Intensity Transformations and Spatial Filtering

3.1 – 3.3

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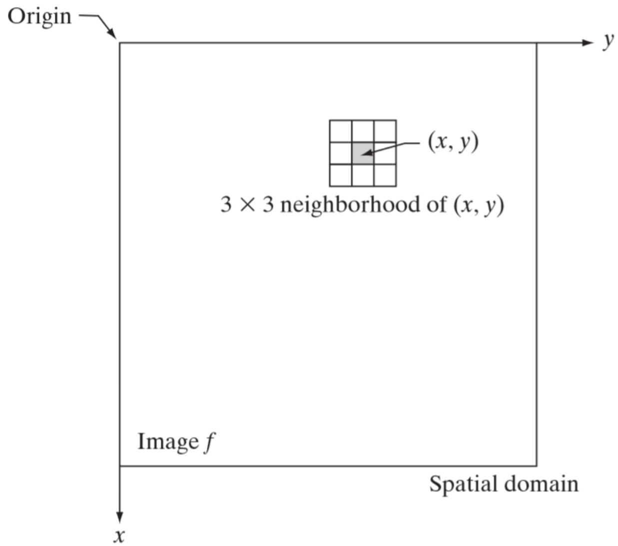
3.1 Background

- All image processing techniques in this chapter are implemented in *Spatial Domain*
- *Spatial Domain* is simply the plane containing the pixels of an image.
- *Spatial Domain* processes we discuss in this chapter can be denoted by the expression

$$g(x, y) = T[f(x, y)]$$

- f(x,y) is the input image g(x,y) is the output image, and T is an operator on f defined over a neighborhood of point(x,y)

3.1 Background



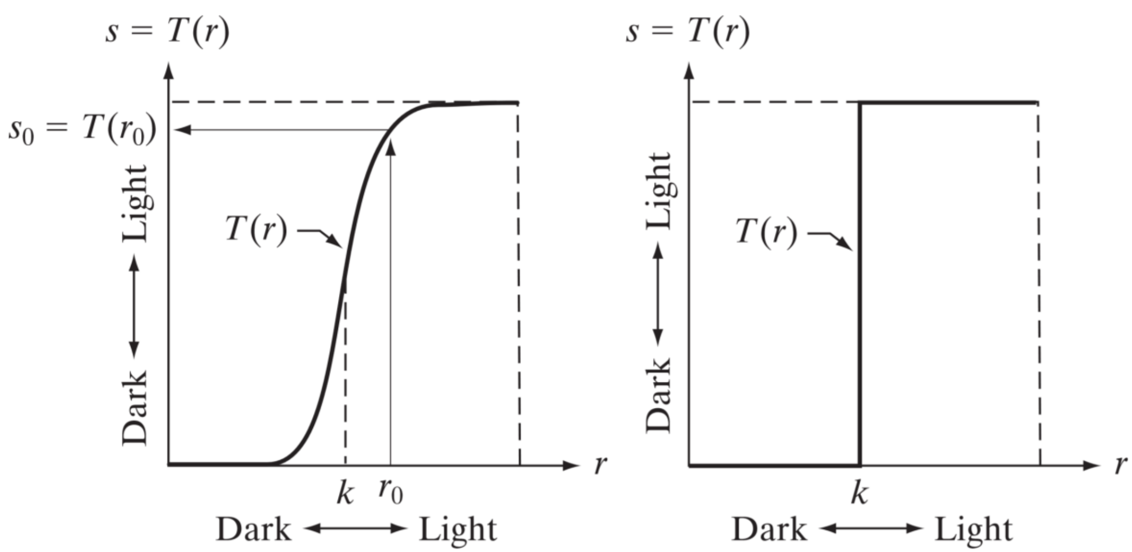
- This Figure illustrates moving the origin of the neighborhood from pixel to pixel and applying the operator *T* to the pixels in neighborhood
- The procedure is called *Spatial Filtering* and the operation *T* is called *Spatial Filter*

- The smallest possible neighborhood is of size 1x1.

$$g(x, y) = T[f(x, y)] \Rightarrow s = T(r)$$

- s and r are variables denoting the intensity of g and f at any point(x,y)

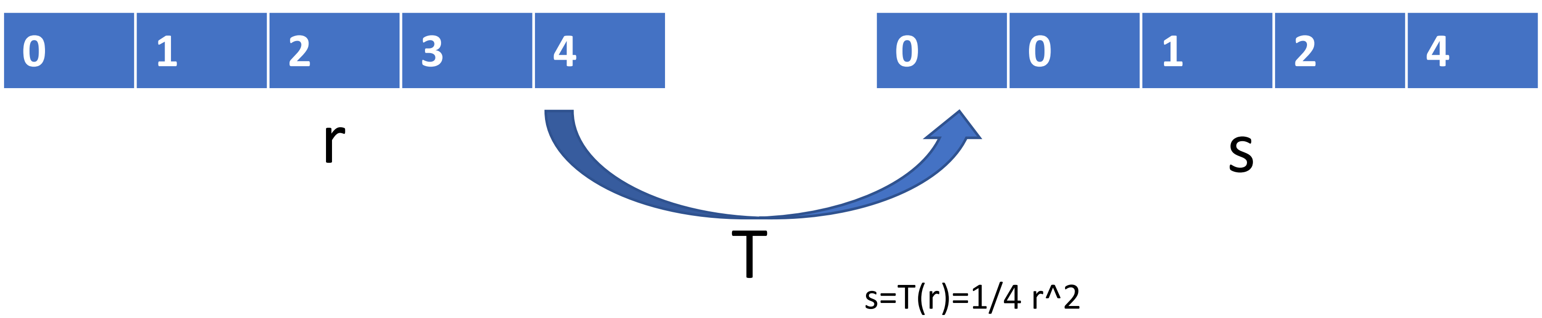
Point processing techniques



Contrast-stretching function Thresholding function

3.2 Some basic Intensity transformation Functions

- The transformation function used in this section can be denoted as $s = T(r)$
- Because we are dealing with digital quantities, values of a transformation function typically are stored in a one-dimensional array.

