid (A \Theta kB) \neq \varnothing\} \tag{9-31}$$

A can be reconstructed from skeleton subsets S_k

$$A = \bigcup_{k=0}^{K} (S_k(A) \oplus kB) \tag{9-32}$$

$$(S_k(A) \oplus kB) = (\ldots((S_k(A) \oplus B) \oplus B) \oplus \ldots) \oplus B \tag{9-33}$$

9.5 Basic Morphological Algorithms

9.5.7 Skeleton

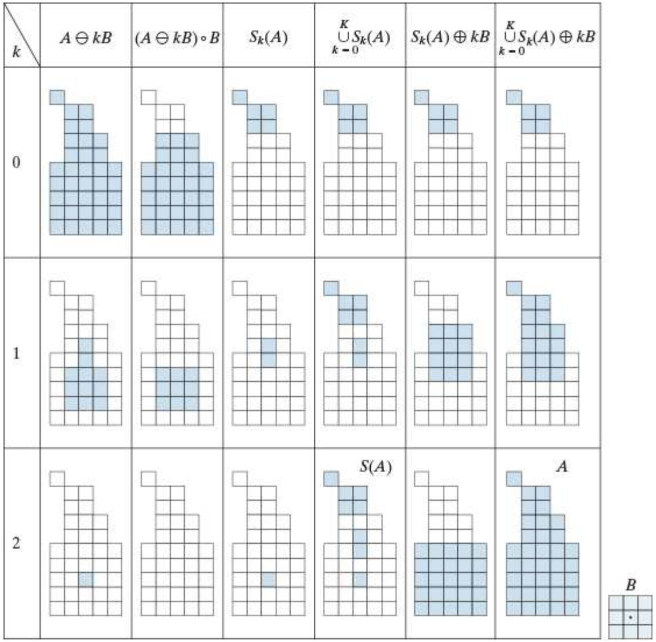
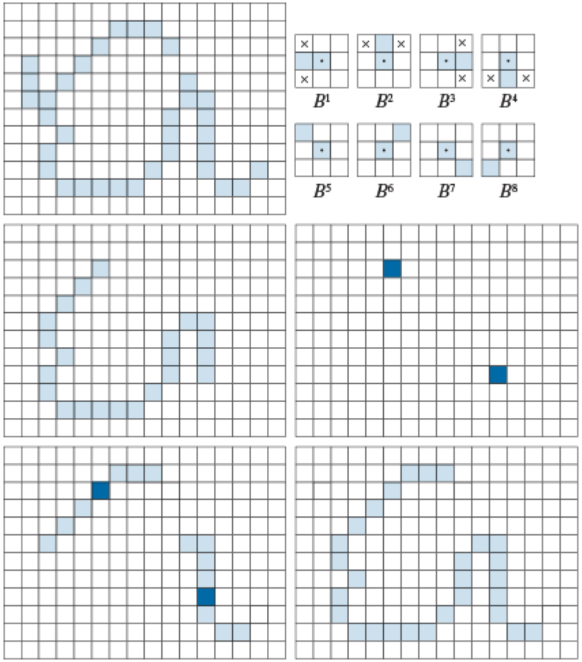


FIGURE 9.26 Implementation of Eqs. (9-28) through (9-33). The original set is at the top left, and its morphological skeleton is at the bottom of the fourth column. The reconstructed set is at the bottom of the sixth column.

9.5 Basic Morphological Algorithms

9.5.8 Pruning



$$X_1 = A \otimes \{B\} \qquad (9\text{-}34)$$

$$X_2 = \bigcup_{k=1}^{8} (X_1 \circledast B^k) \qquad (9\text{-}35)$$

$$X_3 = (X_2 \oplus H) \cap A \qquad (9\text{-}36)$$

$$X_4 = X_1 \cup X_3 \qquad (9\text{-}37)$$

a	b
c	d
e	f

FIGURE 9.27 (a) Set A of foreground pixels (shaded). (b) SEs used for deleting end points. (c) Result of three cycles of thinning. (d) End points of (c). (e) Dilation of end points conditioned on (a). (f) Pruned image.

9.6 Morphological Reconstruction

9.6.1 Geodesic Dilation

$$D_G^{(1)}(F) = (F \oplus B) \cap G \qquad (9\text{-}38)$$

$$D_G^{(n)}(F) = D_G^{(1)}\left[D_G^{(n-1)}(F)\right] \qquad (9\text{-}39)$$

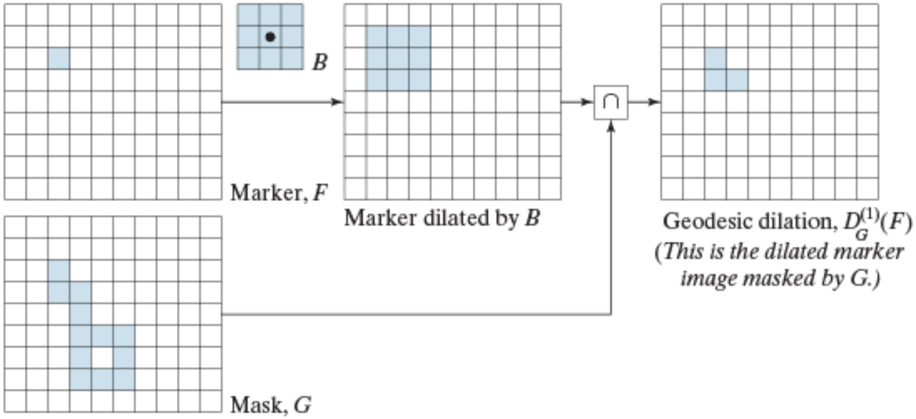


FIGURE 9.28 Illustration of a geodesic dilation of size 1. Note that the marker image contains a point from the object in G. If continued, subsequent dilations and maskings would eventually result in the object contained in G.

