Computer Graphics and Image Processing Laboratory. Seoul National University

Digital Image Processing

- Intensity Transformation & Spatial Filtering -

SNU Computer Graphics & Image Processing LAB
YongJin Jeon

Intensity Transformation & Spatial Filtering

Spatial Domain

Image Processing

Intensity Transformation & Spatial Filtering

Spatial Domain

Image Processing

Image Plane

Intensity Transformation & Spatial Filtering

Spatial Domain

Image Processing

Manipulate pixels in image

Intensity Transformation & Spatial Filtering

Spatial Domain + Image Processing

→ Spatial Processing

Intensity Transformation

Operate on single pixel of an Image

Spatial Filtering

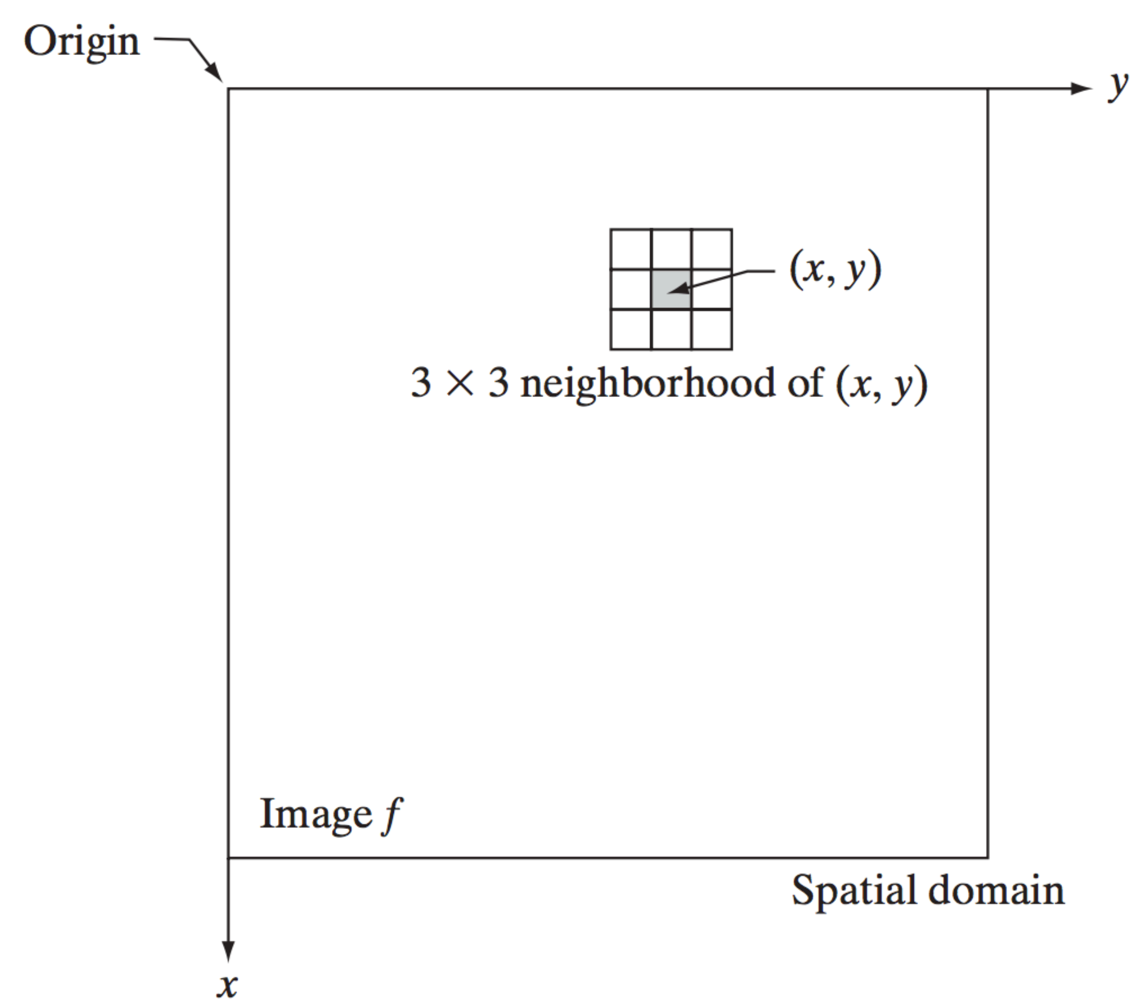
Deals with operations, by working in an neighborhood of every pixel in an image

Basics

- Spatial Domain Process

-> Expression of spatial domain process

$$g(x,y) = T\,[f(x,y)]$$



Basics

- Spatial Domain Process

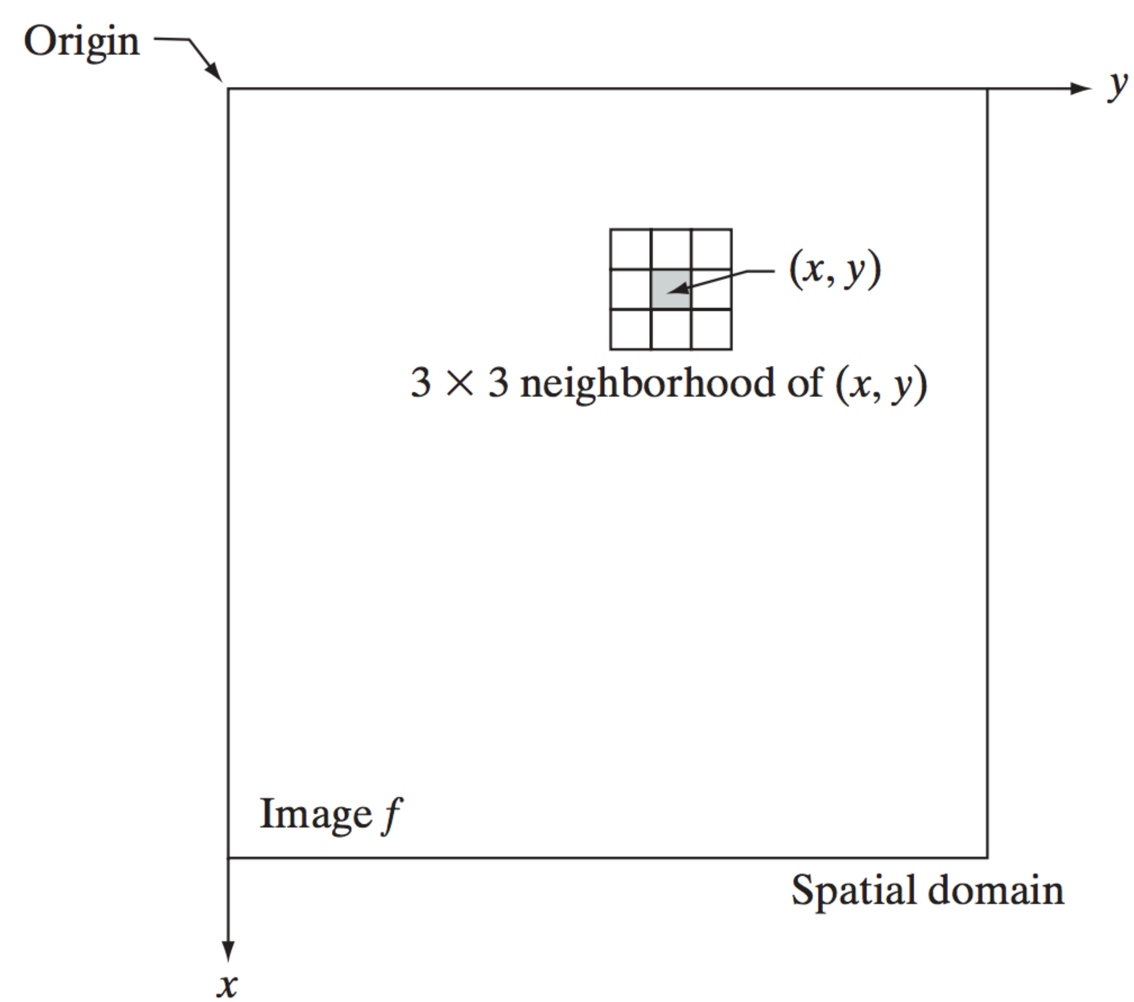
-> Expression of spatial domain process

$$\boxed{g(x,y)} = \boxed{T}\left[\boxed{f(x,y)}\right]$$

output image

input image

operator on f defined over
a neighborhood of point(x,y)



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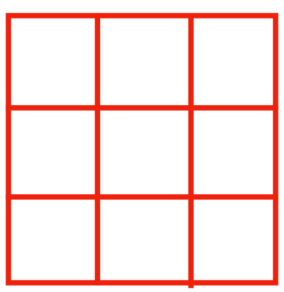
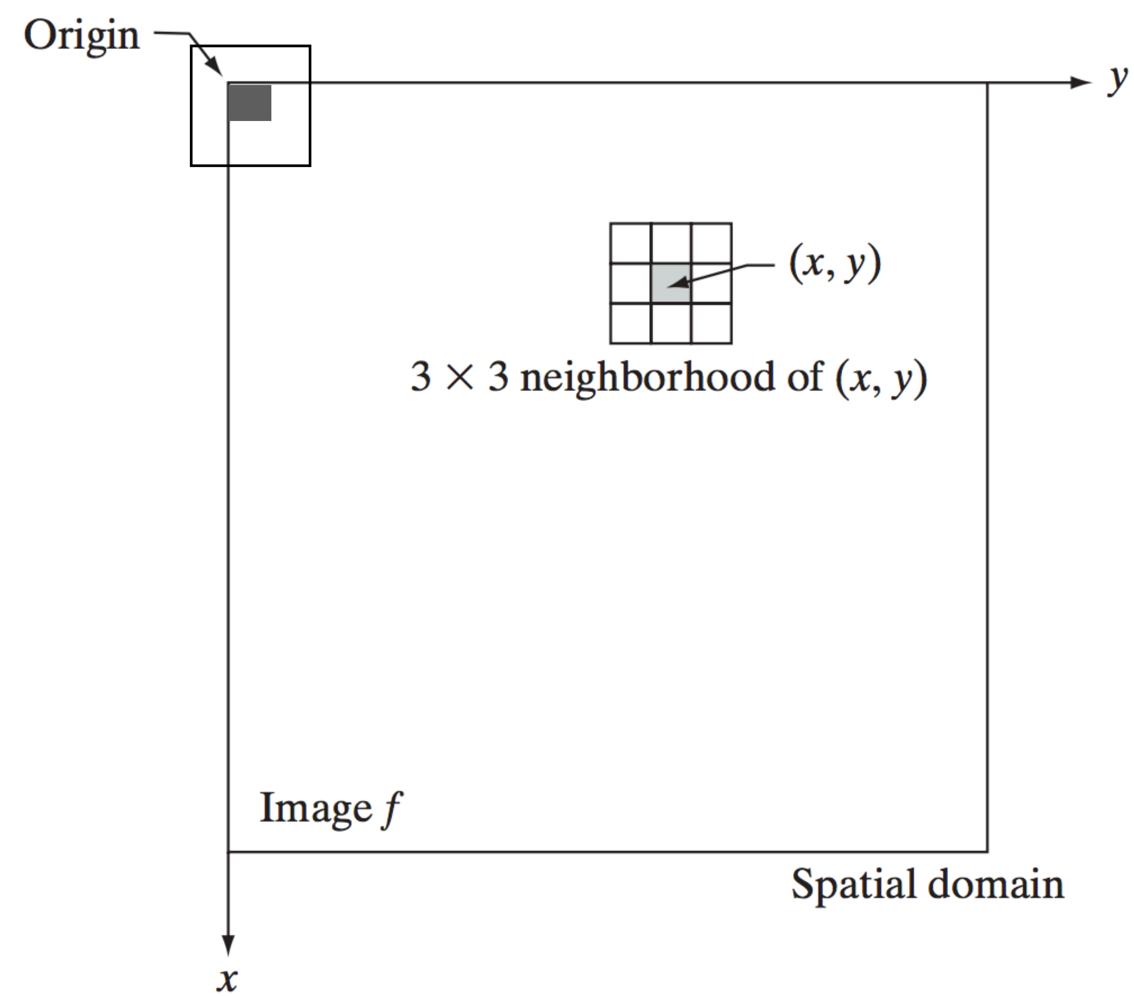
=> ex) Compute the average intensity of the neighborhood



suppose neighborhood is a square of size 3x3



Basics

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$$g(x,y) = T\,[f(x,y)]$$

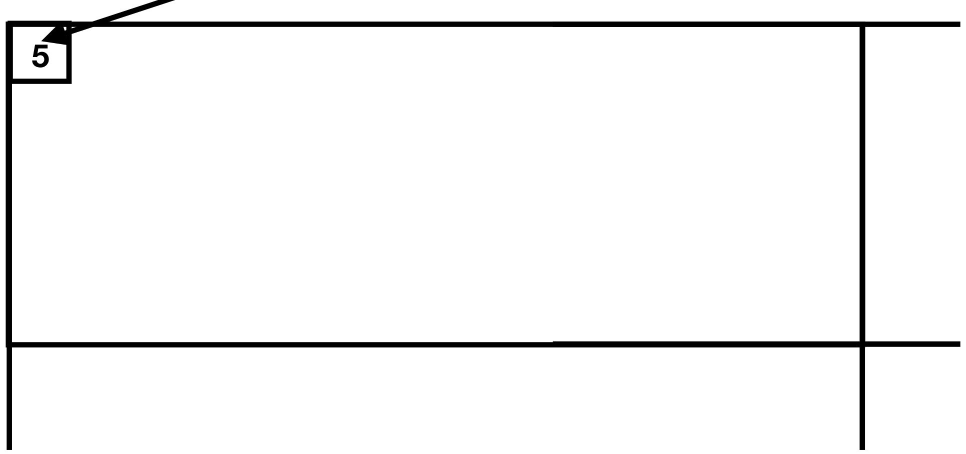
=> ex) Compute the average intensity of the neighborhood

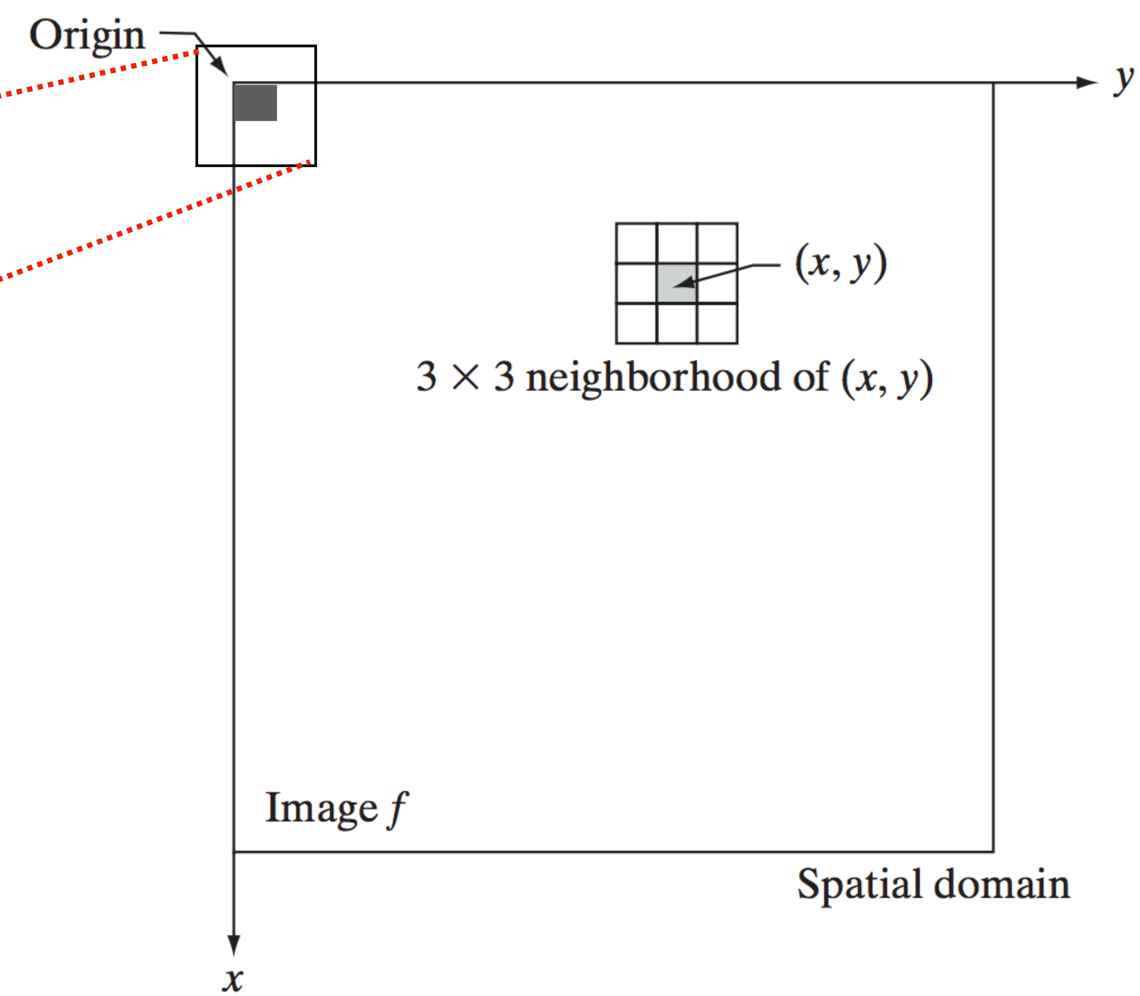
zero padding

0	0	0
0	11	15
0	20	3

$$\rightarrow \frac{11+15+20+3}{9} = 5.4$$

suppose neighborhood is a square of size 3x3





Basics

- Spatial Domain Process

-> Expression of spatial domain process

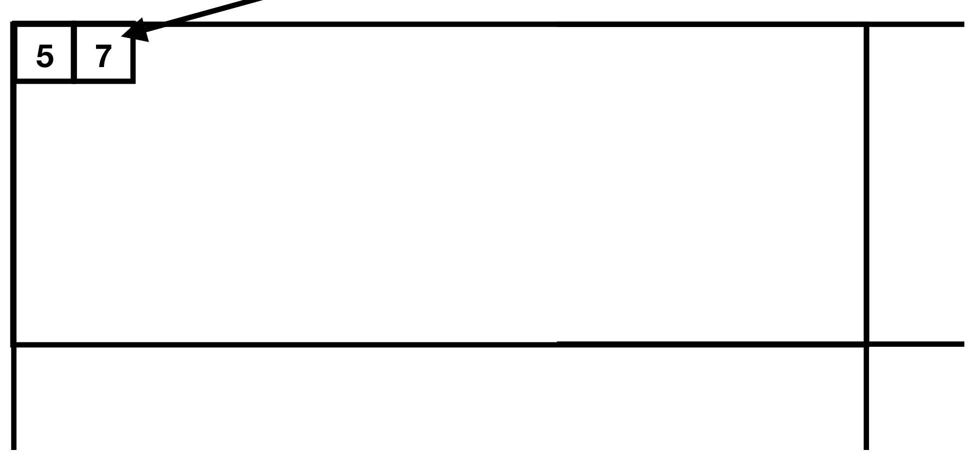
$$g(x,y) = T\ [f(x,y)]$$

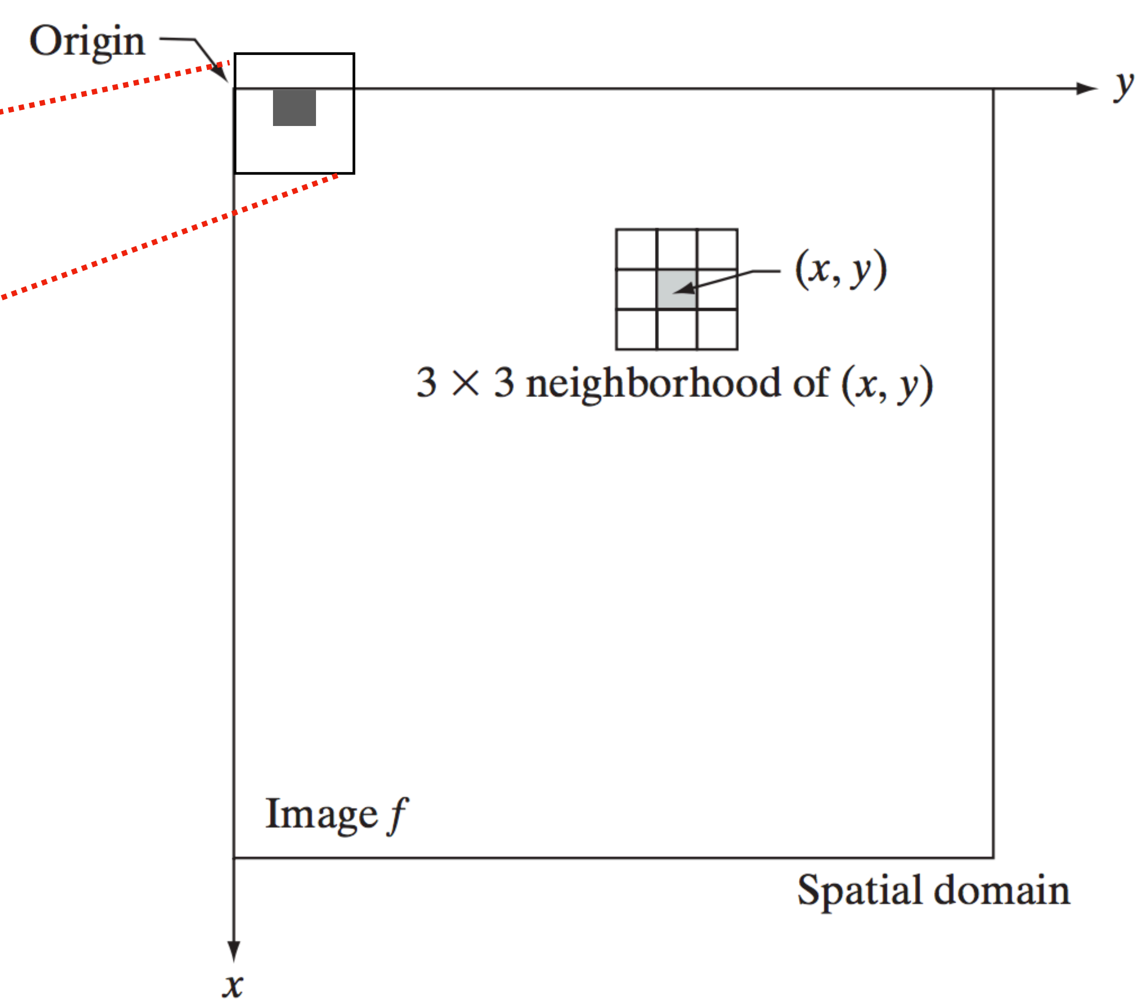
=> ex) Compute the average intensity of the neighborhood

0	0	0
11	15	12
20	3	1

$$\rightarrow \frac{11+15+20+3+12+1}{9} = 6.8$$

suppose neighborhood is a square of size 3x3





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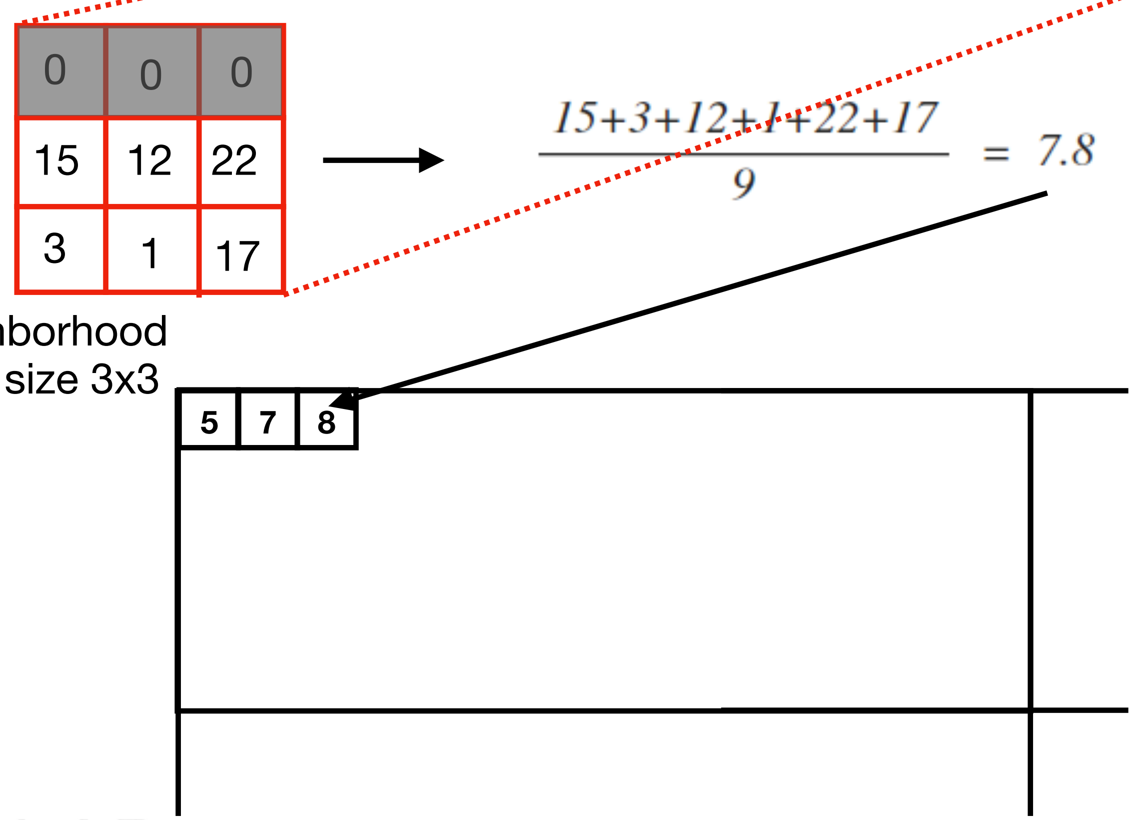

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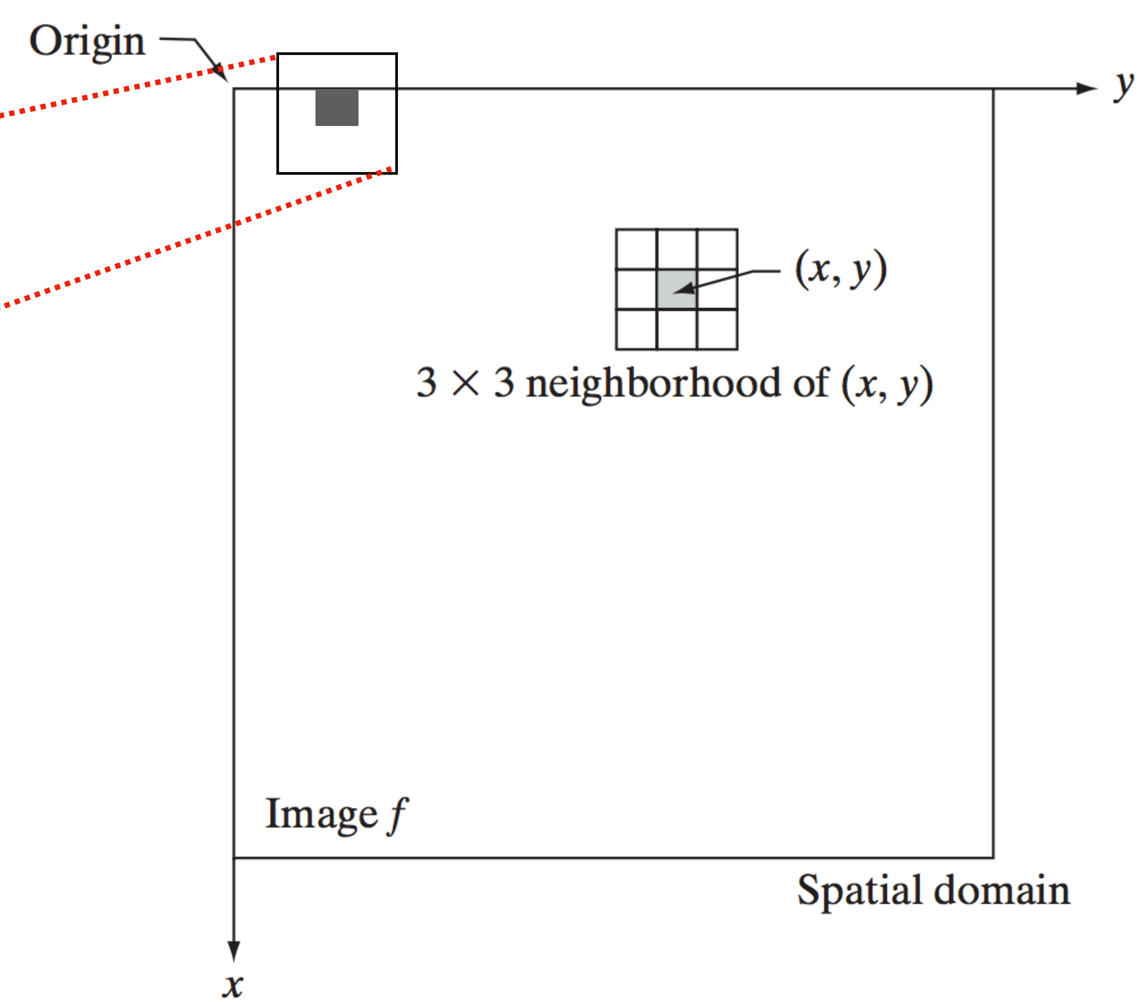

