HW3 Jui-Hung Lu 933293709 .

1. $\sum_{i=1}^{k=2n-1} i!$ where i in the range $1 \sim 2n-1$ step 2

So the times is $1+3+\cdots+2n-3+2n-1$

$= \frac{2n(2n-1)}{2\times 2} = n^2 - \frac{n}{2}$ times ※

$X_n = \Theta(n^2)$

2. (a.) $GCD(233,144)$
$=GCD(144,89)$
$=GCD(89,55)$
$=GCD(55,34)$
$=GCD(34,21)$
$=GCD(21,13)$
$=GCD(13,8)$
$=GCD(8,5)$
$=GCD(5,3)$
$=GCD(3,2)$
$=GCD(2,1)$
$=1$

(b)

$f_n = f_{n-1} + f_{n-2}$

$gcd(f_n, f_{n-1}) = gcd(f_{n-1}+f_{n-2}, f_{n-1})$
$= gcd(f_{n-1}, f_{n-2})$
$\vdots$
and so on

Therefore we can know $gcd(f_n, f_{n-1}) = gcd(f_1, f_0)$
$= 1$

(c)

$$GCD(f_n, f_{n-1}) = \begin{cases} 1 & n=1 \\ GCD(f_n, f_{n-1}) & n>1 \end{cases}$$

(d) $GCD(f_n, f_{n-1}) = \alpha\lambda^n$,

$\Rightarrow GCD(f_{n-1}, f_{n-2}) = \alpha\lambda^{n-1}$

$\because GCD(f_n, f_{n-1}) = GCD(f_{n-1}, f_{n-2})$

$\therefore \quad \alpha\lambda^n = \alpha\lambda^{n-1}$

$\lambda = 1$ or $\underset{(\times)}{0}$

$\Rightarrow \because GCD(f_1, f_0) = 1$

$\therefore \alpha = 1$

$\Rightarrow GCD(f_n, f_{n-1}) = 1$ ※

3 (picture is in next page)

(a)

$\deg(P)=0 \Rightarrow V_n=0$

$\lambda_1=3$

$\Rightarrow x_n=a_1 3^n$

$x_0=a_1 3^0=1$

$\Rightarrow a_1=1$

$\Rightarrow x_n=3^n \Rightarrow x_n=O(3^n)$

in nonnegative DE, $\lim_{n\to\infty}\left(\frac{x_n}{\lambda_1^n}\right)^n=\lim_{n\to\infty}\left(\frac{3^n}{3^n}\right)^n=1$

$\Rightarrow x_n=O(3^n)$

$\therefore$ same as predicted

(b) $x_n=2x_{n-1}-1,\ x_0=0$

$\lambda_n=2$

$\Rightarrow x_n=a2^n+\lambda p=a\cdot 2^n+bn+c$

$\begin{cases} x_0=a+c=0 \\ x_1=2a+b+c=-1 \\ x_2=4a+2b+c=-3 \end{cases} \Rightarrow$ $a=-1,\ b=0,\ c=1$ $\Downarrow$ $x_n=-2^n+1$

$O(x_n)=O(-2^n)$

(c) $x_n=2x_{n-1}-1,\ x_0=1$

$\lambda_n=2$

$\Rightarrow x_n=a\cdot 2^n+bn+c$

$\begin{cases} x_0=a+c=1 \\ x_1=2a+b+c=1 \\ x_2=4a+2b+c=1 \end{cases} \Rightarrow$ $a=0,\ b=0,\ c=1$ $\Downarrow$ $x_n=1$

$O(x_n)=O(1)$

(d) $x_n=5x_{n-1}-4n+1,\ x_0=1$

$\lambda_n=5$

$\Rightarrow x_n=a\cdot 5^n+bn+c$

$\begin{cases} x_0=a+c=1 \\ x_1=5a+b+c=2 \\ x_2=25a+2b+c=3 \end{cases} \Rightarrow$ $a=0,\ b=1,\ c=1$ $\Downarrow$ $x_n=n+1$