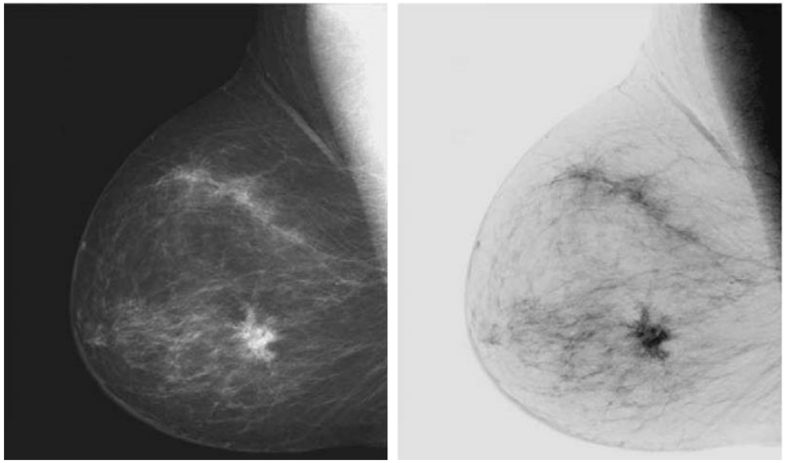


- This section introduces three basic functions used frequently: linear , logarithmic, power-law

3.2 Some basic Intensity transformation Functions

Image Negatives

- The negative of an image in the range [0,L-1] is obtained by using the Negative transformation



Original image Negative image

- Negative transformation is given by the expression

$$s = L - 1 - r$$

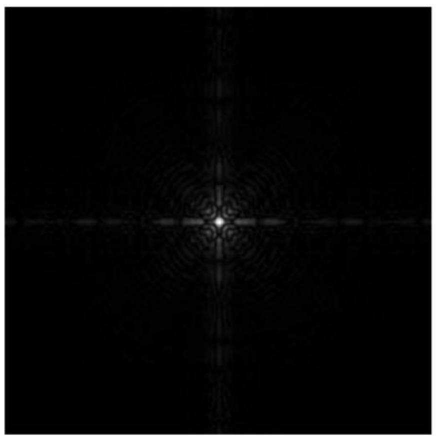
- This type of processing is particularly suited for enhancing white or gray detail embedded in dark region

3.2 Some basic Intensity transformation Functions

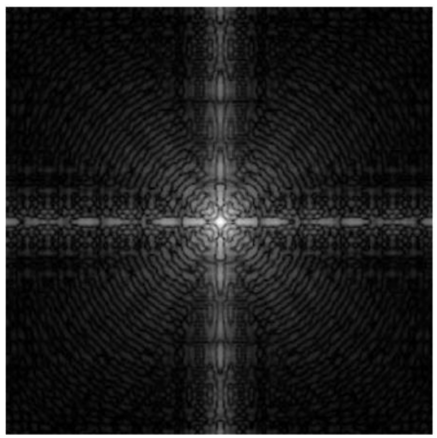
Log transformation

- The general form of the log transformation is

$$s = c \log(1 + r)$$



Original image (Fourier Spectrum)



Log transformed image

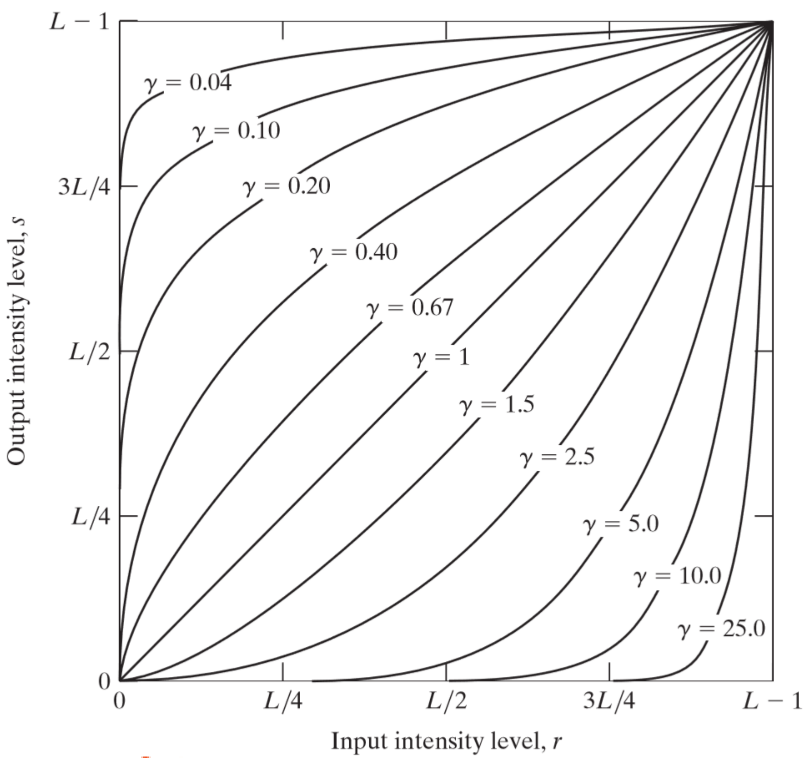
- This transformation maps a narrow range of low intensity values into a wider range of output levels
- Higher values of input is compressed to narrow range of input levels
- The opposite is true of the inverse log transformations