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Basics

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$$g(x,y) = T\,[f(x,y)]$$

=> ex) Compute the average intensity of the neighborhood

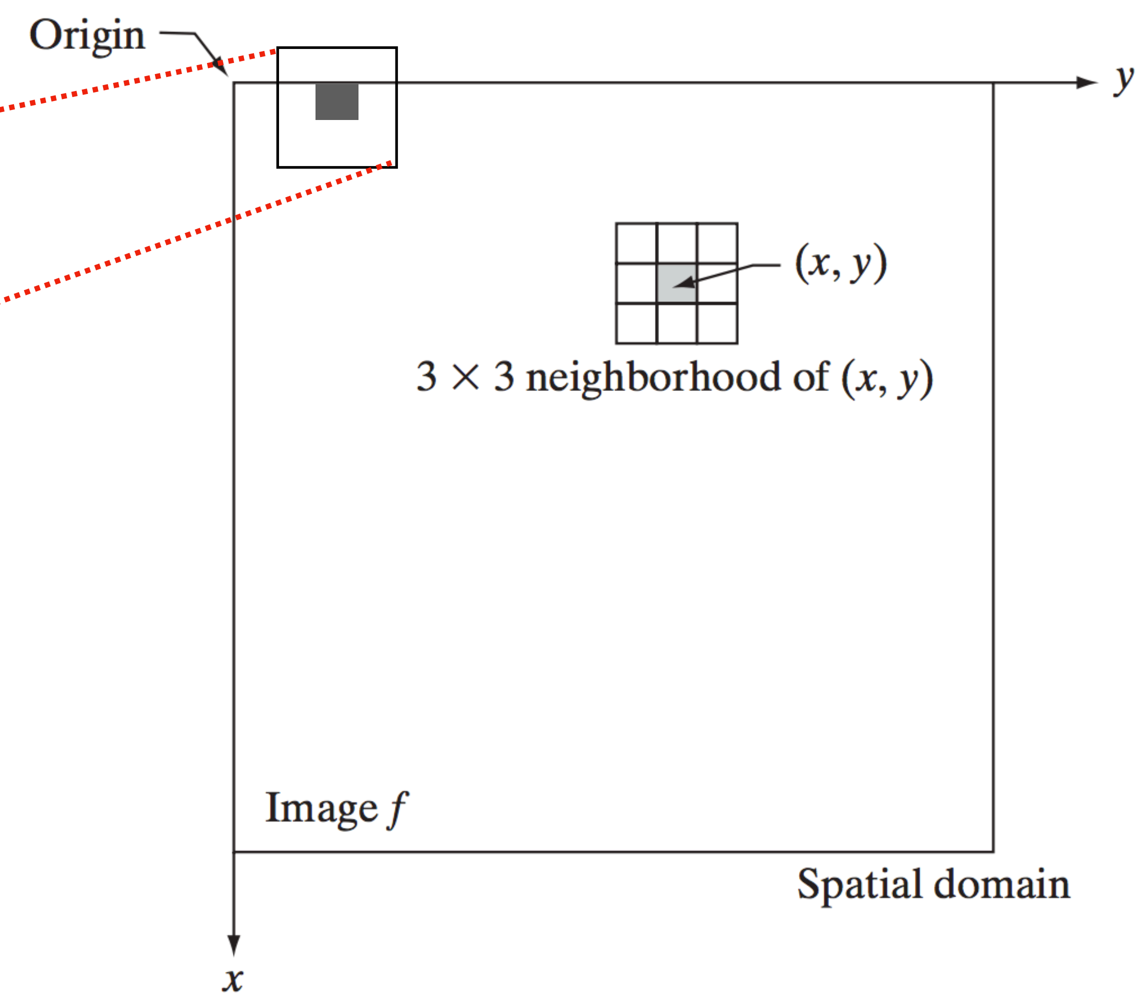
0	0	0
15	12	22
3	1	17

$$\frac{15+3+12+1+22+17}{9} = 7.8$$

suppose neighborhood is a square of size 3x3

Procedure : Spatial Filtering

Operation : Spatial Filter



Basics

- Spatial Domain Process

-> Expression of spatial domain process

$$g(x,y) = T\,[f(x,y)]$$

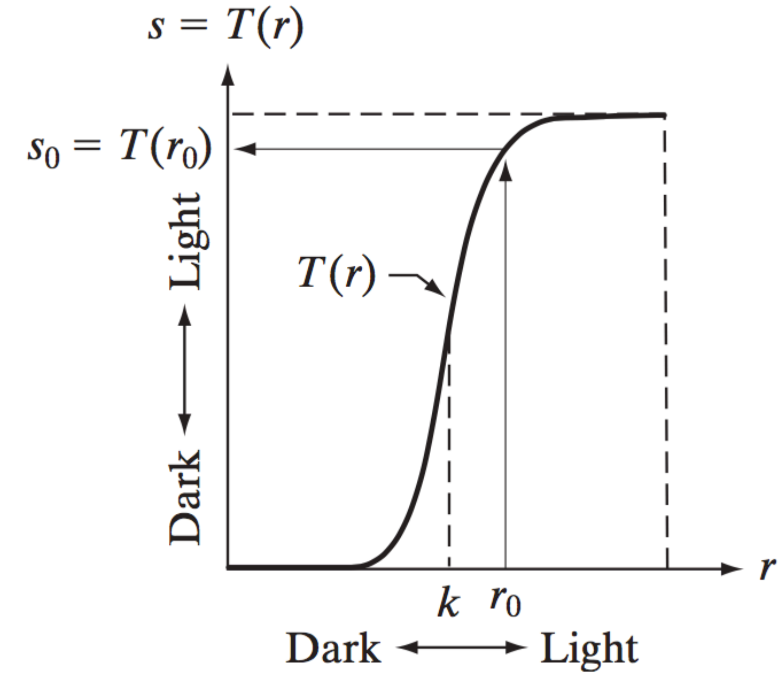
-> In case of smallest possible neighbor -hood (size 1x1) expression becomes an **Intensity transformation function**

$$\boxed{s} = T\boxed{(r)}$$

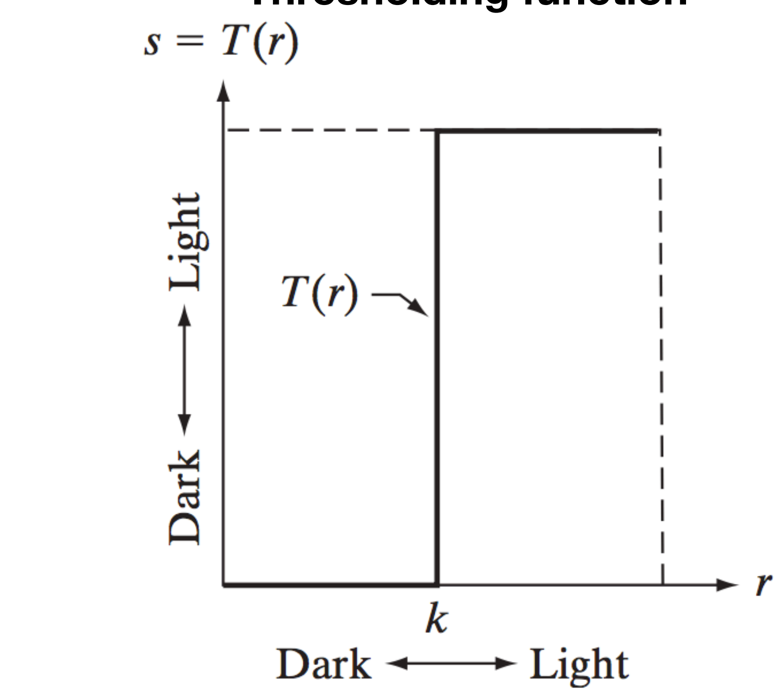
intensity of f

intensity of g

contrast stretching function



Thresholding function



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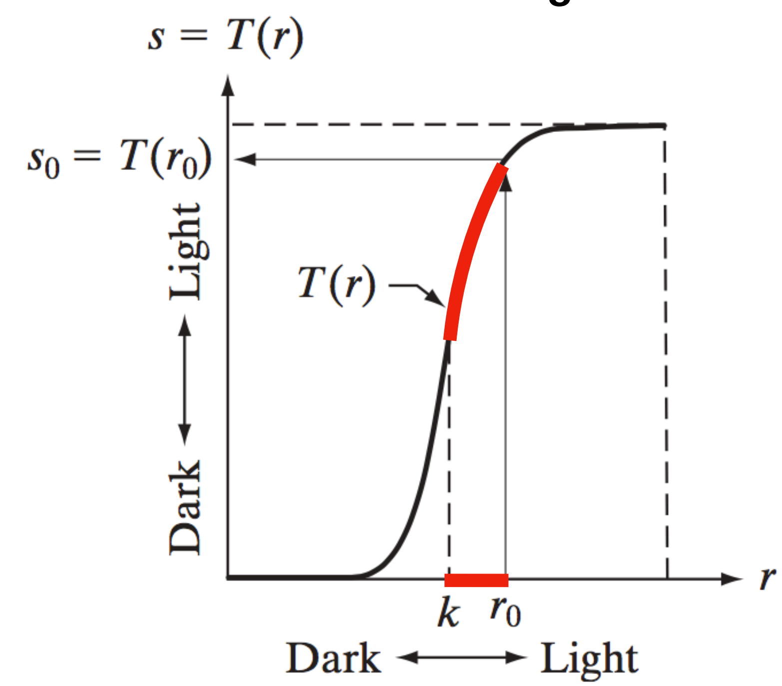

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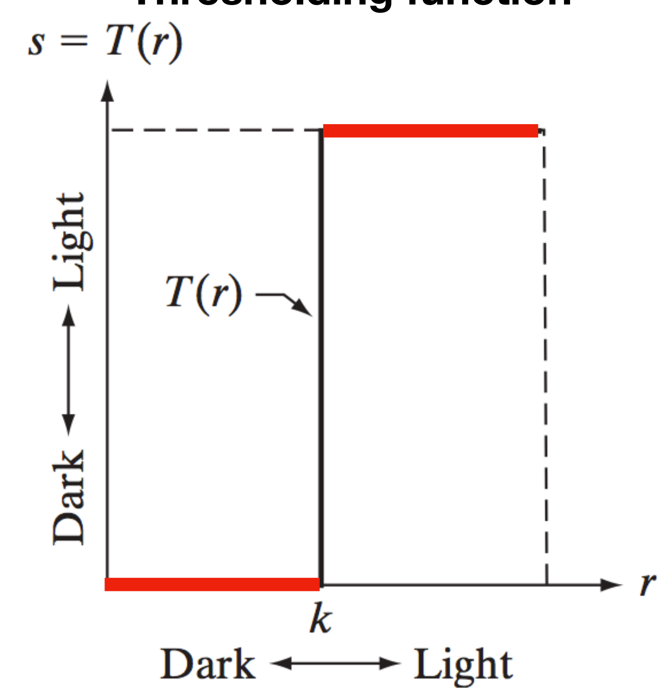

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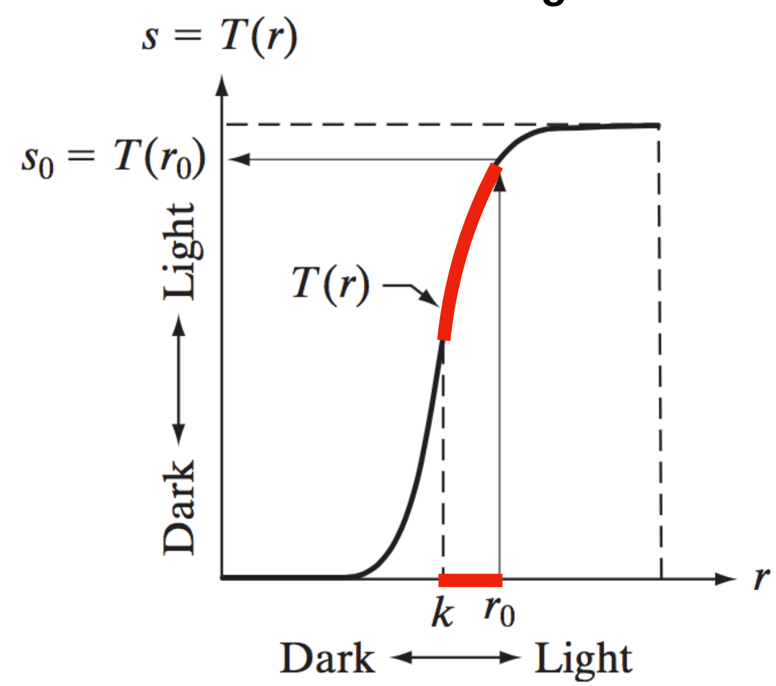
intensity of g

Point Processing ⟷ **Neighborhood Processing**

contrast stretching function



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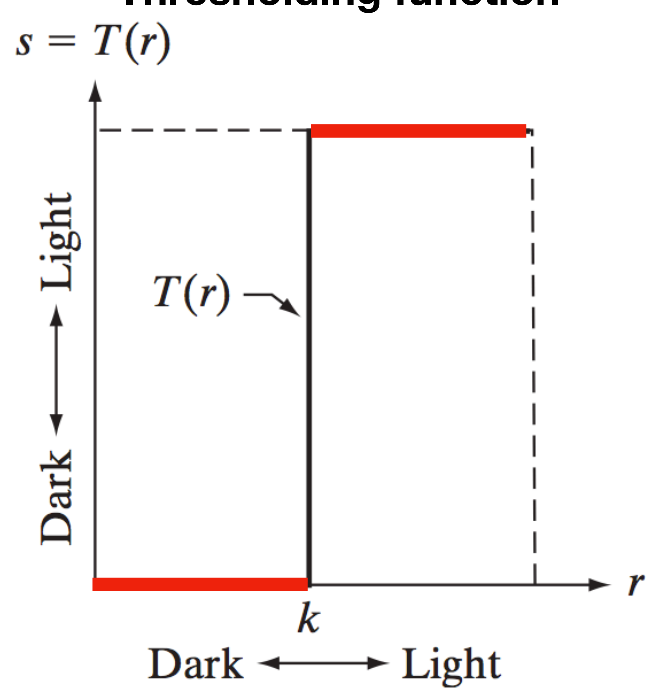


Basic Intensity Transformation Functions

- Intensity Transformation

$$\boxed{s} = T\boxed{(r)}$$

intensity of f (Before processing)

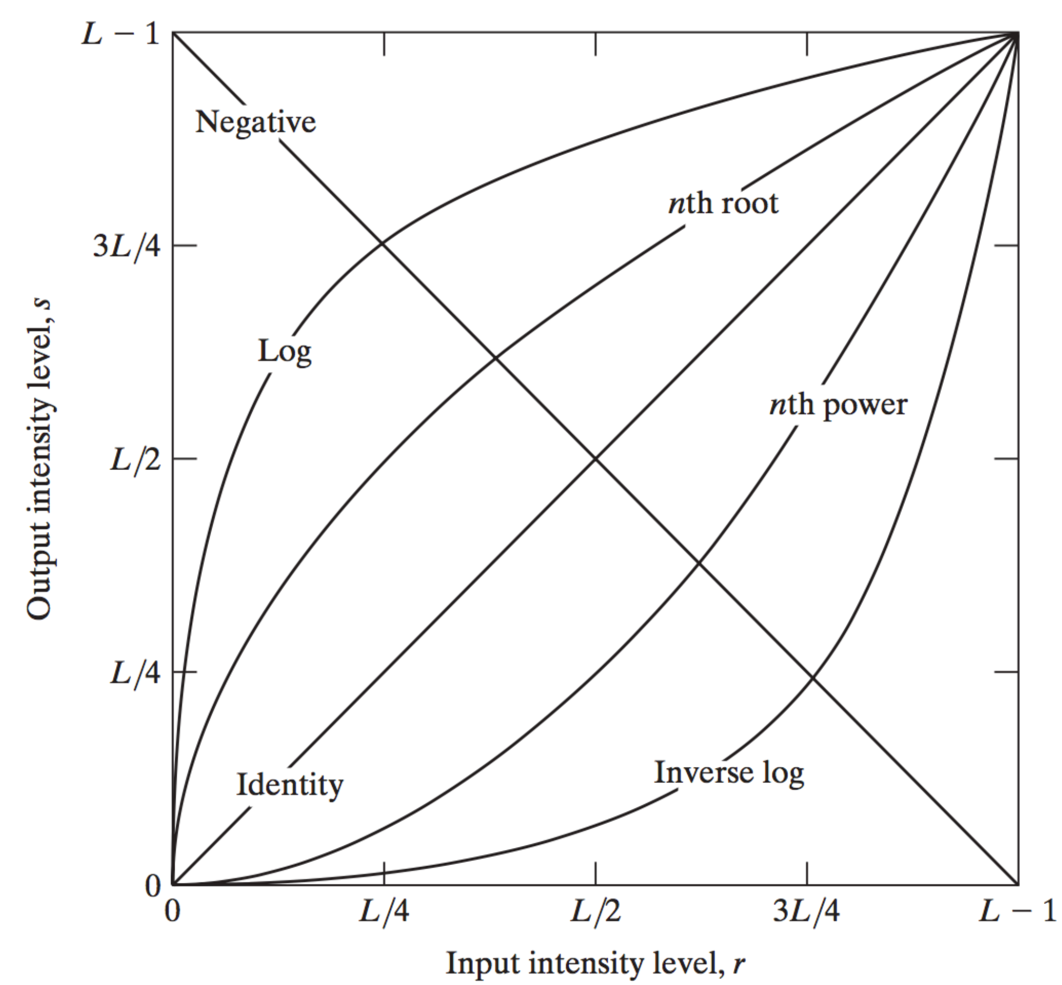
intensity of g (After processing)

* Stored in 1-D array
* Mappings are implemented via look-up table

Linear
- Negative Transformations
- Identity Transformations

Logarithmic
- Log Transformations
- Inverse-log Transformations

Power-law
- n th power transformation
- n th root transformation



Basic Intensity Transformation Functions

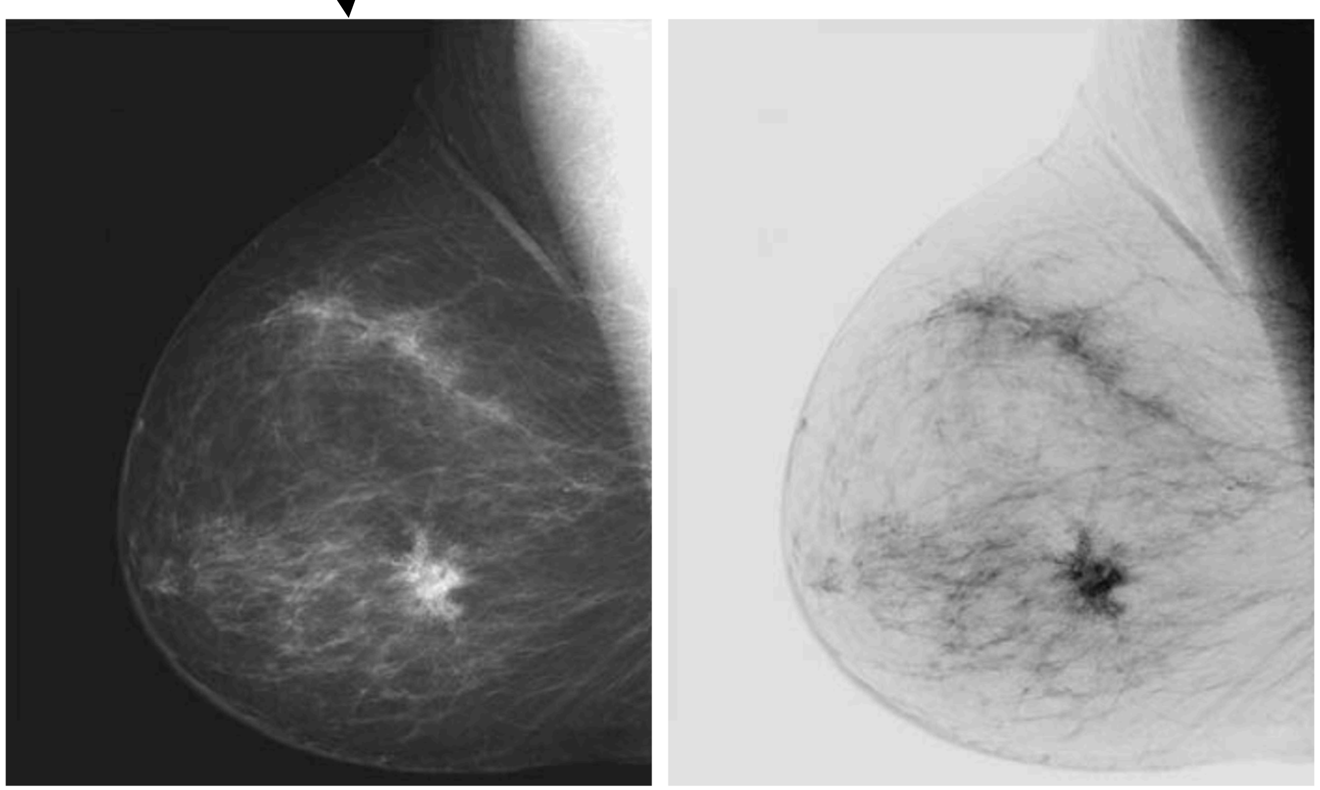
- Negative Transformation

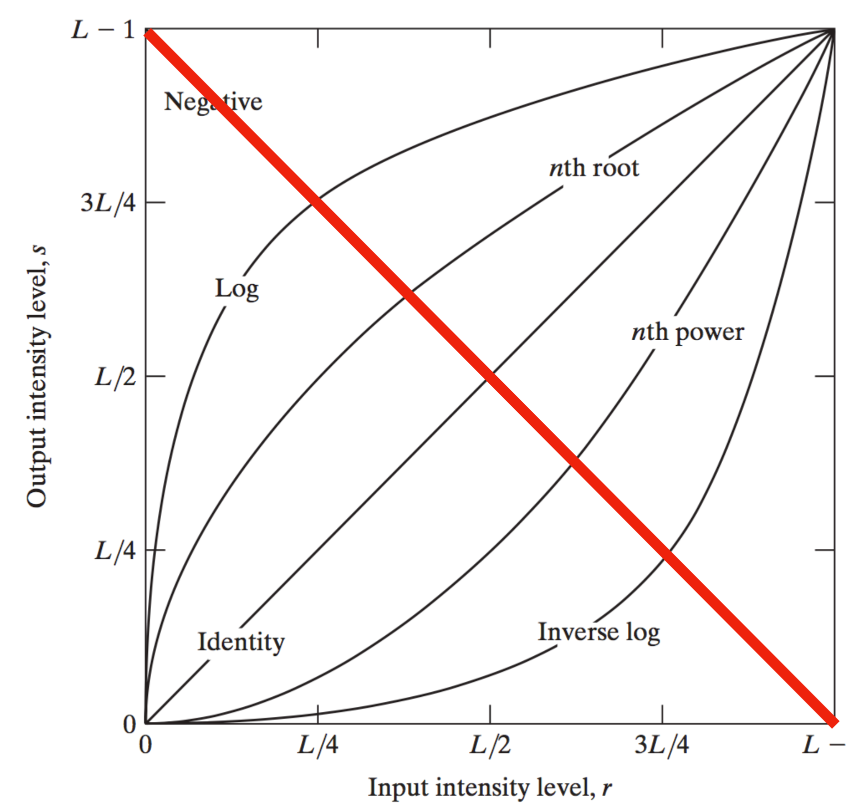
$$s = \boxed{L - 1 - r}\ ,\ \boxed{[0,\ L-1]}$$

Range of intensity level

Reverse of intensity level

-> suit for enhancing white or grey detail embedded in dark regions of image especially when the **<u>black areas are dominant in size</u>**





Basic Intensity Transformation Functions

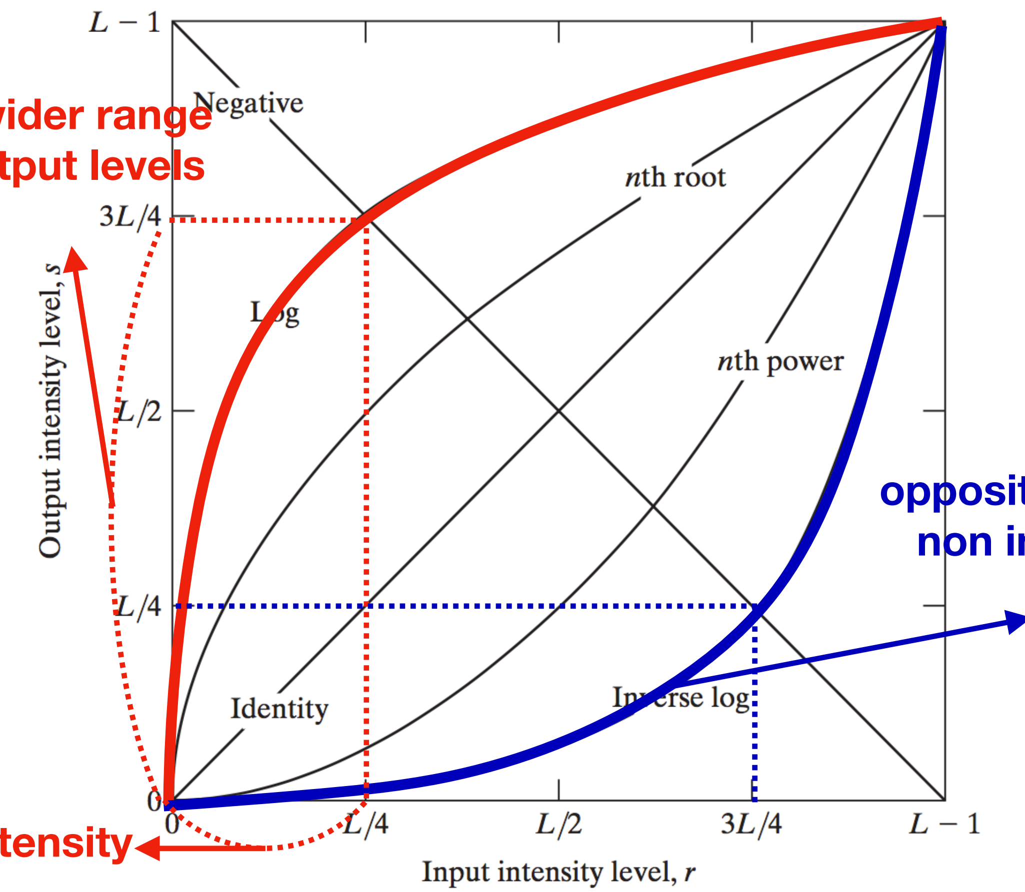
- Log Transformation

$$s = \boxed{c}\,\log(1 + \boxed{r})$$

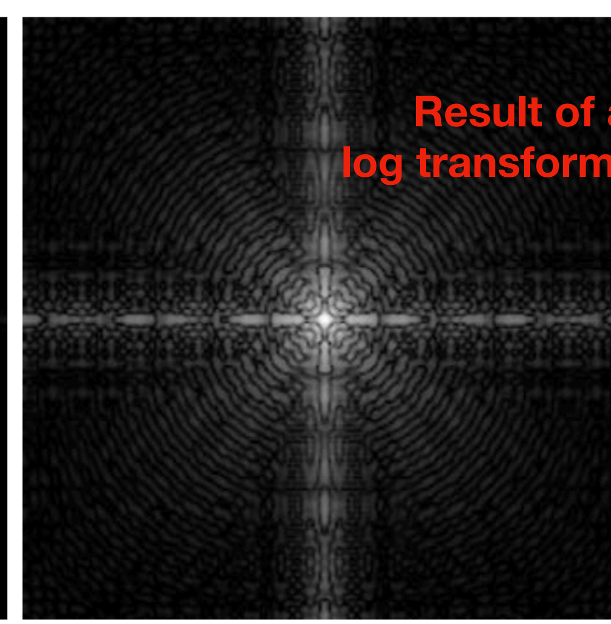
constant   same or larger than zero

-> Use for spreading and compressing of intensity levels an image

-> Log function compresses the dynamic range of images with large variations in pixel values