$O(x_n)=O(n)$

(e) Non-homo DE

$g(n)=2n+1,\ \lambda=1$

$\Rightarrow g(n)=\Theta(n^1\cdot 1^n)$

$\Rightarrow x_n=\Theta(n^{1+1}\cdot 1^n)=\Theta(n^2)$ same as predicted

$x_n=x_{n-1}+2n+1,\ x_0=1$

$\lambda_n=1$

$V_n=V_{n-1}+2n+1$

$V_n=n(an+b)$

$an^2+bn=(n-1)(an-a+b)+2n+1$

$(-2a+2)n+(a-b+1)=0$

$a=1,\ b=2$

$\Rightarrow x_n=n^2+2n+a1^n$

$x_0=1=a\cdot 1$

$a=1$ $\Rightarrow x_n=n^2+2n+1 \Rightarrow O(n^2)$

(f) non-homo DE

$g(n)=3^n,\ \lambda=2$

$g(n)=\Theta(3^n)$ with $r=3>\lambda=2$

$\Rightarrow x_n=\Theta(r^n)=\Theta(3^n)$

same as predict

$x_n=2x_{n-1}+3^n,\ x_0=7$

$\lambda_0=2$

$\lambda_n=a2^n$

$V_n=2V_{n-1}+3^n$

$V_n=b3^n$

$b3^n=2b3^{n-1}+3^n$

$(b-3)3^n=0$

$b=3$

$\Rightarrow x_n=a2^n+3\cdot 3^n$

$7=a+3 \Rightarrow a=4$

$x_n=4\cdot 2^n+3\cdot 3^n$

$O(x_n)=O(3^n)$

(g) non-homo DE

$g(n)=2^n,\ \lambda=2$

$g(n)=(n^0\cdot 2^n)$

$\Rightarrow x_n=\Theta(n^{d+1}\cdot\lambda^n)$

$=\Theta(n\cdot 2^n)$ $\Rightarrow$ same as predict

$x_n=2x_{n-1}+2^n,\ x_0=7$

$\lambda_0=2$

$\lambda_n=a2^n$

$V_n=2V_{n-1}+2^n$

$V_n=bn2^n$

$bn2^n=2b(n-1)2^{n-1}+2^n$

$bn=bn-b+1$

$b=1$

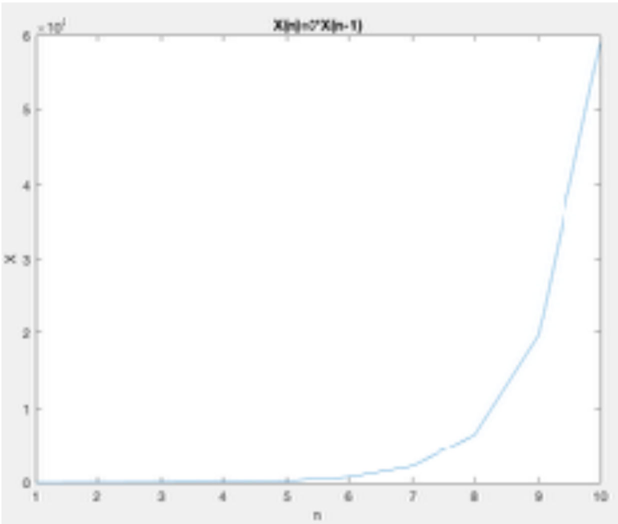
$\Rightarrow x_n=a2^n+n2^n$

$7=a$

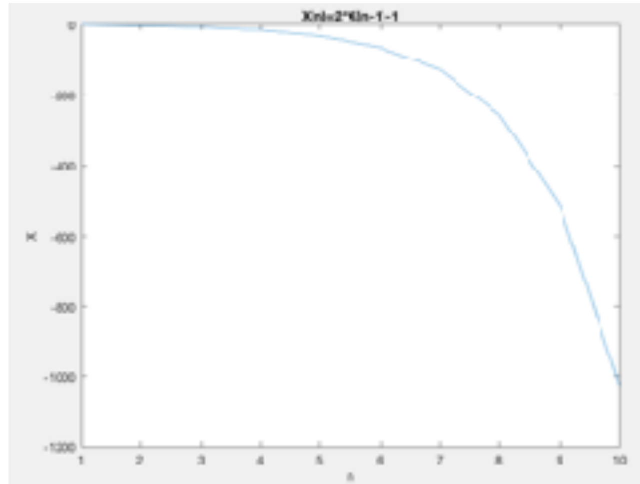
$\Rightarrow x_n=7\cdot 2^n+n2^n \Rightarrow O(x_n)=O(n2^n)$

3.