9.6 Morphological Reconstruction

9.6.2 Geodesic Erosion

$$E_G^{(1)}(F) = (F \Theta B) \cup G \qquad (9\text{-}40)$$

$$E_G^{(n)}(F) = E_G^{(1)}\left[E_G^{(n-1)}(F)\right] \qquad (9\text{-}41)$$

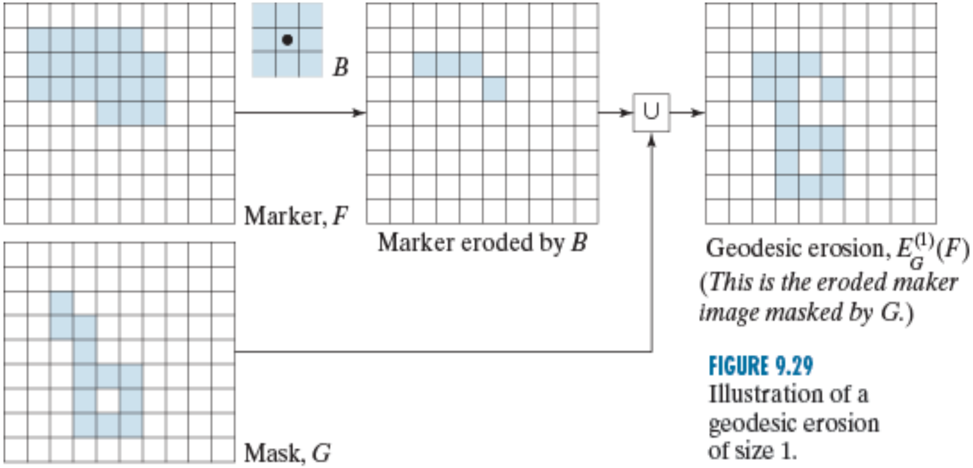


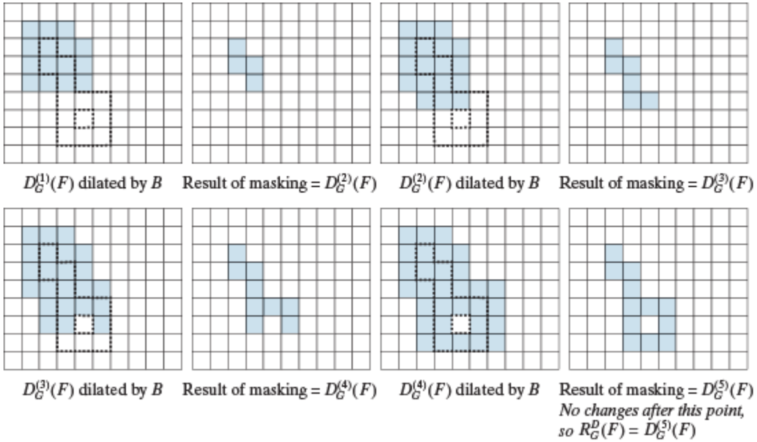
FIGURE 9.29 Illustration of a geodesic erosion of size 1.

9.6 Morphological Reconstruction

9.6.3 Morphological Reconstruction by Dilation and Erosion

$$R_G^D(F) = D_G^{(k)}(F) \qquad (9\text{-}42)$$

$$R_G^E(F) = E_G^{(k)}(F) \qquad (9\text{-}43)$$



a b c d
e f g h

FIGURE 9.30 Illustration of morphological reconstruction by dilation. Sets $D_G^{(1)}(F)$, G, B and F are from Fig. 9.28. The mask (G) is shown dotted for reference.

