Problem 1 $P(X|Yes) = P(Age<30|Yes) \cdot P(income=m|Yes) \times P(student=yes|Yes) \times P(credit=F|Yes) \times P(purchase=Yes)$

$= \frac{2}{9} \times \frac{4}{9} \times \frac{6}{9} \times \frac{6}{9} \times \frac{9}{14} = 0.028$

$P(X|No) = P(Age<30|No) \cdot P(income=m|No) \times P(student=yes|No) \times P(credit=F|No) \times P(purchase=No)$

$= \frac{3}{5} \cdot \frac{2}{5} \cdot \frac{1}{5} \cdot \frac{2}{5} \cdot \frac{5}{14} = 0.0069$

$P(X|Yes) > P(X|No)$

$\Rightarrow$ The person will buy a computer #

Problem 2.

(a) $y = ax+b$

(X_1, X_2)

$a = \frac{2-1}{3-1} = 1, \quad a' = \frac{-1}{1} = -1, \quad Q = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$

$\Rightarrow y = -x+b, \quad b = 4 \Rightarrow x+y = 4$

(X_1, X_3)

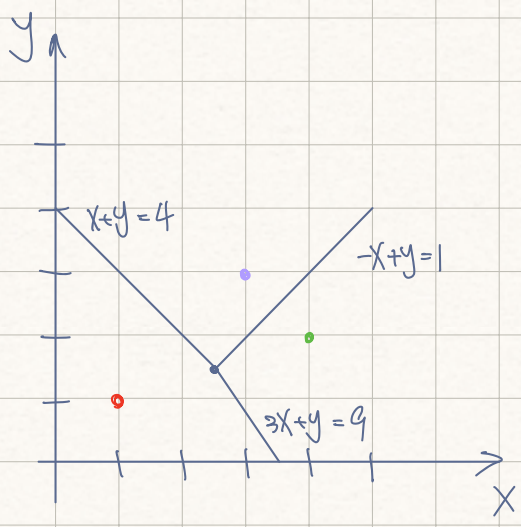
$a = \frac{2-1}{4-1} = \frac{1}{3}, \quad a' = \frac{-1}{\frac{1}{3}} = -3, \quad Q = \begin{bmatrix} 2.5 \\ 1.5 \end{bmatrix}$

$\Rightarrow y = -3x+b, \quad b = 9 \Rightarrow 3x+y = 9$

(X_2, X_3)

$a = \frac{2-3}{4-3} = -1, \quad a' = \frac{-1}{-1} = 1, \quad Q = \begin{bmatrix} 3.5 \\ 3.5 \end{bmatrix}$

$\Rightarrow y = x+b, \quad b = -1 \Rightarrow -x+y = -1$



(b). $d_M(X_1, X_2) = \sqrt{\frac{1}{2}\left(X_1^{(1)} - X_2^{(1)}\right)^2 + \left(X_1^{(2)} - X_2^{(2)}\right)^2}$

$(X_1, X_2) \Rightarrow \sqrt{\frac{1}{2}(x-1)^2 + (y-1)^2} = \sqrt{\frac{1}{2}(x-3)^2 + (y-3)^2}$

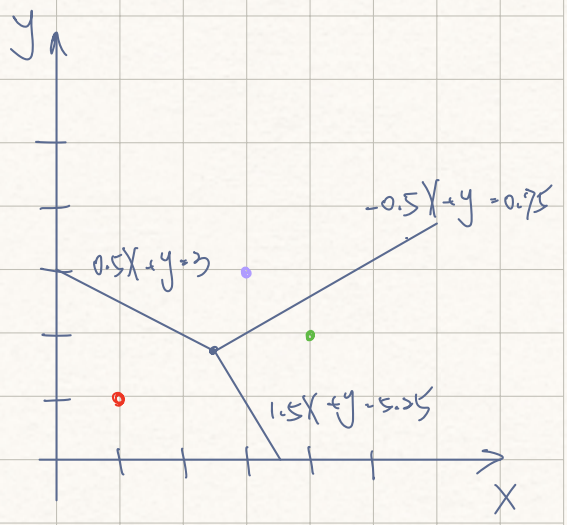
$\Rightarrow 0.5x + y = 3$

$(X_1, X_3) \Rightarrow \sqrt{\frac{1}{2}(x-1)^2 + (y-1)^2} = \sqrt{\frac{1}{2}(x-4)^2 + (y-2)^2}$