3.2 Some basic Intensity transformation Functions

Power-Law (Gamma) Transformations

- The Power-Law transformation have the basic form

$$s = cr^{\gamma}$$

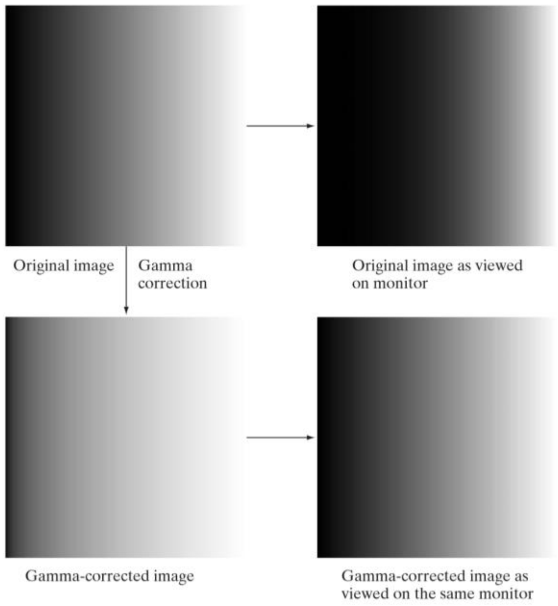


- Power-Law curves with fractional values of γ maps a narrow range of low intensity values into a wider range of output levels
- The opposite is true to γ >1
- Compared to Log transformations, Power-Law transformation is more versatile for this purpose

3.2 Some basic Intensity transformation Functions

Gamma Correction

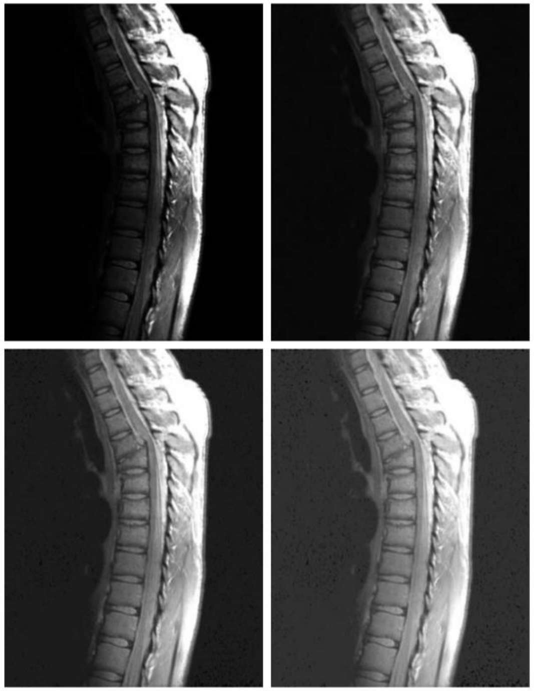
- Variety of devices used for image capture, printing, and display respond according to a Power-Law(gamma)
- The process used to correct these power law response phenomena is called ***gamma correction***



- This picture gives an example of a monitor with a gamma of 2.5
- The monitor produces images darker than intended
- By performing gamma correction $s = r^{1/2.5} = r^{0.4}$ we can see close appearance to original image

3.2 Some basic Intensity transformation Functions

- In addition to gamma correction, Power-Law transformation are useful for general-purpose contrast manipulation



Gamma<1
(0.6,0.4,0.3)

Gamma>1
(3.0,4.0,5.0)