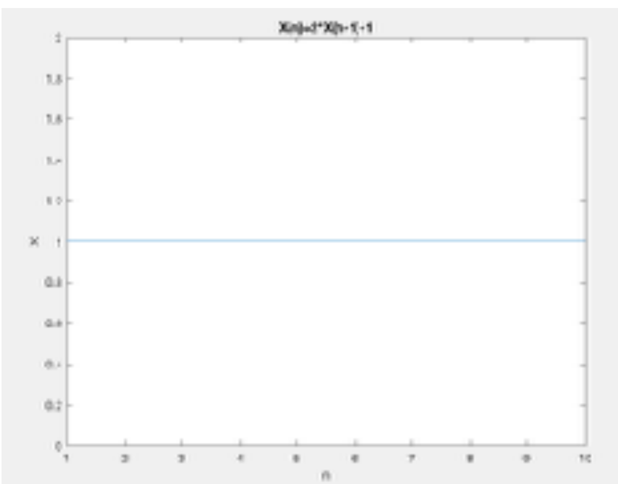
(a)



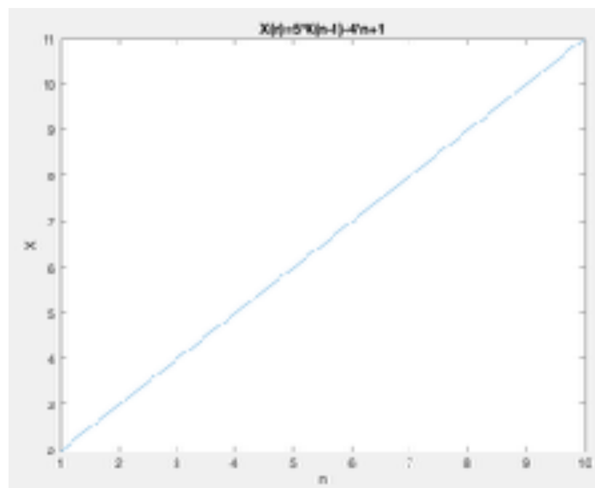
(b)



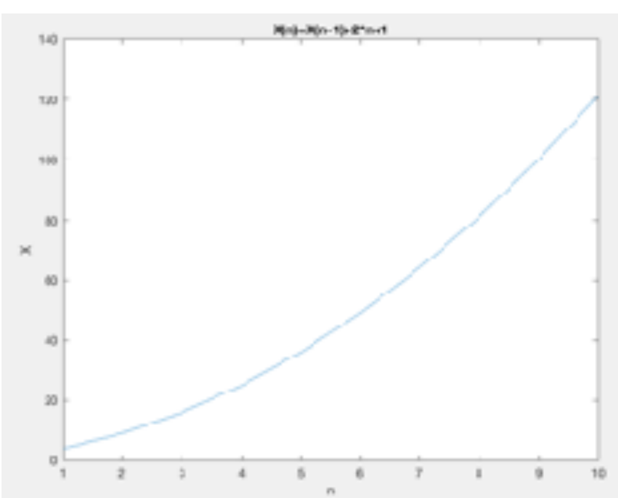
(c)



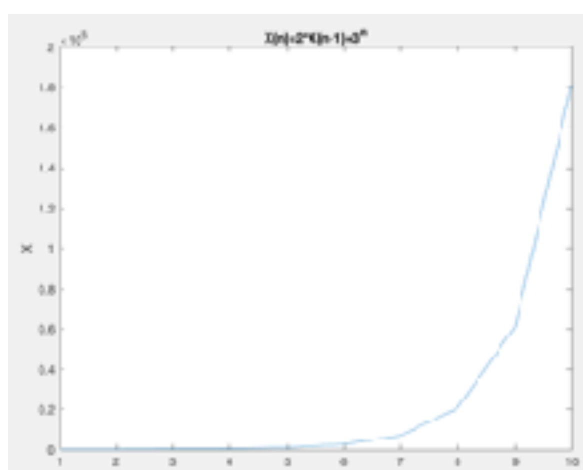
(d)



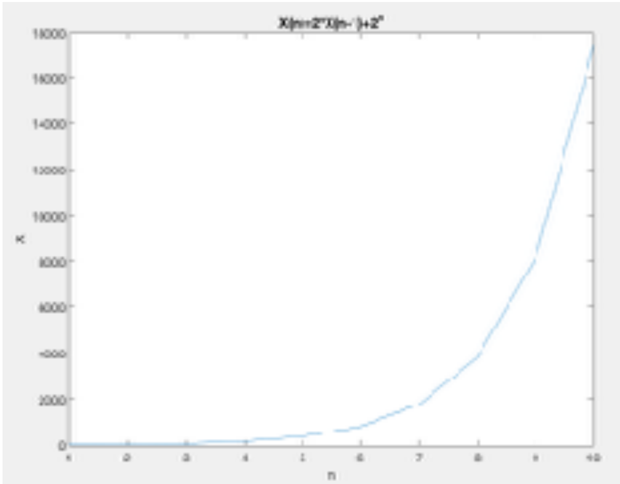
(e)



