PY511 HW1

Raymond Kil

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Problem 1

(a)

i)

$\cdots$ measuring the meter.

ii)

$\cdots$ measuring the kilogram.

iii)

$\cdots$ measuring the mole.

iv)

$\cdots$ measuring the Kelvin.

v)

$\cdots$ measuring the Amp.

(b)

Year of 2018 is before the SI units were redefined. Before the redefinition, the definitions of SI units were basically the other way around. For example, International Prototype of the Kilogram used to define a kilogram, and Plank's constant was expressed in terms of the definition of a kilogram.

Problem 2

(a)

i)

$$\sigma_x \sigma_y = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} \tag{1}$$

ii)

The commutator is defined as following:

$$[\sigma_y, \sigma_z] = \sigma_y \sigma_z - \sigma_z \sigma_y = 2i \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{2}$$

iii)

$$\sigma_x \sigma_y \sigma_z = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = i \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{3}$$

(b)

$$\sigma_+ = \sigma_x + i\sigma_y = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + i \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = \begin{pmatrix} 0 & 2 \\ 0 & 0 \end{pmatrix} \tag{4}$$

$$\sigma_- = \sigma_x - i\sigma_y = \begin{pmatrix} 0 & 0 \\ 2 & 0 \end{pmatrix} \tag{5}$$

We can straightforwardly see that the conjugate transpose of σ_+ and σ_- are σ_- and σ_+, respectively. Therefore, σ_+ and σ_- are NOT Hermitian.

(c)

i)

$$\det[\sigma_x - \lambda \mathbb{1}] = \det \begin{pmatrix} -\lambda & 1 \\ 1 & -\lambda \end{pmatrix} = \lambda^2 - 1 = 0 \Rightarrow \lambda_\pm = \pm 1 \tag{6}$$

Eigenvector can be found using the following: $\sigma_x \vec{v} = \lambda \vec{v}$. For $\lambda_+ = +1$,

$$\sigma_x \vec{v} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} \Rightarrow \begin{matrix} b = a \\ a = b \end{matrix} \Rightarrow \vec{v}_+ = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{7}$$

Similarly, for λ_-,

$$\vec{v}_- = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{8}$$

ii)

$$\det[\sigma_y - \lambda \mathbb{1}] = \det \begin{pmatrix} -\lambda & -i \\ i & -\lambda \end{pmatrix} = \lambda^2 - 1 = 0 \Rightarrow \lambda_\pm = \pm 1 \tag{9}$$

The eigenvector for $\lambda_+ = +1$ is given by:

$$\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} \Rightarrow \begin{pmatrix} 1 \\ i \end{pmatrix}, \text{ and normalization gives } \vec{v_+} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix} \tag{10}$$

The eigenvector for $\lambda_- = -1$ is given by:

$$\vec{v_-} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \end{pmatrix} \tag{11}$$

iii)

$$\det[\sigma_z - \lambda \mathbb{1}] = \det \begin{pmatrix} 1-\lambda & 0 \\ 0 & -1-\lambda \end{pmatrix} = \lambda^2 - 1 = 0 \Rightarrow \lambda_\pm = \pm 1 \tag{12}$$

The eigenvectors are given by:

$$\vec{v_+} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \text{ and } \vec{v_-} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{13}$$

iv)

The square of each Pauli matrix is an identity matrix!

$$\sigma_x^2 + \sigma_y^2 + \sigma_z^2 = 3\mathbb{1} \tag{14}$$

The eigenvalues of identity matrix are:

$$\det[(\sigma_x^2 + \sigma_y^2 + \sigma_z^2) - \lambda \mathbb{1}] = \det[3\mathbb{1} - \lambda \mathbb{1}] = (3-\lambda)^2 \Rightarrow \lambda_\pm = 3 \tag{15}$$

The eigenvector are given by:

$$\vec{v_\pm} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \text{ and } \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{16}$$

d)

Jacobi identity. We are to prove

$$[\sigma_x,[\sigma_y,\sigma_z]]+[\sigma_y,[\sigma_z,\sigma_x]]+[\sigma_z,[\sigma_x,\sigma_y]]=0 \tag{17}$$