9.6 Morphological Reconstruction

- Example：Opening by Reconstruction

$$O_R^{(n)}(F) = R_F^D\left[(F \Theta nB\right] \qquad (9\text{-}44)$$

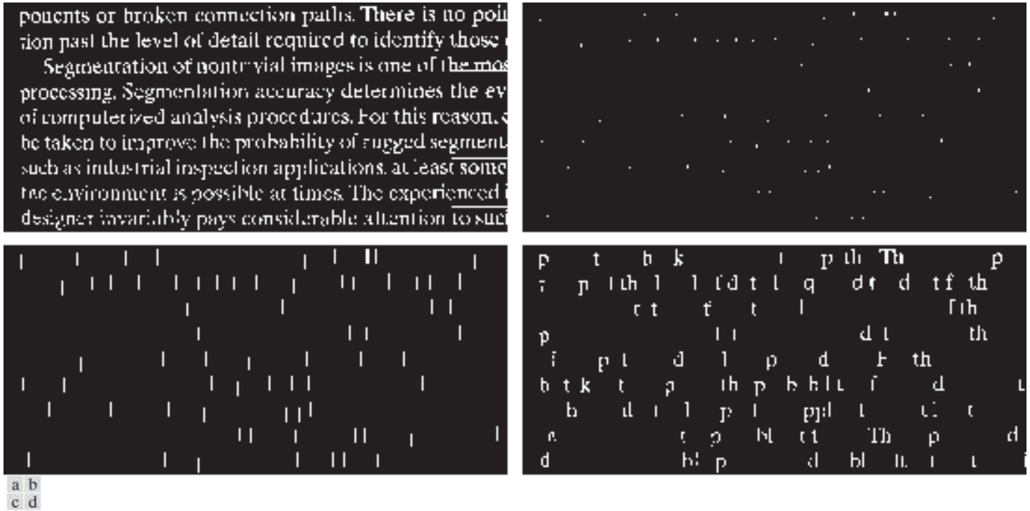


FIGURE 9.31 (a) Text image of size 918×2018 pixels. The approximate average height of the tall characters is 51 pixels. (b) Erosion of (a) with a structuring element of size 51×1 elements (all 1's). (c) Opening of (a) with the same structuring element, shown for comparison. (d) Result of opening by reconstruction.

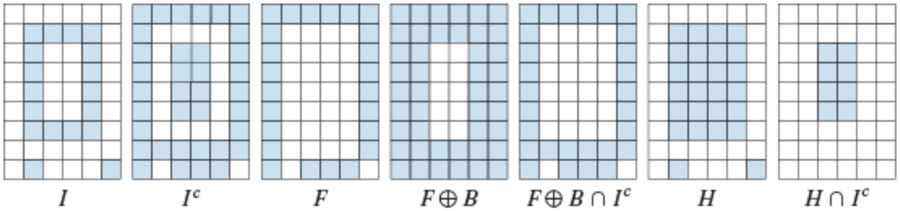
9.6 Morphological Reconstruction

- Example：Filling Holes

$$F(x,y)=\begin{cases}1-I(x,y) & \textit{if } (x,y) \textit{ is on the border of } I \\ 0 & \textit{otherwise}\end{cases} \qquad (9\text{-}45)$$

Then

$$H=\left[R_{I^c}^{D}(F)\right]^c \qquad (9\text{-}46)$$

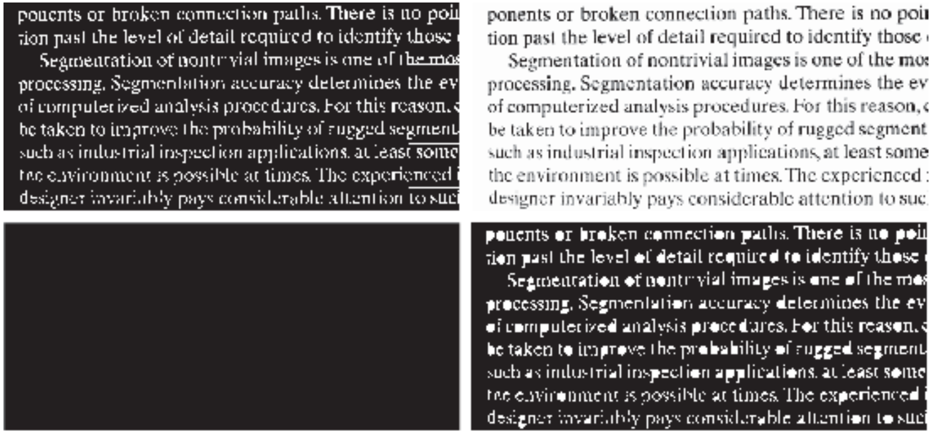


a b c d e f g

FIGURE 9.32 Hole filling using morphological reconstruction.

9.6 Morphological Reconstruction

- Example：Filling Holes



a b
c d

FIGURE 9.33
(a) Text image of size 918 × 2018 pixels.
(b) Complement of (a) for use as a mask image.
(c) Marker image.
(d) Result of hole-filling using Eqs. (9-45) and (9-46).

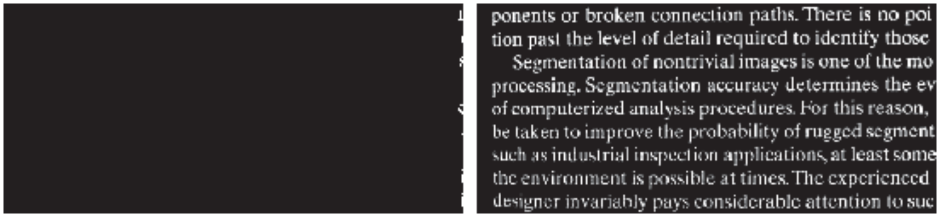
9.6 Morphological Reconstruction

- Example：Border Clearing

$$F(x,y)=\begin{cases} I(x,y) & \text{if } (x,y) \text{ is on the border of } I \\ 0 & \text{otherwise} \end{cases} \qquad (9\text{-}47)$$

Then

$$X = 1 - R_I^D(F) \qquad (9\text{-}48)$$



a b
FIGURE 9.34 (a) Reconstruction by dilation of marker image. (b) Image with no objects touching the border. The original image is Fig. 9.31(a).