Fourier Spectrum range (0 ~ 1.5 * 10^6)



Result of applying the log transformation with c = 1

Basic Intensity Transformation Functions

- Power-Law Transformation (Gamma Transformation)

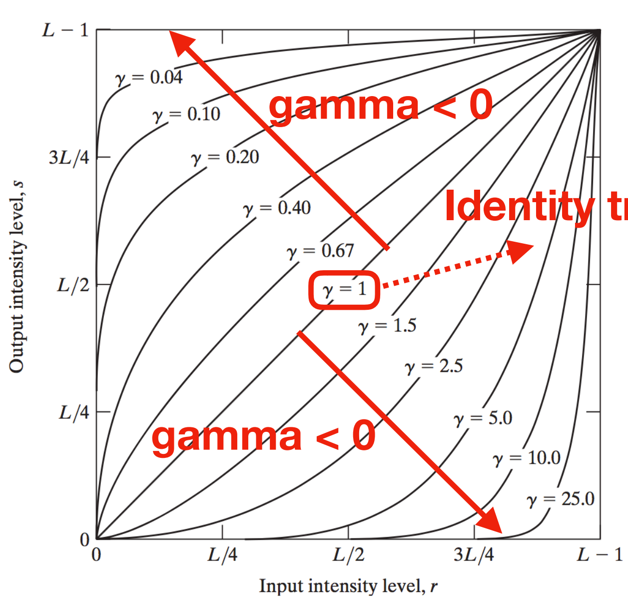
$$s = \boxed{c}\; r^{\boxed{\gamma}}$$

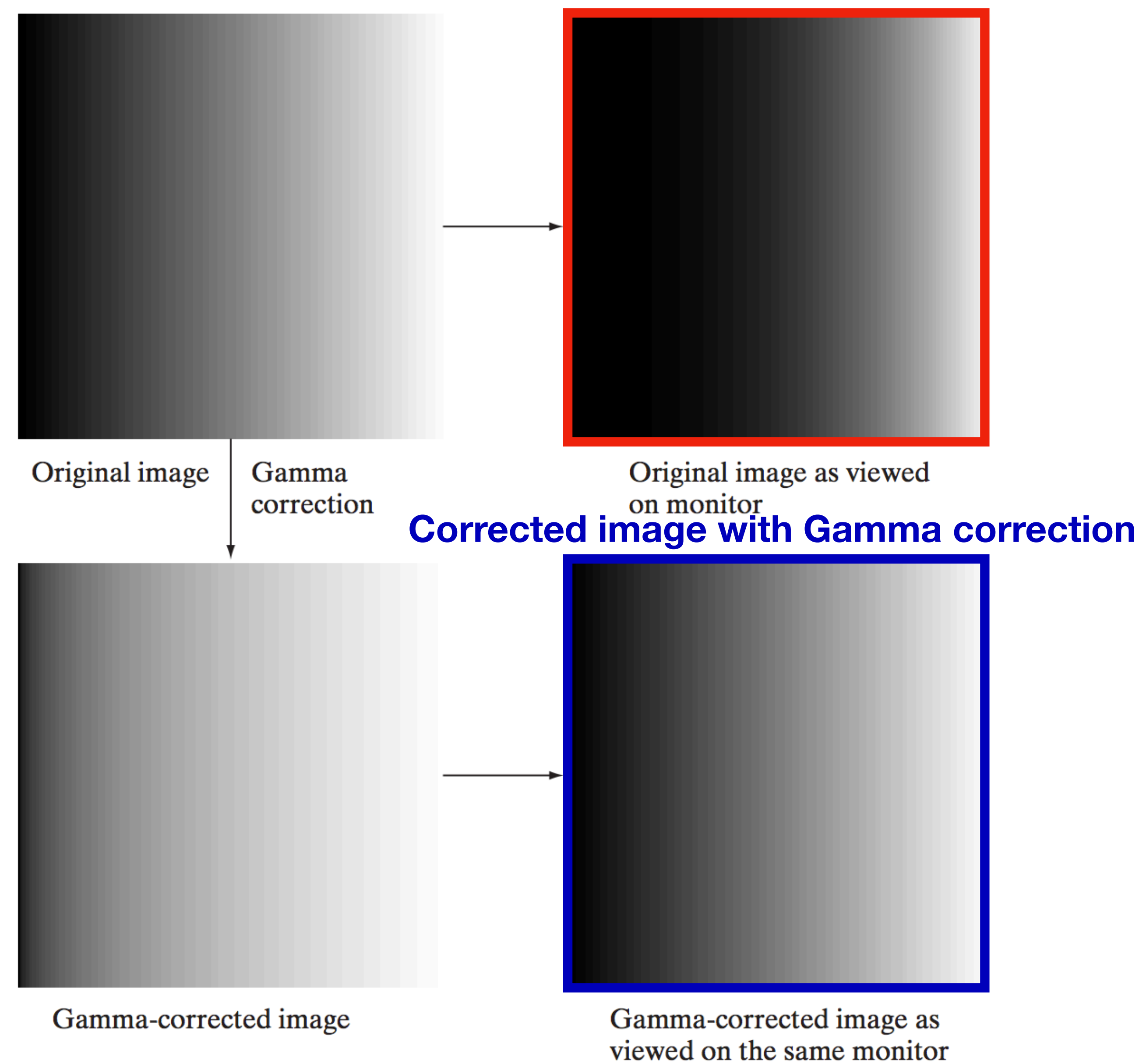
positive

-> Shows similar effects as log transformation

-> Correcting process using power-law response phenomena is called **Gamma correction**

=> [Image capture, Printing, Display]





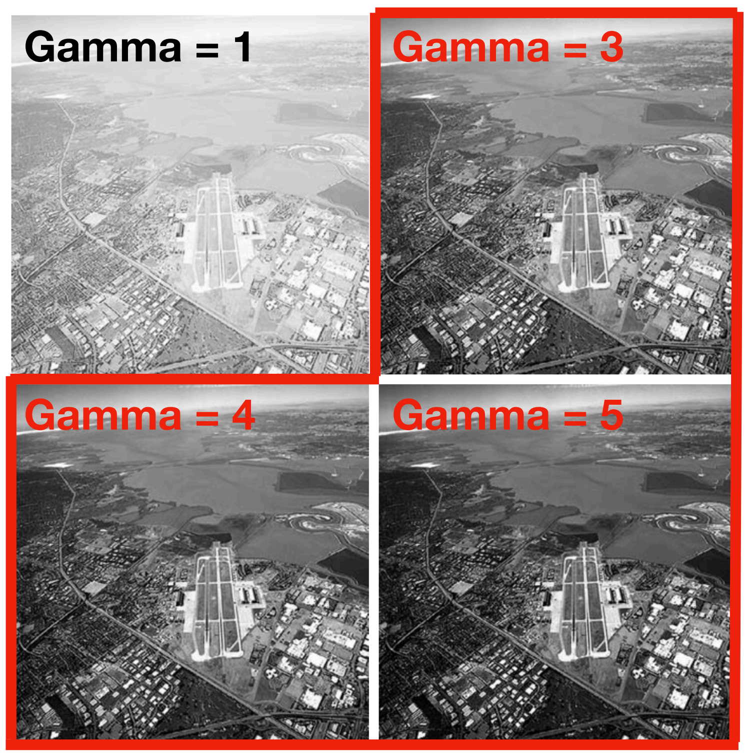
Basic Intensity Transformation Functions

- Importance of Gamma Correction

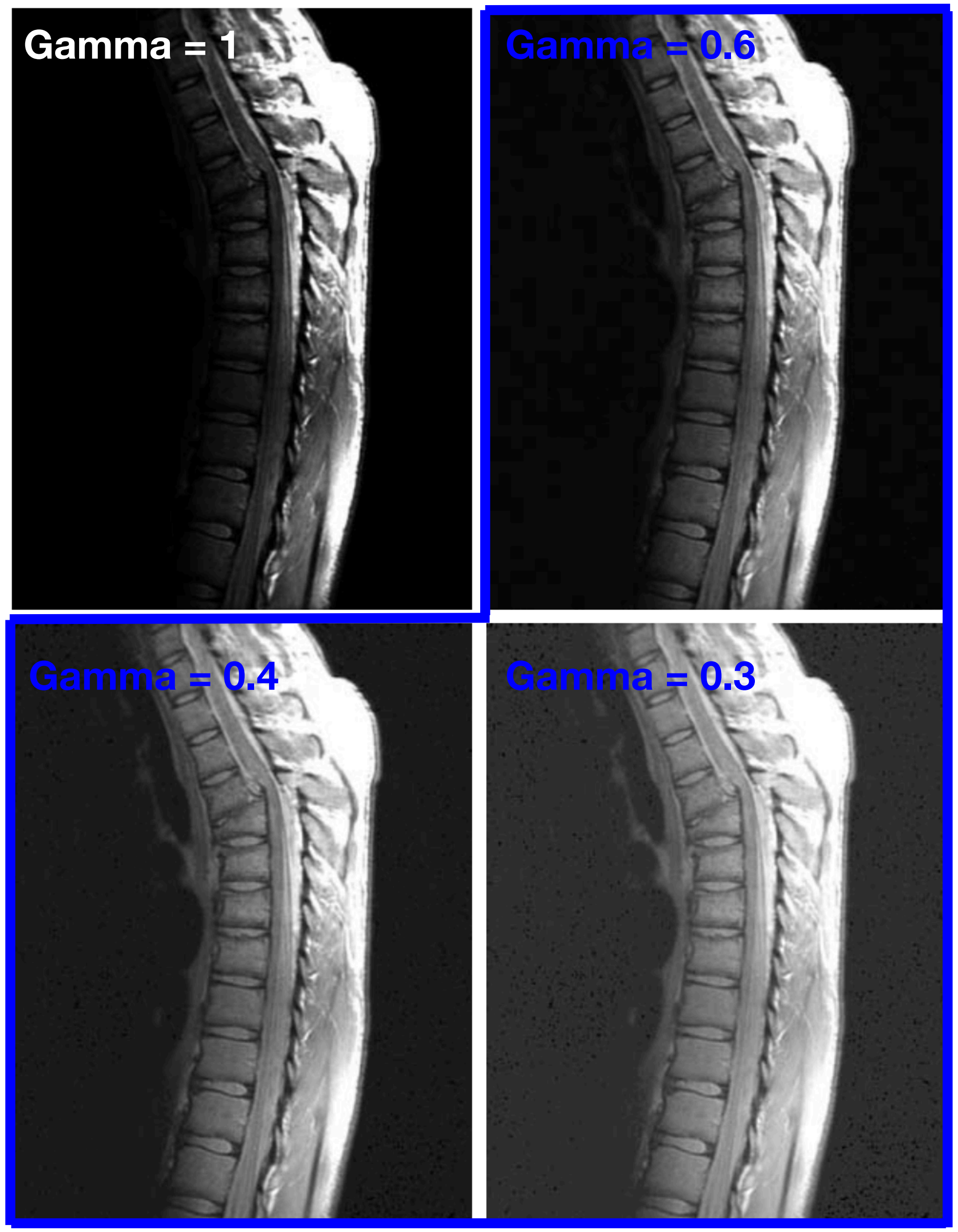
-> Used in displaying image accurately
on the computer screen
(Gamma correction reproduce color accurately)

-> Useful for general purpose contrast
manipulation
(MRI, Arial Image, etc..)

Gamma > 1



Gamma < 1



Basic Intensity Transformation Functions

- Piecewise-Linear Transformation Functions

  -> Real-valued function defined on real numbers or segments

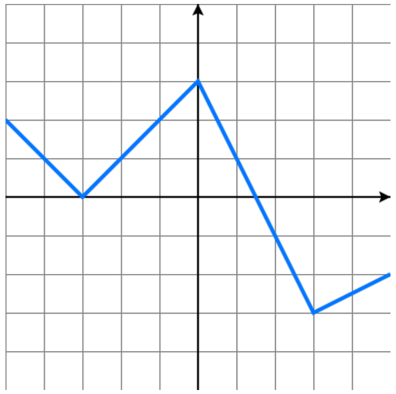
  -> Advantages

    * The form of piecewise functions **can be arbitrarily complex**
    * Practical implementation of some important transformation **can be formulated only as piecewise functions**

  -> Disadvantages

    * Specification of piecewise function **requires considerably more user input**

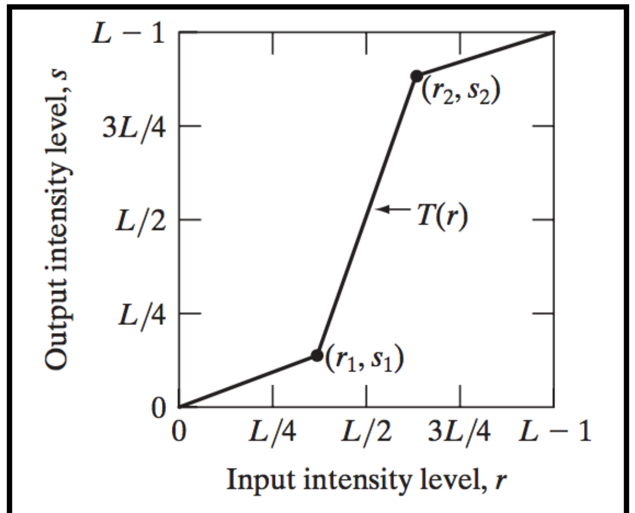
- Contrast Stretching
- Intensity-level Slicing
- Bit-plane Slicing

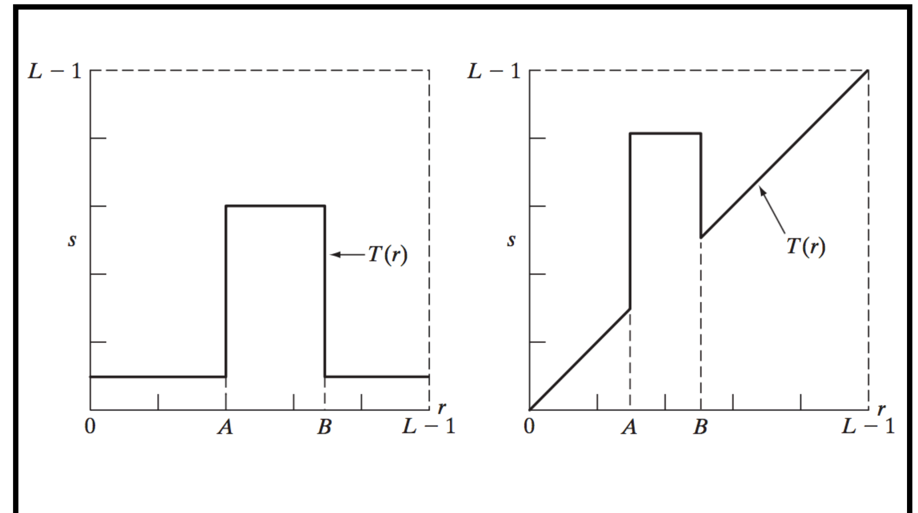


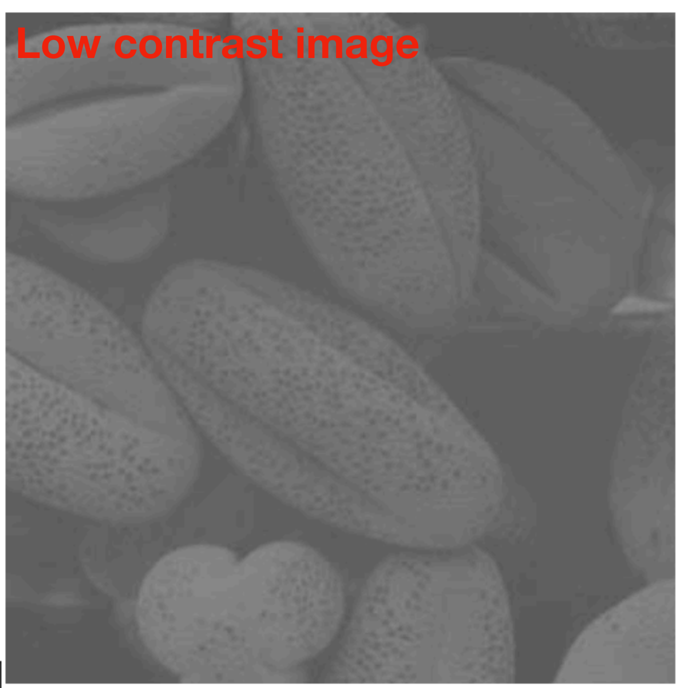
$$f(x) = \begin{cases} -x-3 & \text{if } x \leq -3 \\ x+3 & \text{if } -3 < x < 0 \\ -2x+3 & \text{if } 0 \leq x < 3 \\ 0.5x-4.5 & \text{if } x \geq 3 \end{cases}$$

Basic Intensity Transformation Functions

- Contrast Stretching & Intensity-level Slicing (Thresholding)







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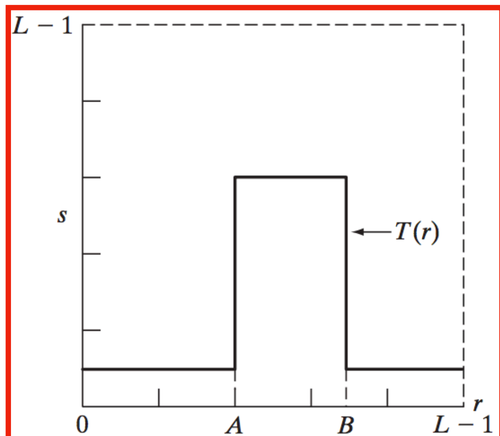
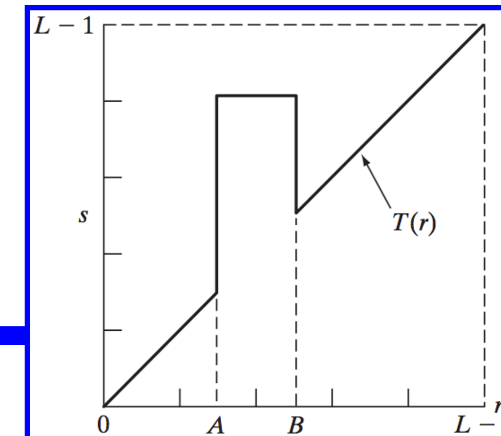




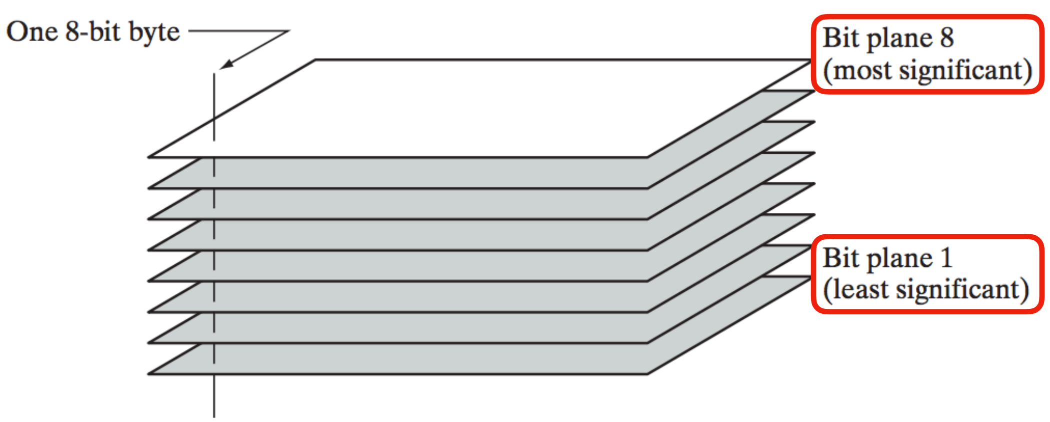
Binary image

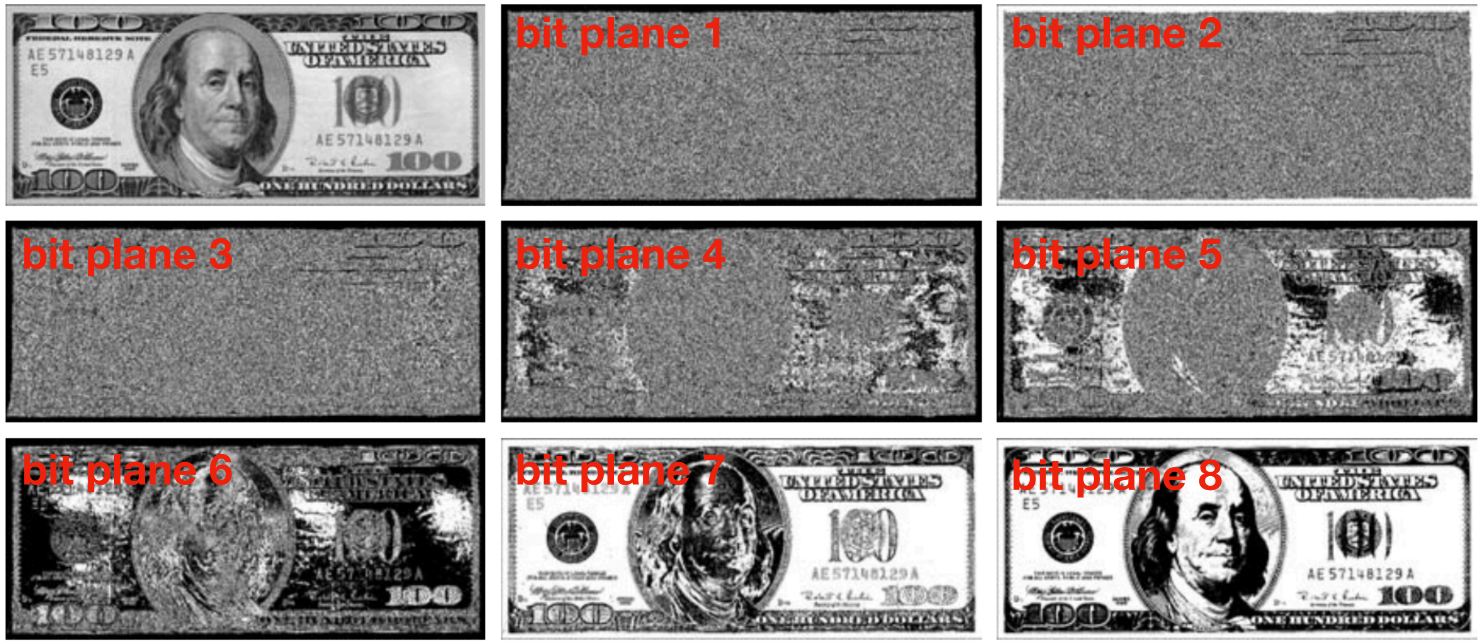
Intensity levels except specific range in an image are preserved

Basic Intensity Transformation Functions

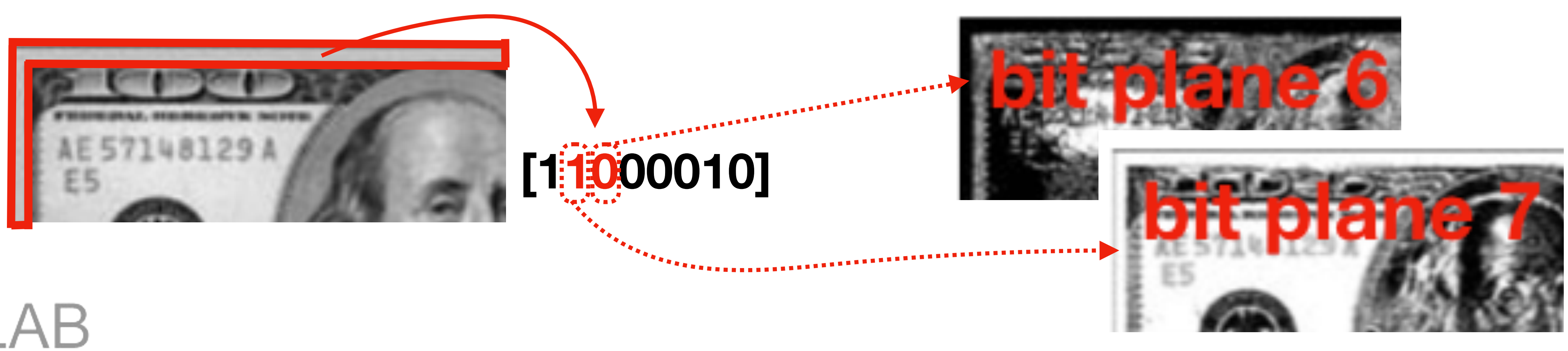
- Bit-plane Slicing

  -> Instead of highlighting intensity level range, highlighting the contributions made to total image appearance by specific bits





  -> Lower-order planes contribute to more subtle intensity details in the image



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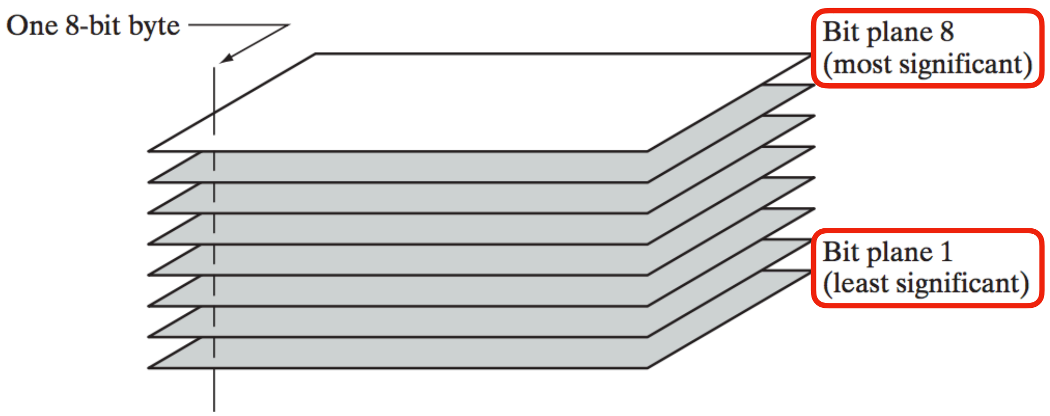
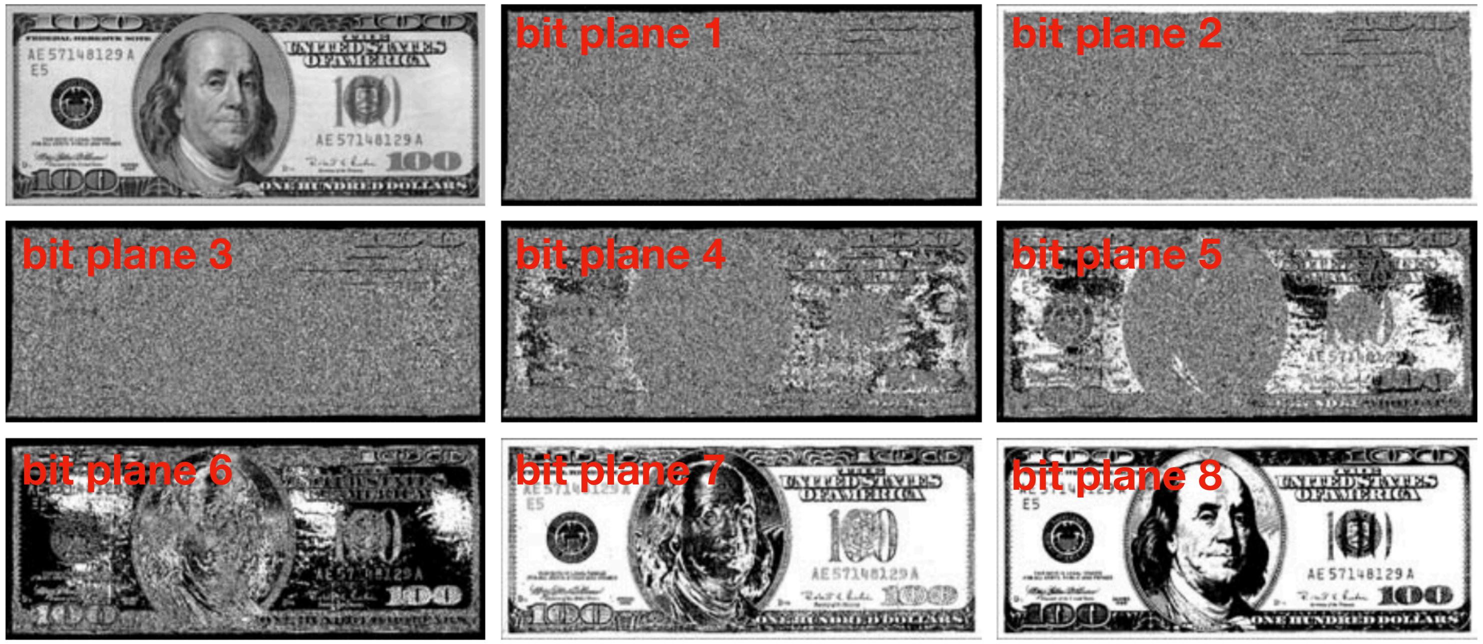




