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1. $f(n) = \Theta(g(n))$

$\Rightarrow$ there must be C_1, C_2 & n_0 let $C_1 \times g(n) \leq f(n) \leq C_2 \times g(n)$, $\forall\, n \geq n_0$

in this case, $f(n) = \log_b n = \frac{\log_2 n}{\log_2 b} \leq C_2 \log_2 n$

$\Rightarrow \because 0 \leq C_2 \leq \infty$ & $0 \leq \frac{1}{\log_2 b} \leq \infty$

$\therefore \log_b n \leq c \log_2 n \Rightarrow g(n) = \log_2 n$

$\Rightarrow \log_b n = \Theta(g(n)) = \Theta(\log_2 n)$ ※

2. For upper bound: $1^{15} + 2^{15} + \cdots + n^{15} \leq n^{15} + n^{15} + \cdots + n^{15} = n n^{15} = n^{16}$

For lower bound: $1^{15} + 2^{15} + \cdots + \left(\frac{n}{2}\right)^{15} + \cdots + n^{15} \leq \left(\frac{n}{2}\right)^{15} + \left(\frac{n}{2}\right)^{15} + \cdots + \left(\frac{n}{2}\right)^{15} \geq \frac{n}{2}\left(\frac{n}{2}\right)^{15} = \left(\frac{n}{2}\right)^{16}$

$\Rightarrow f(n) = \sum_{i=1}^{n} i^{15}$, $g(n) = n^{16}$

$\Rightarrow \left(\frac{1}{2}\right)^{16} g(n) \leq f(n) \leq g(n)$

$\Rightarrow f(x) = \Theta(g(n)) = \Theta(n^{16})$

3. $f(n) = n! = 1 \times 2 \times \cdots \times n \leq n \times n \times \cdots \times n = n^n = g(n)$

$\Rightarrow f(n) = O(g(n)) = O(n^n)$

$n^n \neq \Theta(n!)$ if $n^n \neq O(n!)$ or $n^n \neq \Omega(n!)$

if $n^n = n \times n \times \cdots \times n \leq C_2 n!$ for $C_2 > 0$

we cannot find such C_2

$\Rightarrow n^n \neq O(n!)$

$\Rightarrow n^n \neq \Theta(n!)$

4. $F_n = \begin{cases} 0 & \text{if } n=0 \\ 1 & \text{if } n=1 \\ F(n-1)+F(n-2) & \text{if } n>1 \end{cases}$

for $n > 1$, $2F_{n-2} < F_n < 2F_{n-1}$

$\Rightarrow 2 \cdot 2F_{n-4} < F_n < 2 \cdot 2F_{n-2}$

$\Rightarrow 4 \cdot 2F_{n-6} < F_n < 4 \cdot 2F_{n-3}$

$\Rightarrow 8 \cdot 2F_{n-8} < F_n < 8 \cdot 2F_{n-4}$

lower bound rate

$\Rightarrow 2^{k_1} F_{n-2k_1} < F_n < 2^{k_2} \cdot F_{n-k_2}$

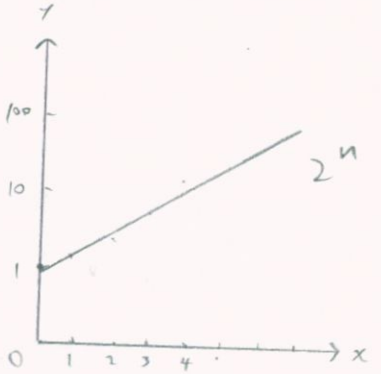
$\Rightarrow 2^{\frac{n-1}{2}} \cdot F_1 < F_n < 2^{n-1} \cdot F_1$ upper bound rate

Therefore $\Rightarrow$ Fibonacci numbers grow exponentially

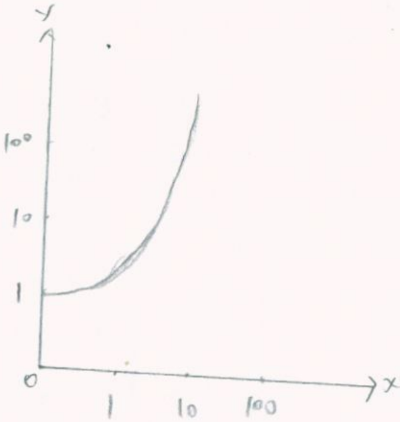
5.

