a.) $f(x,y,z) = (x+y+z)^2$, constraint : $g(x,y,z): x^2+2y^2+3z^2 = 1$

$= x^2+y^2+z^2+2xy+2xz+2yz$ $\quad \hookrightarrow x^2+2y^2+3z^2-1=0$

$$\begin{cases} \dfrac{\partial f}{\partial x} = 2x+2y+2z = \lambda 2x \\ \dfrac{\partial f}{\partial y} = 2y+2x+2z = \lambda 4y \\ \dfrac{\partial f}{\partial z} = 2z+2x+2y = \lambda 6z \\ g(x,y,z) = x^2+2y^2+3z^2-1 = 0 \end{cases}$$

$$\Rightarrow \begin{cases} x+y+z = 2\lambda x \\ x+y+z = 4\lambda y \\ x+y+z = 6\lambda z \end{cases}$$, 重合 $\Rightarrow$ ∞ 解 #

(b). $f(x,y,z) = xy + z^2$ ①, $g(x,y,z) = x^2+y^2+z^2-1=0$ ②, find min.

$$\begin{array}{ll} \frac{df}{dx}: & y = \lambda\cdot(2x) \\ \frac{df}{dy}: & x = \lambda(2y) \Rightarrow y = \frac{x}{2\lambda} \\ \frac{df}{dz}: & 2z = \lambda(2z) \Rightarrow z = \lambda z \Rightarrow z=0 \\ g(x,y,z) & x^2+y^2+z^2-1=0 \end{array}$$

$\Rightarrow x^2+y^2+z^2=1$

$\Rightarrow x^2+y^2=1$

$\Rightarrow (2\lambda x)^2 + \left(\frac{x}{2\lambda}\right)^2 = 1$

$\Rightarrow 4\lambda x^2 + \frac{x^2}{4\lambda^2} = 1$

$\Rightarrow \lambda = \frac{1}{2}$

$\begin{cases} x = -y \\ x^2+y^2=1 \end{cases} \Rightarrow \begin{cases} x=-y \\ x^2 = 0.5 \end{cases}, \quad x = \pm\frac{1}{\sqrt{2}}$

$\Rightarrow f(x,y,z) = xy + z^2$

$= \frac{1}{\sqrt{2}}\cdot\left(-\frac{1}{\sqrt{2}}\right) = -\frac{1}{2}$ #