$\Rightarrow 1.5x + y = 5.5$

$(X_2, X_3) \Rightarrow \sqrt{\frac{1}{2}(x-3)^2 + (y-3)^2} + \sqrt{\frac{1}{2}(x-4)^2 + (y-2)^2}$

$\Rightarrow -0.5x + y = 0.75$ #



Problem 3

(a.) K-NN

In [ ]:
```
import numpy as np
from sklearn.neighbors import KNeighborsClassifier

# from sklearn.metrics import confusion_matrix
from sklearn.metrics import ConfusionMatrixDisplay
import matplotlib.pyplot as plt
```

In [ ]:
```
def plot_digit(data, label):
    plt.figure()
    plt.title('True label = ' + str(label))
    plt.imshow(np.reshape(data, (28, 28)), cmap='gray')
```

In [ ]:
```
train = np.loadtxt('10HW3_train.txt')
val = np.loadtxt('10HW3_validate.txt')
test = np.loadtxt('10HW3_test.txt')

train_x = train[:, 0:784]  # 0 ~ 783
train_y = train[:, 784]  # last one (784)
val_x = val[:, 0:784]
val_y = val[:, 784]
test_x = test[:, 0:784]
test_y = test[:, 784]
k = [1, 3, 5, 11, 16, 21]

# plot_digit(train_x[555], train_y[555])
```

In [ ]:
```
train_err = []
test_err = []
val_err = []
for i in k:
    knn = KNeighborsClassifier(n_neighbors=i)
    knn.fit(train_x, train_y)
    pre_train = knn.predict(train_x)
    pre_test = knn.predict(test_x)
    pre_val = knn.predict(val_x)

    train_err_i = round(1 - knn.score(train_x, train_y), 3)
    val_err_i = round(1 - knn.score(val_x, val_y), 3)
```

