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1. Redutions

(a) $x \times y = \frac{(x+y)^2 - x^2 - y^2}{2}$ => multiplcation ≤ squaring

(b) $y^2 = y \times y$ => squaring ≤ multiplcation

(c) $\frac{1}{x} - \frac{1}{y} = \frac{y-x}{xy}$ => if $y = x+1$

$\frac{1}{x} - \frac{1}{x+1} = \frac{1}{x^2+x}$ => $x^2 = \frac{1}{\frac{1}{x} - \frac{1}{x+1}} - x$

=> squaring ≤ reciprocal

(d) according (a) (b)

multiplcation & squaring are equal

but we can't reduce reciprocal from squaring

2. Lucas Numbers

$L_n = L_{n-1} + L_{n-2}$, $L_0 = 2$, $L_1 = 1$ => $L_2 = 3$, $L_3 = 4$, ...

$F_n = 0\ 1\ 1\ 2\ 3\ 5\ 8 \cdots$

Fibonacci is in $P(O(1.618)^n)$

$L_n = 2\ 1\ 3\ 4\ 7\ 11 \cdots$

L_n is reducible to F_n => L_n is in P ∵ F_n is in P

=> L_n runs poly time in K

3. (a) assume x is positive integer

$x^2 - x - 1 = 0$

$x(x-1) = 1$ => we cannot find a positive integer that can satisfy both $x=1$ & $(x-1)=1$

=> no positive integer solutions

(b) assume $x = \frac{p}{q}$

=> $\frac{p^2}{q^2} - \frac{p}{q} - 1 = 0$ ∵ $GCD(p,q) = 1$

$p^2 - pq - q^2 = 0$ ∴ p & q cannot be even in the same time

$p^2 - pq - q^2$

(p, q) = (odd, odd) => odd − odd − odd => odd

(p, q) = (odd, even) => odd − even − even => odd

(p, q) = (even, odd) => even − even − odd => odd

=> contradiction ⇓ no rational solution

4. $A(n) = A(n-1) + 1 + \frac{2}{n}$

$\Rightarrow A(n) = (n-1) + \sum_{j=1}^{n} \frac{2}{j}$

$= (n-1) + 2\log n + const$

best : $n-1$

worst : $2(n-1)$ $\Rightarrow$ near best.

5. F(x, n, MAX, MIN)
   if n=1, MAX = X[0], MIN = X[0]
   if n=2, if X[0] > X[1]
           MAX = X[0], MIN = X[1]
       else MAX = X[1], MIN = X[0]
   if n>2, split x into A, B
       F(A, $\frac{n}{2}$, MAXA, MINA)
       F(B, $\frac{n}{2}$, MAXB, MINB)
       Compare(MAXA, MAXB)
       Compare(MINA, MINB)

$T(n) = 2T(\frac{n}{2}) + 2$

$= 2^k T(\frac{n}{2^k}) + \frac{2(1-2^k)}{1-2}$

$= 2^k T(\frac{n}{2^k}) + 2 \cdot 2^k - 2$

let $2^k = \frac{n}{2}$, $2^{k+1} = n$, $k = \log n - 1$

$\Rightarrow T(n) = \frac{n}{2} \cdot T(2) + 2 \cdot \frac{n}{2} - 2$

$\because T(2) = 1$

$\therefore T(n) = \frac{3}{2}n - 2$ #

6. (a) assume $E(x_1, x_2, \cdots, x_n)$ is satisfiable

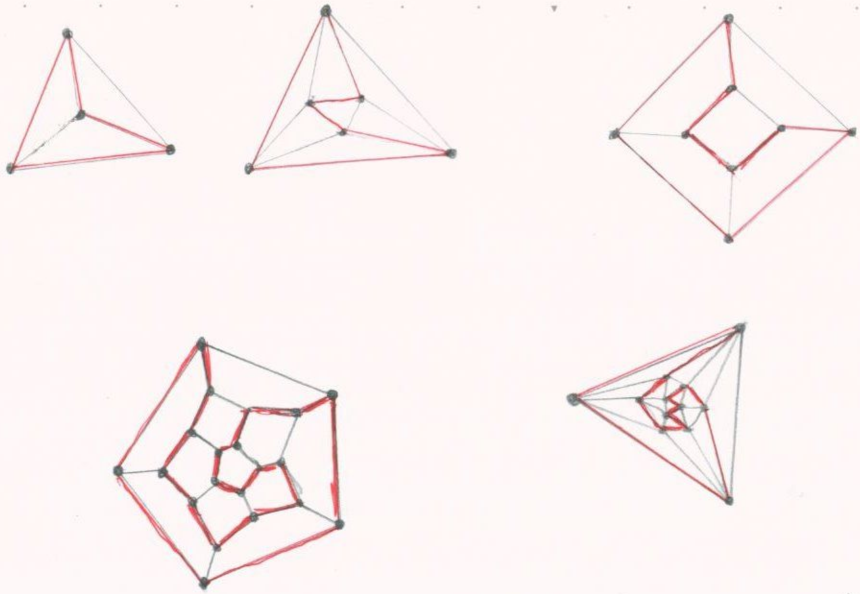
FIND(D($x_1, x_2, \cdots x_n$), i){
    set $D(x_i) = 1$
    if YS(D) = YES
        $D(x_i) = 1$
    else $D(x_i) = 0$
    call FIND(D, i+1)}

(b) $FIND(n) = FIND(n-1) + O(n^k)$

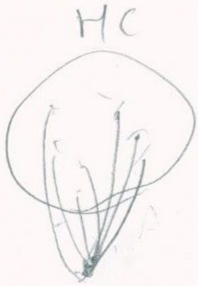
$= FIND(1) + n \cdot O(n^k)$

$\Rightarrow FIND(n) = O(n^{k+1})$ #

7.