Let's break this down:

$$[\sigma_y,\sigma_z]=\sigma_y\sigma_z-\sigma_z\sigma_y=\begin{pmatrix}0&-i\\i&0\end{pmatrix}\begin{pmatrix}1&0\\0&-1\end{pmatrix}-\begin{pmatrix}1&0\\0&-1\end{pmatrix}\begin{pmatrix}0&-i\\i&0\end{pmatrix}=\begin{pmatrix}0&2i\\2i&0\end{pmatrix}=2i\sigma_x \tag{18}$$

Generally, the commutators of Pauli matrices of cyclic permutation are given by $\sigma_i\sigma_j = 2i\sigma_k$, where i, j, k are cyclic permutation of x, y, z. In addition, Pauli matrices are commutative to itself. That is, $[\sigma_i, \sigma_i] = 0$. Therefore, the expression in eq. 17 reduces to:

$$2i\big([\sigma_x,\sigma_x]+[\sigma_y,\sigma_y]+[\sigma_z,\sigma_z]\big)=2i*0=0\text{ Q.E.D.} \tag{19}$$

Problem 3

(a)

The control photograph is showing the silver atoms on the screen when magnetic field is not applied. The collimated beam is not deflected and thus shown as a single blob on the screen.

(b)

By inspection, the maximum deflection is about three lines, which is equivalent to $\approx 150\mu$m. The magnetic moment of the electron is thus given by $\mu_e \approx -9.28\times10^{-24}$ (Python attached at the end). The magnetic moment of the Silver atom is just the magnetic moment of one electron because of spherical symmetry. 46 electrons form spherically symmetric e^- cloud, which results in 0 net angular momentum. This results in only the 47th electron contributing to the magnetic moment of overall Silver atom.

(c)

i)

The deflection of the atom results from the magnetic moment. The magnetic moment of the Silver atom is given by $\mu_e = g_2\frac{1}{2}\mu_B$, which is independent of the mass of the atom. Therefore, two different isotopes of Silver atom will produce same deflection.

ii)

Answered in part (b)!

(d)

Charged particles deflect when traveling through magnetic field. This effect is more dominant than the deflection due to the spin, which obscures the gist of the experiment. Therefore, carrying out the experiment with neutral Silver atoms make more sense.

(e)

From reading off the photograph, we get the value of g-factor to be:

$$\vec{\mu_e}=g\frac{q}{2m_e}\vec{S}\Rightarrow g=\frac{2\mu_e m_e}{eS}\approx-2.18 \tag{20}$$

The accepted value of the g-factor is $g \approx -2$. The percent error is about 9%.

(f)

Being more massive means that it can potentially interact with heavier unknown particles better. Since muon is about 200 times more massive than the electron, it would be more sensitive to beyond Standard Model physics.

Problem 4

(a)

Hadamard gate can be expressed as a linear combination of Pauli matrices:

$$M_H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \frac{1}{\sqrt{2}}\left[\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\right] = \frac{1}{\sqrt{2}}\sigma_z + \frac{1}{\sqrt{2}}\sigma_x \tag{21}$$

Hadamard gate is a quantum logic gate that acts on single cubit. It returns an equal superposition of the cubit in its computational basis.

$$M_H \left|\uparrow\right\rangle = \frac{\left|\uparrow\right\rangle + \left|\downarrow\right\rangle}{\sqrt{2}} \text{ , and } M_H \left|\downarrow\right\rangle = \frac{\left|\uparrow\right\rangle - \left|\downarrow\right\rangle}{\sqrt{2}} \tag{22}$$

In quantum computing, we carry out operations in states in superposition. Thus, we need a method to make a superposition out of a pure state. Hadamard gate does the job, and is therefore a very useful tool in quantum computing.

(b)

$$\det[M_H] = \frac{1}{2}(-1-1) = -1 \tag{23}$$

The inverse of Hadamard gate is given by:

$$M_H^{-1} = \frac{1}{\sqrt{2}}\frac{1}{-1}\begin{pmatrix} -1 & -1 \\ -1 & 1 \end{pmatrix} = M_H \tag{24}$$

Hadamard gate is involutory.

