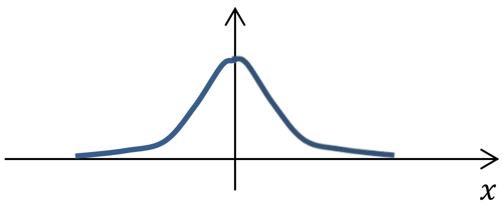
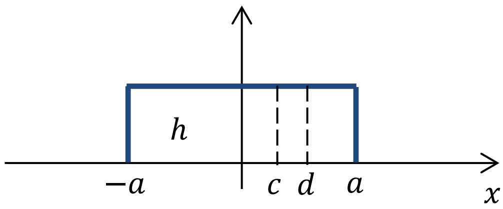
Homework Set 1

Problem 1 (Probability density function)



(A)



(B)

a) Figure (A) shows a Cauchy probability density function whose density is given by $\frac{1}{K}\frac{1}{1+x^2}$. What is the value of K for the density to be a probability density function? What are the mean and variance of the distribution?

b) Figure (B) shows a uniform probability density function. What is the height h of the density function? What are the mean and variance of the distribution? What is the probability that x lies in the interval between the vertical lines marked by c and d?

c) Consider two fair dice with six sides marked with the usual numbers 1 through 6. What is the probability that a throw of the dice results in a score of 7? What is the mean of the numbers that arise when two dice are thrown? What is the variance?

Problem 2 (Sample statistics and confidence intervals)
In characterizing the noise in an amplifier, which is normally distributed, we have the following noise voltages in micro Volts (μV):

-0.4326 -1.6656 0.1253 0.2877 -1.1465 1.1909 1.1892 -0.0376 0.3273 0.1746

a) Estimate the mean of the noise voltages and the variance of the mean.

b) Calculate the 95% and 99% confidence intervals of the mean (of noise voltages).

c) How confident are we that the noise voltage at any time lies between $1\mu V$ and $1.1\mu V$?

Problem 3 (Calculus)
Let $f(x, y) = 3x^2 + y^2 - xy - 11x$

a) Find $\frac{\partial f}{\partial x}$, the partial derivative of f with respect to x. Also find $\frac{\partial f}{\partial y}$.

b) Find the pair $(x, y) \in \Re^2$ that minimizes f.

c) Show that the pair (x, y) you found in b. is a minimizer instead of a maximizer.

Problem 4 (Vector Norms)
Compute the 0, 1, 2, and ∞ norms for $[3\ -1\ 3\ 5\ 0\ 2\ ]^T$.