3.2 Some basic Intensity transformation Functions

Piecewise-Linear Transformation Functions

- Advantage- can be arbitrary complex
- Disadvantage- requires more user input

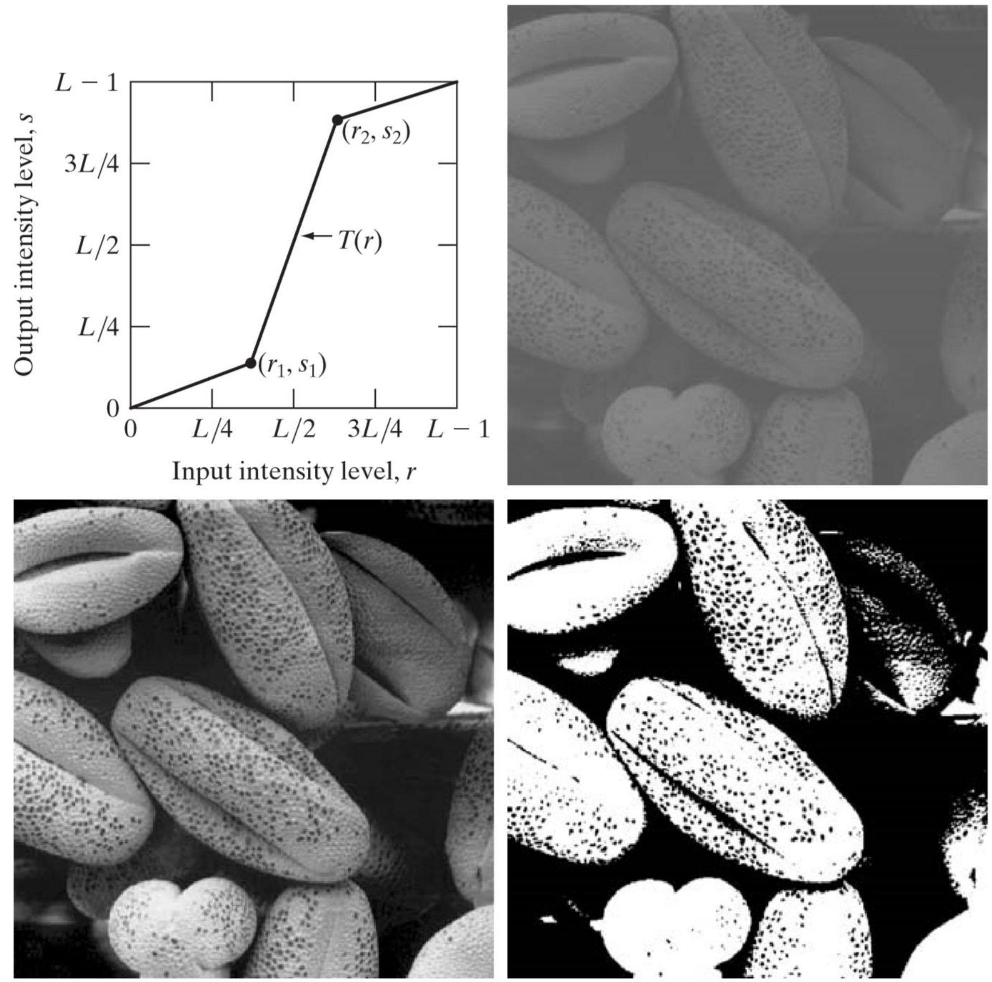
Contrast stretching

- Process that expands the range of intensity levels in an image.
- The locations of points (r_1, s_1) and (r_2, s_2) control the shape of the transformation
- If $r_1 = s_1$ and $r_2 = s_2$ then the transformation produces no change
- If $r_1 = r_2, s_1 = 0$ and $s_2 = L - 1$, the transformation becomes a thresholding function

3.2 Some basic Intensity transformation Functions

Contrast Stretching

a	b
c	d

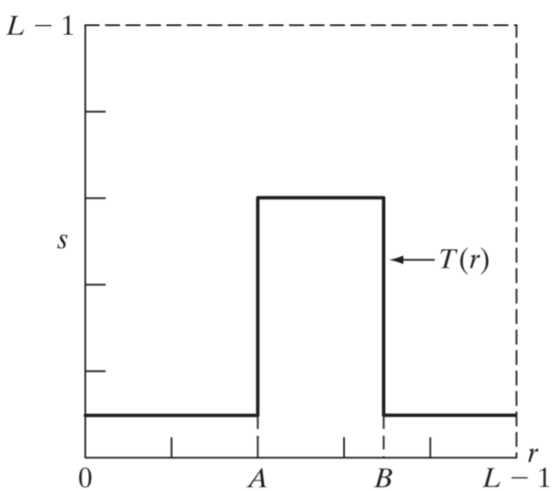


- (b) original image
- (c)image transformed
- by (a)
- (d) Binary image

3.2 Some basic Intensity transformation Functions

Intensity level slicing

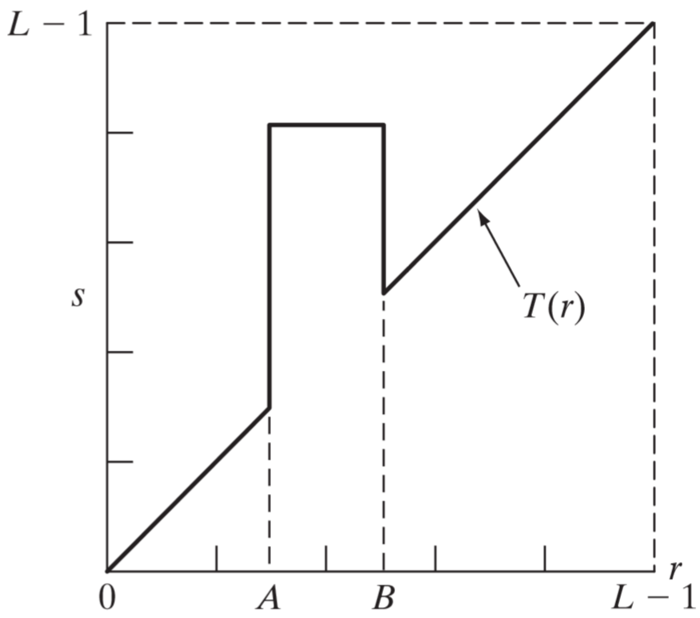
- Can be implemented in several ways but most are variations of two basic themes.
- One approach is to display all the values in the range of interest in one value and other intensities to other value.



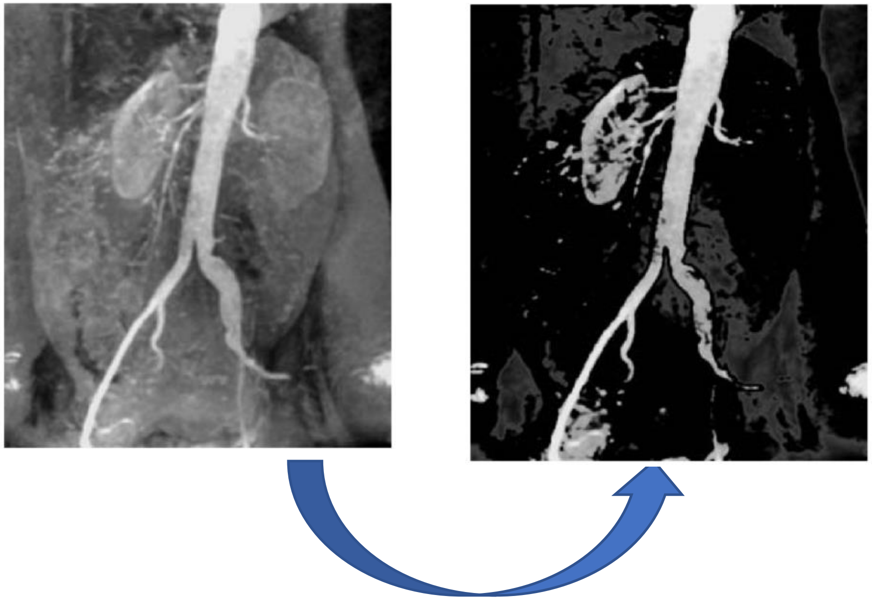
3.2 Some basic Intensity transformation Functions