(f)



(g)


X(n)=2*X(n-1)+2^n
18000
16000
14000
12000
10000
8000
6000
4000
2000
0
x
1
2
3
4
5
6
7
8
9
10
n

4. assume $T(n) = \alpha\lambda_0^n$

$\Rightarrow \alpha\lambda_0^n = \alpha\lambda_0^{n-1} + \alpha\lambda_0^{n-2} + \alpha\lambda_0^{n-3}$

$\lambda_0^3 = \lambda_0^2 + \lambda_0^1 + 1$

$\because T(1) > 0,\ T(2) > 0,\ T(3) > 0$

so root $T(1) = \Theta(\lambda_0^1)$

Therefore we can assume

$T(n) = \Theta(\lambda_0^n)$

$T(n+1) = T(n) + T(n-1) + T(n-2)$

$\Rightarrow T(n+1) = \lambda_0^n + \lambda_0^{n-1} + \lambda_0^{n-2} = \lambda_0^{n-2}(\lambda_0^2 + \lambda_0 + 1)$

$= \lambda_0^{n+1}$

$\Rightarrow T(n) = \Theta(\lambda_0^n)$

$\Rightarrow$ assumption is right

※

5.

min: $X_1 = 1,\ X_2 = 2,\ X_3 = 4,\ X_4 = 7,\ X_5 = 11 \ldots X_n = X_{n-1} + X_{n-2} + 1$

max: $X_1 = 1,\ X_2 = 3,\ X_3 = 7,\ X_4 = 15,\ \cdots\ X_n = 2^{n+1} - 1$

$X_n = X_{n-1} + X_{n-2} + 1$

$X_1 = 1$

$X_2 = 2$

$\lambda_n^2 - \lambda_n - 1 = 0$

$\lambda = \frac{1 \pm \sqrt{5}}{2}$

$\Rightarrow f_n = a\left(\frac{1+\sqrt{5}}{2}\right)^n + b\left(\frac{1-\sqrt{5}}{2}\right)^n + cn + d$

$\lambda_p = cn + d \Rightarrow c(n-1) + d + c(n-2) + d + 1$

$c = 0,\ d = -1$

$$\begin{cases} 1 = a + b - 1 \\ 2 = a \cdot \frac{1+\sqrt{5}}{2} + b \cdot \frac{1-\sqrt{5}}{2} - 1 \end{cases}$$

$\Rightarrow a = 1 + \frac{2}{5}\sqrt{5},\ b = 1 - \frac{2}{5}\sqrt{5}$

$n(h) = \left(1 + \frac{2}{5}\sqrt{5}\right)\left(\frac{1+\sqrt{5}}{2}\right)^h + \left(1 - \frac{2}{5}\sqrt{5}\right)\left(\frac{1-\sqrt{5}}{2}\right)^h - 1 \Rightarrow \min$

$2^{h+1} - 1 \Rightarrow \max$

DE of max:

$= 2^{h+1} - 1 = 2(2^{h+1-1}) + 1$

$\Rightarrow X_n = 2X_{n-1} + 1,\ X(0) = 1$

DE of min:

$X_h = X_{h-1} + X_{h-2} + 1,\ X(0) = 1$

$\Rightarrow \left(1 + \frac{2}{5}\sqrt{5}\right)\left(\frac{1+\sqrt{5}}{2}\right)^h + \left(1 - \frac{2}{5}\sqrt{5}\right)\left(\frac{1-\sqrt{5}}{2}\right)^h - 1 \leq V(h) \leq 2^{h+1} - 1$

※

6.
$F(n)=((1+\sqrt{5})/2)^n+((1-\sqrt{5})/2)^n \Rightarrow O(((1+\sqrt{5})/2)^n+((1-\sqrt{5})/2)^n)=O(((1+\sqrt{5})/2)^n)$

There are n stack frame
We use recursive to do
In each time, before we make f(n), we have f(n-2)+f(n-3) notches
When we make f(n), we need add [f(n-1)+ f(n-2)]-[f(n-2)+f(n-3)]=2f(n-2)+f(n-3)-[f(n-2)+f(n-3)]=f(n-2) notches

Therefore
The DE can be f(n)=f(n-1)+2F(n-2) T(n)

$=O(2^n+(-1)^n)=O(1+\sqrt{2})^n=O(2)$

Therefore, space complexity is O(2n)