  -> Lower-order planes contribute to more subtle intensity details in the image

  -> Decomposing into bit-plane is useful for analyzing relative importance of each bit in the image

  => Useful in [**Quantizing image / Image Compression**]

Histogram Processing

- Histogram of a Digital Image

$$h(r_k) = n_k$$

Histogram Processing

- Histogram of a Digital Image

estimate of the probability
of occurrence of intensity level
in an image

$$h(\boxed{r_k}) = \boxed{n_k} \xrightarrow{\text{Normalize}} \boxed{p(r_k)} = n_k/\boxed{M}\boxed{N}$$

k th intensity value

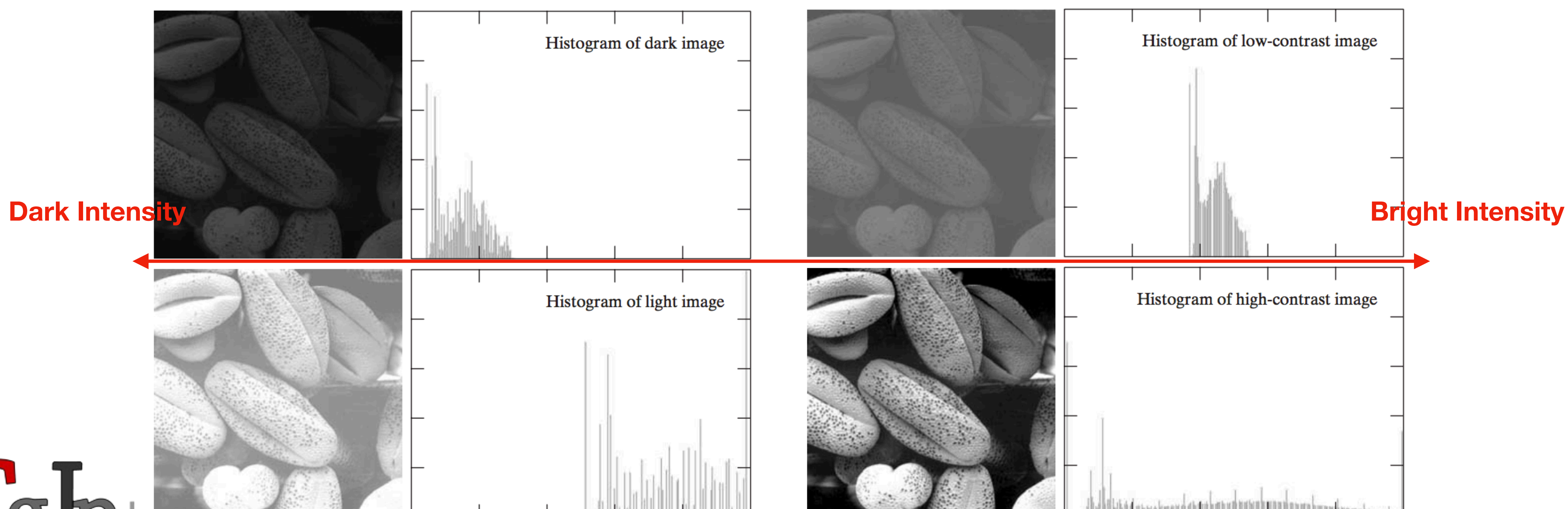
number of pixels
in the image
with intensity rk

Row dimension of the image

Column dimension

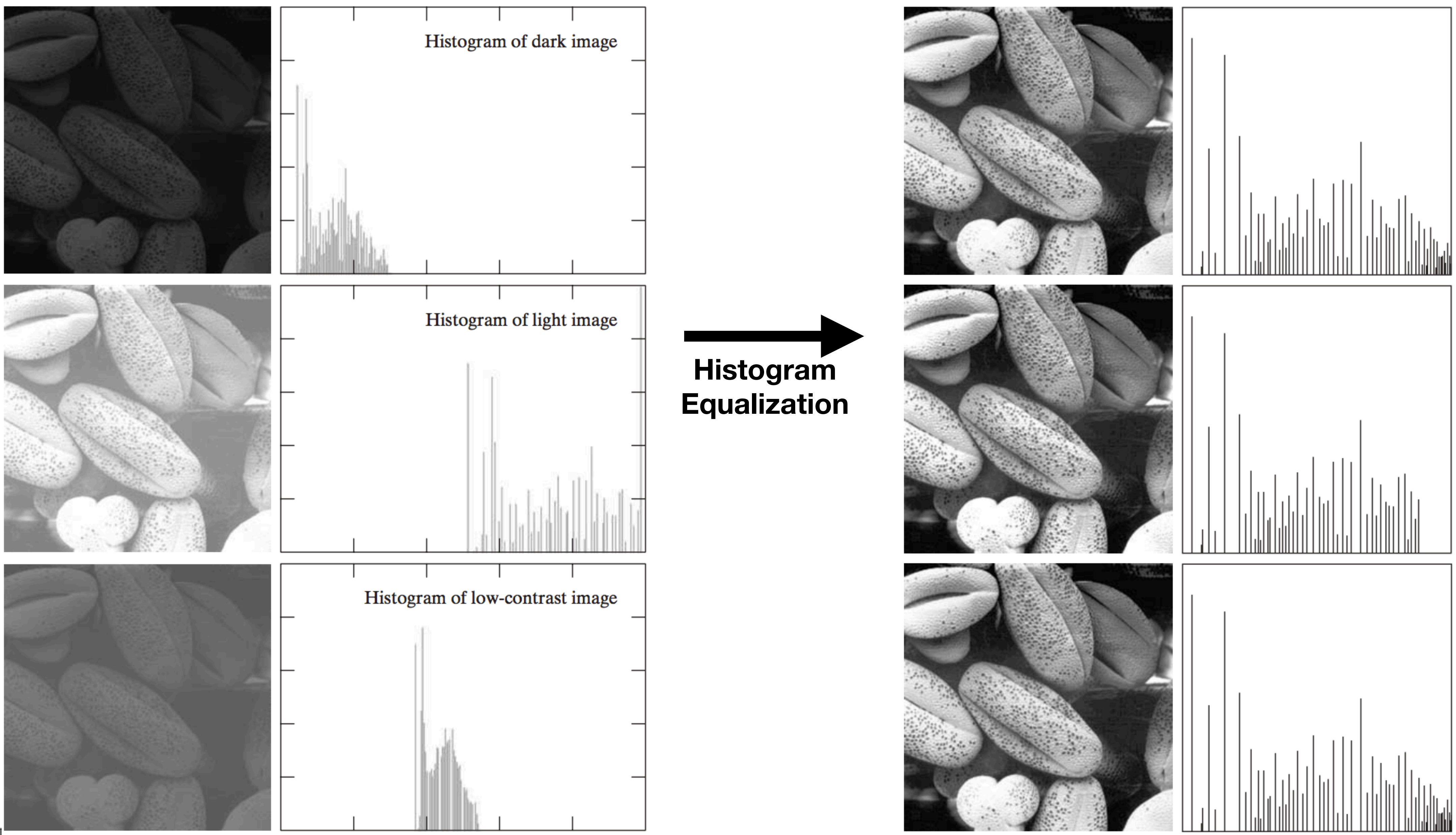
-> Histograms are the basis for numerous spatial domain processing

-> Histogram manipulation can be used for image enhancement



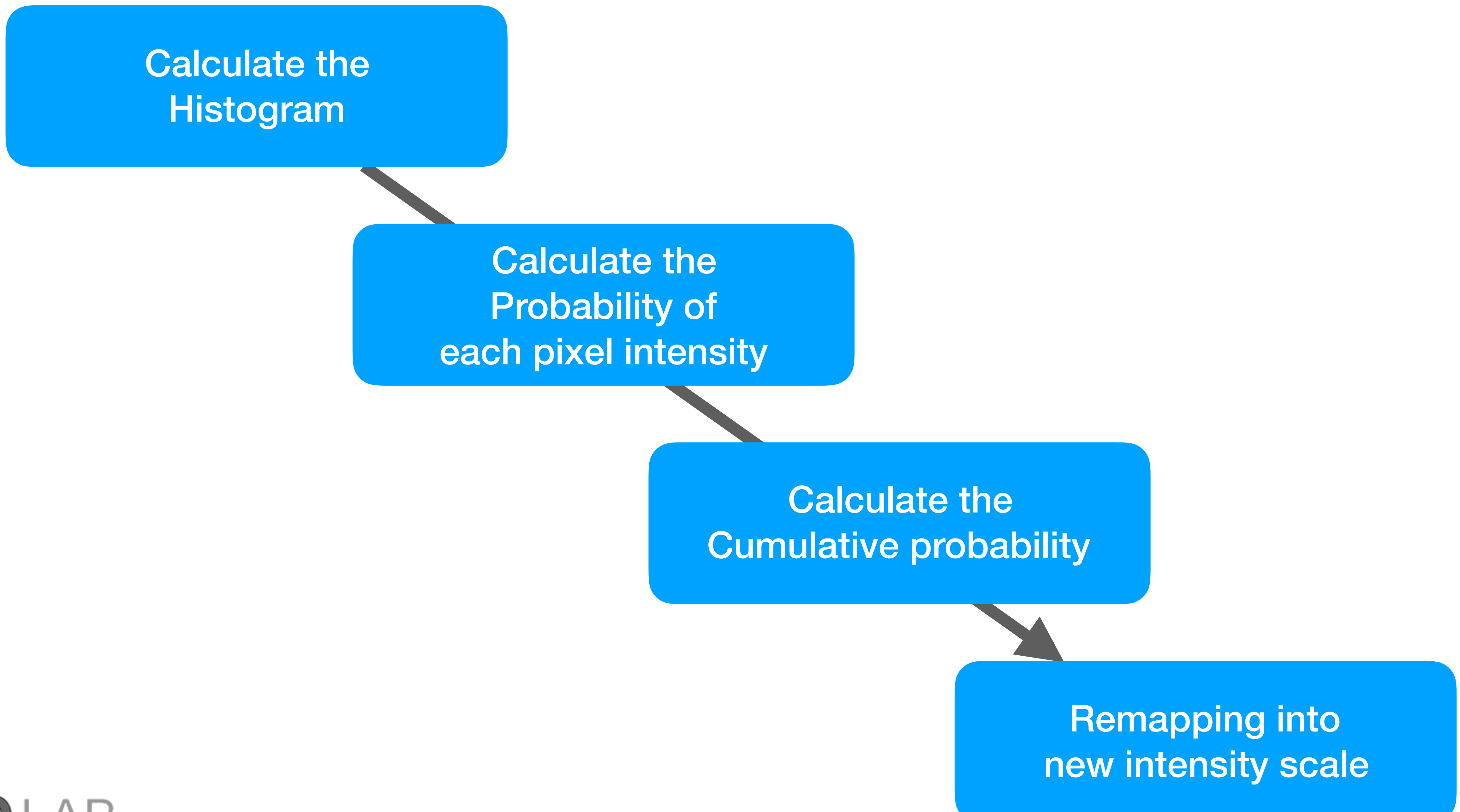
Histogram Processing

- Histogram Equalization
  -> Span the intensity scale to enhance the image



Histogram Processing

- Histogram Equalization
  -> Process of Histogram Equalization



Histogram Processing

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  -> Calculate the Probability of each pixel intensity
    => Assume Transformation function is strictly monotonic increasing function

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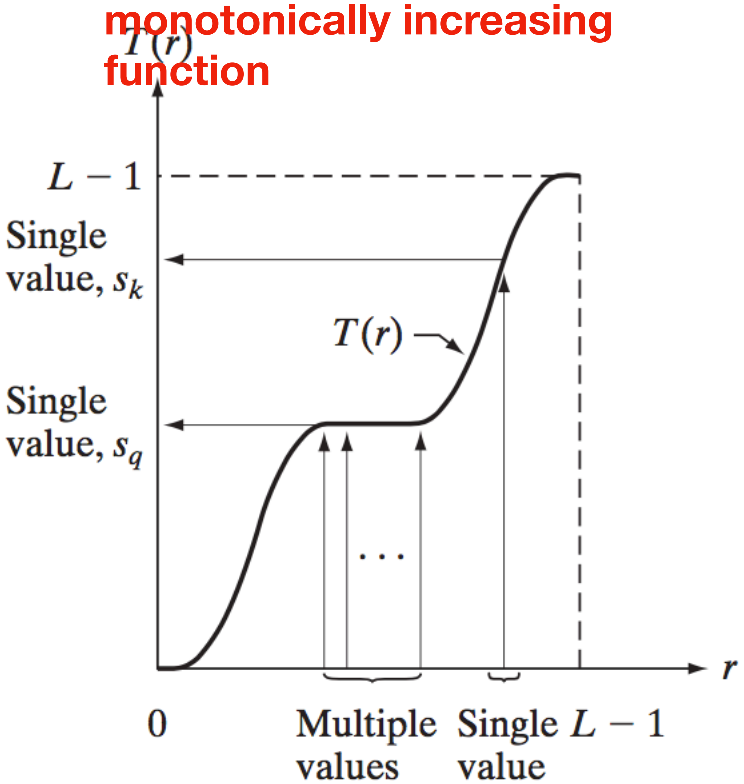
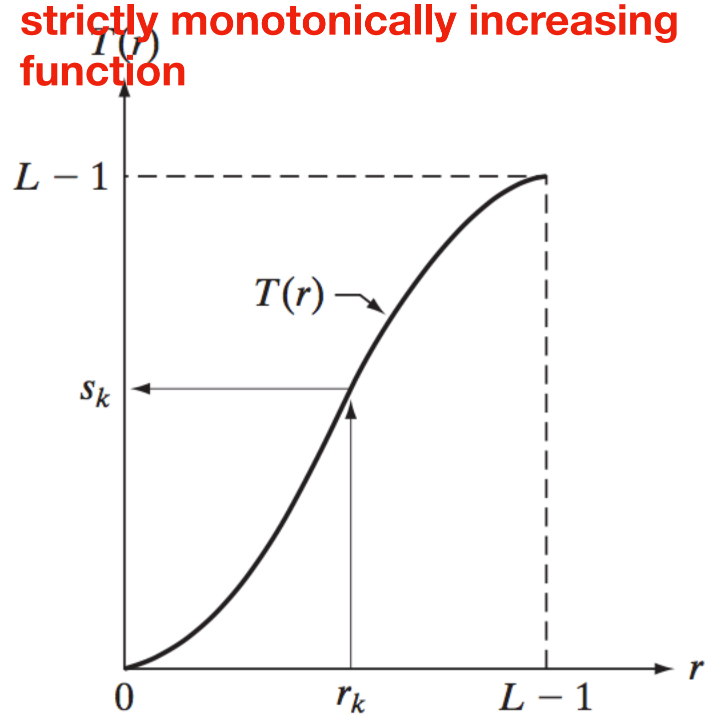
why?

monotonically increasing function



strictly monotonically increasing function



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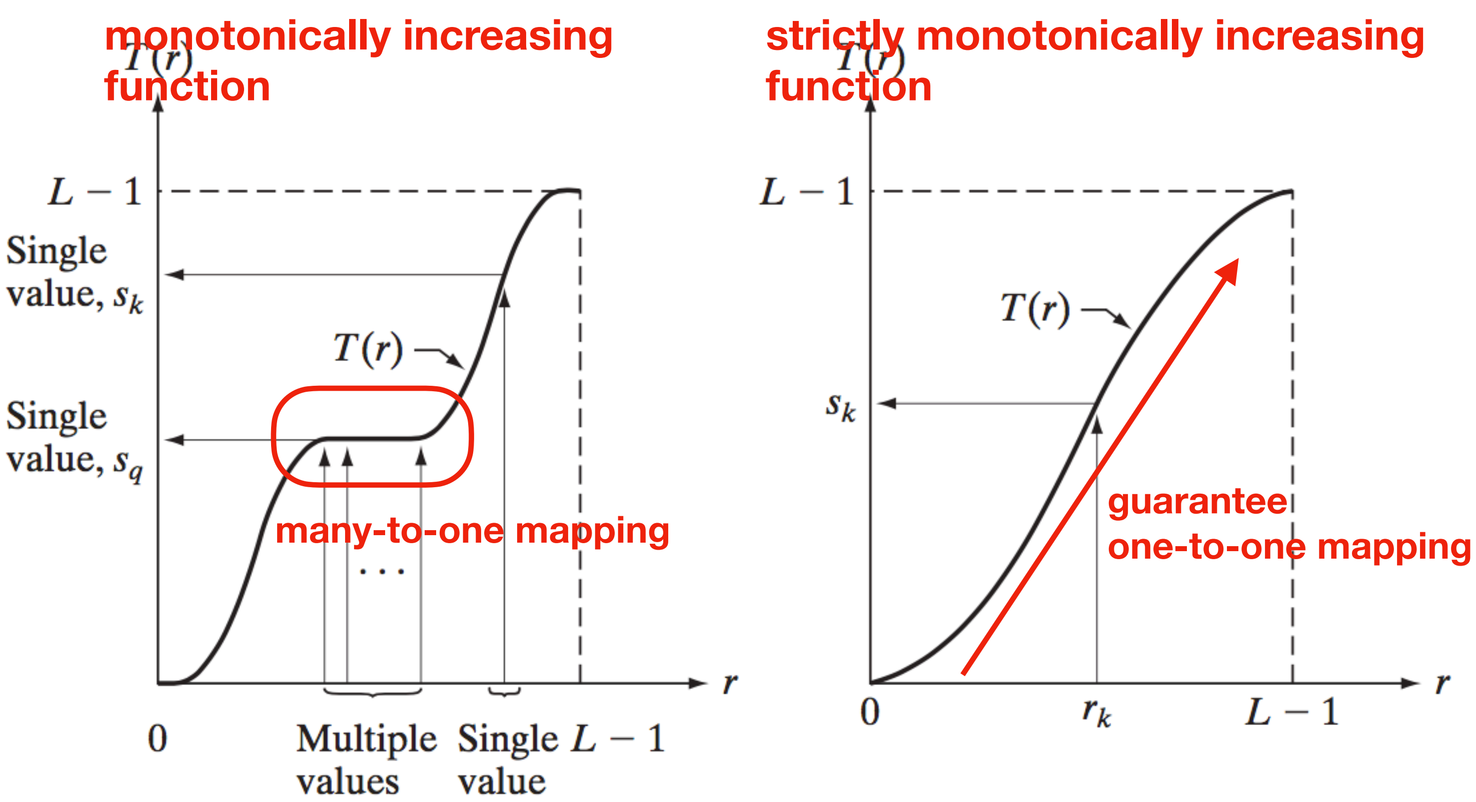

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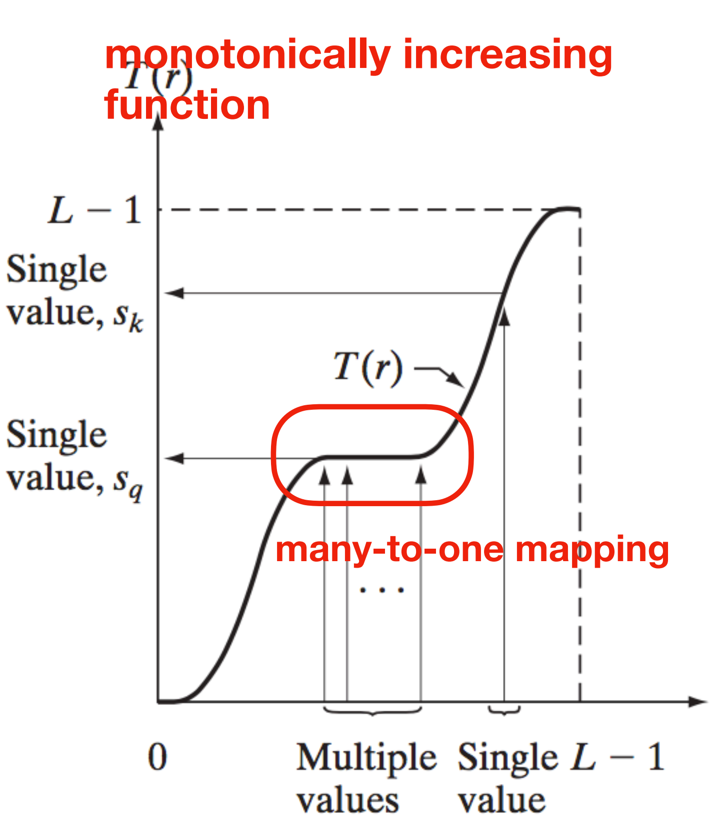

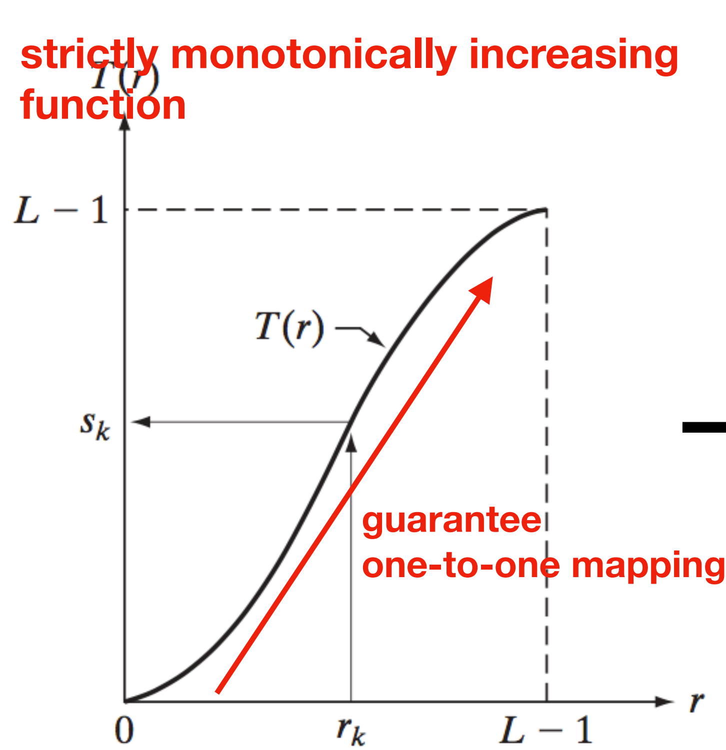

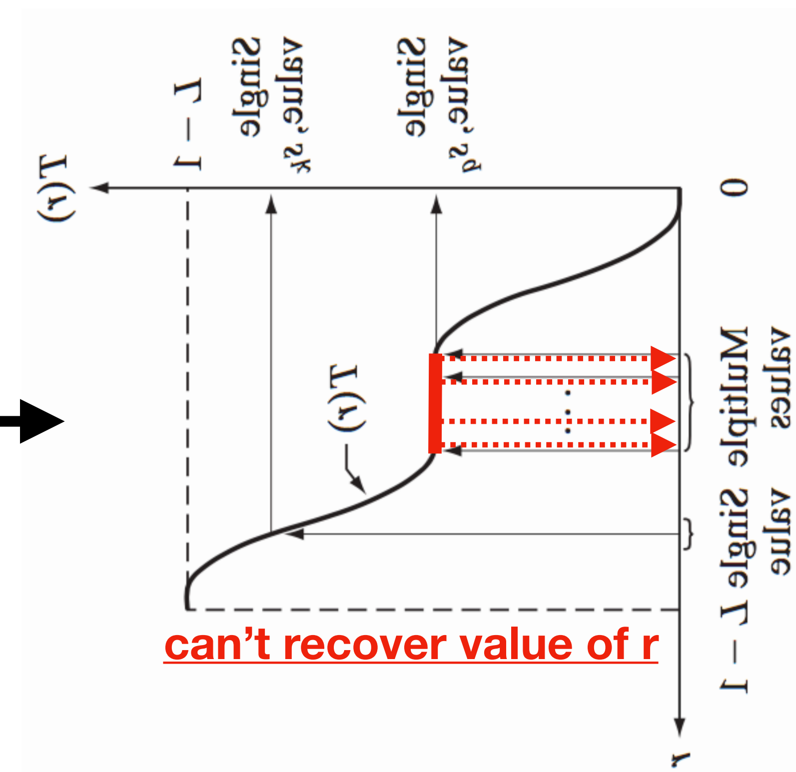

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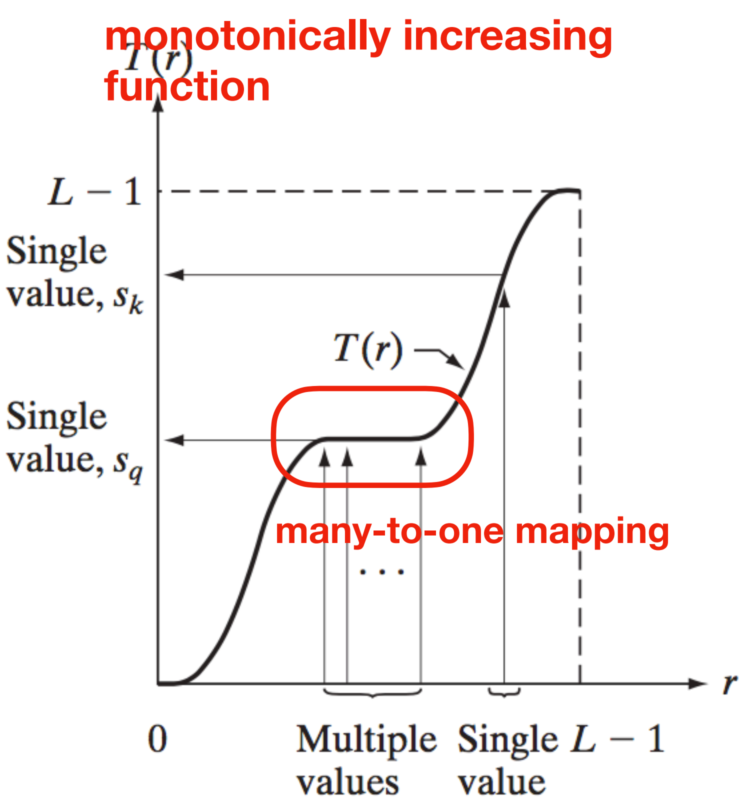

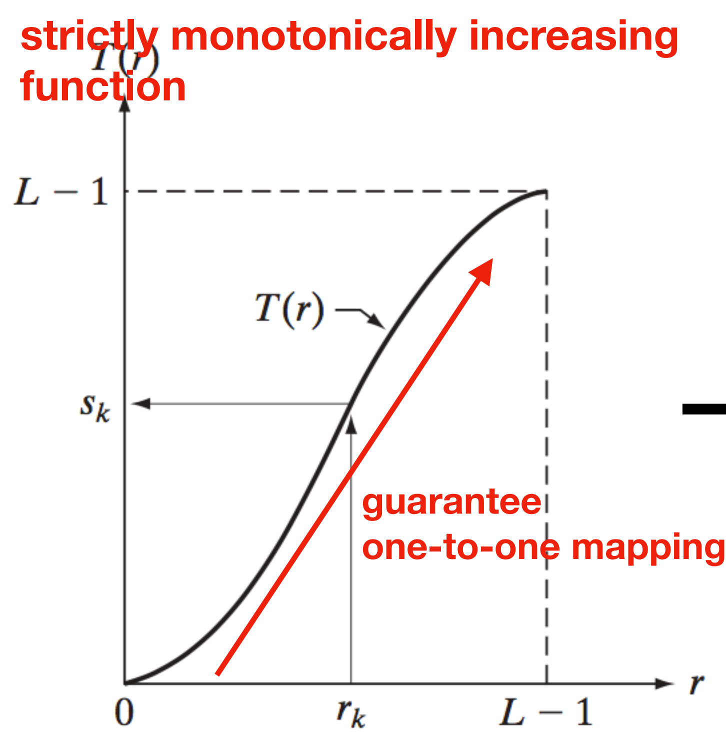

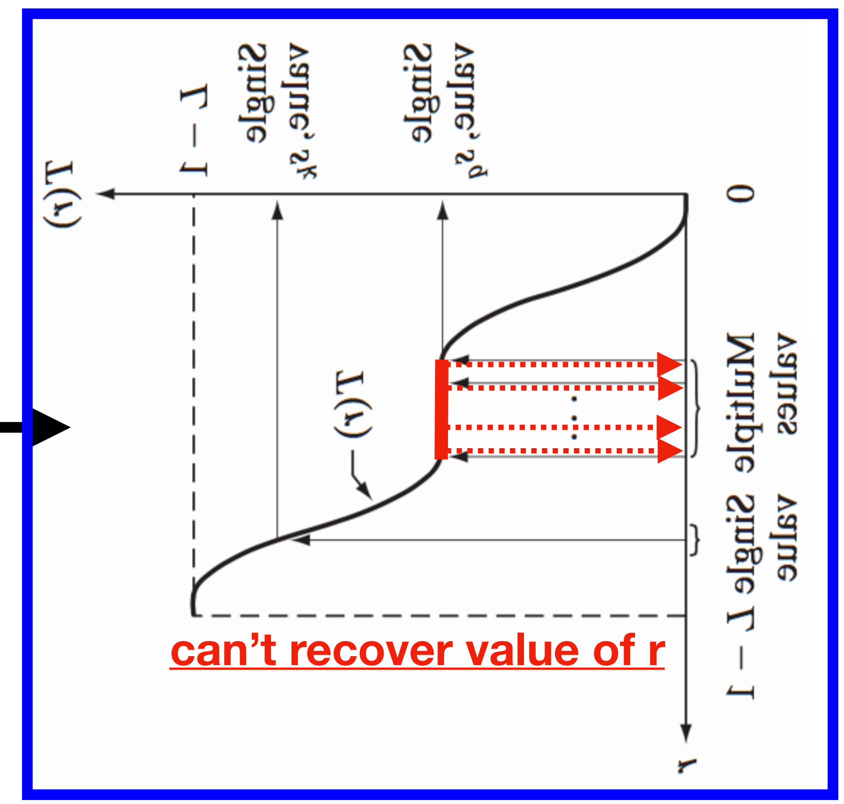

Handle this problem by Histogram Matching Technique

Histogram Processing

- Histogram Equalization
  -> Calculate the Probability of each pixel intensity
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    => Viewing Intensity levels in image as random variable in the interval [0, L-1]