Problem 5

(a)

i)

To derive the following matrix representation, it is worth noticing the following:

$$\sigma_z^n = \begin{cases} \mathbb{1} & n = \text{even} \\ \sigma_z & n = \text{odd} \end{cases} \tag{25}$$

$$e^{i\theta} = 1 + i\theta + \frac{1}{2!}i^2\theta^2 + \frac{1}{3!}i^3\theta^3 + \cdots \tag{26}$$

$$e^{-i\theta} = 1 - i\theta + \frac{1}{2!}i^2\theta^2 - \frac{1}{3!}i^3\theta^3 + \cdots \tag{27}$$

Now, Taylor expanding $e^{i\theta\sigma_z}$ gives:

$$\begin{aligned} e^{i\theta\sigma_z} &= \sum_{n=0}^{\infty}\frac{1}{n!}(i\theta\sigma_z)^n = \mathbb{1} + i\theta\sigma_z + \frac{1}{2!}i^2\theta^2\sigma_z^2 + \frac{1}{3!}i^3\theta^3\sigma_z^3 + \frac{1}{4!}i^4\theta^4\sigma_z^4 + \cdots \\ &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} i\theta & 0 \\ 0 & -i\theta \end{pmatrix} + \frac{1}{2!}\begin{pmatrix} i^2\theta^2 & 0 \\ 0 & i^2\theta^2 \end{pmatrix} + \frac{1}{3!}\begin{pmatrix} i^3\theta^3 & 0 \\ 0 & -i^3\theta^3 \end{pmatrix} + \frac{1}{4!}\begin{pmatrix} i^4\theta^4 & 0 \\ 0 & i^4\theta^4 \end{pmatrix} + \cdots \end{aligned} \tag{28}$$

Substituting eq.26 and eq.27 to eq.28 directly gives:

$$e^{i\theta\sigma_z} = \begin{pmatrix} e^{i\theta} & 0 \\ 0 & e^{-i\theta} \end{pmatrix} \tag{29}$$

ii)

Similar logic applies here as well. Useful things to notice are:

$$\sigma_x^n = \begin{cases} \mathbb{1} & n = \text{even} \\ \sigma_x & n = \text{odd} \end{cases} \tag{30}$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots \tag{31}$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots \tag{32}$$

Now, expanding $e^{i\theta\sigma_x}$ gives:

$$\begin{aligned} e^{i\theta\sigma_x} &= \mathbb{1} + i\theta\sigma_x + \frac{i^2\theta^2\sigma_x^2}{2!} + \frac{i^3\theta^3\sigma_x^3}{3!} + \frac{i^4\theta^4\sigma_x^4}{4!} + \frac{i^5\theta^5\sigma_x^5}{5!} + \frac{i^6\theta^6\sigma_x^6}{6!} \cdots \\ &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & i\theta \\ i\theta & 0 \end{pmatrix} + \frac{1}{2!}\begin{pmatrix} i^2\theta^2 & 0 \\ 0 & i^2\theta^2 \end{pmatrix} + \frac{1}{3!}\begin{pmatrix} 0 & i^3\theta^3 \\ i^3\theta^3 & 0 \end{pmatrix} + \frac{1}{4!}\begin{pmatrix} 0 & i^4\theta^4 \\ i^4\theta^4 & 0 \end{pmatrix} + \cdots \\ &= \begin{pmatrix} 1 + \frac{i^2\theta^2}{2!} + \frac{i^4\theta^4}{4!} + \frac{i^6\theta^6}{6!} + \cdots & i\theta + \frac{i^3\theta^3}{3!} + \frac{i^5\theta^5}{5!} + \frac{i^7\theta^7}{7!} + \cdots \\ i\theta + \frac{i^3\theta^3}{3!} + \frac{i^5\theta^5}{5!} + \frac{i^7\theta^7}{7!} + \cdots & 1 + \frac{i^2\theta^2}{2!} + \frac{i^4\theta^4}{4!} + \frac{i^6\theta^6}{6!} + \cdots \end{pmatrix} \\ &= \begin{pmatrix} 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \cdots & i\left(\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \cdots\right) \\ i\left(\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \cdots\right) & 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \cdots \end{pmatrix} \\ &= \begin{pmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{pmatrix} \end{aligned} \tag{33}$$

iii)

$e^{i\theta\sigma_y}$ is equivalent to multiplying $-i$ and i to σ_{x01} and σ_{x10} respectively. This gives

$$e^{i\theta\sigma_y} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \tag{34}$$

(b)

i)