9.7 Summary of Morphological Operation on Images

- Basic types of structuring elements

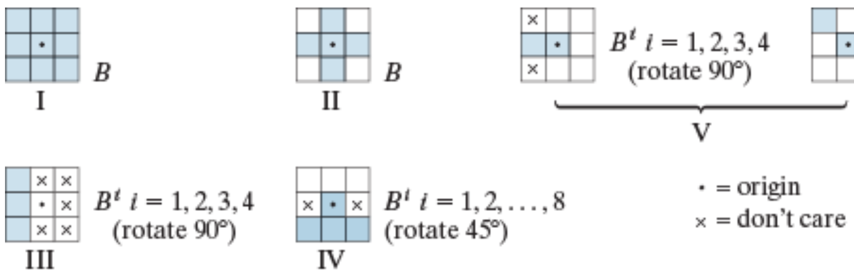


FIGURE 9.35 Five basic types of structuring elements used for binary morphology.

9.7 Summary of Morphological Operation on Images

TABLE 9.1
Summary of binary morphological operations and their properties. A is a set of foreground pixels contained in binary image I, and B is a structuring element. I is a binary image (containing A), with 1's corresponding to the elements of A and 0's elsewhere. The Roman numerals refer to the structuring elements in Fig. 9.35.

Operation	Equation	Comments
Translation	$(B)_z = \{c \mid c = b + z, \text{ for } b \in B\}$	Translates the origin of B to point z.
Reflection	$\hat{B} = \{w \mid w = -b, \text{ for } b \in B\}$	Reflects B about its origin.
Complement	$A^c = \{w \mid w \notin A\}$	Set of points not in A.
Difference	$A - B = \{w \mid w \in A, w \notin B\} = A \cap B^c$	Set of points in A, but not in B.
Erosion	$A \ominus B = \{z \mid (B)_z \subseteq A\}$	Erodes the boundary of A. (I)
Dilation	$A \oplus B = \{z \mid (\hat{B})_z \cap A \neq \varnothing\}$	Dilates the boundary of A. (I)
Opening	$A \circ B = (A \ominus B) \oplus B$	Smoothes contours, breaks narrow isthmuses, and eliminates small islands and sharp peaks. (I)
Closing	$A \bullet B = (A \oplus B) \ominus B$	Smoothes contours, fuses narrow breaks and long thin gulfs, and eliminates small holes. (I)
Hit-or-miss transform	$I \circledast B = \{z \mid (B)_z \subseteq I\}$	Finds instances of B in image I. B contains *both* foreground and background elements.
Boundary extraction	$\beta(A) = A - (A \ominus B)$	Set of points on the boundary of set A. (I)
Hole filling	$X_k = (X_{k-1} \oplus B) \cap I^c \quad k = 1, 2, 3, \ldots$	Fills holes in A. X_0 is of same size as I, with a 1 in each hole and 0's elsewhere. (II)
Connected components	$X_k = (X_{k-1} \oplus B) \cap I \quad k = 1, 2, 3, \ldots$	Finds connected components in I. X_0 is a set, the same size as I, with a 1 in each connected component and 0's elsewhere. (I)
Convex hull	$X_k^i = (X_{k-1}^i \circledast B^i) \cup X_{k-1}^i$; $i = 1, 2, 3, 4 \quad k = 1, 2, 3, \ldots$ $X_0^i = I$; $D^i = X_{conv}^i$; $C(A) = \bigcup_{i=1}^{4} D^i$	Finds the convex hull, $C(A)$, of a set, A, of foreground pixels contained in image I. X_{conv}^i means that $X_k^i = X_{k-1}^i$. (III)

9.7 Summary of Morphological Operation on Images

TABLE 9.1 (*Continued*)

Operation	Equation	Comments
Thinning	$A \otimes B = A - (A \circledast B)$ $= A \cap (A \circledast B)^c$ $A \otimes \{B\} =$ $\left(\left(\ldots\left(\left(A \otimes B^1\right) \otimes B^2\right)\ldots\right) \otimes B^n\right)$ $\{B\} = \{B^1, B^2, B^3, \ldots, B^n\}$	Thins set A. The first two equations give the basic definition of thinning. The last two equations denote thinning by a sequence of structuring elements. This method is normally used in practice. (IV)
Thickening	$A \odot B = A \cup (A \circledast B)$ $A \odot \{B\} =$ $\left(\left(\ldots\left(\left(A \odot B^1\right) \odot B^2\right)\ldots\right) \odot B^n\right)$	Thickens set A using a sequence of structuring elements, as above. Uses (IV) with 0's and 1's reversed.
Skeletons	$S(A) = \bigcup_{k=0}^{K} S_k(A)$ $S_k(A) = (A \ominus kB)$ $- (A \ominus kB) \circ B$ Reconstruction of A: $A = \bigcup_{k=0}^{K} (S_k(A) \oplus kB)$	Finds the skeleton $S(A)$ of set A. The last equation indicates that A can be reconstructed from its skeleton subsets $S_k(A)$. K is the value of the iterative step after which the set A erodes to the empty set. The notation $(A \ominus kB)$ denotes the kth iteration of successive erosions of A by B. (I)
Pruning	$X_1 = A \otimes \{B\}$ $X_2 = \bigcup_{k=1}^{8} \left(X_1 \circledast B^k\right)$ $X_3 = (X_2 \oplus H) \cap A$ $X_4 = X_1 \cup X_3$	X_4 is the result of pruning set A. The number of times that the first equation is applied to obtain X_1 must be specified. Structuring elements (V) are used for the first two equations. In the third equation H denotes structuring element. (I)
Geodesic dilation–size 1	$D_G^{(1)}(F) = (F \oplus B) \cap G$	F and G are called the *marker* and the *mask* images, respectively. (I)
Geodesic dilation–size n	$D_G^{(n)}(F) = D_G^{(1)}\left(D_G^{(n-1)}(F)\right)$	Same comment as above.
Geodesic erosion–size 1	$E_G^{(1)}(F) = (F \ominus B) \cup G$	Same comment as above.
Geodesic erosion–size n	$E_G^{(n)}(F) = E_G^{(1)}\left(E_G^{(n-1)}(F)\right)$	Same comment as above.
Morphological reconstruction by dilation	$R_G^D(F) = D_G^{(k)}(F)$	With k is such that $D_G^{(k)}(F) = D_G^{(k+1)}(F)$.