Histogram Processing

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  -> Calculate the Probability of each pixel intensity

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$$p_r(r),\ p_s(s)\ :\ PDF(Probability\ Density\ Function)\ of\ r\ and\ s$$

  => if, PDF of r and T(r) are Known and T(r) is continuous and differentiable over the range of value of interest

Histogram Processing

- Histogram Equalization
  -> Calculate the Cumulative probability
    => Assume Transformation function is strictly monotonic increasing function
    => Viewing Intensity levels in image as random variable in the interval [0, L-1]

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$$p_s(s)\ =\ p_r(r)\left|\frac{dr}{ds}\right|$$

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  -> Calculate the Cumulative probability

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PDF of output intensity variable s

$$p_s(s) = p_r(r)\left|\frac{dr}{ds}\right|$$

$$\boxed{p(r_k) = n_k/MN}$$

: PDF of s is determined by PDF of r and transformation function

  => Transformation function of particular importance in image processing has the form

Histogram Processing

- Histogram Equalization

  -> Calculate the Cumulative probability

  => Assume Transformation function is strictly monotonic increasing function

  => Viewing Intensity levels in image as random variable in the interval [0, L-1]

$$p_r(r),\ p_s(s)\ :\ PDF(Probability\ \ Density\ \ Function)\ \ of\ \ r\ \ and\ \ s$$

  => if, PDF of r and T(r) are Known and T(r) is continuous and differentiable over the range of value of interest

$$\boxed{p(r_k) = n_k/MN}$$

$$p_s(s) = p_r(r)\left|\frac{dr}{ds}\right|$$

: PDF of s is determined by PDF of r and transformation function

  => Transformation function of particular importance in image processing has the form

$$s = T(r) = (L-1)\int_0^r p_r(w)dw$$

Histogram Processing

- Histogram Equalization
  -> Calculate the Cumulative probability
  => Assume Transformation function is strictly monotonic increasing function
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$$p_r(r),\ p_s(s)\ :\ PDF(Probability\ Density\ Function)\ of\ r\ and\ s$$

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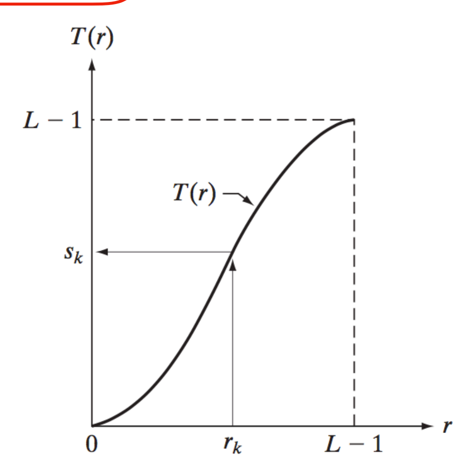
: PDF of s is determined by PDF of r and transformation function

=> Transformation function of particular importance in image processing has the form

$$\boxed{s = T(r) = \underbrace{(L-1)}_{scaling}\int_0^r p_r(w)\,dw}$$

→ Cumulative Distribution Function (CDF)



Histogram Processing

- Histogram Equalization

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derivative of a definite integral with respect to its upper limit is the integrand evaluated at the limit

$$\frac{ds}{dr} = \frac{dT(r)}{dr} = (L\text{-}1)\frac{d}{dr}\left[\int_0^r p_r(w)\,dw\right] = (L\text{-}1)p_r(r)$$

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- Histogram Equalization

  -> Calculate the Cumulative probability

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$$\boxed{p(r_k)\ =\ n_k/MN}$$

PDF of output intensity variable s

$$p_s(s) = p_r(r)\left|\frac{dr}{ds}\right|$$

: PDF of s is determined by PDF of r and transformation function

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$$s = T(r) = (L-1)\int_0^r p_r(w)\,dw$$

  => To find the Ps(s), use Leibniz's rule and this equation

derivative of a definite integral with respect to its upper limit is the integrand evaluated at the limit

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$$p_s(s) = p_r(r)\left|\frac{dr}{ds}\right| = p_r(r)\left|\frac{1}{(L-1)p_r(r)}\right| = \frac{1}{L-1},\ (0 \le s \le L-1)$$

Histogram Processing

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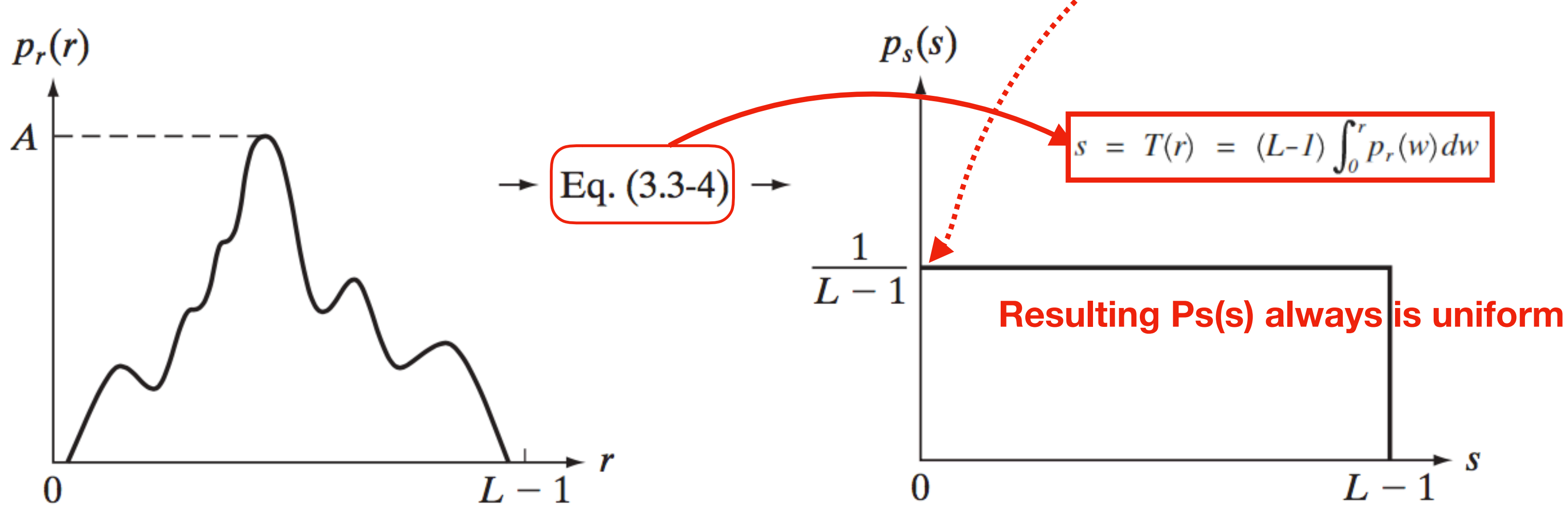

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  => Suppose intensity values in an image have the PDF

$$p_r(r) = \frac{2r}{(L-1)^2}, \quad (0 \le r \le L-1) \;\; / \;\; 0, \; otherwise$$

Histogram Processing

- Histogram Equalization

  -> Calculate the Cumulative probability

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$$p_r(r) = \frac{2r}{(L-1)^2}, \quad (0 \le r \le L-1) \;\; / \;\; 0, \; otherwise$$

$$s = T(r) = (L-1)\int_0^r p_r(w)dw = \frac{2}{L-1}\int_0^r w dw = \frac{r^2}{L-1}$$

Histogram Processing

- Histogram Equalization

  -> Calculate the Cumulative probability

  => Assume Transformation function is strictly monotonic increasing function

  => Viewing Intensity levels in image as random variable in the interval [0, L-1]

  => if, PDF of r and T(r) are Known and T(r) is continuous and differentiable over the range of value of interest

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  => Suppose intensity values in an image have the PDF and recalculate the image processing form and Ps(s)

$$p_r(r) = \frac{2r}{(L-1)^2}, \quad (0 \le r \le L-1) \quad / \quad 0, \ otherwise$$

$$s = T(r) = (L-1)\int_0^r p_r(w)dw = \frac{2}{L-1}\int_0^r wdw = \frac{r^2}{L-1}$$

Uniform PDF

$$p_s(s) = p_r(r)\left|\frac{dr}{ds}\right| = \frac{2r}{(L-1)^2}\left|\left[\frac{ds}{dr}\right]^{-1}\right| = \frac{2r}{(L-1)^2}\left|\left[\frac{d}{dr}\frac{r^2}{L-1}\right]^{-1}\right| = \frac{2r}{(L-1)^2}\left|\left[\frac{(L-1)}{2r}\right]\right| = \boxed{\frac{1}{L-1}}$$

Histogram Processing

- Histogram Equalization
  -> Remapping into new intensity scale
    => Assume Transformation function is strictly monotonic increasing function
    => Viewing Intensity levels in image as random variable in the interval [0, L-1]
    => if, PDF of r and T(r) are Known and T(r) is continuous and differentiable over the range of value of interest
   => Transformation function of particular importance in image processing has the form
   => To find the Ps(s), use Leibniz's rule and this equation
   => Discrete form of the transformation equation function is

$$s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j) = \frac{(L-1)}{MN}\sum_{j=0}^{k} n_j, \quad (k = 0,1,2,\ldots,L-1)$$

$$\boxed{p(r_k) = n_k/MN}$$

Row & Column of the image dimension, Scale factor to normalize

Histogram Processing

- Histogram Equalization
  -> Remapping into new intensity scale
    => Assume Transformation function is strictly monotonic increasing function
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Example

r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

$$s_0 = T(r_0) = 7\sum_{j=0}^{0} p_r(r_j) = 7p_r(r_0) = 1.33$$

$$s_1 = T(r_1) = 7\sum_{j=0}^{1} p_r(r_j) = 7p_r(r_0) + 7p_r(r_1) = 3.08$$

$$s_2 = 4.55,\ s_3 = 5.67,\ s_4 = 6.23,\ s_5 = 6.65,\ s_6 = 6.86,\ s_7 = 7.00$$

Histogram Processing

- Histogram Equalization
  -> Remapping into new intensity scale
  => Assume Transformation function is strictly monotonic increasing function
  => Viewing Intensity levels in image as random variable in the interval [0, L-1]
  => if, PDF of r and T(r) are Known and T(r) is continuous and differentiable over the range of value of interest
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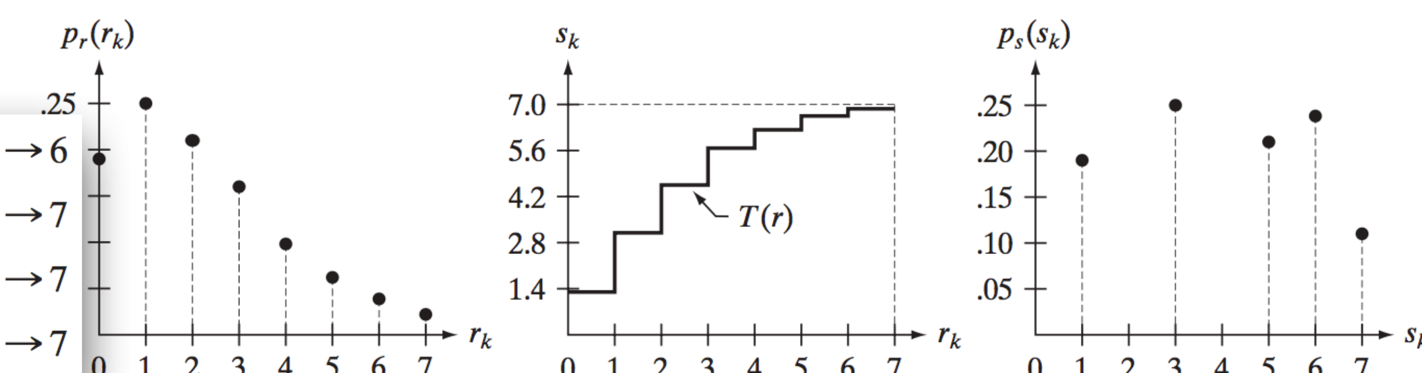
$$s_2 = 4.55,\ s_3 = 5.67,\ s_4 = 6.23,\ s_5 = 6.65,\ s_6 = 6.86,\ s_7 = 7.00$$

$s_0 = 1.33 \rightarrow 1$	$s_4 = 6.23 \rightarrow 6$
$s_1 = 3.08 \rightarrow 3$	$s_5 = 6.65 \rightarrow 7$
$s_2 = 4.55 \rightarrow 5$	$s_6 = 6.86 \rightarrow 7$
$s_3 = 5.67 \rightarrow 6$	$s_7 = 7.00 \rightarrow 7$



Histogram Processing

- Histogram Matching

Histogram Equalization →

Result from Histogram Equalization is Predictable

Method is simple to implement

Histogram Processing

- Histogram Matching

Histogram Equalization →

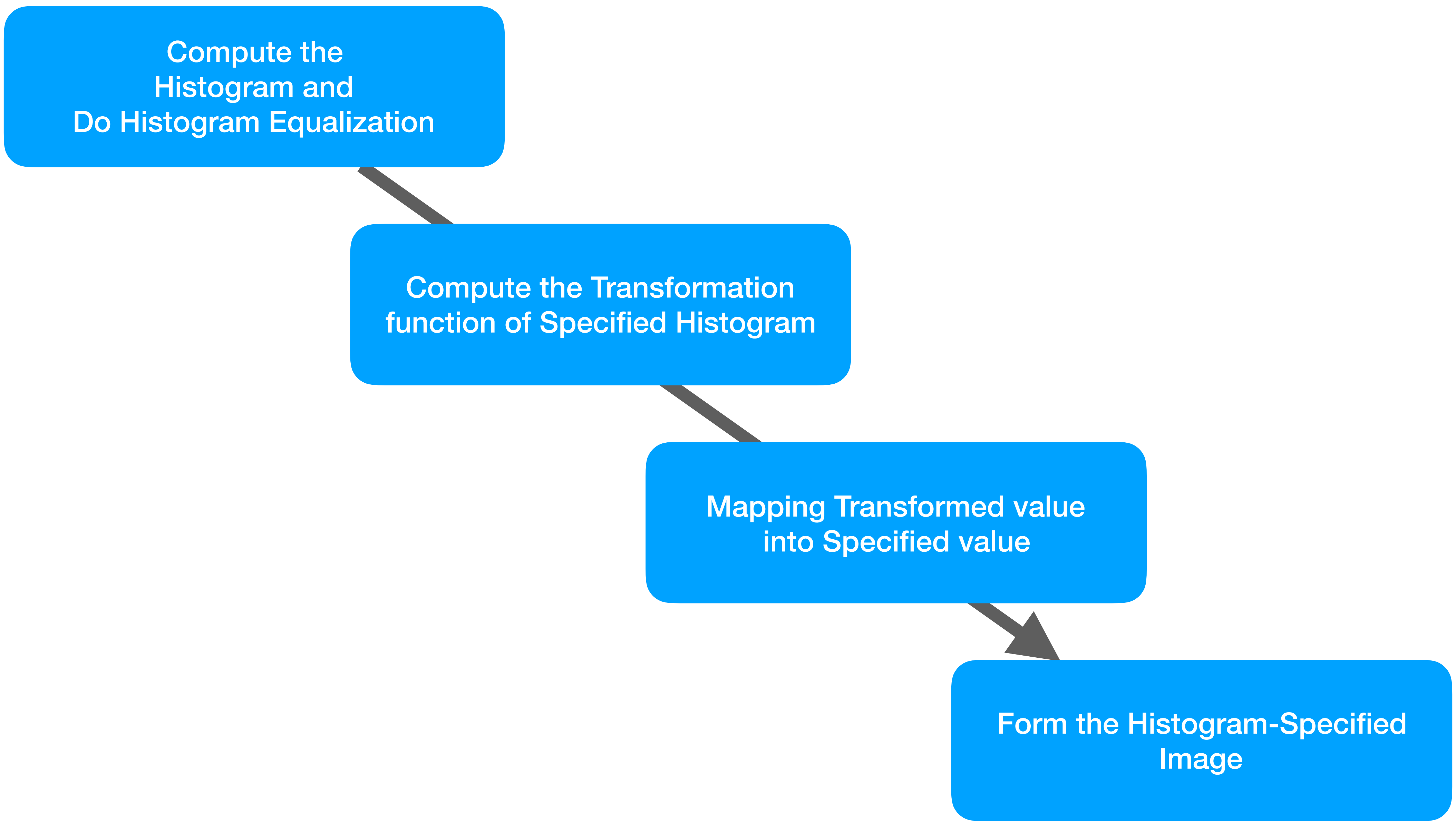
Result from Histogram Equalization is Predictable

Method is simple to implement

Produce output image that has a uniform histogram

Histogram Processing

- Histogram Matching
  -> Method to generate a processed image that has specified desired histogram



Histogram Processing

- Histogram Matching
  -> Compute the histogram and do histogram-equalization

$$s = T(r) = (L-1)\int_0^1 p_r(w)\,dw$$

Histogram Processing

- Histogram Matching
  -> Compute the histogram and do histogram-equalization
  -> Compute the Transformation function of Specified Histogram

  * New continuous random variable : z
  * Pr(r) : Continuous PDF of r , Pz(z) : Continuous PDF of z

$$s = T(r) = (L-1)\int_0^1 p_r(w)\,dw$$

$$G(z) = (L-1)\int_0^z p_z(t)\,dt = s$$

Histogram Processing

- Histogram Matching
  -> Compute the histogram and do histogram-equalization
  -> Compute the Transformation function of Specified Histogram
  -> Mapping Transformed value into Specified value

* New continuous random variable : z
* Pr(r) : Continuous PDF of r , Pz(z) : Continuous PDF of z

$$s = T(r) = (L-1)\int_0^1 p_r(w)\,dw$$

$$G(z) = (L-1)\int_0^z p_z(t)\,dt = s$$

$$z = G^{-1}[T(r)] = G^{-1}(s)$$

Histogram Processing

- Histogram Matching
  -> Compute the histogram and do histogram-equalization
  -> Compute the Transformation function of Specified Histogram
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$$\boxed{z = G^{-1}[T(r)] = G^{-1}(s)}$$

z is obtained from s
=> This process is a mapping from s to z, being the desired values

Histogram Processing

- Histogram Matching

-> Compute the histogram and do histogram-equalization
-> Compute the Transformation function of Specified Histogram
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z is obtained from s
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-> Obtain the output image

1) **Equalizing the input image** using transformation function of input variable r
2) **Perform Inverse mapping for every each pixel** with value s in the equalized image
=> PDF of output image will be equal to the specified PDF

Histogram Processing

- Histogram Matching
  -> Compute the histogram and do histogram-equalization
  -> Compute the Transformation function of Specified Histogram
  -> Mapping Transformed value into Specified value
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$$G(z) = (L-1)\int_0^{z} p_z(t)\,dt = s$$

$$z = G^{-1}[T(r)] = G^{-1}(s)$$

Discretize

$$s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j) = \frac{(L-1)}{MN}\sum_{j=0}^{k} n_j, \quad k=0,1,2,\ldots,L-1$$

$$G(z_q) = (L-1)\sum_{i=0}^{q} p_z(z_i) = s_k$$

$$z_q = G^{-1}(s_k)$$

Histogram Processing

- Histogram Matching
  -> Compute the histogram and do histogram-equalization
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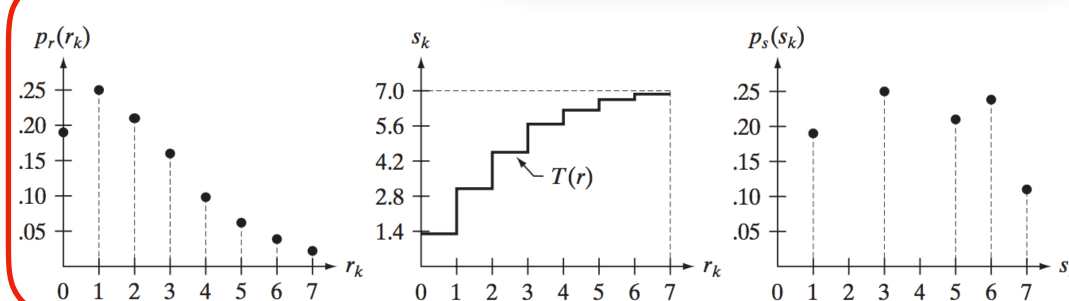
Compute G



Equalized

CgIp LAB
Computer Graphics and Image Processing Laboratory. Seoul National University

Histogram Processing

- Histogram Matching
  -> Compute the histogram and do histogram-equalization
  -> Compute the Transformation function of Specified Histogram
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  -> Obtain the output image

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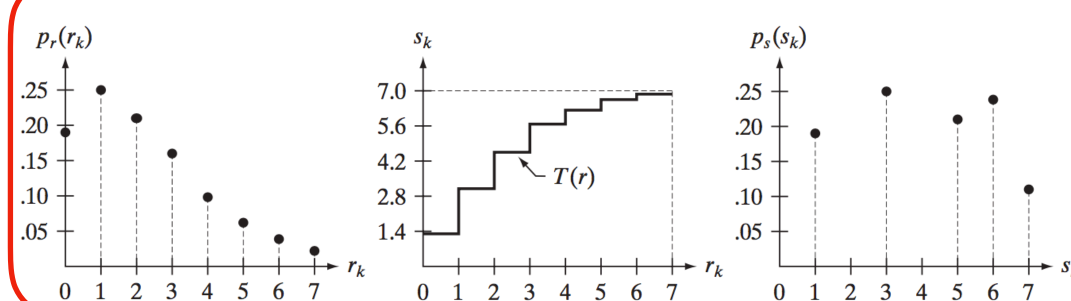
Compute G



Equalized

Convert to Integer

$$G(z_0) = 0.00 \rightarrow 0 \qquad G(z_4) = 2.45 \rightarrow 2$$

$$G(z_1) = 0.00 \rightarrow 0 \qquad G(z_5) = 4.55 \rightarrow 5$$

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Histogram Processing

- Histogram Matching
  -> Compute the histogram and do histogram-equa
  -> Compute the Transformation function of Specifi
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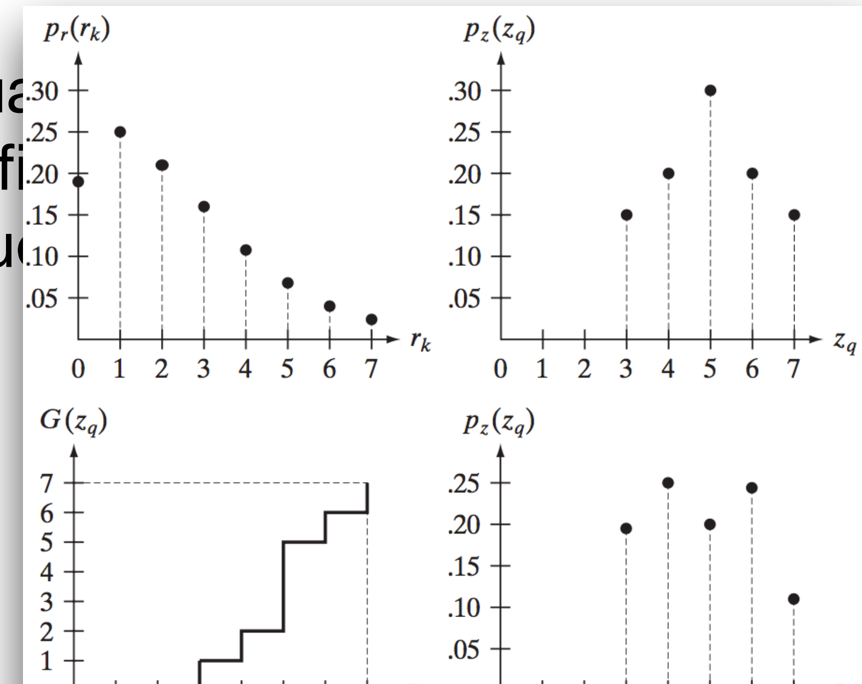

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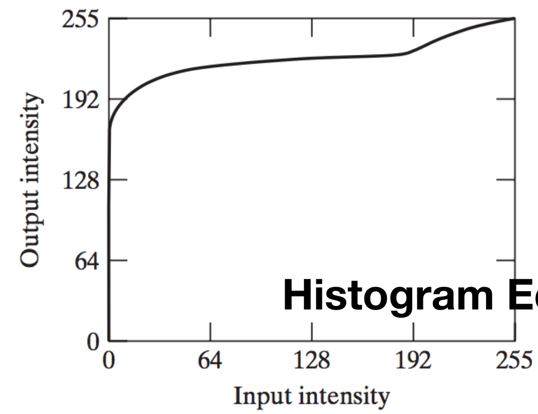
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

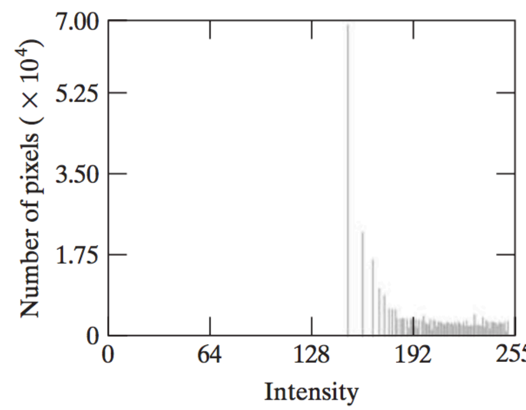
z_q	Specified $p_z(z_q)$	Actual $p_z(z_k)$
$z_0 = 0$	0.00	0.00
$z_1 = 1$	0.00	0.00
$z_2 = 2$	0.00	0.00
$z_3 = 3$	0.15	0.19
$z_4 = 4$	0.20	0.25
$z_5 = 5$	0.30	0.21
$z_6 = 6$	0.20	0.24
$z_7 = 7$	0.15	0.11

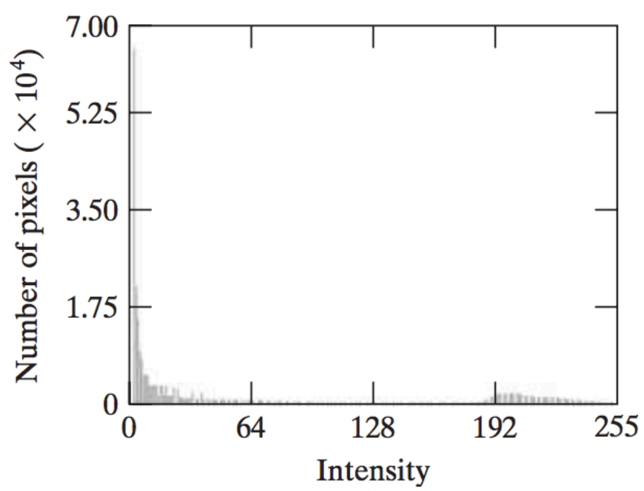
z_q	$G(z_q)$
$z_0 = 0$	0
$z_1 = 1$	0
$z_2 = 2$	0
$z_3 = 3$	1
$z_4 = 4$	2
$z_5 = 5$	5
$z_6 = 6$	6
$z_7 = 7$	7