8.
(a)
① HP is NP



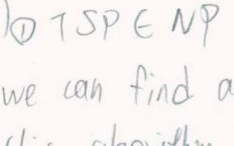
② $HC \leq HP$
$G : u\ v_1 v_2 \cdots v_n$
$G' : u\ v_1 v_2 \cdots v_n w$

(b)
① $HC \in NP$

② $HP \leq HC$
$G : v_1, v_2, \cdots v_m$
$G' : u\ v_1, v_2, \cdots, v_m$

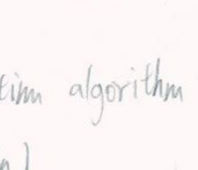
(c) ① $TSP \in NP$
we can find a poly time algorithm according to www.slideshare.net/emrecankucukoglu/tsp-isnpcomplet
this algorithm is $O(n)$

② $HC \leq_p TSP$



$G(V,E)$

=>



$G'(V,E')$

Convert G to $G' = (V, E' = V \times V, d)$,
$B = 0$ of TSP

$d_{c_i, c_j} = \{ 0$ if edge $(c_i, c_j) \in E$,
1 otherwise$)$

Time of reduction is $O(n^2)$: poly time

9.
let x be G_1 & G_2 and y be $\{i_1, i_2, \cdots i_n\}$

An algorithm $A(x,y)$ verifies graph-isomorphism by:

(a) check y is a permutation of $\{1, 2, \cdots, n\}$

if no, return false,

else continue

(b) permute the vertices of G_1

verify the permuted G_1 is identical of G_2

$$\left.\begin{array}{l} \text{step (a)} = O(V^2) \\ \text{step (b)} = O(V+E) \end{array}\right\} \Rightarrow A \text{ runs } O(V^2) \Rightarrow \text{graph-isomorphism} \in NP$$

10.

(a)

	V_1	V_2	V_3
V_1	0	1	0
V_2	1	0	1
V_3	0	1	0

	V_1	V_2	V_3
V_1	0	0	1
V_2	0	0	1
V_3	1	1	0

	V_1	V_2	V_3
V_1	0	1	1
V_2	1	0	0
V_3	1	0	0

$C_N = 001, 001, 110 = 78$

(b) $\because$ C_N is easy

$C_N(G_1) = C_N(G_2)$ is easy, too.

The 2 C_N is different & not isomorphic

if I know C_N

$\Rightarrow$ we can know where they are isomorphic $\rightarrow$ If find the C_N of a graph is easy

$\Rightarrow$ graph isomorphism is easy

(c) if x is the number after unroll G and is binary

if $I > x$, I must bigger then C_N

$\because x \geq C_N$ & $I \leq x$ & $I \leq C_N$

$\therefore$ it is easy to say "No" that $I < C_N$. $\rightarrow$ $\because$ x is n^2 bits

$\Rightarrow$ the unroll operation can be finished in poly time

$\Rightarrow$ Is-Canonical is in Co-NP

11. ① show $T \in CoNP$ ② show $\overline{SAT} \leq_p T$

①

$T \in CoNP$ if $\overline{T} \in NP$

$\overline{T} = \{ E(x_1 \cdots x_n)$, s.t. there is some assignment to the variable such that $E(x_1 = a_1, \cdots, x_n = a_n) = FALSE \}$

$\overline{T} \in NP$ because the "magic" guess will provide the assignment, that we can verify in poly time

②

$\overline{SAT} \in CoNP$-Complete

$\overline{SAT} = \{ E \mid E$ is not satisfiable, (or FALSE $\forall$ assignment$\}$

we want $\overline{SAT} \leq_p T$

$E(x_1, \cdots x_n) \rightarrow \overset{\text{"negate"}}{\boxed{\text{not } E}} \longrightarrow$ instance of T

De Morgan Law: $\overline{A \cup B} = \bar{A} \cap \bar{B}$

$\overline{A \cap B} = \bar{A} \cup \bar{B}$

$\overline{(x \cup y \cup z \cup w)} = \bar{x} \wedge \overline{(y \cup z \cup w)} = \bar{x} \wedge \bar{y} \wedge \overline{(z \cup w)} = \bar{x} \wedge \bar{y} \wedge \bar{z} \wedge \bar{w}$

① + ② $\Rightarrow$ T is CoNP-Complete.

e.

12.

① 3 SAT is NP since any assignment of variables can be vertified in poly time.

② SAT is NP-Complete

$SAT \leq_p 3SAT$

SAT problem $(x \vee \bar{y} \vee z \vee \bar{w} \vee u)$ ...

$(x \vee \bar{y} \vee a_1) \wedge (\bar{a}_1 \vee z \vee a_2) \wedge (\bar{a}_2 \vee \bar{w} \vee u)$

$\begin{cases} m \text{ variables } (m > 3) \\ m-2 \text{ clauses} \\ m-3 \text{ dummy variable} \end{cases}$

① if the SAT instance is staisfiable then corresponding 3SAT instance is satisfiable

② if 3SAT instance is satisf. => SAT inst is satisfiable.