9.7 Summary of Morphological Operation on Images

TABLE 9.1 *(Continued)*

Operation	Equation	Comments
Morphological reconstruction by erosion	$R_G^E(F) = E_G^{(k)}(F)$	With k such that $E_G^{(k)}(F) = E_G^{(k+1)}(F)$.
Opening by reconstruction	$O_R^{(n)}(F) = R_F^D(F \ominus nB)$	$F \ominus nB$ indicates n successive erosions by B, starting with F. The form of B is application-dependent.
Closing by reconstruction	$C_R^{(n)}(F) = R_F^E(F \oplus nB)$	$F \oplus nB$ indicates n successive dilations by B, starting with F. The form of B is application-dependent.
Hole filling	$H = \left[R_{F^c}^D(F)\right]^c$	H is equal to the input image I, but with all holes filled. See Eq. (9-45) for the definition of marker image F.
Border clearing	$X = I - R_I^D(F)$	X is equal to the input image I, but with all objects that touch (are connected to) the boundary removed. See Eq. (9-47) for the definition of marker image F.

9.8 Gray-Scale Morphology

- Structuring elements in gray-scale morphology are used as “probes” to examine a given image for specific properties.
- Structuring elements in gray-scale morphology belong to one of two categories: nonflat and flat.

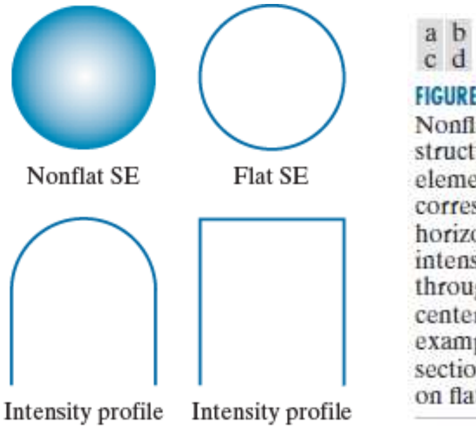


FIGURE 9.36 Nonflat and flat structuring elements, and corresponding horizontal intensity profiles through their centers. All examples in this section are based on flat SEs.

9.8 Gray-Scale Morphology

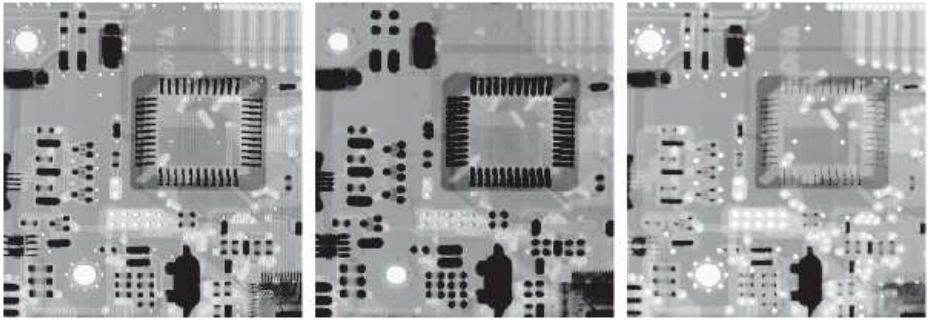
9.6.1 Grayscale Erosion and Dilation

Erosion by a flat structuring element

$$[f \Theta b](x,y) = \min_{(s,t)\in b}\{f(x+s, y+t)\} \qquad (9\text{-}49)$$

Dilation by a flat structuring element

$$[f \oplus b](x,y) = \max_{(s,t)\in b}\{f(x-s, y-t)\} \qquad (9\text{-}50)$$



a b c

FIGURE 9.37 (a) Gray-scale X-ray image of size 448 × 425 pixels. (b) Erosion using a flat disk SE with a radius of 2 pixels. (c) Dilation using the same SE. (Original image courtesy of Lixi, Inc.)