- Histogram Matching



Histogram Equalization

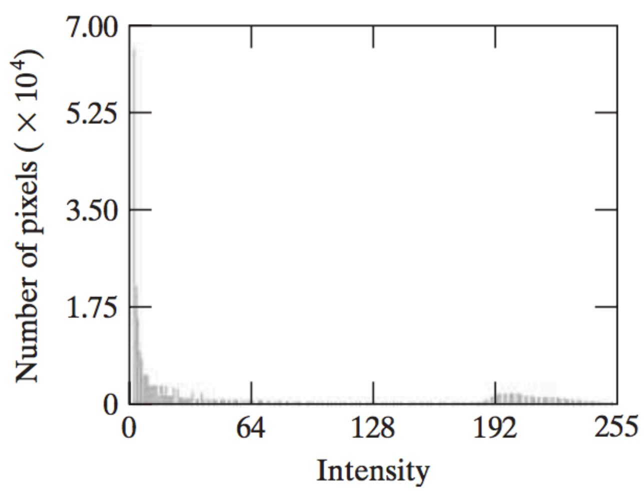




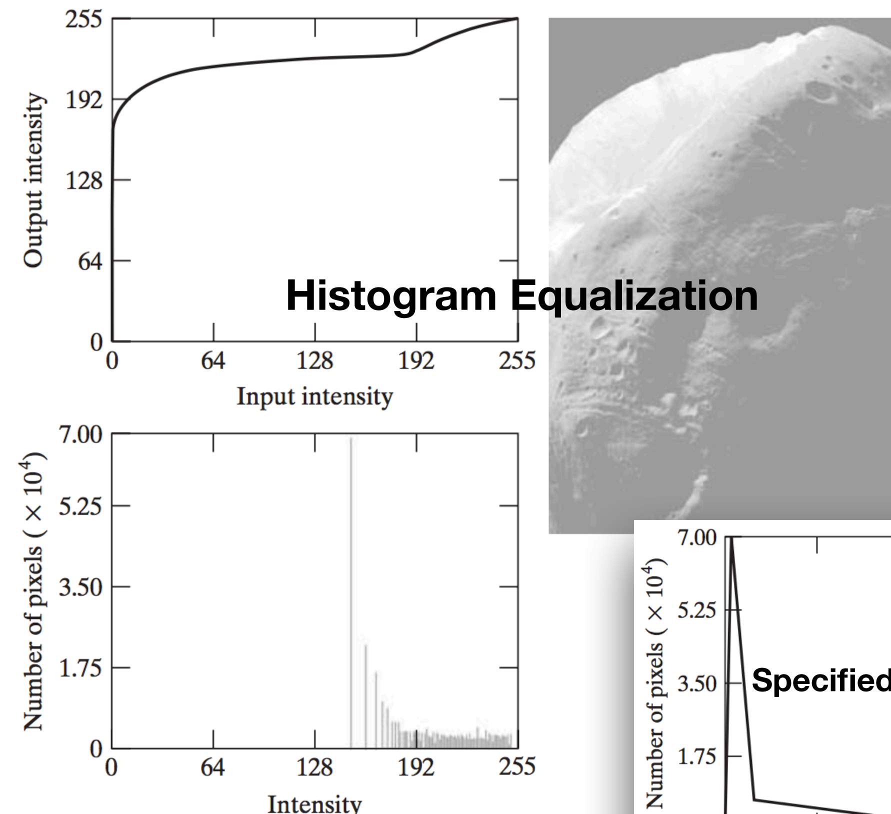
Original Image

Histogram Processing

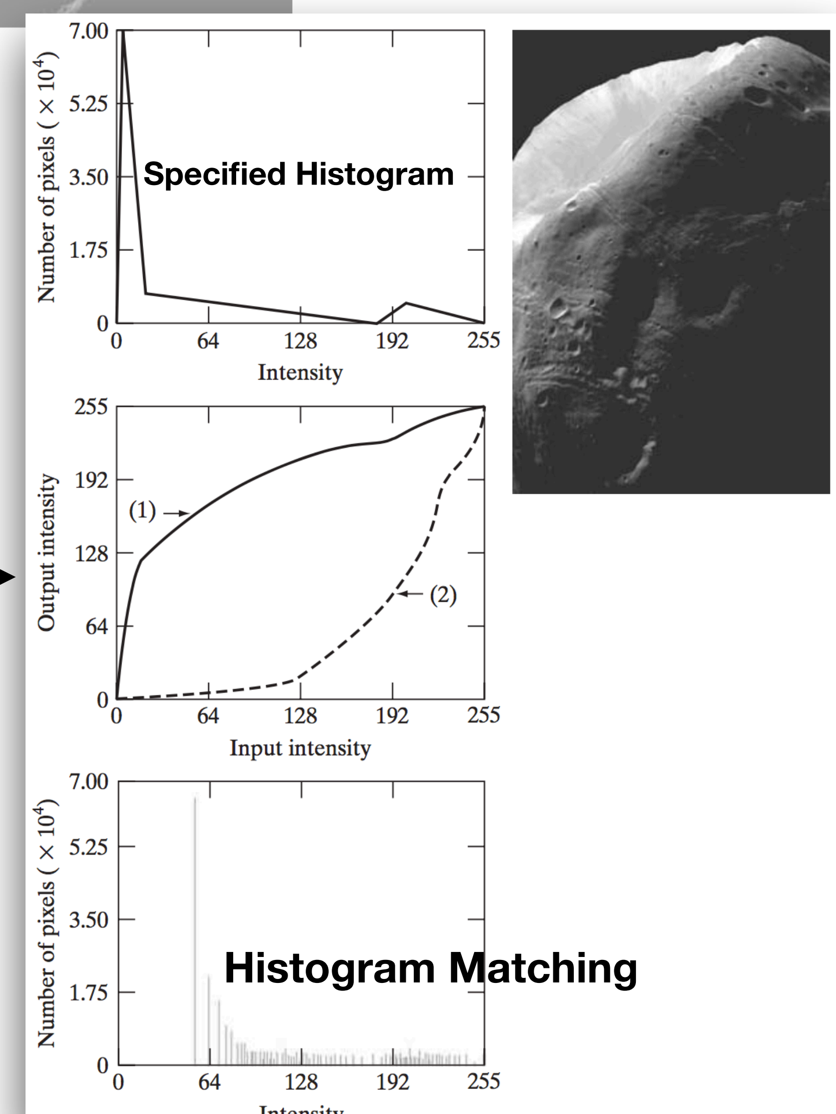
- Histogram Matching



Original Image



Histogram Equalization



Specified Histogram

Histogram Matching

Histogram Processing

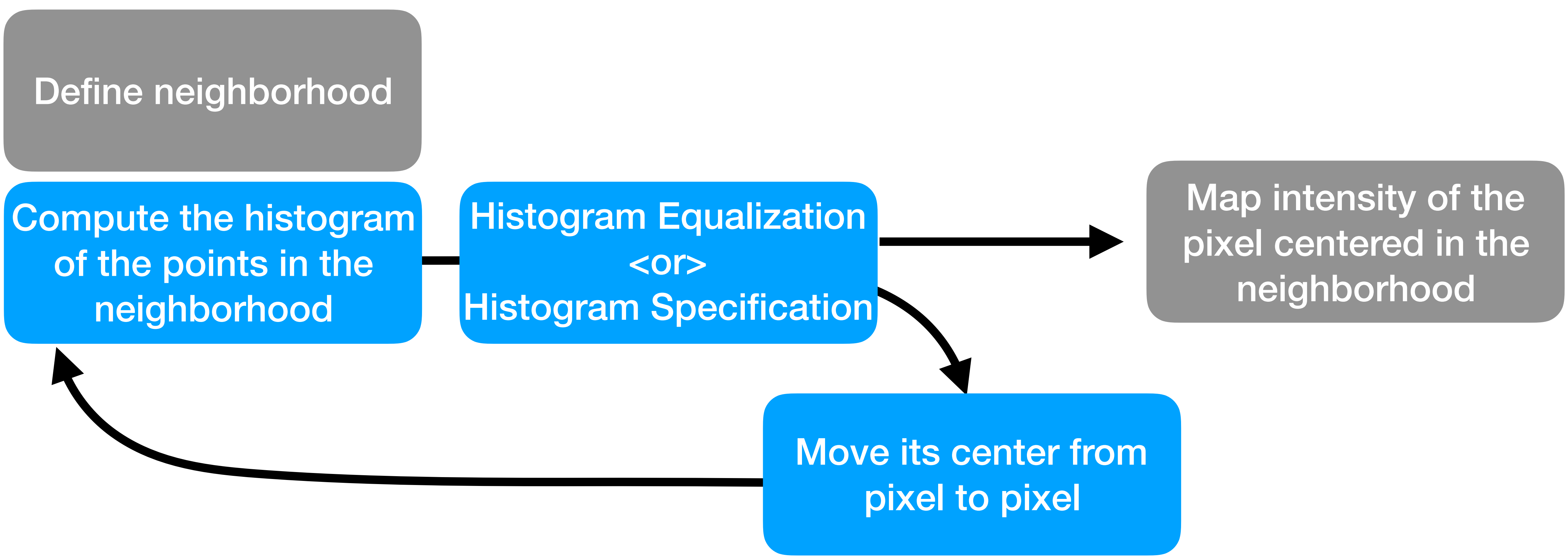
- Local Histogram Processing

-> Modify pixels by transformation function : Global approach

- Histogram Equalization
- Histogram Specification

-> Sometimes enhancing details over small area is necessary

=> Devise transformation function based on the intensity distribution in a neighborhood of every pixel in the image



Histogram Processing

- Local Histogram Processing

  -> <u>Modify pixels by transformation function : Global approach</u>

  - Histogram Equalization
  - Histogram Specification

  -> Sometimes enhancing details over small area is necessary

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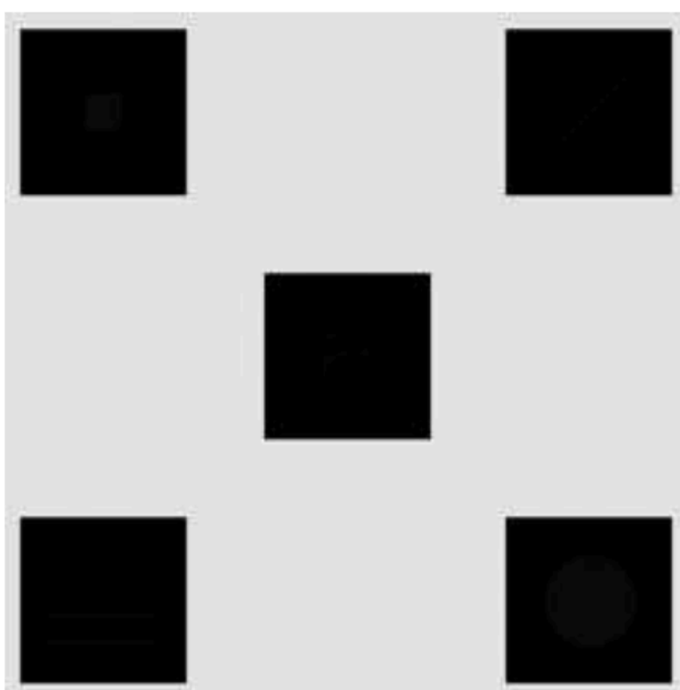
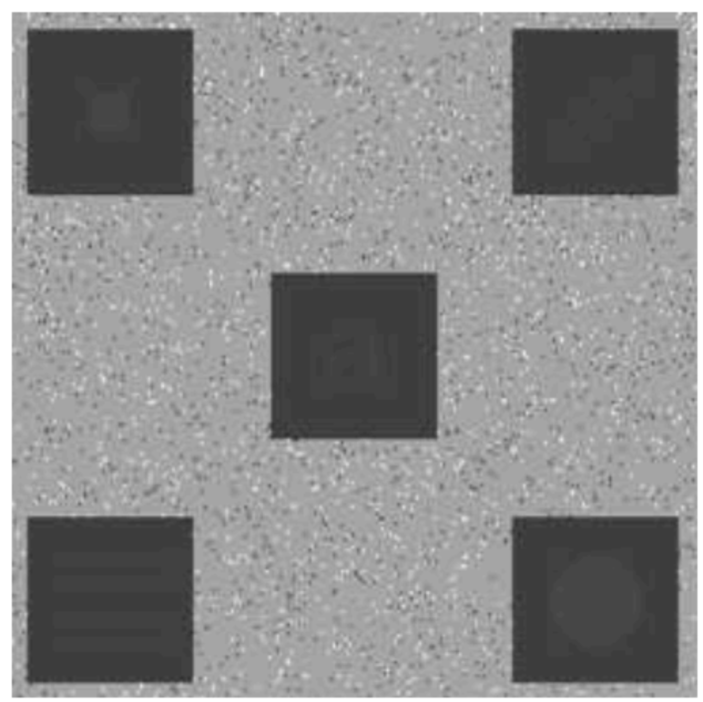
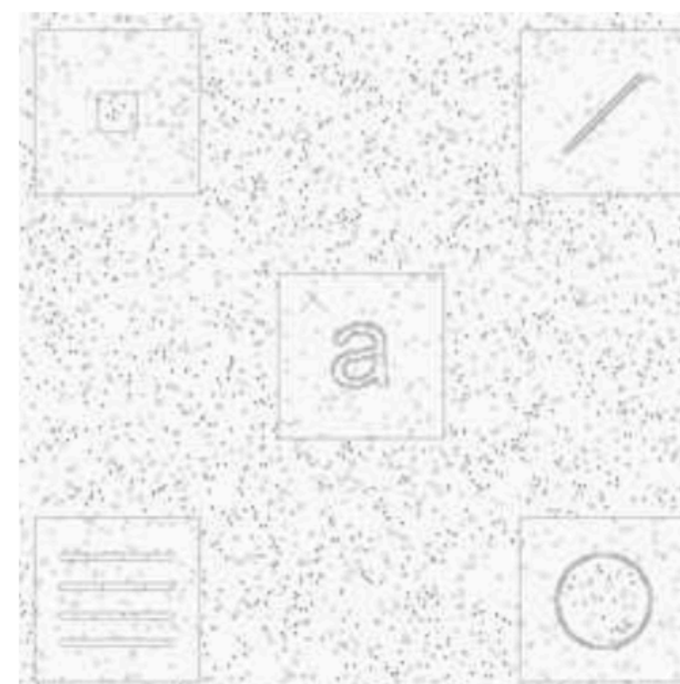




Global Histogram Equalization



Local Histogram Equalization

Histogram Processing

- Using Histogram Statistics for Image Enhancement

  -> Statistics obtained directly from image can be used for
  image enhancement

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- Using Histogram Statistics for Image Enhancement

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* r : Discrete random variable [0,L-1]
* P(ri) : Normalized histogram (estimate of the probability that intensity ri occurs in the image)

Histogram Processing

- Using Histogram Statistics for Image Enhancement

  -> Statistics obtained directly from image can be used for image enhancement

  * r : Discrete random variable [0,L-1]
  * P(ri) : Normalized histogram (estimate of the probability that intensity ri occurs in the image)

$$\boxed{m = \sum_{i=0}^{L-1} r_i p(r_i)}, \quad \boxed{\mu_2(r) = \sum_{i=0}^{L-1} (r_i - m)^2 p(r_i)} \quad \longleftarrow \textbf{With Histogram}$$

mean **variance**

Estimate mean & variance from samples directly without computing the histogram

$$\boxed{m = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y)}, \quad \boxed{\sigma^2 = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} [f(x,y) - m]^2} \quad \longleftarrow \textbf{With pixel Intensity}$$

sample mean **sample variance**

Reference Example 3.11 ⟶ **Histogram based, Intensity based results are same**

Histogram Processing

- Using Histogram Statistics for Image Enhancement

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mean **variance** ← **With Histogram**

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sample mean **sample variance** ← **With pixel Intensity**

-> Powerful in local enhancement, where local mean and variance are used

Compute from value of pixel's neighborhoods

$$m_{s_{xy}} = \sum_{i=0}^{L-1} r_i p_{s_{xy}}(r_i), \quad \sigma^2_{s_{xy}} = \sum_{i=0}^{L-1} (r_i - m_{s_{xy}})^2 p_{s_{xy}}(r_i)$$

pixels in the neighborhood

Histogram Processing

- Using Histogram Statistics for Image Enhancement

-> Statistics obtained directly from image can be used for
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* r : Discrete random variable [0,L-1]
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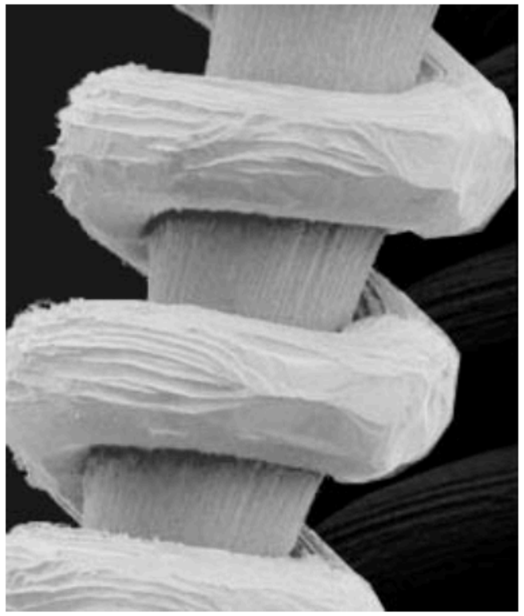
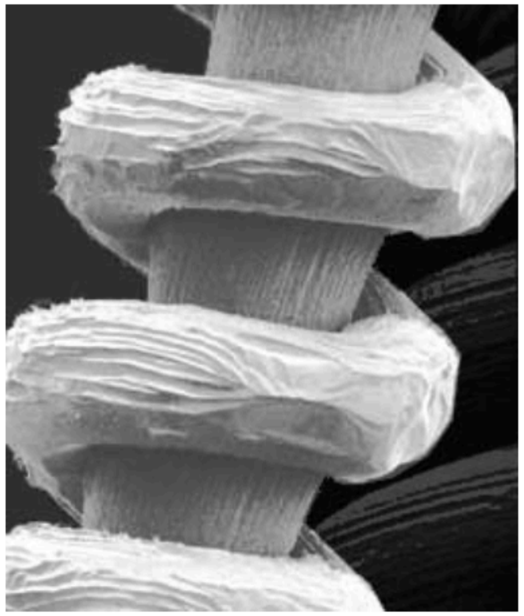


original



Global
Histogram Equalization

Enhanced Image using
Local Histogram Statistics

Fundamentals of Spatial Filtering

- Mechanics of Spatial Filtering
  -> Spatial Filter
    => Neighborhood (small rectangle)
    => Predefined Operation
      * Performed on the image pixels encompassed by neighborhood

Linear Spatial Filtering (Image of size MxN, filter of size mxn)

$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t)$$

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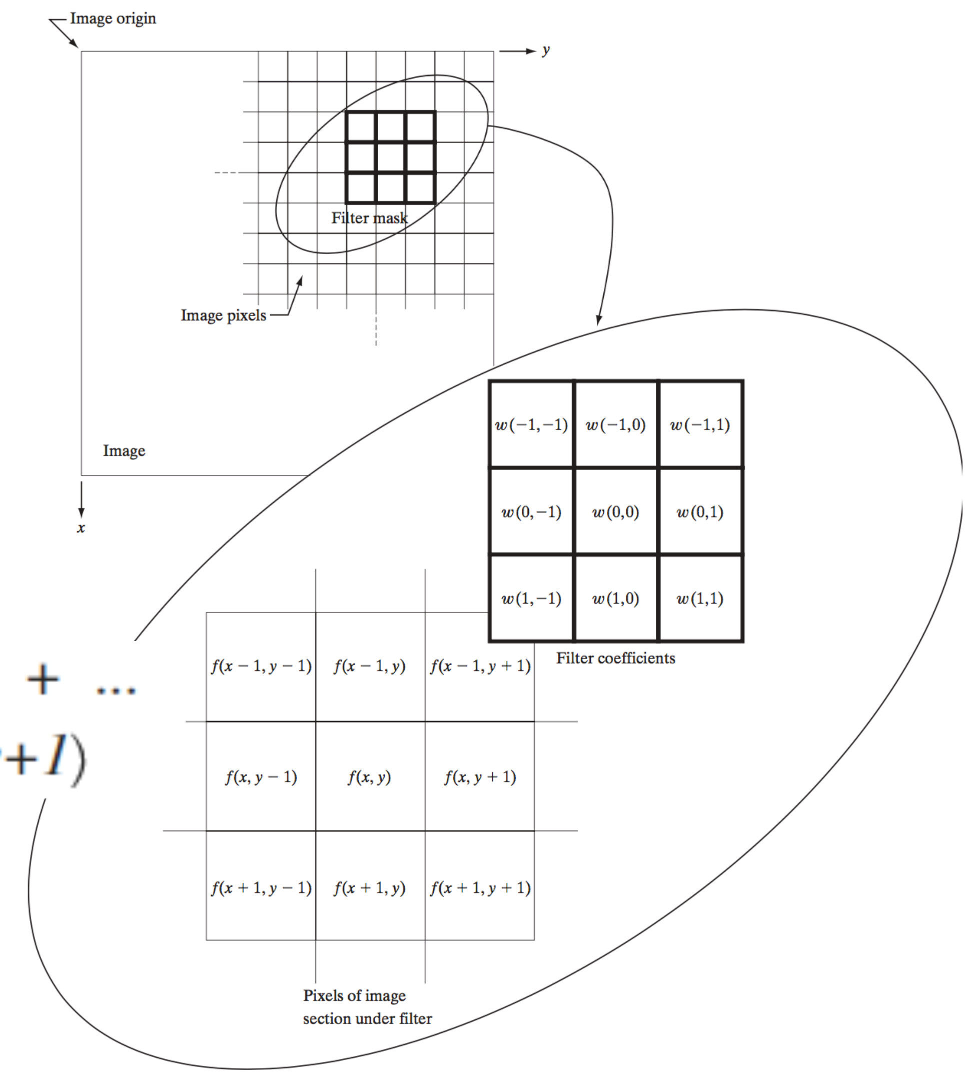


Linear Spatial Filtering (using a 3x3 neighborhood)

$$g(x,y) = w(-1,-1) f(x-1,y-1) + w(-1,0) f(x-1,y) + \ldots$$
$$+ w(0,0) f(x,y) + \ldots + w(1,1) f(x+1,y+1)$$

Fundamentals of Spatial Filtering

- Spatial Correlation and Convolution

  -> Correlation

  => Process of moving filter mask over the image and computing the sum of product

  -> Convolution

  => Mechanics are same with correlation but filter rotates by 180 degrees first

$$\boxed{w(x,y) * f(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x-s, y-t)}$$

Convolution $\left(\begin{bmatrix} X_{11} & X_{12} & X_{13} \\ X_{21} & X_{22} & X_{23} \\ X_{31} & X_{32} & X_{33} \end{bmatrix}, \boxed{\begin{matrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{matrix}} \right)$

$\Updownarrow$

$$\boxed{w(x,y) \star f(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t)}$$

Correlation $\left(\begin{bmatrix} X_{11} & X_{12} & X_{13} \\ X_{21} & X_{22} & X_{23} \\ X_{31} & X_{32} & X_{33} \end{bmatrix}, \boxed{\begin{matrix} F_{22} & F_{21} \\ F_{12} & F_{11} \end{matrix}} \right)$

Fundamentals of Spatial Filtering

- Vector Representation of Linear Filtering

$$R = w_1 z_1 + w_2 z_2 + \dots + w_{mn} z_{mn} = \sum_{k=1}^{mn} w_k z_k = \mathbf{w}^{\mathsf{T}} \mathbf{z}$$

Fundamentals of Spatial Filtering

- Vector Representation of Linear Filtering

$$R = w_1 z_1 + w_2 z_2 + \ldots + w_{mn} z_{mn} = \sum_{k=1}^{mn} w_k z_k = \mathbf{w}^{\mathsf{T}} \mathbf{z}$$

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

$$\begin{aligned} R &= w_1 z_1 + w_2 z_2 + \ldots + w_9 z_9 \\ &= \sum_{k=1}^{9} w_k z_k \\ &= \mathbf{w}^T \mathbf{z} \end{aligned}$$

Fundamentals of Spatial Filtering

- Generating Spatial Filter Masks
  -> Generating *m x n* linear spatial filter requires that
  specified *mn* number of mask coefficients

Linear Filtering

- Coefficients are implements the desired average
- Results in image Smoothing

Continuous Function with two variables

- Objective is to obtain a spatial filter mask based on continuous function
- Use Gaussian function

$$h(x, y) = e^{-\frac{x^2+y^2}{2\sigma^2}}$$

Non-linear Filtering

- Requires specified size of neighborhood and operations
- Use Max, Min, Median filter

Smoothing Spatial Filters

- Smoothing Spatial Filters
  -> Used for **blurring** and **noise reduction**
  => Blurring : For preprocessing task (Removing small details from an image)
  => Noise Reduction : Accomplished by blurring with linear or nonlinear filtering

  -> Smoothing Linear Filters
  => Simply average the pixels contained in the neighborhood of the filter mask

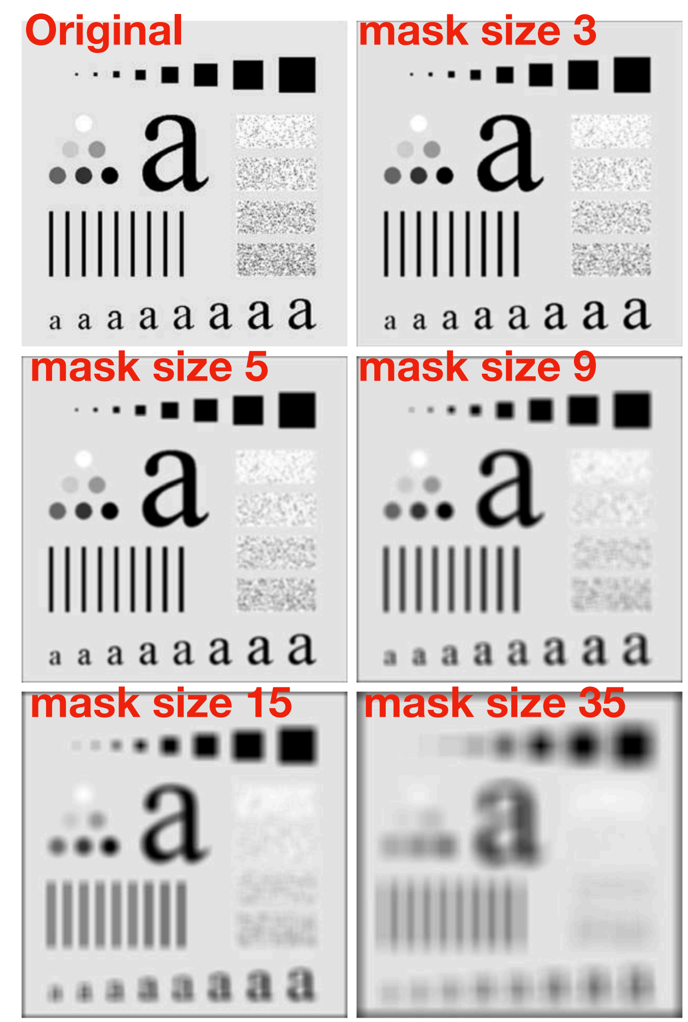
[Averaging Filters]

$$R = \frac{1}{MN}\sum_{i=1}^{MN} z_i$$

$$\frac{1}{9} \times \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \qquad \frac{1}{16} \times \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

[Weighted average]

$$g(x,y) = \frac{\sum_{s=-a}^{a}\sum_{t=-b}^{b} w(s,t) f(x+s, y+t)}{\sum_{s=-a}^{a}\sum_{t=-b}^{b} w(s,t)}$$



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  -> Used for **blurring** and **noise reduction**

