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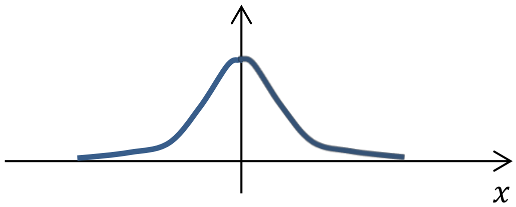
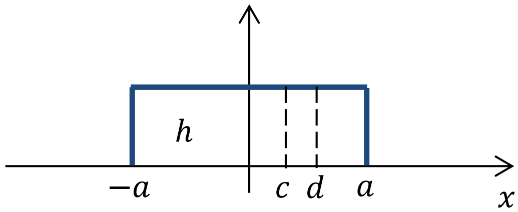
Homework Set 1

Problem 1 (Probability density function)



(A)



(B)

a) Figure (A) shows a Cauchy probability density function whose density is given by $\frac{1}{K}\frac{1}{1+x^2}$. What is the value of K for the density to be a probability density function? What are the mean and variance of the distribution?

b) Figure (B) shows a uniform probability density function. What is the height h of the density function? What are the mean and variance of the distribution? What is the probability that x lies in the interval between the vertical lines marked by c and d?

c) Consider two fair dice with six sides marked with the usual numbers 1 through 6. What is the probability that a throw of the dice results in a score of 7? What is the mean of the numbers that arise when two dice are thrown? What is the variance?

Problem 2 (Sample statistics and confidence intervals)
In characterizing the noise in an amplifier, which is normally distributed, we have the following noise voltages in micro Volts (μV):

-0.4326 -1.6656 0.1253 0.2877 -1.1465 1.1909 1.1892 -0.0376 0.3273 0.1746

a) Estimate the mean of the noise voltages and the variance of the mean.

b) Calculate the 95% and 99% confidence intervals of the mean (of noise voltages).

c) How confident are we that the noise voltage at any time lies between $1\mu V$ and $1.1\mu V$?

Problem 3 (Calculus)
Let $f(x,y) = 3x^2 + y^2 - xy - 11x$

a) Find $\frac{\partial f}{\partial x}$, the partial derivative of f with respect to x. Also find $\frac{\partial f}{\partial y}$.

b) Find the pair $(x,y) \in \Re^2$ that minimizes f.

c) Show that the pair (x,y) you found in b. is a minimizer instead of a maximizer.

Problem 4 (Vector Norms)
Compute the 0, 1, 2, and ∞ norms for $[3\ -1\ 3\ 5\ 0\ 2\ ]^T$.

Prob 1.

a. ①

若為 PDF, $\int f(x)\,dx = 1$

$\Rightarrow \int_{-\infty}^{\infty} \frac{1}{k}\cdot\frac{1}{1+x^2}\,dx = 1$

$\Rightarrow \frac{1}{k}\cdot \tan^{-1}(x)\Big|_{-\infty}^{\infty} = 1$

$\Rightarrow k = \tan^{-1}(x)\Big|_{-\infty}^{\infty}$

$= \frac{\pi}{2} - (-\frac{\pi}{2})$

$= \pi$ #

②

mean $= \mu$

$= \int x\, P(x)$

$= \int_{-\infty}^{\infty} x \cdot \frac{1}{\pi}\cdot\frac{1}{1+x^2}\,dx$

$= \frac{1}{\pi}\int \frac{x}{1+x^2}\,dx$

$= \frac{1}{\pi}\cdot\frac{1}{2}\ln(1+x^2)\Big|_{-\infty}^{\infty}$

$= 0$ #

③ varience $= \sigma^2$

$= \int (x-\mu)^2 P(x)$

$= \int (x-0)^2\cdot\frac{1}{\pi}\cdot\frac{1}{1+x^2}\,dx$

$= \frac{1}{\pi}\int \frac{x^2}{1+x^2}\,dx$

$= \frac{1}{\pi}\cdot\int (\frac{1+x^2}{1+x^2} - \frac{1}{1+x^2})\,dx$

$= \frac{1}{\pi}(x - \tan^{-1}x)\Big|_{-\infty}^{\infty}$

b. ①

若為 PDF, $\int f(x)\,dx = 1$

$\Rightarrow \int f(x)\,dx = 1$

$\Rightarrow h(a-(-a)) = 1$

$\Rightarrow h = \frac{1}{2a}$ #

②

$f(x) = \begin{cases} h, & -a \le x \le a \\ 0, & x \le -a,\ x \ge a \end{cases}$

$\Rightarrow \begin{cases} \mu = \\ \sigma^2 = \end{cases}$

③

c. ①

all chances $= 6\times 6 = 36$

score 7 =
1 − 6
2 − 5
3 − 4
4 − 3
5 − 2
6 − 7

$\Big\}$ 6

$P = \frac{6}{36} = \frac{1}{6}$ #

②

$E[x] = \sum x\, \frac{1+2+3+4+5+6}{6}$

$= 7$ #

③ σ^2

Prob 2.

-0.4326 -1.6656 0.1253 0.2877 -1.1465 1.1909 1.1892 -0.0376 0.3273 0.1746

a. mean $= (-0.4326 - 1.6656 + 0.1253 + 0.2877 - 1.1465 + 1.1909 + 1.1892 - 0.0376 + 0.3273 + 0.1746)/10$

$= 0.0127/10 = 0.00127\ (\mu V)$ #

variance $= \frac{1}{10-1}\sum_{i=1}^{10}(X_i - 0.00127)^2$

$= 0.816194$ #

b. $\sigma^2 = 0.816194$

$\Rightarrow \sigma = 0.903$

$$\Rightarrow \begin{cases} 95\%\ CI = \mu \pm 1.96\left(\frac{\sigma}{\sqrt{n}}\right) = 0.00127 \pm 1.96\left(\frac{0.903}{3.162}\right) = \begin{cases} 0.561 \\ -0.558 \end{cases} \# \\ 99\%\ CI = \mu \pm 2.58\left(\frac{\sigma}{\sqrt{n}}\right) = 0.00127 \pm 2.58\left(\frac{0.903}{3.162}\right) = \begin{cases} 0.738 \\ -0.735 \end{cases} \# \end{cases}$$

c. $1\ \mu V$

$\mu +$

Prob 3. $f(x,y) = 3x^2 + y^2 - xy - 11x$

a. $\frac{\partial f}{\partial x} = 6x - y - 11$ #

$\frac{\partial f}{\partial y} = 2y - x$ #

b.

c.

Prob 4.

$[3, -1, 3, 5, 0, 2]^T = \begin{bmatrix} 3 \\ -1 \\ 3 \\ 5 \\ 0 \\ 2 \end{bmatrix}$

L_0 norm : cause there are 5 non-zero elements in the vector,

so the L_0 norm equal 5 #

L_1 norm : $3 + 1 + 3 + 5 + 0 + 2 = 14$ #

L_2 norm $= \sqrt{3^2 + 1^2 + 3^2 + 5^2 + 0^2 + 2^2} = \sqrt{48} = 4\sqrt{3}$ #

L_∞ norm $= \sqrt{3^\infty + 1^\infty + 3^\infty + 5^\infty + 0^\infty + 2^\infty} = \sqrt[\infty]{5^\infty} = 5$ #