9.8 Gray-Scale Morphology

9.6.1 Grayscale Erosion and Dilation

Erosion by a nonflat structuring element

$$[f \Theta b_N](x, y) = \min_{(s,t)\in b_N} \{f(x+s, y+t) - b_N(s,t)\} \qquad (9\text{-}51)$$

Dilation by a nonflat structuring element

$$[f \oplus b_N](x, y) = \max_{(s,t)\in b_N} \{f(x-s, y-t) + b_N(s,t)\} \qquad (9\text{-}52)$$

Erosion and dilation are duals with respect to function complement ation and reflection

$$(f \Theta b)^c = (f^c \oplus \hat{b}) \qquad (9\text{-}54)$$

$$(f \oplus b)^c = (f^c \Theta \hat{b}) \qquad (9\text{-}55)$$

9.8 Gray-Scale Morphology

9.6.2 Grayscale Opening and Closing

$$f \circ b = (f \ominus b) \oplus b \qquad (9\text{-}56)$$

$$f \bullet b = (f \oplus b) \ominus b \qquad (9\text{-}57)$$

$$(f \bullet b)^c = f^c \circ \hat{b} \qquad (9\text{-}58)$$

$$(f \circ b)^c = f^c \bullet \hat{b} \qquad (9\text{-}59)$$

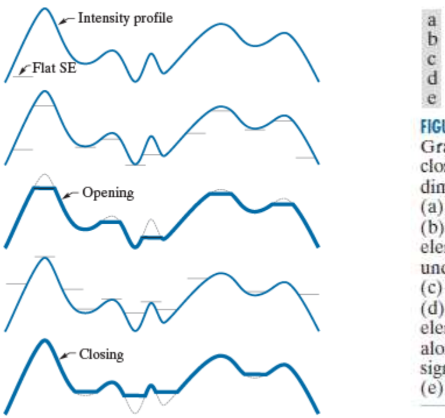


FIGURE 9.38 Grayscale opening and closing in one dimension. (a) Original 1-D signal. (b) Flat structuring element pushed up underneath the signal. (c) Opening. (d) Flat structuring element pushed down along the top of the signal. (e) Closing.

9.8 Gray-Scale Morphology

9.6.2 Grayscale Opening and Closing

Property of Opening

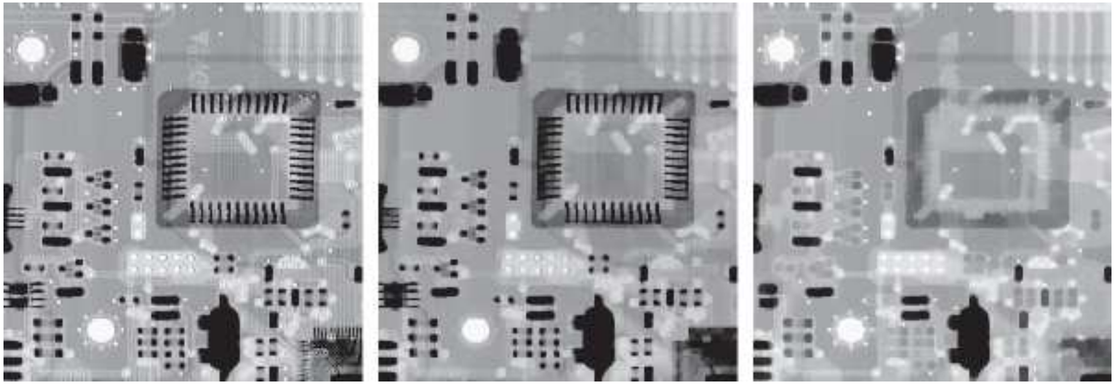
1. $(f \circ b) \dashv f$
2. If $f_1 \dashv f_2$, then $(f_1 \circ b) \dashv (f_2 \circ b)$
3. $(f \circ b) \circ b = f \circ b$

Property of Closing

1. $f \dashv (f \bullet b)$
2. If $f_1 \dashv f_2$, then $(f_1 \bullet b) \dashv (f_2 \bullet b)$
3. $(f \bullet b) \bullet b = f \bullet b$

9.8 Gray-Scale Morphology

9.6.2 Grayscale Opening and Closing



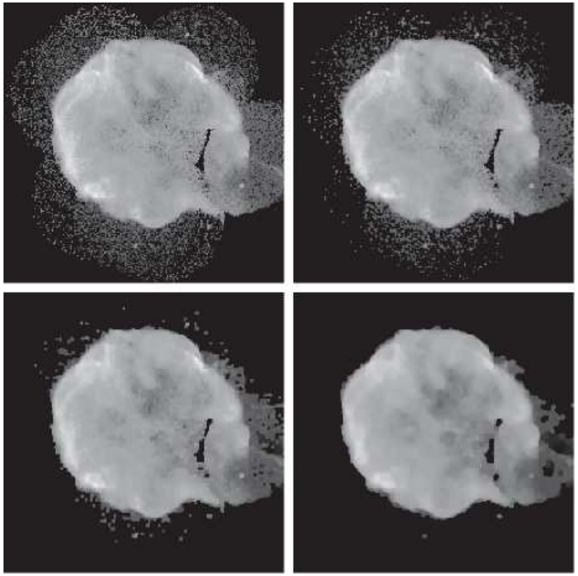
a b c

FIGURE 9.39
(a) A grayscale X-ray image of size 448 × 425 pixels.
(b) Opening using a disk SE with a radius of 3 pixels.
(c) Closing using an SE of radius 5.