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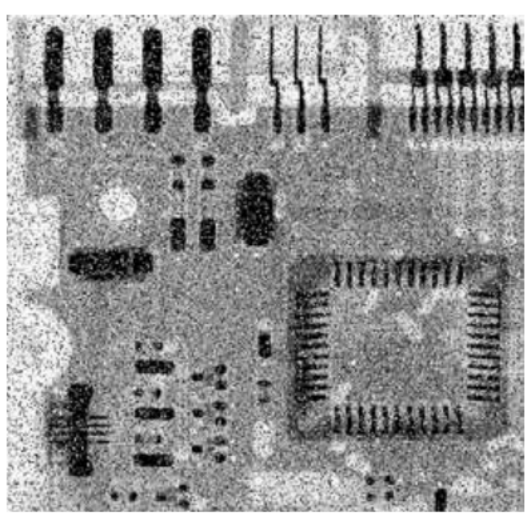
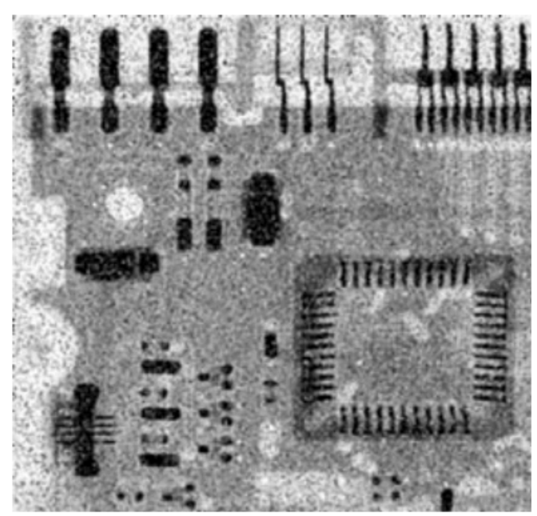
  -> NonLinear Filters (Order-Statistic Filters)

  => Spatial filters whose response is based on ordering(ranking) the pixels contained in the image area encompassed by the filter, and replacing the value of center pixel with the value determined by ranking result

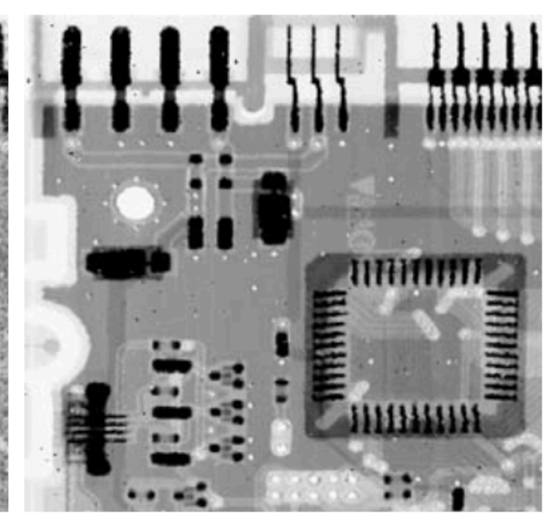
  [Max, Min, Median Filter]



Original image (Noised)



Noise reduction with averaging mask



Noise reduction with median filter

Sharpening Spatial Filters

- Sharpening Spatial Filters
  -> To highlight transitions in intensity
    => used in electronic printing, medical imaging, autonomous guidance in military systems

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Smoothing $\longrightarrow$ **Averaging** $\longrightarrow$ **Integration**

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Smoothing $\longrightarrow$ **Averaging** $\longrightarrow$ **Integration**

Sharpening $\longrightarrow$ **Differentiation**

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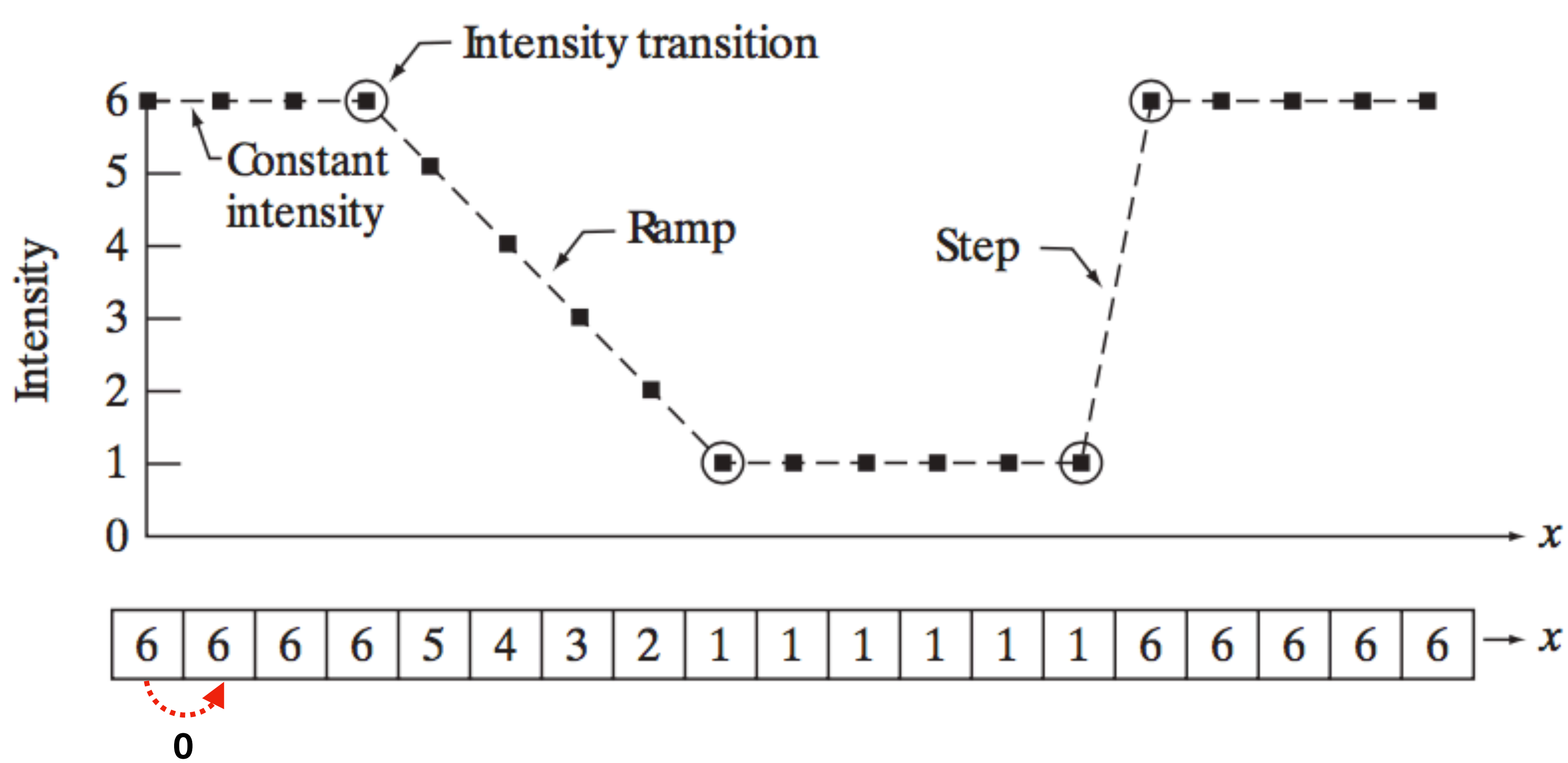
Smoothing ⟶ **Averaging** ⟶ **Integration**

Sharpening ⟶ **Differentiation**

First order partial derivative

$$\boxed{\frac{\partial f}{\partial x} = f(x+1) - f(x)}$$

Use partial derivative to keep notation the same as consider the image



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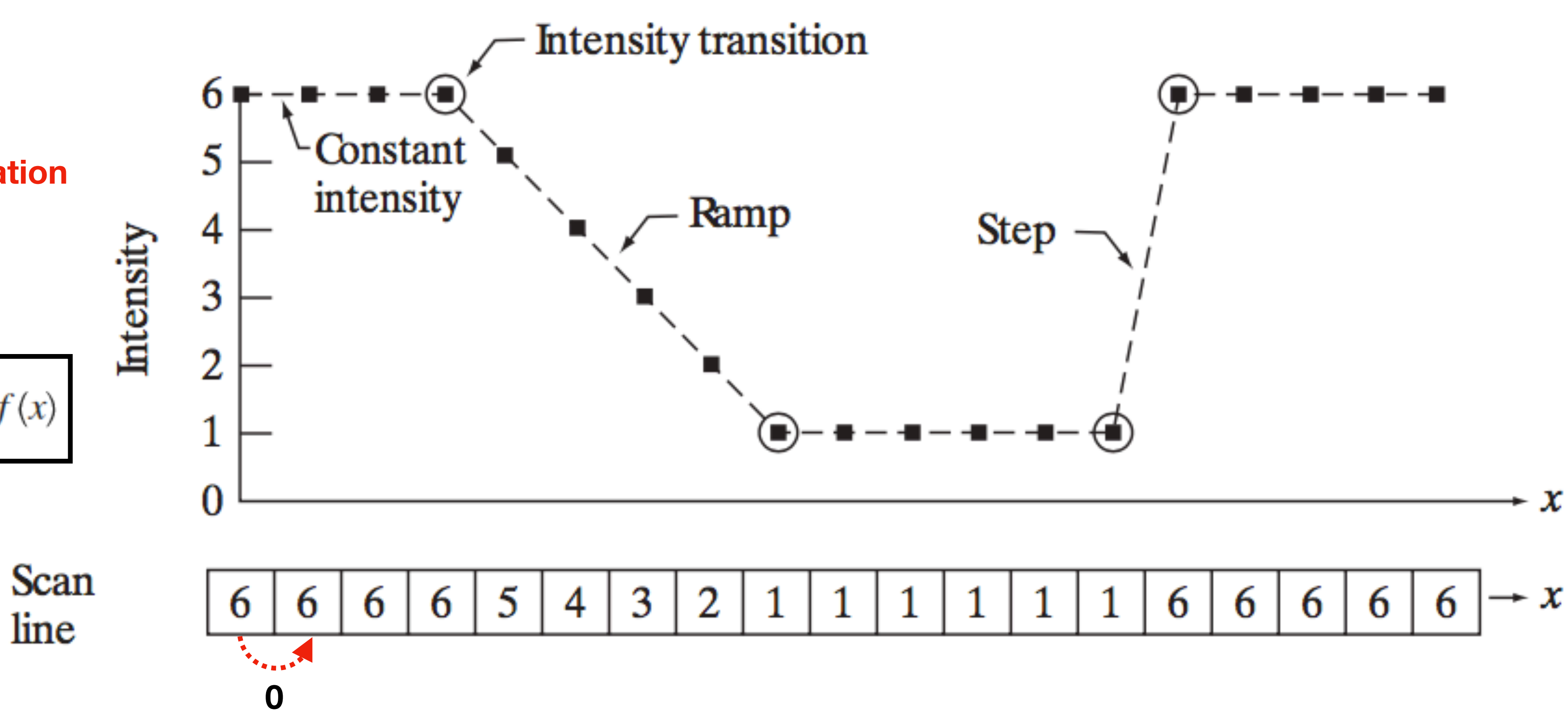
Smoothing → **Averaging** → **Integration**

Sharpening → **Differentiation**

look ahead operation

First order partial derivative

$$\frac{\partial f}{\partial x} = \boxed{f(x+1)} - f(x)$$



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Sharpening → **Differentiation**

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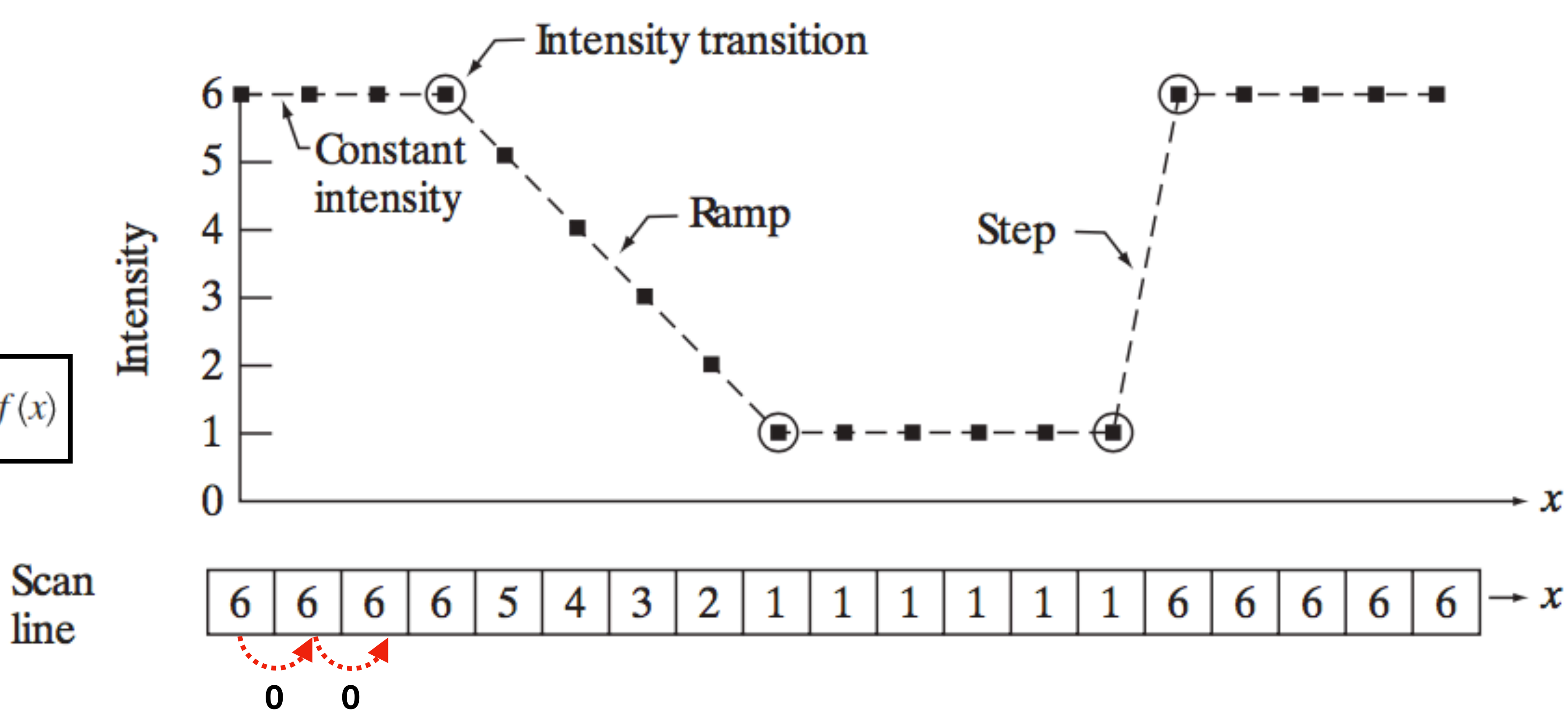

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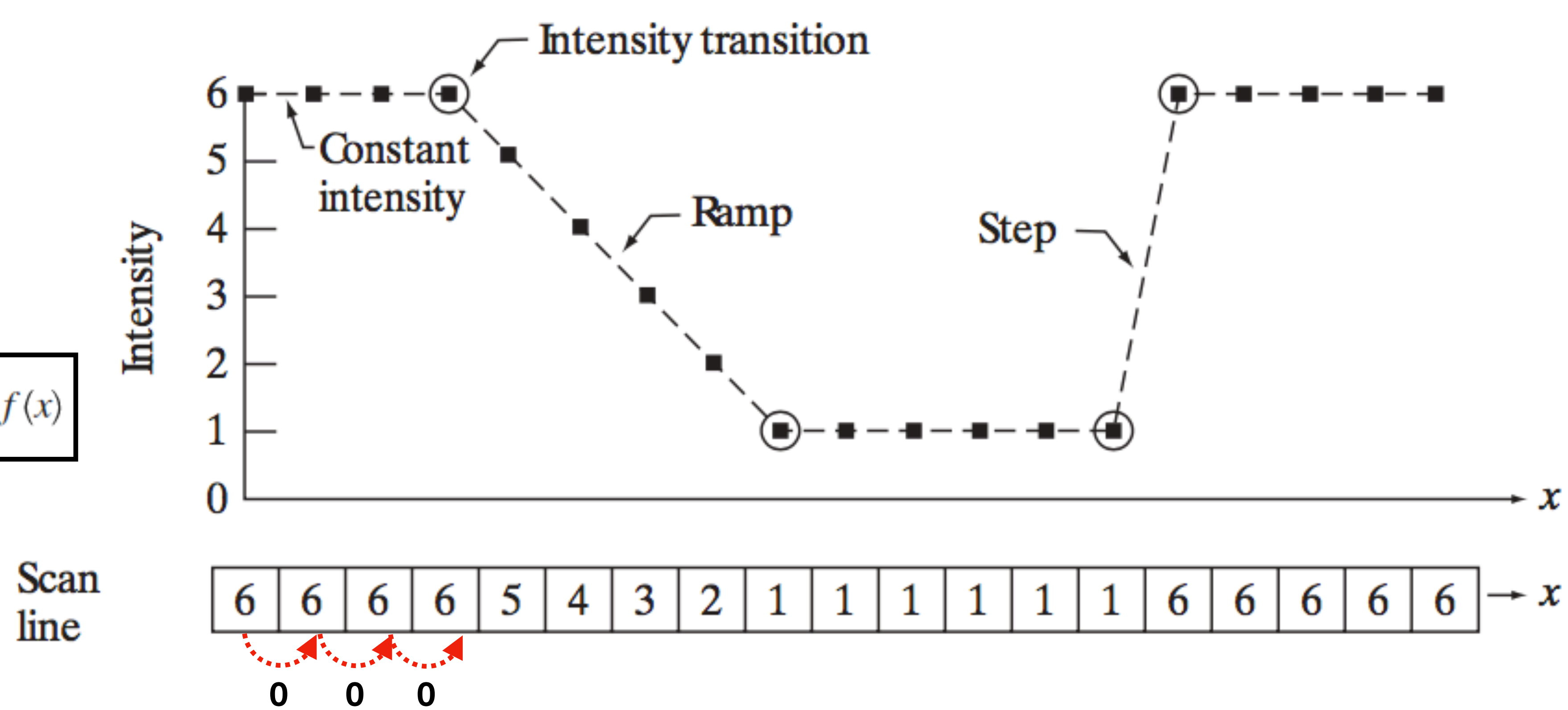

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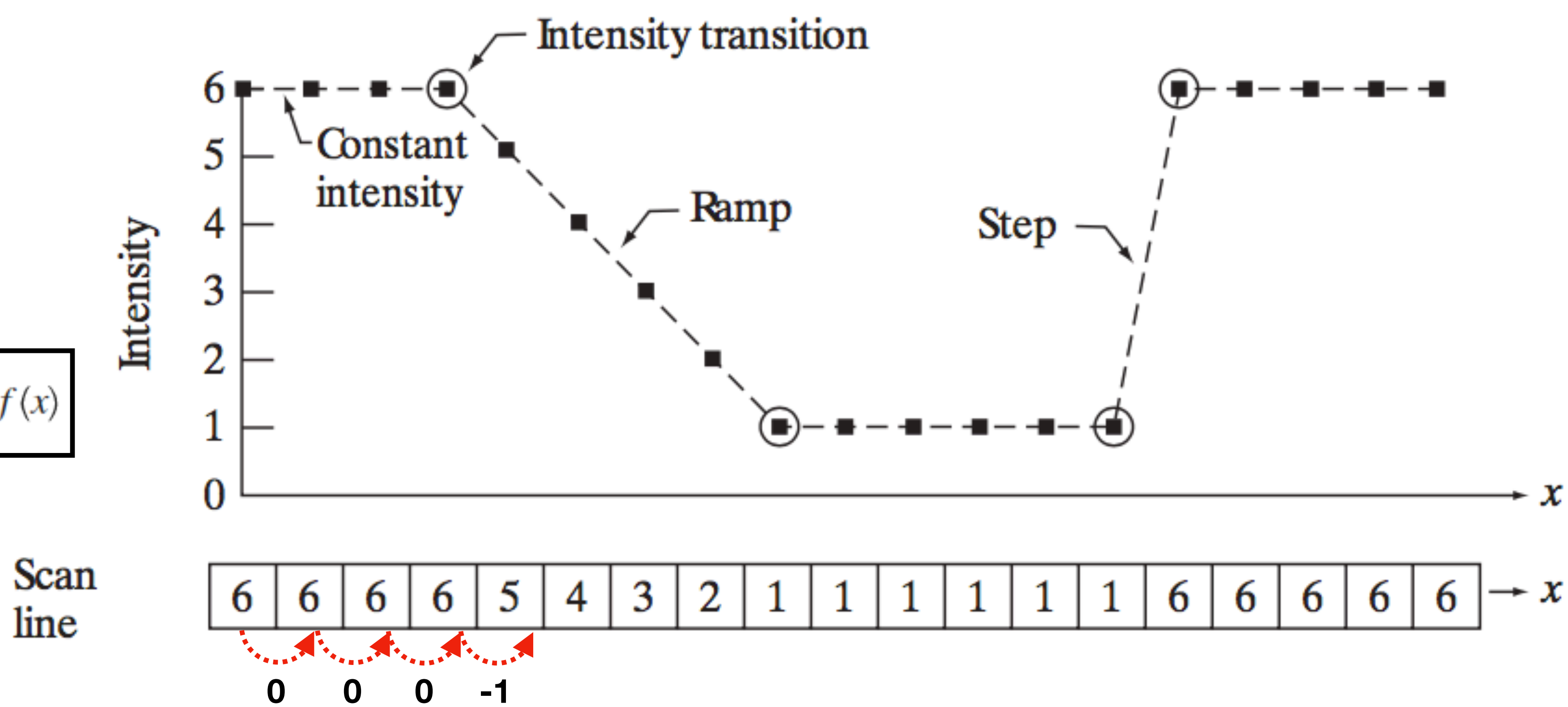

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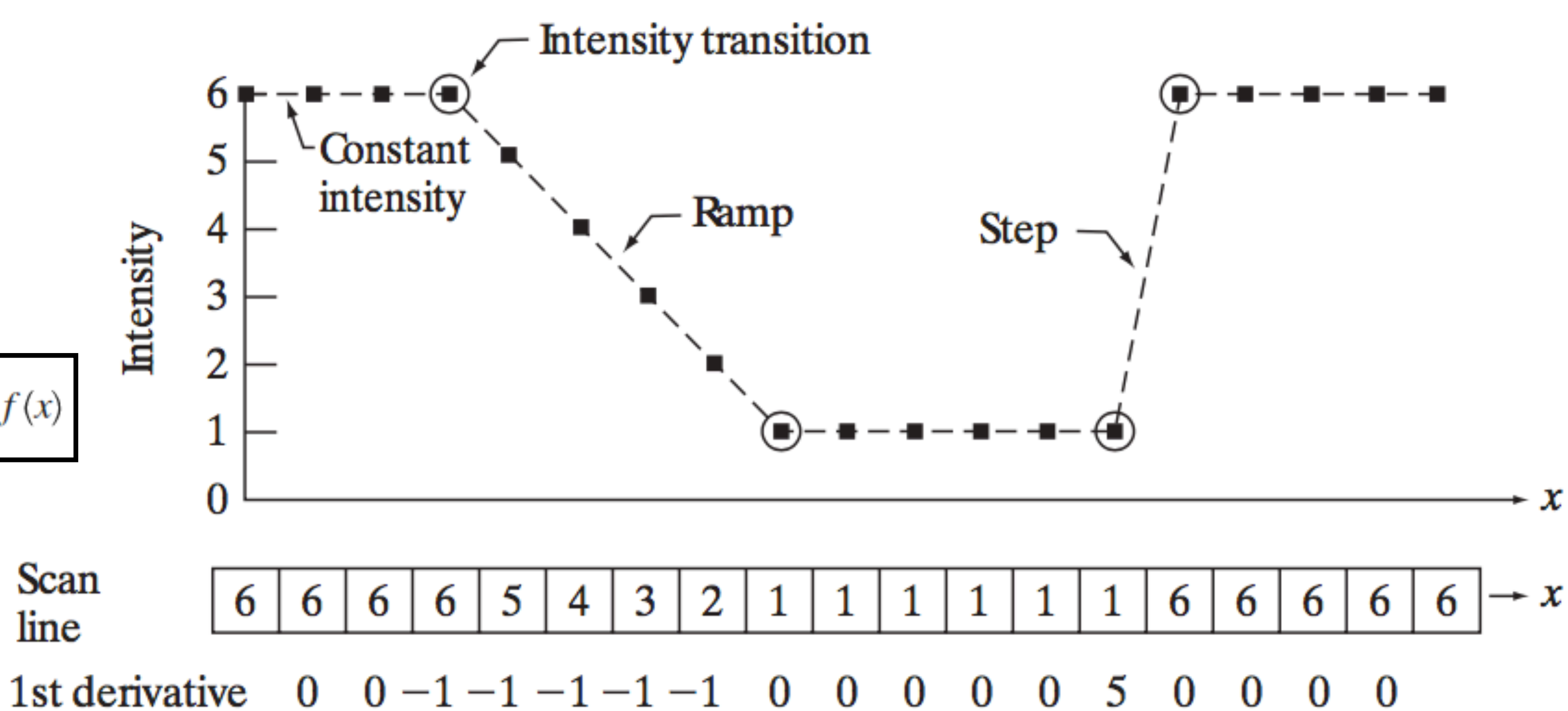
Smoothing → **Averaging** → **Integration**

Sharpening → **Differentiation**

First order partial derivative

$$\boxed{\frac{\partial f}{\partial x} = f(x+1) - f(x)}$$



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    => used in electronic printing, medical imaging, autonomous guidance in military systems

Smoothing → **Averaging** → **Integration**

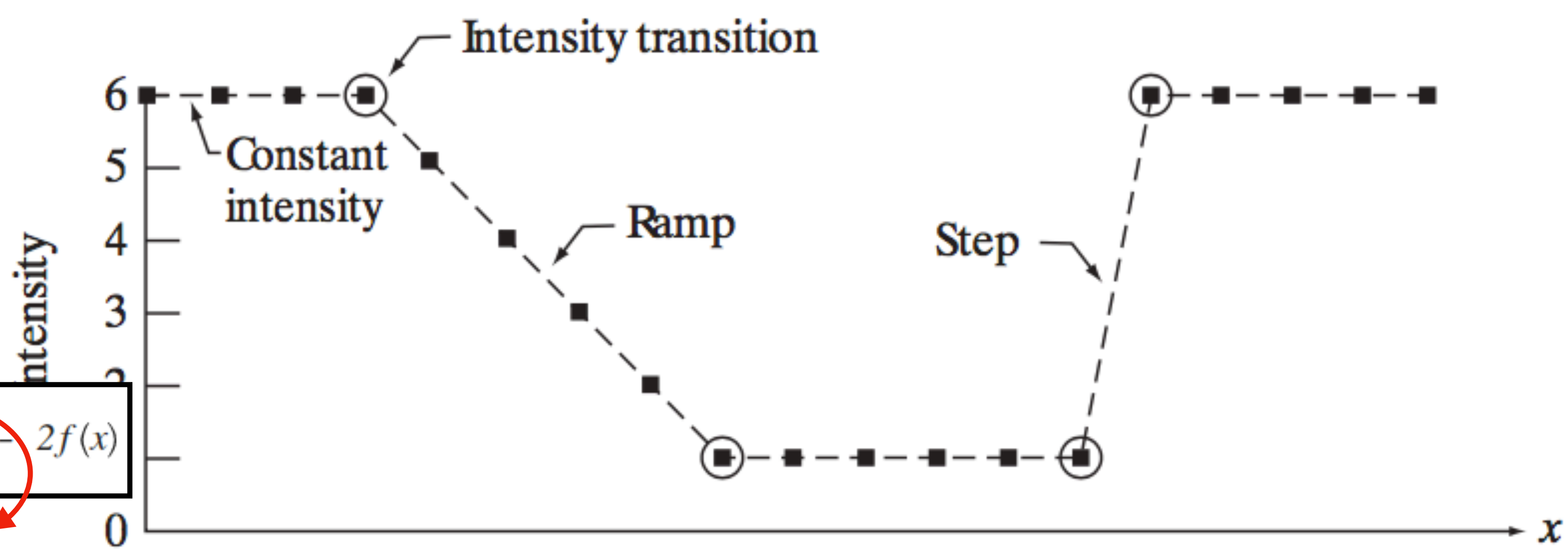
Sharpening → **Differentiation**

Second order partial derivative

look next step

$$\frac{\partial^2 f}{\partial x^2} = \boxed{f(x+1)} + \boxed{f(x-1)} - 2f(x)$$

look previous step



Scan line	6	6	6	6	5	4	3	2	1	1	1	1	1	1	6	6	6	6	6
1st derivative		0	0	−1	−1	−1	−1	−1	0	0	0	0	0	5	0	0	0	0	
2nd derivative		0	0	−1	0	0	0	0	1	0	0	0	0	5	−5	0	0	0	