Intensity level slicing

- The second approach brightens the desired range of intensities of but leaves other intensity levels in the image unchanged



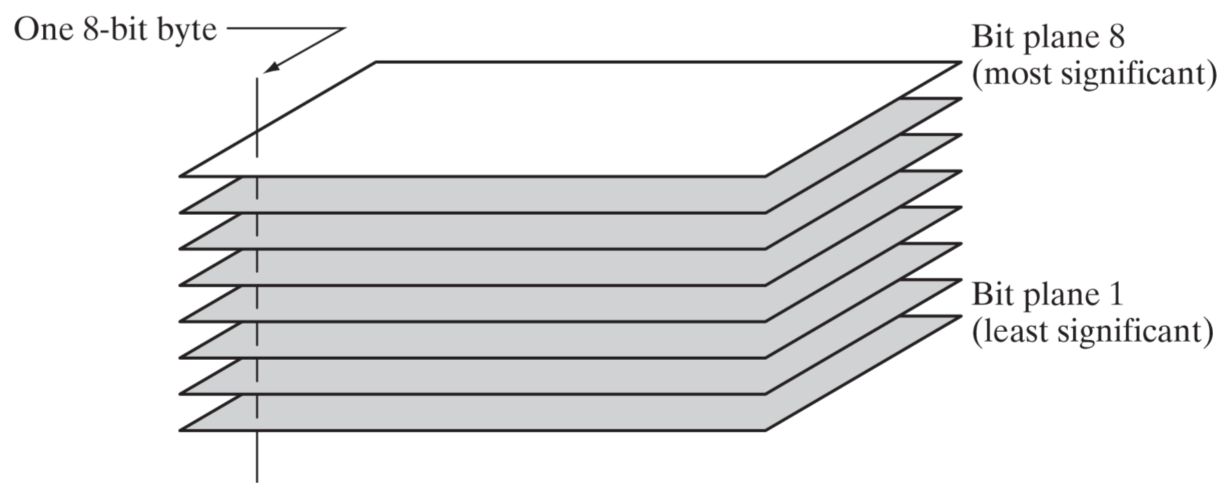
In this case it actually lowered the desired range of intensities



3.2 Some basic Intensity transformation Functions

Bit-plane slicing

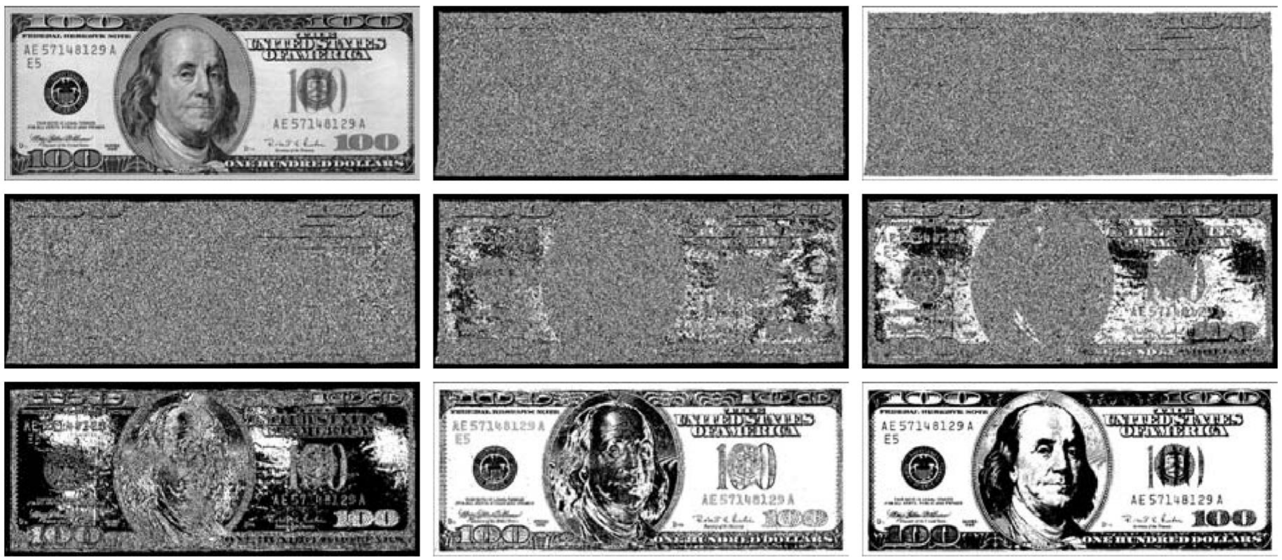
- An 8-bit image may be considered as a being composed of eight 1-bit planes



- Each plane gets binary image. For example, 8th bit plane maps all intensities between 0-127 to 0 and 128-255 to 1

3.2 Some basic Intensity transformation Functions

Bit-plane slicing



a	b	c
d	e	f
g	h	i

- (a) is original image
- (b) through (i) corresponds to bit planes1 through 8

3.2 Some basic Intensity transformation Functions

Bit-plane slicing

- Decomposing an image into its bit planes is useful for analyzing the relative importance in the image.
- This type of decomposition is useful for image compression
- Using 4 bit planes uses 50% storage of original image and 2 bit planes uses 25% of storage