9.8 Gray-Scale Morphology

9.6.3 Basic Gray-Scale Morphological Algorithms

- Morphological Smoothing



a b
c d

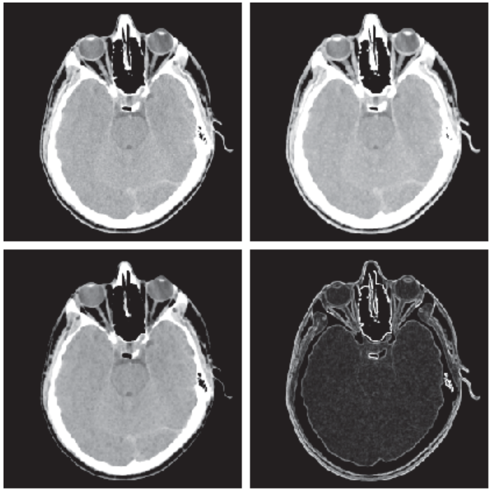
FIGURE 9.40 (a) 566 × 566 image of the Cygnus Loop supernova, taken in the X-ray band by NASA's Hubble Telescope. (b)–(d) Results of performing opening and closing sequences on the original image with disk structuring elements of radii, 1, 3, and 5, respectively. (Original image courtesy of NASA.)

9.8 Gray-Scale Morphology

9.6.3 Basic Gray-Scale Morphological Algorithms

- Morphological Gradient

$$g = (f \oplus b) - (f \ominus b) \qquad (9\text{-}60)$$



a b
c d

FIGURE 9.41 (a) 512×512 image of a head CT scan. (b) Dilation. (c) Erosion. (d) Morphological gradient, computed as the difference between (b) and (c). (Original image courtesy of Dr. David R. Pickens, Vanderbilt University.)

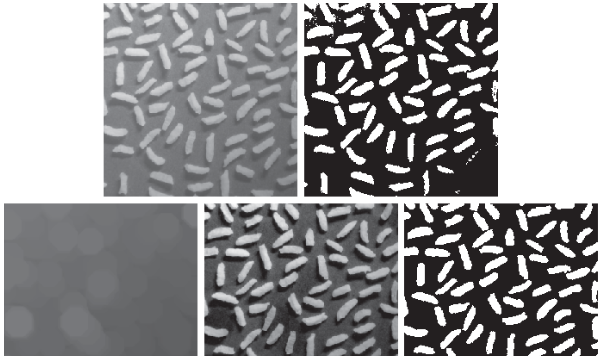
9.8 Gray-Scale Morphology

9.6.3 Basic Gray-Scale Morphological Algorithms

- Top-hat and Bottom-hat Transformation

$$T_{hat}(f) = f - (f \circ b) \qquad (9\text{-}61)$$

$$B_{hat}(f) = (f \bullet b) - f \qquad (9\text{-}62)$$



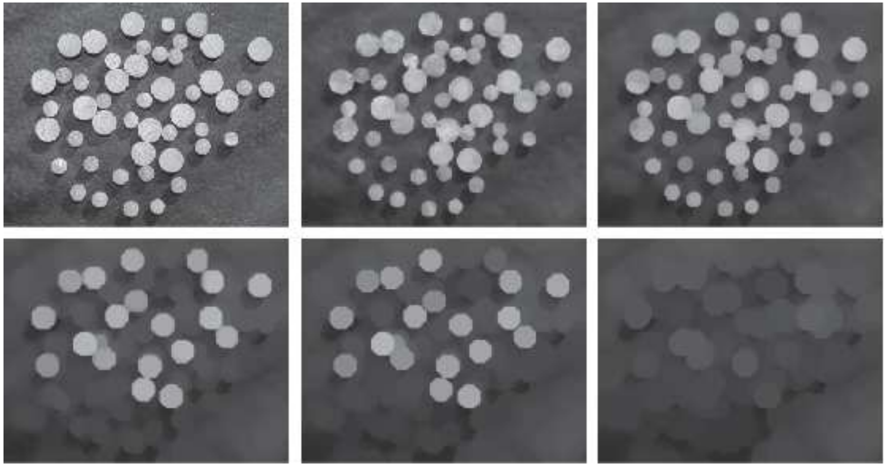
a b
c d e

FIGURE 9.42 Using the top-hat transformation for *shading correction*. (a) Original image of size 600×600 pixels. (b) Thresholded image. (c) Image opened using a disk SE of radius 40. (d) Top-hat transformation (the image minus its opening). (e) Thresholded top-hat image.

9.8 Gray-Scale Morphology

9.6.3 Basic Gray-Scale Morphological Algorithms

- Granulometry



a b c
d e f

FIGURE 9.43 (a) 531 × 675 image of wood dowels. (b) Smoothed image. (c)–(f) Openings of (b) with disks of radii equal to 10, 20, 25, and 30 pixels, respectively. (Original image courtesy of Dr. Steve Eddins, MathWorks, Inc.)

9.8 Gray-Scale Morphology

9.6.3 Basic Gray-Scale Morphological Algorithms

- Granulometry

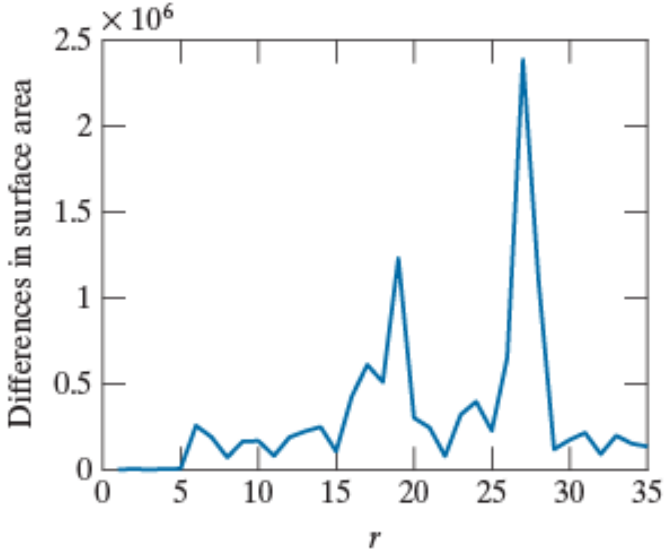


FIGURE 9.44 Differences in surface area as a function of SE disk radius, r. The two peaks indicate that there are two dominant particle sizes in the image.