→ x

Sharpening Spatial Filters

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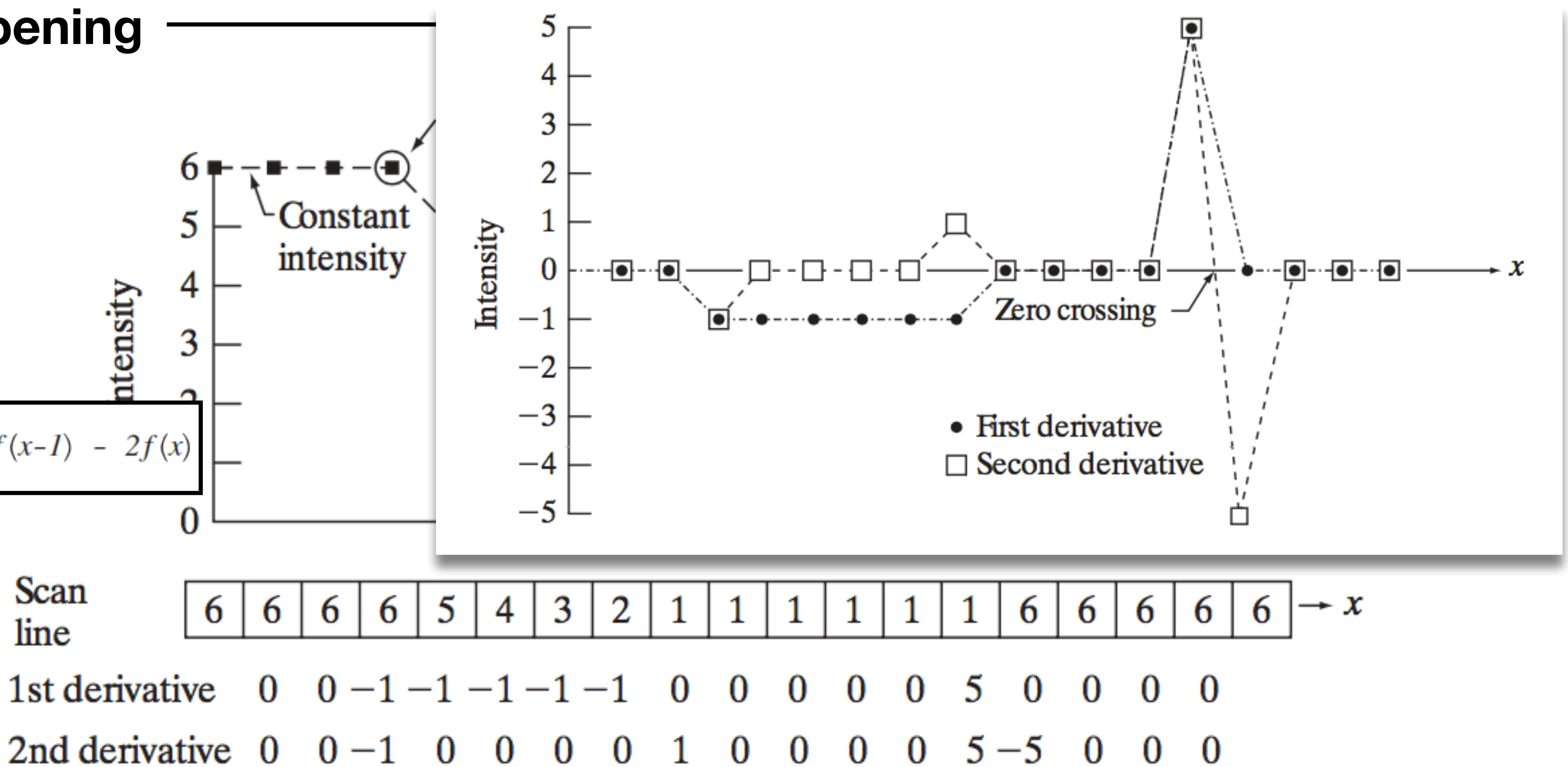
Smoothing ⟶ **Averaging** ⟶ **Integration**

Sharpening

Second order partial derivative

$$\boxed{\frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x)}$$



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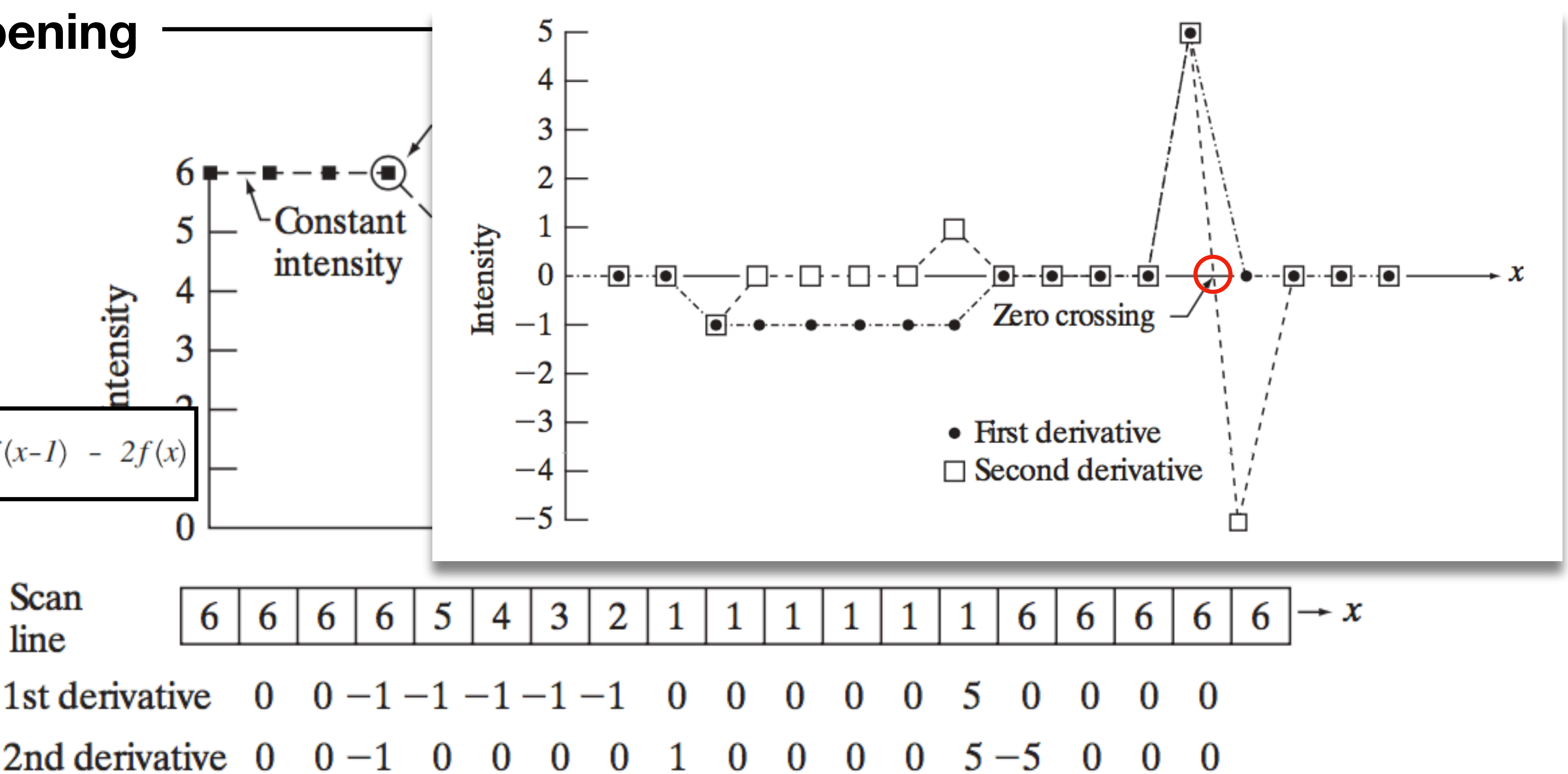
Smoothing ⟶ **Averaging** ⟶ **Integration**

Sharpening ⟶

Second order partial derivative

$$\boxed{\frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x)}$$



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Smoothing ⟶ **Averaging** ⟶ **Integration**

Sharpening ——

Second order partial derivative

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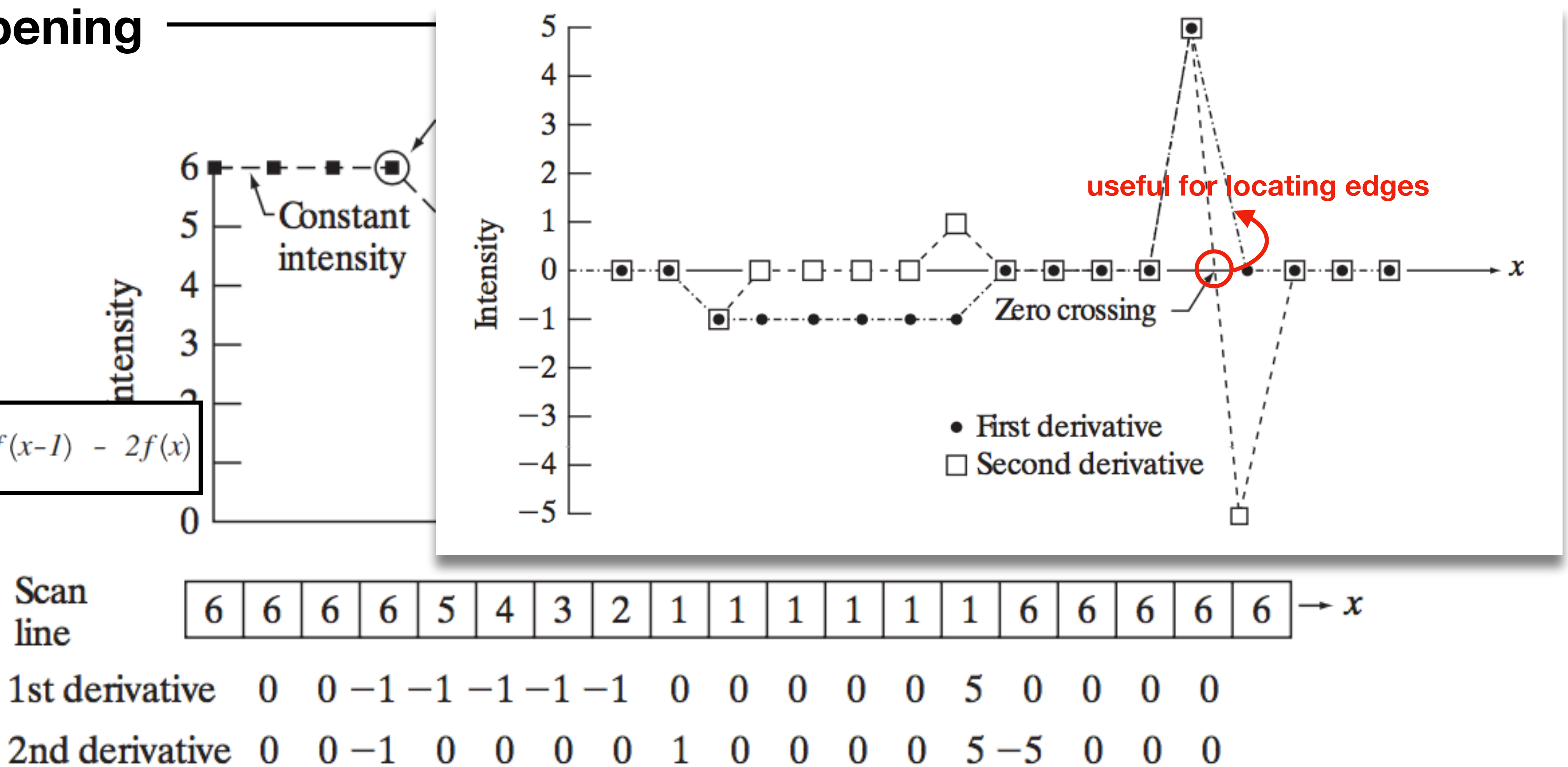


Sharpening Spatial Filters

- Using Second Derivative for Image Sharpening-Laplacian
  -> Isotropic Filter
    => Response is independent on the direction of discontinuities in the image [Rotation Invariant]
    => Simplest isotropic derivative operator is the Laplacian, for a function of two variables f(x,y)

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

$$\frac{\partial^2 f}{\partial x^2} = f(x+1,y) + f(x-1,y) - 2f(x,y)$$

$$\frac{\partial^2 f}{\partial y^2} = f(x,y+1) + f(x,y-1) - 2f(x,y)$$

$$\nabla^2 f = f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1) - 4f(x,y)$$

0	1	0
1	−4	1
0	1	0

Sharpening Spatial Filters

- Using Second Derivative for Image Sharpening-Laplacian

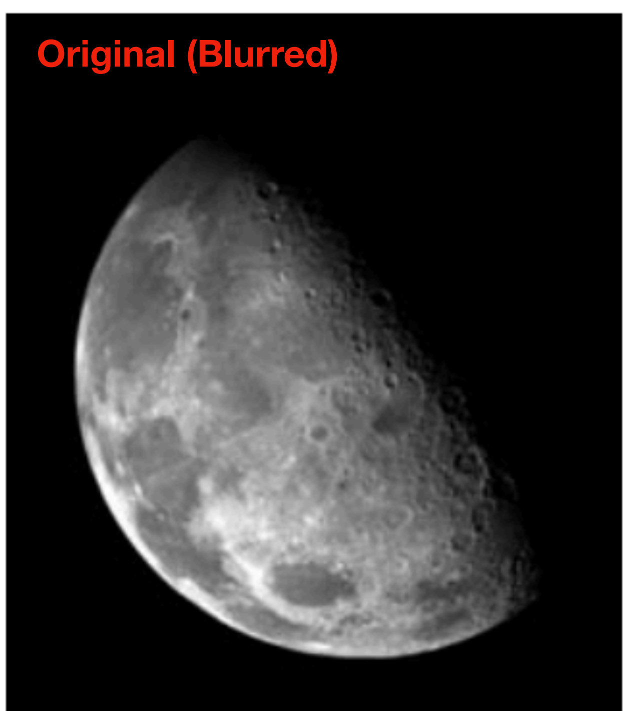
  -> Isotropic Filter

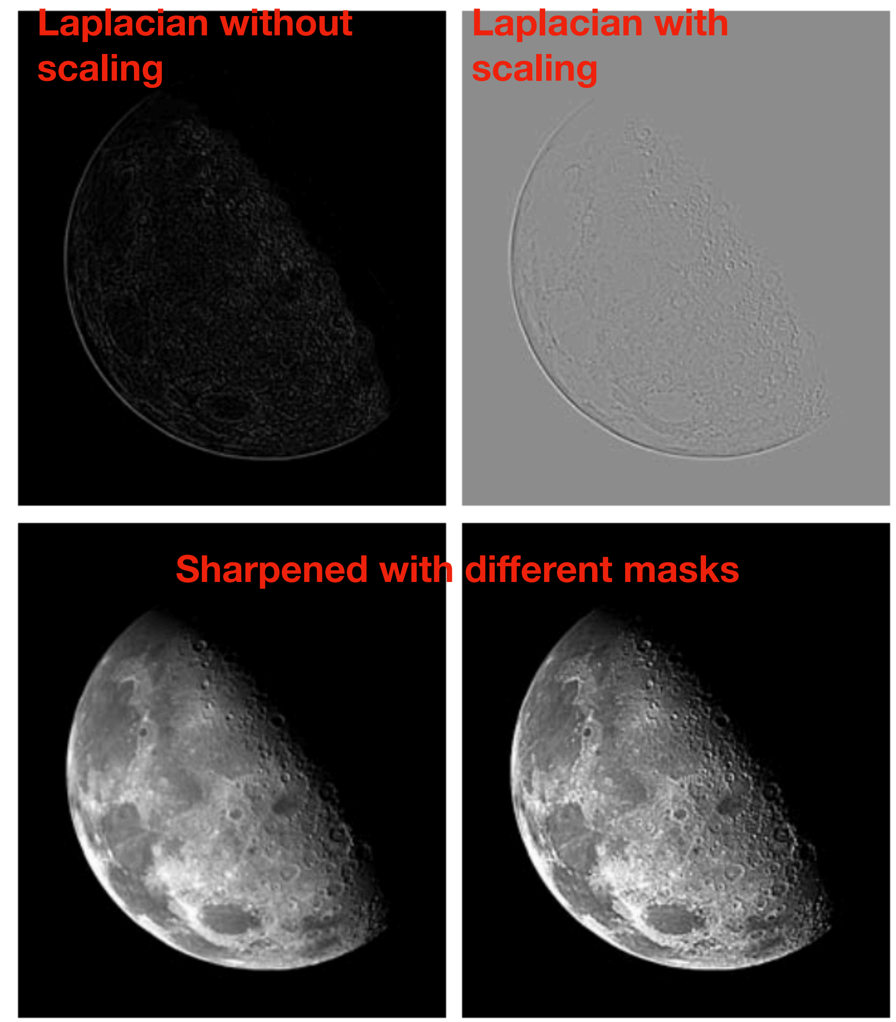
  => Response is independent on the direction of discontinuities in the image [Rotation Invariant]

  => Simplest isotropic derivative operator is the Laplacian, for a function of two variables f(x,y)

  -> Laplacian Filtering for Image Sharpening

$$g(x,y) = f(x,y) + c[\nabla^2 f(x,y)]$$





Sharpening Spatial Filters

- Unsharp Masking and Highboost Filtering
  -> Sharpen images consist of subtracting an unsharp (smoothed) version of an image from the original image

Blur the original image → Subtract the blurred one from the original → Add the subtraction result to the original →

Unsharp masking

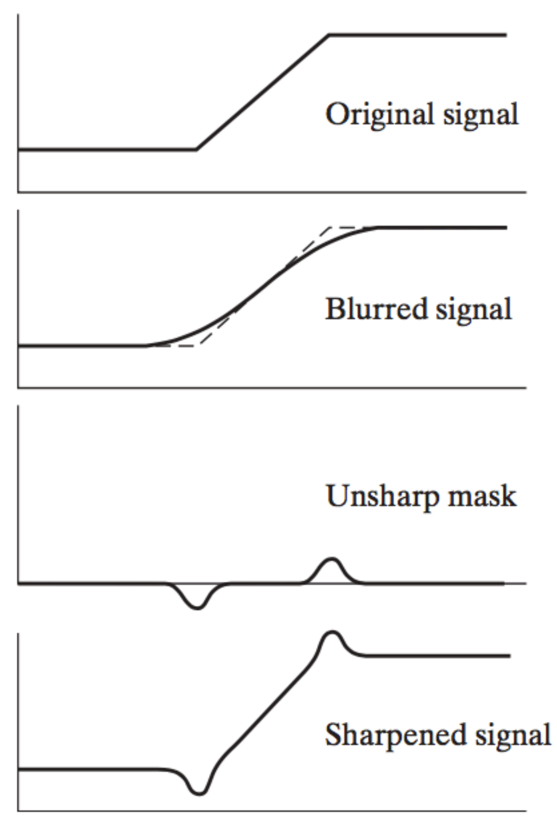
$$g_{mask}(x,y) = f(x,y) - \bar{f}(x,y)$$

Blurred image

Mask back to the original image

$$g(x,y) = f(x,y) + k * g_{mask}(x,y)$$

if K > 1, process is called highboost filtering



Sharpening Spatial Filters

- Unsharp Masking and Highboost Filtering
  -> Sharpen images consist of subtracting an unsharp (smoothed) version of an image from the original image

Blur the original image → Subtract the blurred one from the original → Add the subtraction result to the original

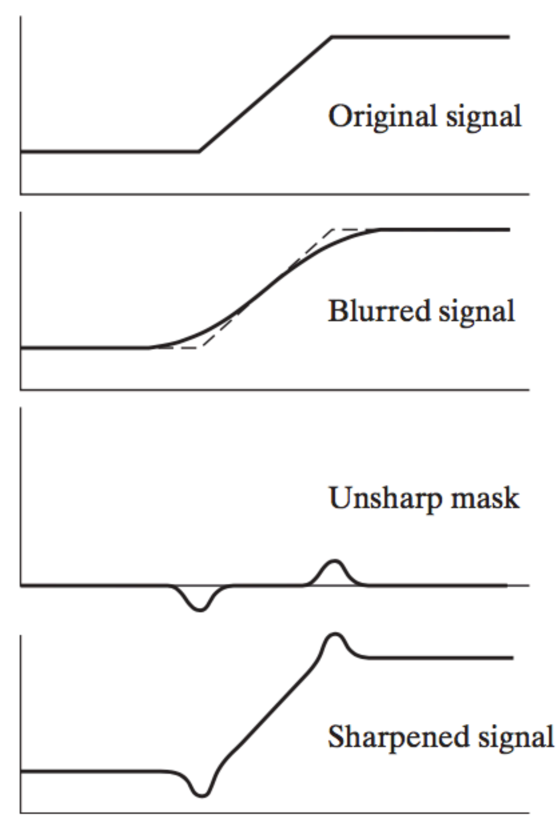
Unsharp masking

$$g_{mask}(x,y) = f(x,y) - \boxed{\overline{f}(x,y)}$$

Blurred image

Mask back to the original image

$$g(x,y) = f(x,y) + k * g_{mask}(x,y)$$



Sharpening Spatial Filters

- Using First Derivatives for Image Sharpening-Gradient
  -> Implement by using the magnitude of the gradient

$$\nabla f \equiv grad(f) \equiv \begin{bmatrix} g_x \\ g_y \end{bmatrix} = \begin{bmatrix} \dfrac{\partial f}{\partial x} \\ \dfrac{\partial f}{\partial y} \end{bmatrix}$$

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z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

-> Robert Cross-gradient Operators

$$g_x = (z_9 - z_5), \quad g_y = (z_8 - z_6)$$

$$M(x,y) = [(z_9 - z_5)^2 + (z_8 - z_6)^2]^{1/2} \approx |z_9 - z_5| + |z_8 - z_6|$$

x

−1	0
0	1

y

0	−1
1	0

-> Sobel Operators

−1	−2	−1
0	0	0
1	2	1

−1	0	1
−2	0	2
−1	0	1

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−1	0
0	1

0	−1
1	0

-> Sobel Operators

$$g_x = (z_7 + 2z_8 + z_9) - (z_1 + 2z_2 + z_3)$$

$$g_y = (z_3 + 2z_6 + z_9) - (z_1 + 2z_4 + z_7)$$

$$M(x,y) \approx |(z_7 + 2z_8 + z_9) - (z_1 + 2z_2 + z_3)| + |(z_3 + 2z_6 + z_9) - (z_1 + 2z_4 + z_7)|$$

x

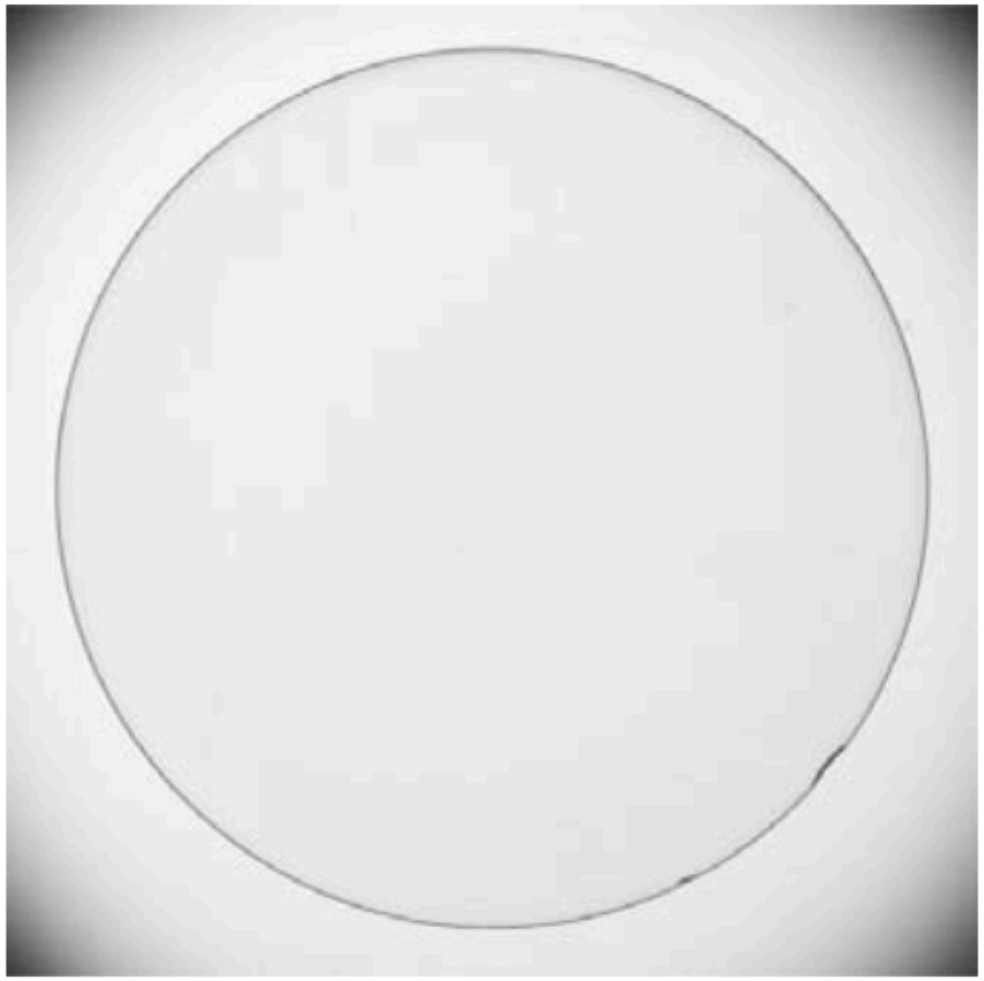
−1	−2	−1
0	0	0
1	2	1

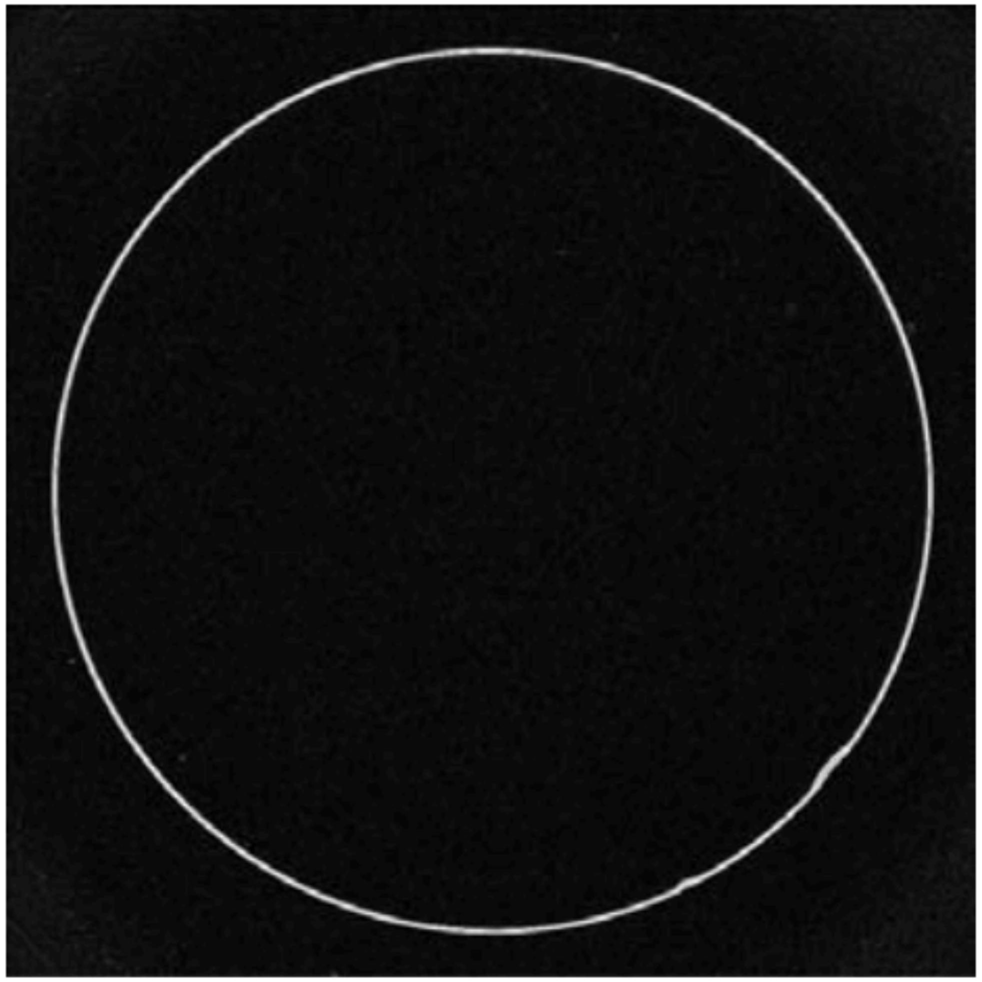
y

−1	0	1
−2	0	2
−1	0	1

Sharpening Spatial Filters

- Using First Derivatives for Image Sharpening-Gradient
  -> Results





Sobel Gradient Result