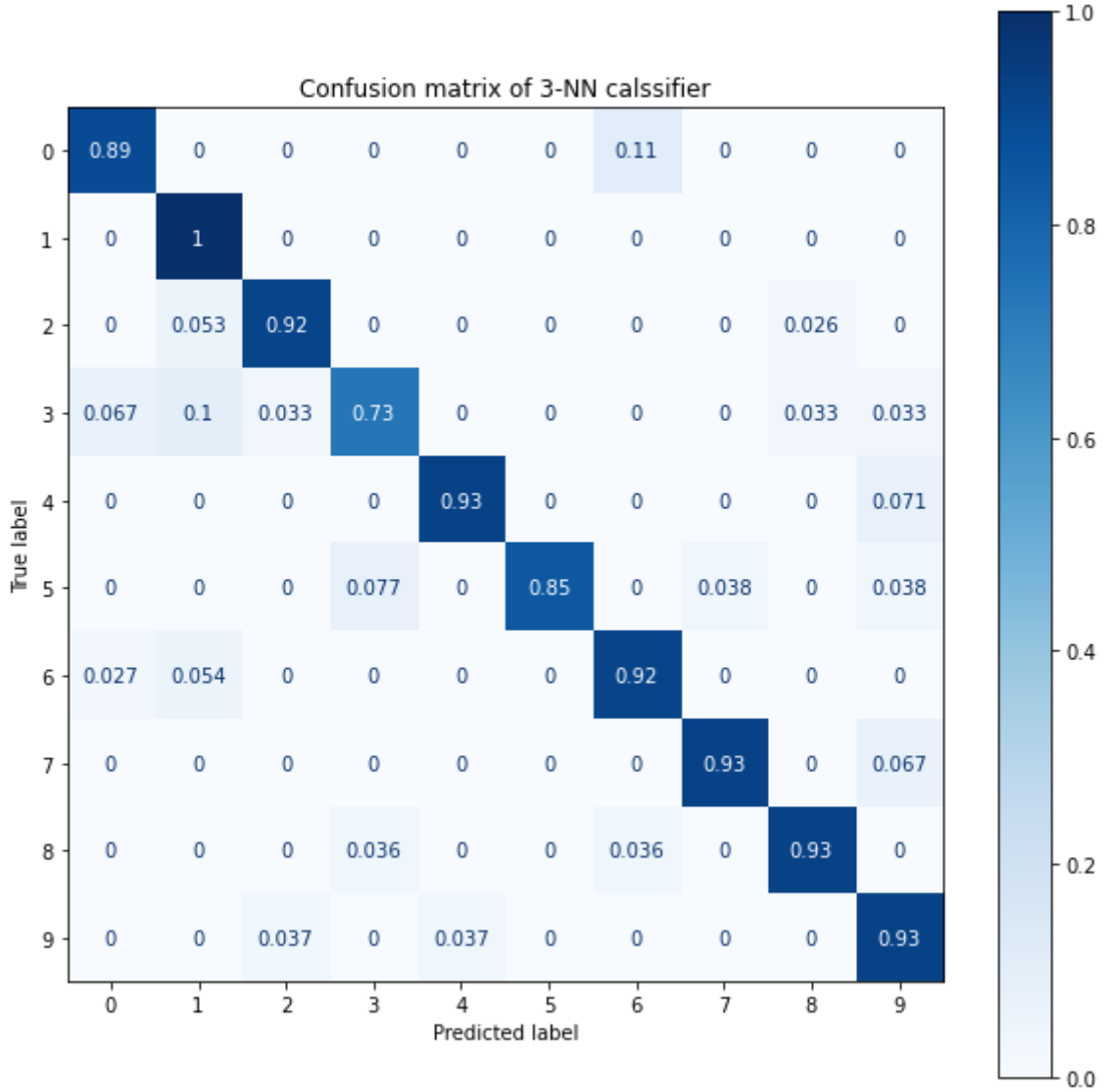
```
    test_err_i = round(1 - knn.score(test_x, test_y), 3)

    train_err.append(test_err_i)
    val_err.append(val_err_i)
    test_err.append(test_err_i)

print("For each of values of k:")
print(f"Training error  : {train_err}")
print(f"Validation error: {val_err}")
print(f"Testing error   : {test_err}")
```

```
For each of values of k
Training error  : [0.107, 0.097, 0.083, 0.127, 0.137, 0.153]
Validation error: [0.127, 0.143, 0.13, 0.173, 0.197, 0.203]
Testing error   : [0.107, 0.097, 0.083, 0.127, 0.137, 0.153]
```

(b.) 3-NN

In [ ]:

```
knn = KNeighborsClassifier(n_neighbors=3)
knn.fit(train_x, train_y)
pre_test = knn.predict(test_x)

class_names = np.arange(0, 10, step=1)
title = "Confusion matrix of 3-NN calssifier"

ax1 = plt.figure(figsize=(10, 10)).subplots()

disp = ConfusionMatrixDisplay.from_estimator(
    estimator=knn,
    X=test_x,
    y=test_y,
    display_labels=class_names,
    cmap=plt.cm.Blues,
    normalize="true",
    ax=ax1,
)
disp.ax_.set_title(title)
plt.show()
```



