Distribución normal

P.e. $X \sim N(175, 10)$

i) Cálculo de probabilidad, p.e. $P(X < 180)$

$P(X < 180)$

175 180

Estandarización: $Z = \dfrac{X - 175}{10}$

$= P(Z < 0,5)$

0   ↑

$\dfrac{180 - 175}{10} = 0,5 = 0,6915$

↑ Tabla

ii) Cálculo de cuantiles

0,05   90%   0,05

$X_{0,05} = ?$ 175 $X_{0,95} = ?$

$Z = \dfrac{X - 175}{10} \iff X = 175 + 10 \cdot Z$

90%

0

$Z_{0,05} = -Z_{0,95}$ $Z_{0,95}$
‖ ← Tabla
1,645
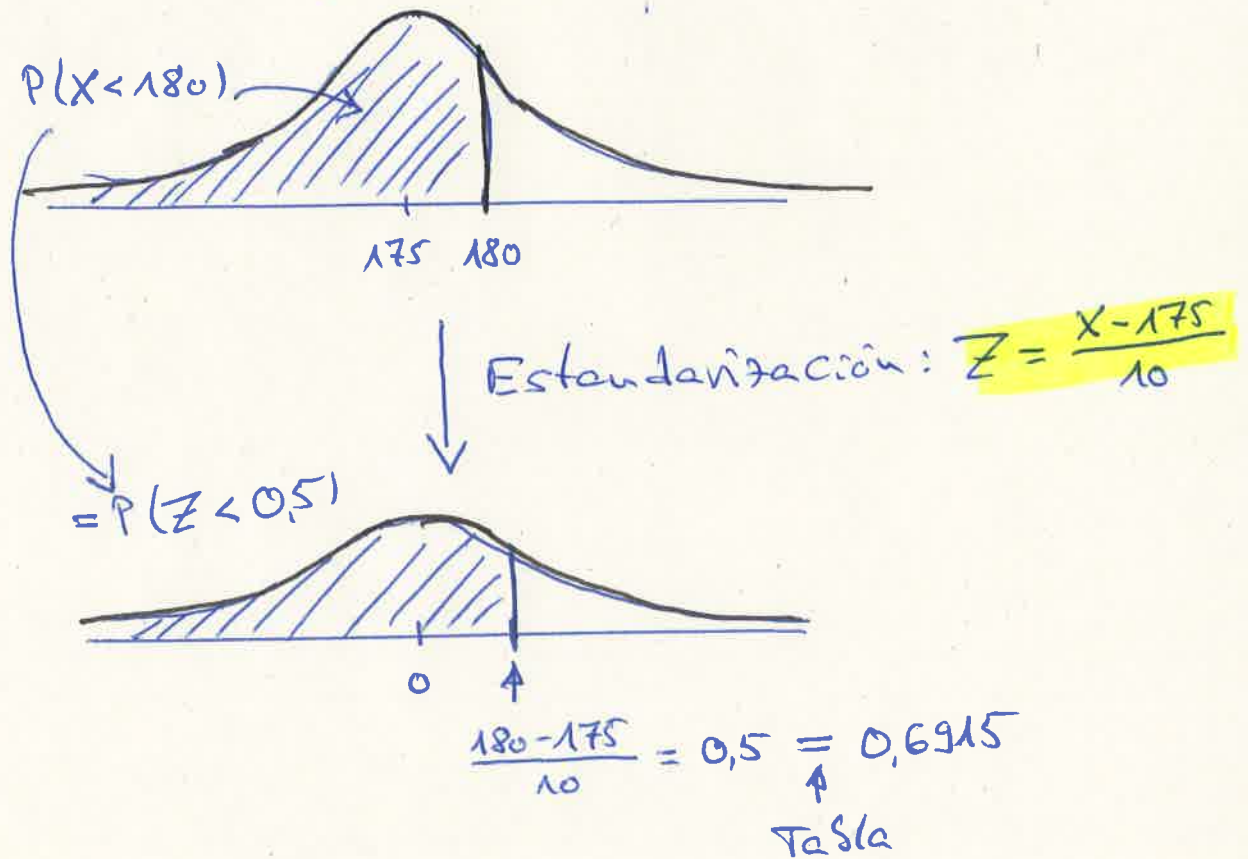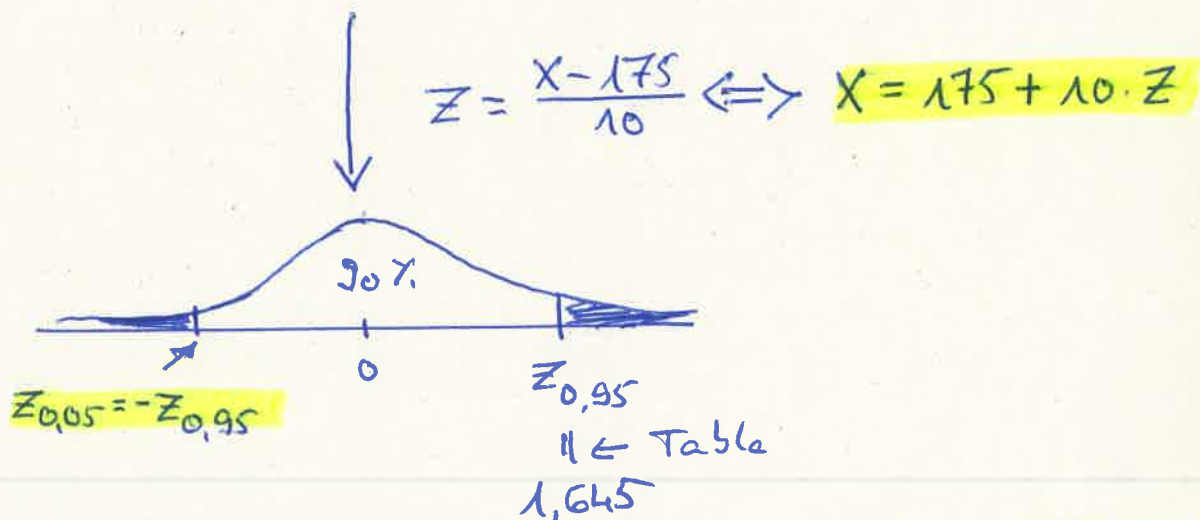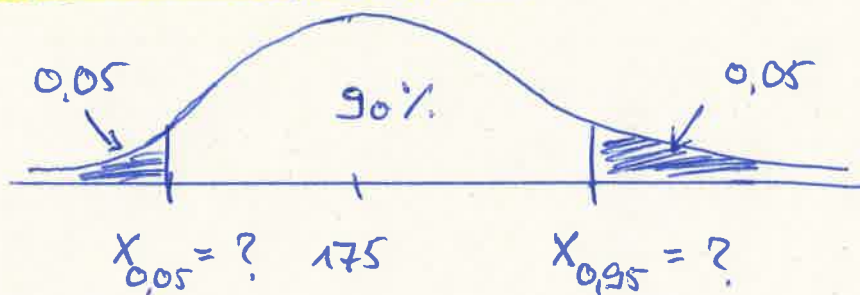
$\Rightarrow X_{0,95} = 175 + 10 \cdot 1,645$ $X_{0,05} = 175 - 10 \cdot 1,645 = 158,55$
$= 191,45$