9.8 Gray-Scale Morphology

9.6.3 Basic Gray-Scale Morphological Algorithms

- Textural Segmentation

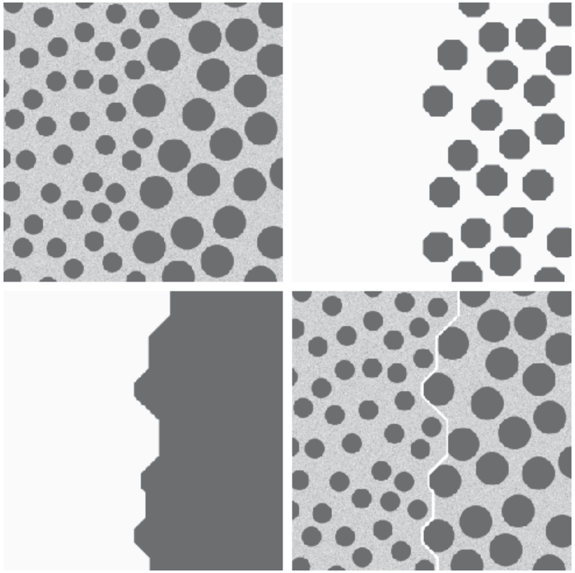


FIGURE 9.45 Textural segmentation. (a) A 600×600 image consisting of two types of blobs. (b) Image with small blobs removed by closing (a). (c) Image with light patches between large blobs removed by opening (b). (d) Original image with boundary between the two regions in (c) superimposed. The boundary was obtained using a morphological gradient.

9.8 Gray-Scale Morphology

9.6.4 Gray-Scale Morphological Reconstruction

Geodesic dilation

$$D_g^{(1)}(f) = (f \oplus b) \wedge g \qquad \text{(9-63)}$$

$$D_g^{(n)}(f) = D_g^{(1)}\left[D_g^{(n-1)}(f)\right] \qquad \text{(9-64)}$$

Geodesic erosion

$$E_g^{(1)}(f) = (f \Theta b) \vee g \qquad \text{(9-65)}$$

$$E_g^{(n)}(f) = E_g^{(1)}\left[E_g^{(n-1)}(f)\right] \qquad \text{(9-66)}$$

Morphological reconstruction by dilation

$$R_g^D(f) = D_g^{(k)}(f) \qquad \text{(9-67)}$$

Morphological reconstruction by erosion

$$R_g^E(f) = E_g^{(k)}(f) \qquad \text{(9-68)}$$

Opening by reconstruction

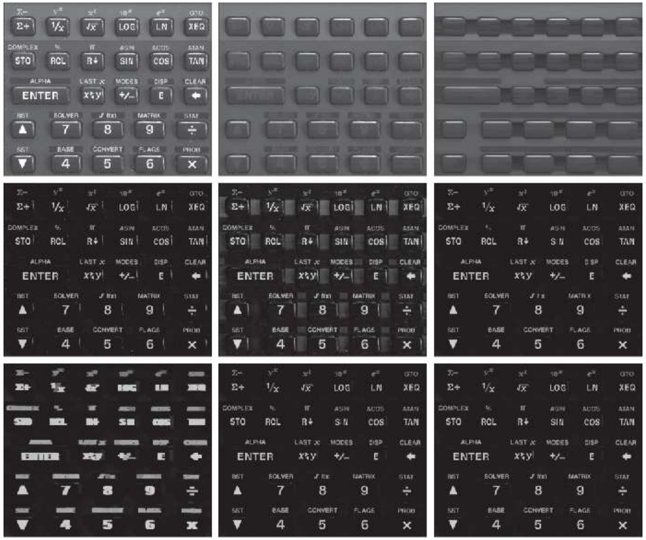
$$O_R^{(n)}(f) = R_f^D[(f \Theta nb)] \qquad \text{(9-69)}$$

Closing by reconstruction

$$C_R^{(n)}(f) = R_f^E[(f \oplus nb)] \qquad \text{(9-70)}$$

9.8 Gray-Scale Morphology

9.6.4 Gray-Scale Morphological Reconstruction



a b c
d e f
g h i

FIGURE 9.46 (a) Original image of size 1134 × 1360 pixels. (b) Opening by reconstruction of (a), using a structuring element consisting of a horizontal line 71 pixels long in the erosion. (c) Opening of (a) using the same SE. (d) Top-hat by reconstruction. (e) Result of applying just a top-hat transformation. (f) Opening by reconstruction of (d), using a horizontal line 11 pixels long. (g) Dilation of (f) using a horizontal line 21 pixels long. (h) Minimum of (d) and (g). (i) Final reconstruction result. (Images courtesy of Dr. Steve Eddins, MathWorks, Inc.)