Let's recall that σ_+ and σ_- are the following:

$$\sigma_+ = \begin{pmatrix} 0 & 2 \\ 0 & 0 \end{pmatrix} \text{ , and } \sigma_- = \begin{pmatrix} 0 & 0 \\ 2 & 0 \end{pmatrix} \tag{35}$$

Therefore,

$$\sigma_+^2 = \sigma_-^2 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \tag{36}$$

and second order and higher terms all vanish. Now, $e^{i\theta\sigma_+}$ is given by:

$$\begin{aligned} e^{i\theta\sigma_+} &= \mathbb{1} + (i\theta\sigma_+) + \frac{1}{2!}(i\theta\sigma_+)^2 + \frac{1}{3!}(i\theta\sigma_+)^3 + \cdots \\ &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & 2i\theta \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 2i\theta \\ 0 & 1 \end{pmatrix} \end{aligned} \tag{37}$$

ii)

Similarly, $e^{i\theta\sigma_-}$ is given by:

$$\begin{aligned} e^{i\theta\sigma_-} &= \mathbb{1} + (i\theta\sigma_-) + \frac{1}{2!}(i\theta\sigma_-)^2 + \frac{1}{3!}(i\theta\sigma_-)^3 + \cdots \\ &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 2i\theta & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 2i\theta & 1 \end{pmatrix} \end{aligned} \tag{38}$$

(c)

First, notice

$$e^{-i\theta\sigma_+} = \begin{pmatrix} 1 & -2i\theta \\ 0 & 1 \end{pmatrix} \text{ , and } e^{-i\theta\sigma_-} = \begin{pmatrix} 1 & 0 \\ -2i\theta & 1 \end{pmatrix} \tag{39}$$

i)

$$\begin{aligned} e^{i\theta\sigma_+}\sigma_z e^{-i\theta\sigma_+} &= \begin{pmatrix} 1 & 2i\theta \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 & -2i\theta \\ 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} 1 & -2i\theta \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 & -2i\theta \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -4i\theta \\ 0 & -1 \end{pmatrix} \end{aligned} \tag{40}$$

ii)

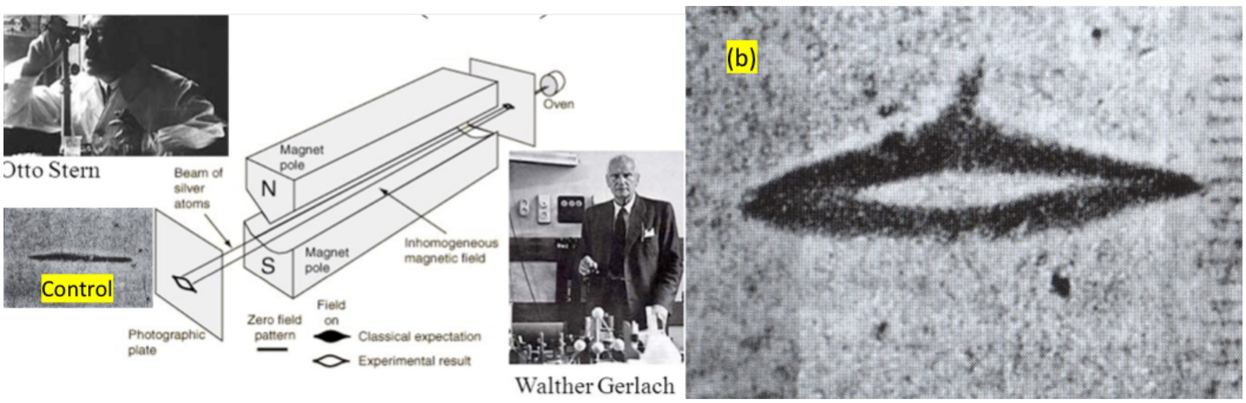
$$\begin{aligned} e^{-i\theta\sigma_-}\sigma_z e^{i\theta\sigma_+} &= \begin{pmatrix} 1 & 0 \\ -2i\theta & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 & 2i\theta \\ 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} 1 & 0 \\ -2i\theta & -1 \end{pmatrix}\begin{pmatrix} 1 & 2i\theta \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2i\theta \\ -2i\theta & 4\theta^2 - 1 \end{pmatrix} \end{aligned} \tag{41}$$

PY511 Fall 2022

Instructor: Shyamsunder Erramilli

Homework 1.3 Cheap Cigars and the Stern-Gerlach Experiment (Helper file)

Calculate the deflection experienced by a neutral silver atom ${}^{107}_{47}\mathrm{Ag}$ in the original Stern-Gerlach experiment. The oven temperature is 1000 C, with the silver atom beam traversing a magnet that is 3.5 cm long, with the field strength is 0.1 Tesla and a field gradient of 10 Tesla/cm. The fine horizontal lines in (b) are 50 microns apart.