Dado: $\mu = E(X) = 3,5$

$\sigma = \sqrt{V(X)} = 1,71$

TCL: $\bar{X}_{100} \sim N\left(3,5, \dfrac{1,71}{\sqrt{100}} = 0,171\right)$

$P(\bar{X}_{100} \geqslant 3,7 \vee \bar{X}_{100} \leqslant 3,3)$

$\underset{\underset{\text{Simetría}}{\uparrow}}{=} 2 \cdot P(\bar{X}_{100} \leqslant 3,3) = 2 \cdot \underset{\underset{\text{Est.}}{\uparrow}}{P}\left(Z \leqslant \dfrac{3,3-3,5}{0,171} = -1,17\right)$
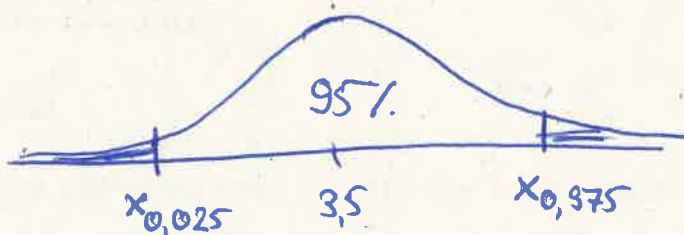
$= 2 \cdot (1 - P(1,17)) = 2 \cdot 0,121 = 0,242$

• $P(\bar{X}_{1000} \leqslant 3,3) = \ldots$

$\bar{X}_{1000} \sim N\left(3,5, \dfrac{1,71}{\sqrt{1000}} = 0,054\right)$

$P(\bar{X}_{1000} \leqslant 3,3) = P\left(Z \leqslant \dfrac{-0,2}{0,054} = -3,7\right) = 0,0001$

$\rightsquigarrow$ Dado trucado

• ¿Qué valores ~~pod~~ de $\bar{X}_{100}$ podemos esperar con una probabilidad de 0,95?

$x_{0,025}$ 3,5 $x_{0,975}$

$x_{0,975} = 3,5 + \underbrace{1,96}_{Z_{0,975}} \cdot 0,171 = 3,84$

$x_{0,025} = 3,5 - 1,96 \cdot 0,171 = 3,16$

X: # Estudiantes que aprueban PE

$X \sim B(282, 0.9)$

$E(X) = 282 \cdot 0.9 = 253.8$

$P(X \lesssim 240) = \ldots$

       i) pbinom $(240, 282, 0.9) \rightsquigarrow 0.006$

       ii) Aproximación con la dist. normal

$X \sim B(n, p) \approx N(n \cdot p, \sigma = \sqrt{n \cdot p(1-p)})$

      ↑

    n grande

$X \approx N(253.8, \sqrt{25.38})$

   $\rightsquigarrow$ pnorm $(\ldots) \Rrightarrow$