Image reconstructed using high bit planes
8,7=>8,7,6=>8,7,6,5

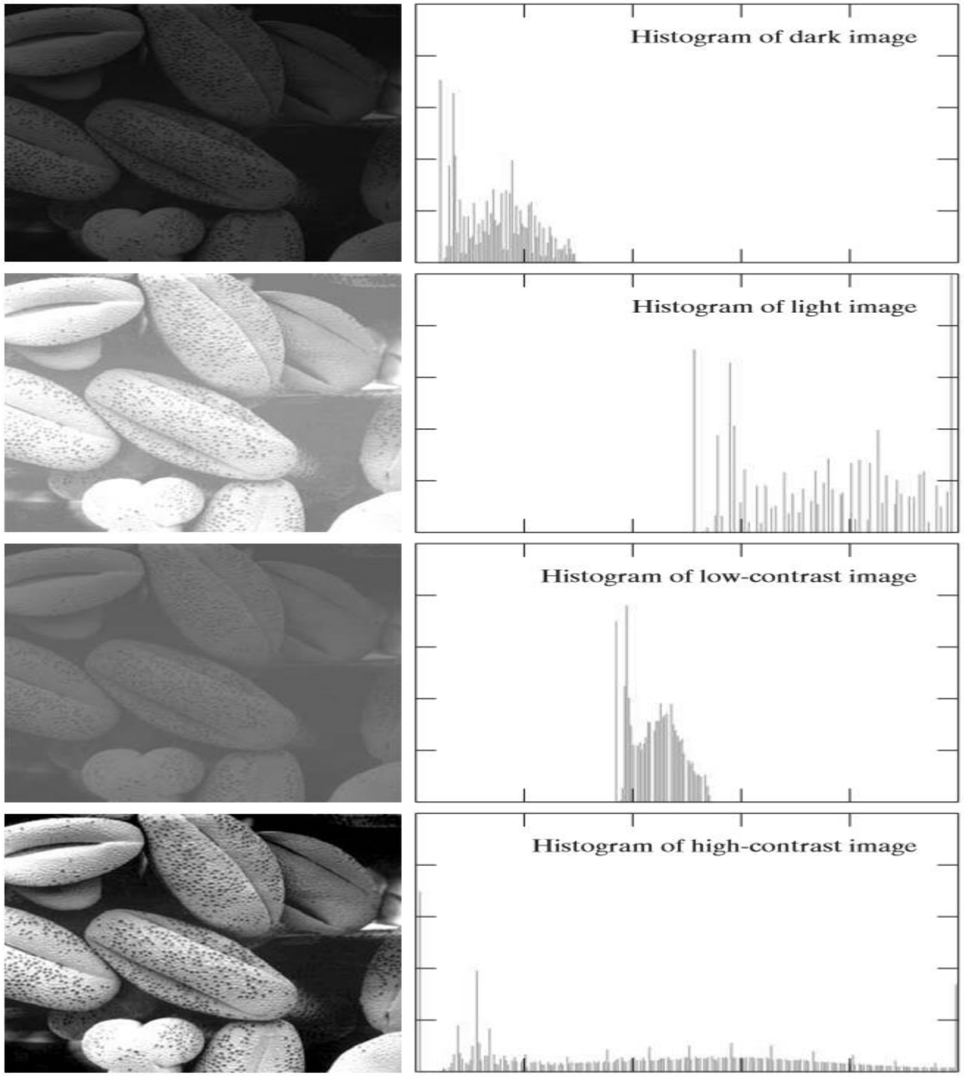
3.3 Histogram Processing

- Histogram of a digital image is a discrete function

$h(r_k)=n_k$

where n_k is the number of pixels in the image with intensity r_k

- Right picture shows 4 basic images types and their corresponding histograms



3.3 Histogram Processing

Histogram Equalization

- Consider a moment continuous intensity values and r representing intensities of an image
- A transformation has formula

$$s = T(r) \quad 0 \le r \le L - 1$$

with two conditions:

(a) T(r) is a monotonically increasing function

(b) $0 \le T(r) \le L-1$ for $0 \le r \le L - 1$

- The condition a(a) guarantees that output intensity values will never be less than corresponding value
- The condition (b) guarantees that the range of output intensities is the same as the input

3.3 Histogram Processing

Histogram Equalization

- A transformation function of particular importance in image has the form

$$s = T(r) = (L-1)\int_0^r p_r(w)\,dw$$

the right side is also recognized as CDF

$$p_s(s) = p_r(r)\left|\frac{dr}{ds}\right|$$

$$\frac{ds}{dr} = \frac{dT(r)}{dr}$$

$$= (L-1)\frac{d}{dr}\left[\int_0^r p_r(w)\,dw\right]$$

$$= (L-1)p_r(r)$$

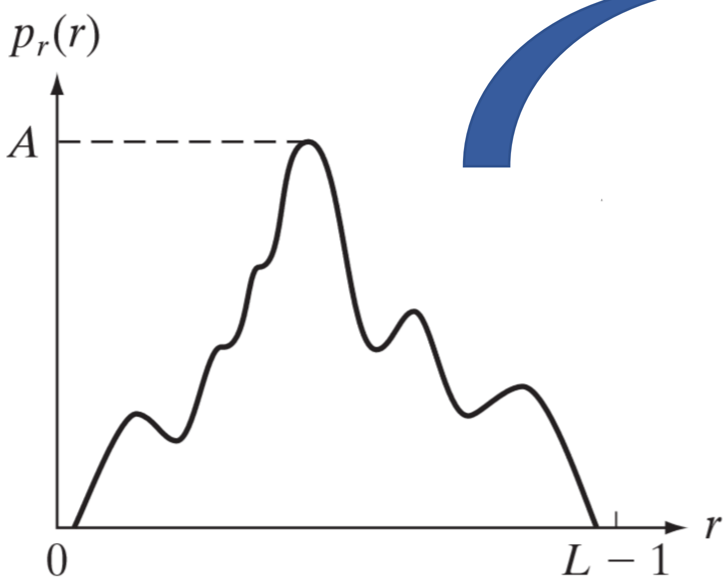
$$p_s(s) = p_r(r)\left|\frac{dr}{ds}\right|$$