In [1]:
```
%pylab inline
```

```
Populating the interactive namespace from numpy and matplotlib
```

Fundamental physical constants

The speed of light, Planck constant, Boltzmann constant, the Gravitational constant have been fixed in SI units to be rational numbers, without errors.

In electromagnetism, the only dimensionless parameter is the fine structure constant (expanded below in SI units).

$$\alpha = \frac{1}{4\pi\epsilon_0}\frac{e^2}{\hbar c}$$

The unit system is arbitrary. We could just as well describe Physics in *Natural Units* where they are all dimensionless

$$c = \hbar = k_B = G = \epsilon_0 = \mu_0 = 1 \text{ all dimensionless}$$

This value of $\approx \frac{1}{137}$ is universal, *independent of the system of units* and is one of about 17 seemingly arbitrary parameters in the Standard Model.

Scipy has a module that contains 2018 CODATA recommended values of physical constants and related constants in SI units.

In [2]:

```
from scipy import constants

# The fundamental Physics constants are now loaded from scipy
# The next few lines defines these constants using more compact fa

c    = constants.c                # speed of light
hbar = constants.hbar             # Planck's constant hbar
m_e  = constants.m_e              # mass of the electron
q_e  = constants.e                # magnitude of electron charge
print(c, hbar, m_e, q_e)          # Sanity check if constants loaded
```

```
299792458.0 1.0545718176461565e-34 9.1093837015e-31 1.602176634e-19
```

Bohr magneton μ_B is a physical constant that expresses the magnetic moment of an electron caused by its angular momentum.

$$\mu_B = \frac{e\hbar}{2m_e} \approx 9.27^{-24}\text{J/T} = \text{mu\_Bohr[0]} = \text{mu\_B}$$

In [7]:

```
# Bohr magneton from scipy.constants

mu_Bohr = constants.physical_constants["Bohr magneton"]
mu_B    = mu_Bohr[0]
print(mu_B)
print('Bohr magneton (SI): {0:4.2g} J/T'.format(mu_B))
```

```
9.2740100783e-24
Bohr magneton (SI): 9.3e-24 J/T
```

Magnetic Moment Undergraduate History review

- **Classical**: Current loop, in terms of angular momentum:

$$\boldsymbol{\mu} = \frac{1}{2}\oint \mathbf{r} \times \mathbf{J} \text{ and } \mathbf{J} = \frac{e}{m}\rho\mathbf{v}$$
$$\mathbf{L} = m\mathbf{v} \times \mathbf{r} \Rightarrow \boldsymbol{\mu} = -\frac{e}{2m}\mathbf{L}$$

- **Bohr Theory (no spin):** Magnitudes

$$L = m_l\hbar \Rightarrow \text{ For } m_l = 1 \text{ we have } \mu = \frac{e\hbar}{2m} \equiv \mu_B$$

μ_B is called the Bohr magneton.

$$\mu_B = 9.2740100783... \times 10^{-24}\mathrm{J} \cdot \mathrm{T}^{-1}$$

Also recall First year UG Physics -- Energy $U = -\boldsymbol{\mu} \cdot \mathbf{B}$, Torque $\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B}$ and Force

$$\mathbf{F} = -\nabla(\boldsymbol{\mu} \cdot \mathbf{B}) \Rightarrow F_y = \mu\frac{\partial B_y}{\partial y}$$

Strategy: Use the deflection in a magnetic field gradient to measure the magnetic moment.