(a) 2^n

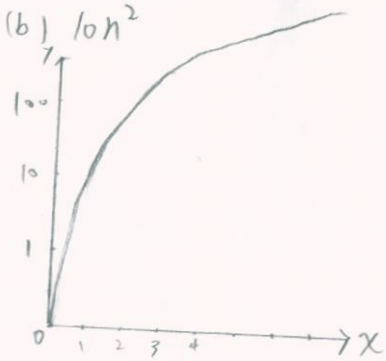
semi-log

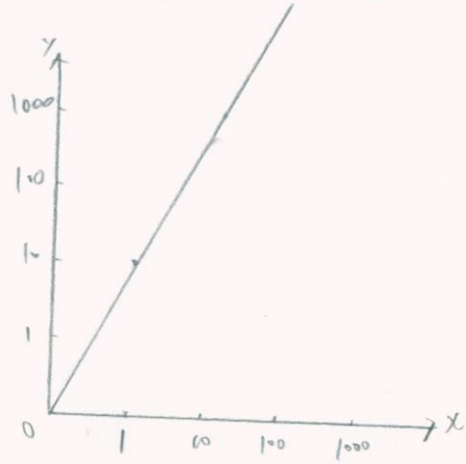


log-log

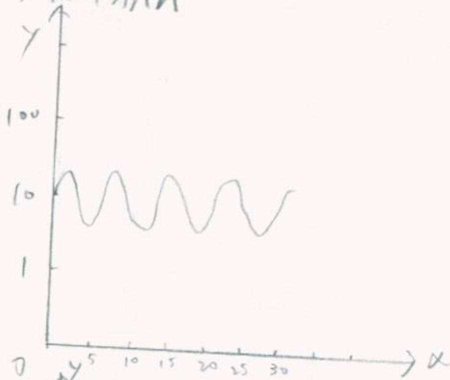


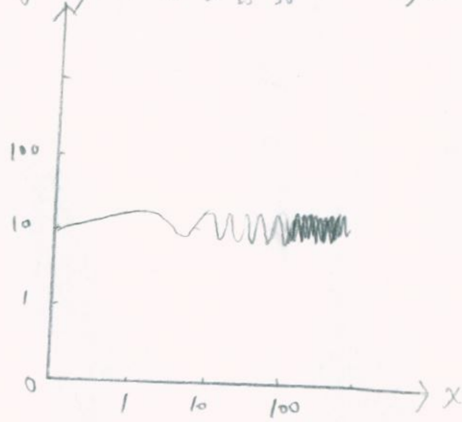
(b) $\log n^2$





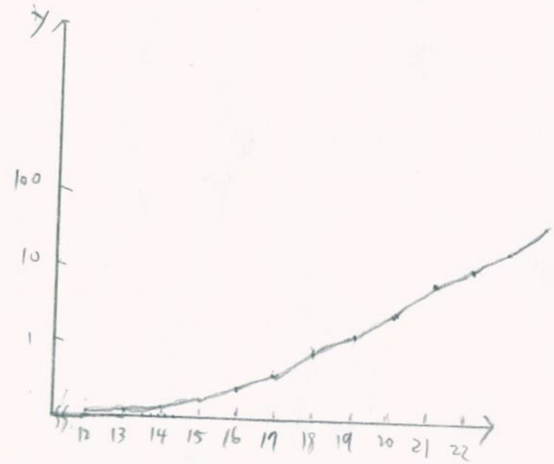
(c) $10+\sin n$





=> exponential function in semi-log plot & polynomial function in log-log plot give straight lines

6.



the best plot is semi-log plot

because the execution time become twic bigger in next input

The complexit of this function is exponential

because the execution time almost become double when input size add one

7. $g(b) = F(A, B)$ => move the round up number from B to A

if $g(0)$, move 0 from B to A => true

if $g(1)$, move 1 from B to A => true

if $m > 2$, move $m-1$ from B to A => true

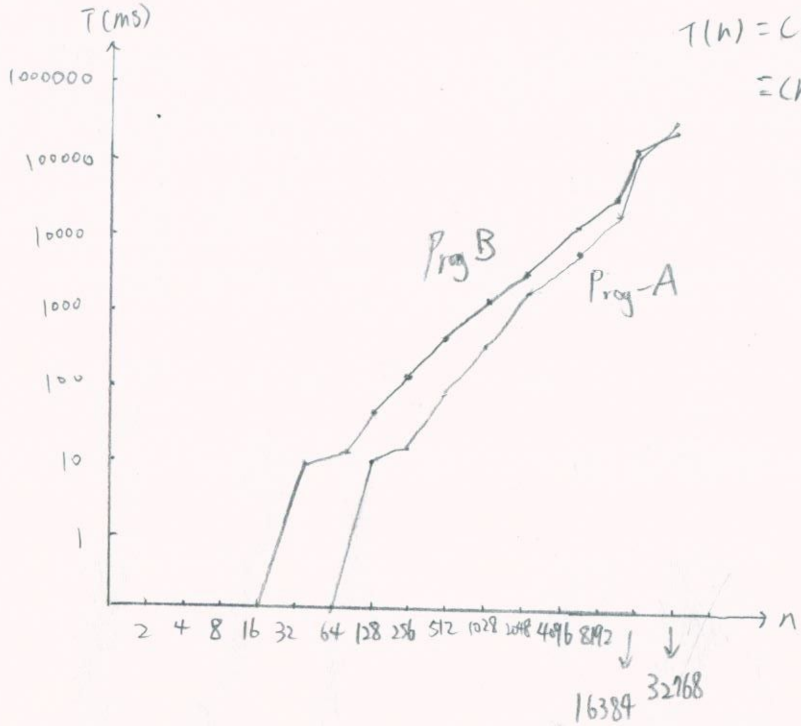
8. when $n > 13$, we can found that the T will double when $n+1$,

then we can know $T(30) = 301652 \times 2^{30-17}$

$= 301652 \times 2^{13}$

$= 2471133184$

9.



$T(n) = c\,g(n)$

$= cn^k$ in A:

8000 $\downarrow \times 4 = 2^2$

32000 $\downarrow \times 4 = 2^2$

128094 $\downarrow \times 4 = 2^2$

512376

$\Rightarrow k_A = 2$

in B:

16812 $\downarrow \times 3 = 2^{1.6}$

50438 $\downarrow \times 3 = 2^{1.6}$

151578 $\downarrow \times 3 = 2^{1.6}$

454734

$\Rightarrow k_B = 1.6$

$\therefore$ Prog $A = \Theta(n^2)$

Prog $B = \Theta(n^{1.6})$

$\therefore$ Prog A will be expected to be asymptotically faster