$$= p_r(r)\left|\frac{1}{(L-1)p_r(r)}\right|$$

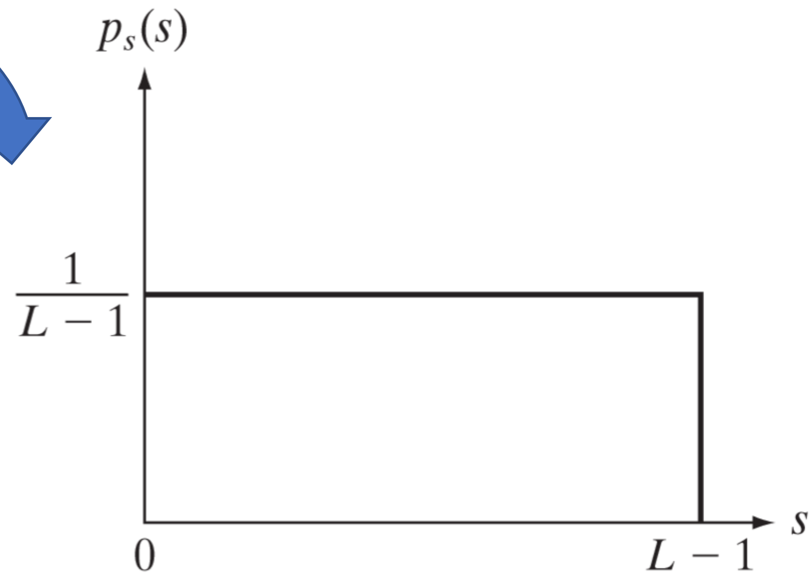
$$= \frac{1}{L-1} \qquad 0 \le s \le L-1$$

3.3 Histogram Processing

$$s = T(r) = (L - 1)\int_0^r p_r(w)\,dw$$



Hist of original image



Hist of equalized image

3.3 Histogram Processing

Histogram Equalization

- Discrete case

Probability of occurrence of intensity level r_k in a digital image is approximated by

$$p_r(r_k) = \frac{n_k}{MN} \qquad k = 0, 1, 2, \ldots, L-1$$

The discrete form of transformation is

$$s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j)$$

$$= \frac{(L-1)}{MN}\sum_{j=0}^{k} n_j \qquad k = 0, 1, 2, \ldots, L-1$$

3.3 Histogram Processing

3 bit case example

r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

$$s_0 = T(r_0) = 7\sum_{j=0}^{0} p_r(r_j) = 7p_r(r_0) = 1.33$$

$$s_1 = T(r_1) = 7\sum_{j=0}^{1} p_r(r_j) = 7p_r(r_0) + 7p_r(r_1) = 3.08$$

$s_0 = 1.33 \rightarrow 1$

$s_1 = 3.08 \rightarrow 3$

$s_2 = 4.55 \rightarrow 5$

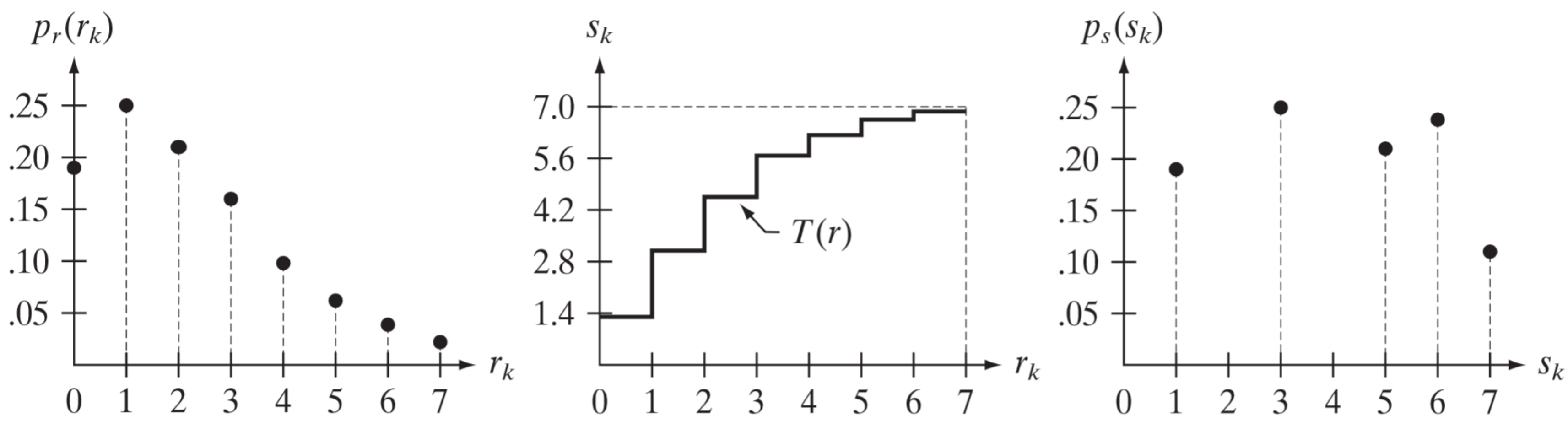
$s_3 = 5.67 \rightarrow 6$

$s_4 = 6.23 \rightarrow 6$

$s_5 = 6.65 \rightarrow 7$

$s_6 = 6.86 \rightarrow 7$

$s_7 = 7.00 \rightarrow 7$



3.3 Histogram Processing

Histogram Matching (Specification)

- Sometimes it is useful to be able to specify the shape of the histogram that we wish the processed image to have
- Formula:

$$s = T(r) = (L - 1)\int_0^r p_r(w)\,dw$$

$$G(z) = (L - 1)\int_0^z p_z(t)\,dt = s$$

Let G(z)=T(r) then,

$$z = G^{-1}[T(r)] = G^{-1}(s)$$

3.3 Histogram Processing

Histogram Matching (Specification)