How big is the electron magnetic moment μ_e? Stern created a betting pool, challenging Theorists to predict what he and Gerlach would measure. Stern-Gerlach experiment:

The actual measured value today is a bit different from the Bohr theory value

In [9]:
```
k_B    = constants.Boltzmann
m_p    = constants.m_p          # Mass of proton
m_n    = constants.m_n          # Mass of neutron
T      = 1000 + 273.15          # Temperature in Kelvin
L_m    = 3.5E-2                 # Length is 3.5 cm  to SI
grad_B = 1E3                    # gradient 10 Tesla/cm to SI
```

In [10]:
```
Delta_y = 1E-4                   # By inspection
mu_SG = (6*k_B*T/(grad_B*L_m**2))*Delta_y

print("Stern-Gerlach measured electron mag moment: ", mu_SG)
```

```
Stern-Gerlach measured electron mag moment:  8.609501751918369e-24
```

Magnetic moment of the electron and the g-factor

The Magnetic moment of the electron is proportional to its Spin angular momentum:

$$\boldsymbol{\mu}_e = -g_e \mu_B \frac{\mathbf{S}}{\hbar}$$

Since

$$S_z = m_s \hbar = \pm \frac{1}{2}\hbar \Rightarrow \mu_e = -g_e \frac{e\hbar}{2m} m_s = -g_e \frac{1}{2}\mu_B$$

Atomic Physicists use the minus sign to keep g_e positive. Nuclear Physicists are more precise, as they deal with both positive and negative charges, and consider the g-factor of the electron to be negative. It's best not to get involved in such religious arguments.

Given this Theological schism it's fine to forget the minus signs and simply write the magnetic moment of the electron in terms of the *Bohr magneton* $\mu_B = \frac{e\hbar}{2m_e}$:

$$\mu_\mathrm{e} = g_e \frac{1}{2}\mu_B$$

My own Religious beliefs forbid the use of Gaussian Units: If you wish to remain sane, do not ever write $g_e e/mc$. You'll be measuring charge in 'statcoulombs' on your way to a padded cell and *Narakam* *('Hell' in Telugu)* . Ignore Sakurai and other theorists who follow our misguided but otherwise excellent textbook.

In [12]:

```
mu_electron = constants.physical_constants["electron mag. mom."]
mu_e = mu_electron[0]
print(mu_e)
```

```
-9.2847647043e-24
```

So what is the g factor for the electron measured to be?

In [13]:

```
# g factor, or dimensionless magnetic moment, quantifies the magne

gFactor = 2*mu_e/mu_B
print(gFactor)
```

```
-2.002319304358999
```

Dirac Theory, QED

Dirac's Relativistic equation leads to

$$g_e = 2$$

QED 1-loop correction (Schwinger)

$$g_e = 2\left(1 + \frac{\alpha}{2\pi} + \ldots\right)$$

Experimental value is (to an astonishing 13 decimal place!)

$$g_e = 2.00231930436182(52)$$

Amazingly, the Standard Model calculation is fully consistent for the g-factor of the electron.

For the muon, however....

The g-2 experiment at Boston University (Fermilab collab)

Deviations from the simple Dirac theory are expressed in terms of the magnetic moment "anomaly" written a_e for the electron and a_μ for the muon

$$a_\mu = \frac{g_\mu - 2}{2}$$

The 1-loop correction is the same for the electron and muon:

$$a_\mu = a_e = \frac{\alpha}{2\pi}$$

Differences appear in higher order corrections. For the muon, the g-2 experiment has revealed some hints of Beyond Standard Model Physics.

Why would the muon be more sensitive than the electron to Beyond Standard Model Physics?

Homework 1 calculations

Start with:

- Force on the silver atom in a field gradient $F_y = \mu_e |\nabla B_z|$
- Acceleration of the silver atom of mass M $a_y = \frac{F_y}{M}$
- Speed of the atom out of the oven $v = \sqrt{\frac{3k_B T}{M}}$
- Estimate time to traverse the S-G apparatus ℓ $\Delta t = \frac{\ell}{v}$
- Displacement $\Delta y = \frac{1}{2} a_y (\Delta t)^2$ Combine all this

$$\mu_e = \frac{6k_B T}{|\nabla B_z| \ell^2} \Delta y$$

By inspection, from the photograph $\Delta y \approx 10^{-4}$ m Also given:

$$T = 1273\text{K}, |\nabla B_z| \text{T} \cdot \text{m}^{-1}$$

Summary:

The S-G value I get from this mumerical estimate is about $??? \times 10^{-24} \text{J} \cdot \text{T}^{-1}$ which is/is not close to the modern value in CODATA.