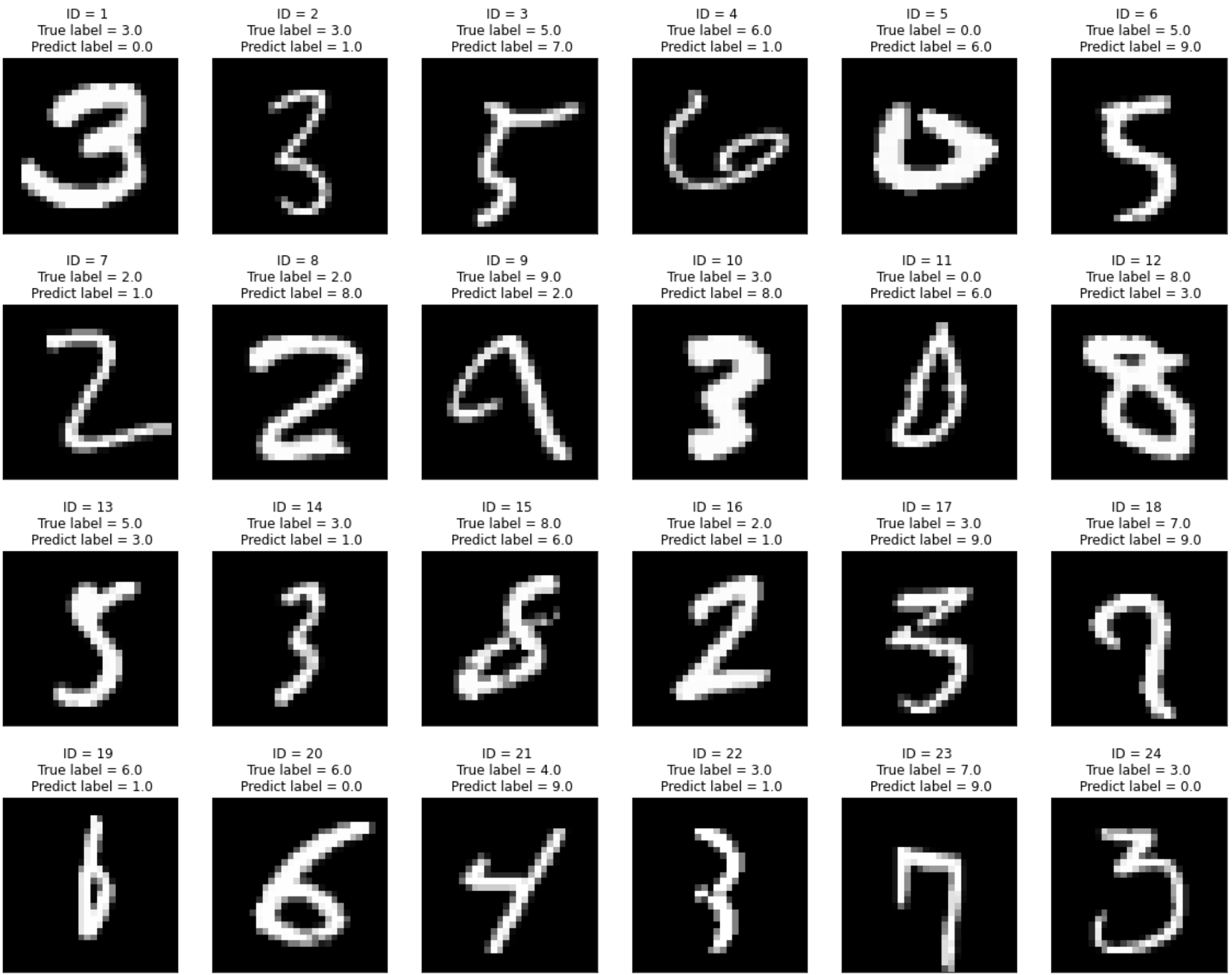
根據上方confusion matrix結果顯示，我認為數字1的分類最為簡單，因其正確率數值最高，並且最難辨識為數字3，其TP值僅有0.73

