

Problem 1.

a) $\phi(x_1) = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$, $\phi(x_2) = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, $\phi(x_3) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $\phi(x_4) = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$

b) $k(x_1, x_1) = [-1 \ 1 \ 1] \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} = 3$

$k(x_1, x_2) = 1$, $k(x_2, x_2) = 1$

$k(x_1, x_3) = 1$, $k(x_2, x_3) = 1$

$k(x_3, x_3) = 3$, $k(x_1, x_4) = 0$

$k(x_2, x_4) = 0$, $k(x_3, x_4) = 0$

$k(x_4, x_4) = 0$

c)
$$L(w, b, \alpha) = \frac{1}{2} \|w\|^2 - \sum_{i=1}^m \lambda_i [w^T x_i + b - 1]$$

$$= \frac{1}{2} \|w\|^2 - \sum_{i=1}^m \lambda_i [y_i (w^T x_i + b)] + \sum_{i=1}^m \lambda_i$$

d) $L = f(\lambda) - \alpha g(\lambda)$

$$= \sum_{i=1}^4 \lambda_i - \frac{1}{2} \sum_{i=1}^4 \sum_{j=1}^4 \lambda_i \lambda_j y_i y_j k(x_i, x_j) - \alpha \sum_{i=1}^4 \lambda_i y_i$$

$$= (\lambda_1 + \dots + \lambda_4) - \frac{1}{2} (3\lambda_1^2 - 2\lambda_1\lambda_2 + 2\lambda_1\lambda_3 + \lambda_2^2 - 2\lambda_2\lambda_3 + 3\lambda_3^2) - \alpha(\lambda_1 - \lambda_2 + \lambda_3 + \lambda_4)$$

$$\Rightarrow \begin{cases} \lambda_1 = 1, \lambda_2 = 4 \\ \lambda_3 = 1, \lambda_4 = 2 \\ \alpha = 1 \end{cases}$$

e)
$$w = \sum_{i=1}^4 \lambda_i y_i \phi(x_i) = 1 \cdot \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} + 4 \cdot (-1) \cdot \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + 1 \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + 2 \cdot \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} -1 \\ -2 \\ 3 \end{bmatrix}$$

$$b = 1 - w^T \phi(x_1) = 1 - \begin{bmatrix} -1 & -2 & 3 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$$

$$= 1$$

