Sphere Generator: Techniques

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October 3, 2012

This is documentation for sphere_generator.py, a program to generate spheres of arbitrary surface area triangulated by equilateral triangles. The sphere is represented as a list of triples, each triple containing 3 vertex id numbers. Matching up the IDs yields an object homeomorphic to a sphere, and with an (ideally) uniform curvature.

This is a description of the ideas used to make the sphere generator program. The README and the programmer's guide offer more details. In this document I will describe how the program selects for standard deviation surface area of a given sphere.

Associated with each vertex is a point curvature:

$$K_p(v) = 2(2 - \theta_t N_T(v)),$$

where $\theta_t = \pi/3$ is the interior angle of an equilateral triangle and $N_t(v)$ is the total number of triangles connected to a given vertex. This description of curvature is given by Regge calculus.

The mean curvature is then

$$K = \frac{1}{N_v} \sum_{vertices,\ v} K_p(v),$$

where N_v is the total number of vertices in the sphere.

The standard deviation of curvature, σ is then given by

$$\sigma^2 = \frac{1}{N_v} \sum_{vertices,\ v} (K - K_p(v))^2 .$$

Let ϵ_a and ϵ_σ be numbers between 0 and 1. Let the target standard deviation—i.e., the standard deviation we want the sphere to have—be σ_t and the target surface area of the sphere—i.e., the surface area we want the sphere to have—be A_t. The *fitness function* for a given sphere S is

$$f(S) = e^{-\epsilon_a(A-A_t)} e^{-\epsilon_\sigma(\sigma-\sigma_t)},$$

where A is the surface area of the sphere at a given time.

The metropolis algorithm does the following:

- Make a small change to the sphere using one of the ergodic moves. Ergodic here means that a composition of changes can bring any sphere to any other sphere.
- Test to see if the fitness function becomes larger or smaller. If the fitness function becomes larger, accept the change. Otherwise, reject it.
- Repeat.

That's it! That's how spheres are generated! For implementation details, see the programmer's guide. If you have any questions, feel free to email me.