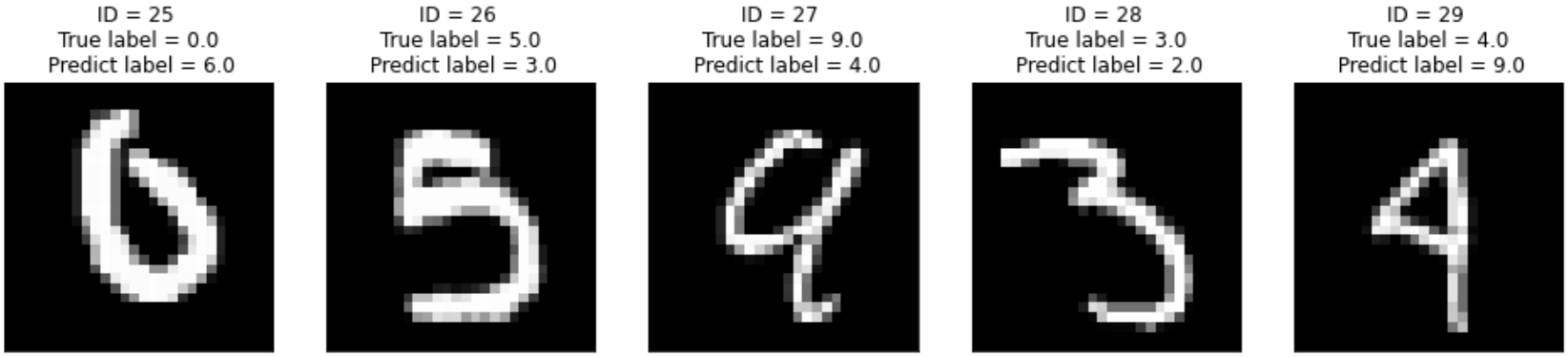
(c.) Error case

In [ ]:
```
err = test_y == pre_test
num = sum(err == False)
print(f"Totle error case: {num}")

# np.where will return tuple, which contain 2 element
index = np.where(err == False)[0]
```

```
plt.figure(figsize=(20, 20), facecolor='white')
for i_index, i_data in enumerate(index):
    plt.subplot(5, 6, (i_index + 1))
    plt.title(
        f"ID = {i_index+1} \n True label = {str(test_y[i_data])} \n Predict label = {str(pre_test[i_data])}"
    )
    plt.imshow(np.reshape(test_x[i_data], (28, 28)), cmap='gray')
    plt.xticks([]), plt.yticks([])

plt.show()
```

```
Totle error case: 29
```


ID = 1
True label = 3.0
Predict label = 0.0
ID = 2
True label = 3.0
Predict label = 1.0
ID = 3
True label = 5.0
Predict label = 7.0
ID = 4
True label = 6.0
Predict label = 1.0
ID = 5
True label = 0.0
Predict label = 6.0
ID = 6
True label = 5.0
Predict label = 9.0
ID = 7
True label = 2.0
Predict label = 1.0
ID = 8
True label = 2.0
Predict label = 8.0
ID = 9
True label = 9.0
Predict label = 2.0
ID = 10
True label = 3.0
Predict label = 8.0
ID = 11
True label = 0.0
Predict label = 6.0
ID = 12
True label = 8.0
Predict label = 3.0
ID = 13
True label = 5.0
Predict label = 3.0
ID = 14
True label = 3.0
Predict label = 1.0
ID = 15
True label = 8.0
Predict label = 6.0
ID = 16
True label = 2.0
Predict label = 1.0
ID = 17
True label = 3.0
Predict label = 9.0
ID = 18
True label = 7.0
Predict label = 9.0
ID = 19
True label = 6.0
Predict label = 1.0
ID = 20
True label = 6.0
Predict label = 0.0
ID = 21
True label = 4.0
Predict label = 9.0
ID = 22
True label = 3.0
Predict label = 1.0
ID = 23
True label = 7.0
Predict label = 9.0
ID = 24
True label = 3.0
Predict label = 0.0



將此次預測結果錯誤的結果全部繪製出來，共29筆資料，可發現有些確實是人眼當下就可以辨識出來的結果，此部分應再將KNN的內部參數調整好即可，如距離計算方式等等。

但同時也可發現如ID:5, 18, 25這幾張圖樣，我自己肉眼判斷也很難決定到底是甚麼數字，難以辨認