Raymond Karyshyn
March 10th, 2021
CMSC-204-33085

CMSC 204 - Activity 6.1
Module 13 - "Searching & Hashing"

In Chapter 23.1 of our textbook, we are taught that "*the cost of a successful search for an entry is the same as the cost of adding that entry*". This is because a successful search for a particular entry follows the same probe sequence as when the entry is first added to a hash table. The terminology "average number of probes" in the Activity 6.1 requirements is synonymous with "cost of adding an entry", "cost of a successful search", and "average number of comparisons". In the rest of Chapter 23, multiple formulas are given for calculating the average number of comparisons for an unsuccessful search as well as a successful search. Given what I've established about the terminology discussed in Chapter 23.1, I know to use the successful search formulas instead of the unsuccessful search formulas.

Each successful search formula in Chapter 23, is given with respect to the load factor (λ), the ratio of the number of entries in a given dictionary to the size of a hash table. In the Activity 6.1 requirements, I am asked to find the maximum λ of a hash table that guarantees that the average number of probes ("cost of adding an entry"/"cost of a successful search"/"average number of comparisons") is less than or equal to 4. This means that I need to set each successful search formula less than or equal to the worst case of the average number of probes, 4, and solve for λ. The hash functions I am tasked with finding the maximum λ for are linear probing, linear-quotient, and separate chaining. The "cost of a successful search" equations for each of these hash function are:

$\frac{1}{2}\{1 + \frac{1}{(1-\lambda)}\}$	*for* linear probing	*(from Chapter 23.4)*
$\frac{1}{\lambda} log(\frac{1}{1-\lambda})$	*for* linear-quotient	*(from Chapter 23.5)*
$1 + \frac{\lambda}{2}$	*for* separate chaining	*(from Chapter 23.6)*

Raymond Karyshyn
March 10th, 2021
CMSC-204-33085

Linear Probing:

$$n_{avg\ \#\ of\ probes} = 4 \geq \frac{1}{2}\{1 + \frac{1}{(1-\lambda)}\}$$

$$8 \geq 1 + \frac{1}{(1-\lambda)}$$

$$7 \geq \frac{1}{(1-\lambda)}$$

$$1 - \lambda \geq \frac{1}{7}$$

$$\lambda \leq 1 - \frac{1}{7}$$

$$\lambda \leq \frac{6}{7}$$

$$\lambda \leq 0.8671$$

Linear-Quotient:

This uses the same equation as quadratic probing and double hashing.

$$n_{avg\ \#\ of\ probes} = 4 \geq \frac{1}{\lambda}log(\frac{1}{1-\lambda})$$

$$4\lambda \geq log(\frac{1}{1-\lambda})$$

$$4\lambda \geq - log(1 - \lambda)$$

$$log(1 - \lambda) \geq - 4\lambda$$

$$\lambda \leq 0.9999$$ *(calculated with Wolfram Alpha)*

Separate Chaining:

$$n_{avg\ \#\ of\ probes} = 4 \geq 1 + \frac{\lambda}{2}$$

$$3 \geq \frac{\lambda}{2}$$

$$\lambda \leq 6$$