- Discrete case

$$s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j)$$

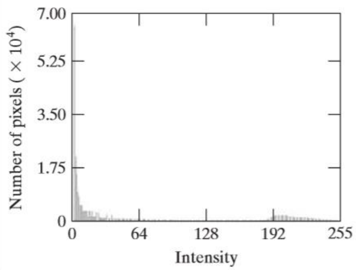
$$= \frac{(L-1)}{MN}\sum_{j=0}^{k} n_j \qquad k = 0, 1, 2, \ldots, L-1$$

$$G(z_q) = (L-1)\sum_{i=0}^{q} p_z(z_i)$$

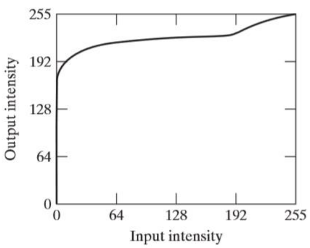
Let $G(z_q) = s_k$

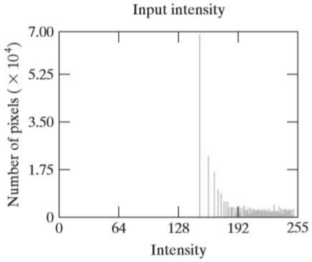
Then, $z_q = G^{-1}(s_k)$

3.3 Histogram Processing

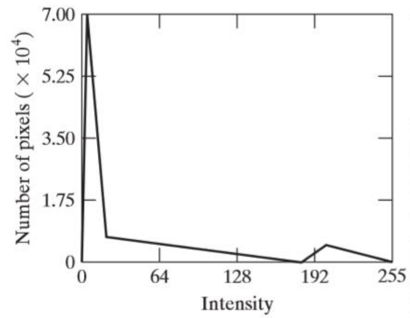


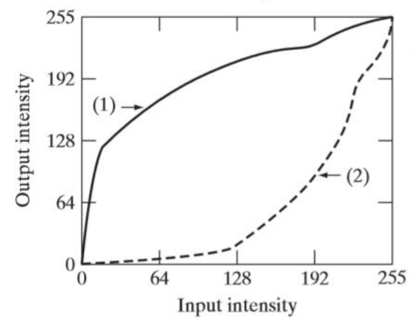
Equalization

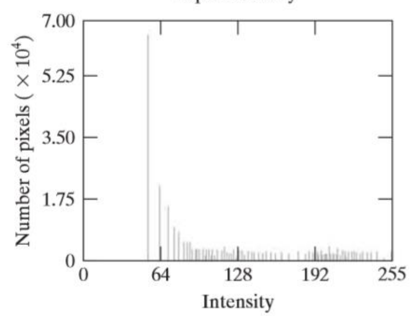




Specification

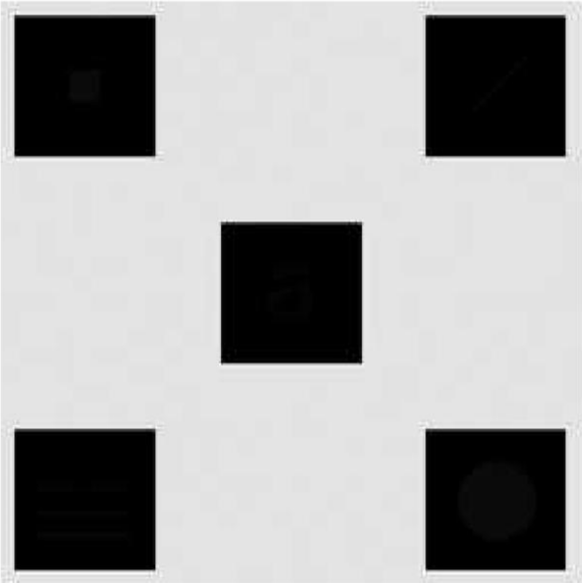




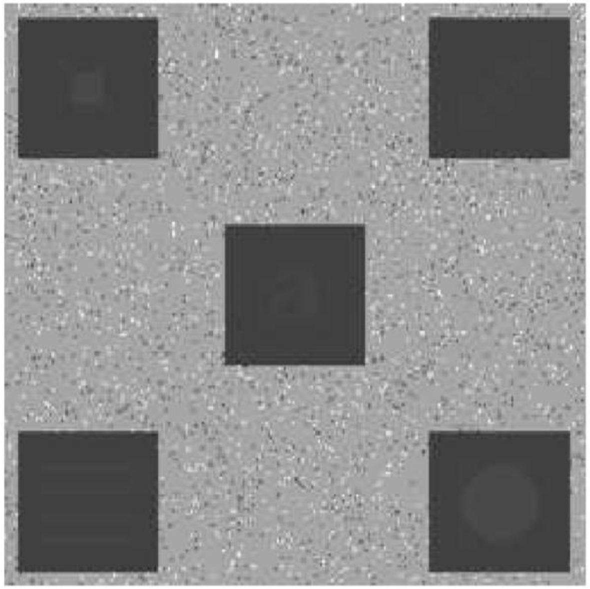


Local Histogram Processing

- There are cases in which it is necessary to enhance details over small areas in an image



Original image



Global histogram equalization

Local histogram equalization using neighborhood 3x3

3. Histogram Processing

Using Histogram Statistics for Image Enhancement

- Mean

$$m = \sum_{i=0}^{L-1} r_i p(r_i)$$

$$m = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y)$$

Variance

$$\mu_2(r) = \sum_{i=0}^{L-1} (r_i - m)^2 p(r_i)$$

$$\sigma^2 = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} \left[f(x, y) - m \right]^2$$

Local mean, variance can be computed the same way using neighborhood intensities

Thank you