Sets

Monday, October 21, 2024 11:47 AM

Set
→ unordered collection of objects

small letter = member
big letter = set

Elements
→ member of set

Roster Method
- order not important!
- listing more than once does not change the set!
  → only member or not!
- ellipses (three dots) to describe set w/o listing all members when pattern is clear!

*Cheat Sheet!

N = natural	=	0, all positive int
Z = integers	=	all neg, 0, all pos, *perfect squares
Z^+ = positive int	=	1, 2, 3 . . .
R = real numbers	=	kasama irrational . . . ? all!
R^+ = real positive	=	
C = Complex	=	combo of real & imaginary
Q = Rational Numbers	=	fraction!

Set - Builder Notation
→ specifies the property / properties must satisfy
$O = \{x \mid x$ is an odd positive
$O = \{x \in Z^+ \mid x$ is odd and
→ a predicate may be used:
$S = \{x \mid P(x)\}$
Ex. $= \{x \mid Prime(x)\}$
→ positive rational numbers:
$Q^+ = \{x \in R \mid x = p/q$ for s
where q is no

Universal & Empty Set
U
→ containing everything currently under consideration
→ collection of sets that formed a bigger set

Empty (∅)
→ set that does not have any value

Interval Notation
[ ] = includes first value
( ) = does not include

$[a, b] = \{x \mid a \le x \le b\}$
$[a, b) = \{x \mid a \le x \le b\}$
$(a, b] = \{x \mid a < x \le b\}$
$(a, b) = \{x \mid a < x < b\}$

Things to Remember!
☆ sets can be elements of sets
- $\{\{1, 2, 3\}, a, \{b, c\}\}$
- $\{N, Z, Q, R\}$

☆ empty set is diff from set containing empty set
$\emptyset \neq \{\emptyset\}$ ← set w/in a set

☆ $\{1, 2\} \neq \{\{1, 2\}\}$

Set Equality "="
→ sets are only equal IF AND ONLY IF they have the same elements

$\forall x (x \in A \leftrightarrow x \in B)$ Ex. $\{1, 3, 5\} = \{3, 5, 1\}$
$\{1, 5, 5, 5, 3, 3, 1\} = \{1, 3, 5\}$

Subsets "$\subseteq$"

↳ set A is subset of B, IF AND ONLY IF every element of A is also an element of B

$A \subseteq B$ → A is a subset of B
→ $B = \{A, \dots\}$

Ex. $\{1, 2\} \subseteq \{1, 2, 3\}$
$\{1, 2\} \subseteq \{1, 2\}$

$\forall x (x \in A \rightarrow x \in B)$

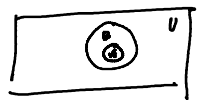
*Cheatsheet!
* $a \in \emptyset$ = false, $\emptyset \subseteq S$ for every set
$a \in S \rightarrow a \in S$, $S \subseteq S$ for every set

Note!
$\forall x (x \in A \leftrightarrow x \in B) = \forall x [(x \in A \rightarrow x \in B) \wedge (x \in B \rightarrow x \in A)]$
$A = B$ $A \subseteq B \wedge B \subseteq A$

Proper Subset "$\subset$"

→ if equal, not proper subset!
→ two sets should not be equal!

$\forall x (x \in A \rightarrow x \in B) \wedge \exists x (x \in B \wedge x \notin A)$



Set Cardinality

↳ number of elements in a set!
→ written as $|A|$
→ finite if n distinct elements
→ infinite, otherwise!

Examples:
- $|\emptyset| = 0$ S = alphabet, $|S| = 26$
- $|\{\emptyset\}| = 1$
- $\{1, 2, 3\} = 3$
- $S = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$; $|S| = 4$

Power Sets $= 2^n$, n = # of elements

↳ all subset that can be derived from given set,
→ $P(S) = \{\emptyset, \{0\}, \{1\}, \{0, 1\}\}$
$P(\emptyset) = \{\emptyset\}$
→ set of all subsets of A, denoted as $P(A)$ = power set of A

Tuples

→ used in 2D space to define coordinates!
→ (x, y) pair to specify a location

↳ ordered n-tuple $(a_1, a_2, \dots a_n)$ = ordered collection that has a_1 as its first element, a_2 as second until a_n as its last element
→ two n-tuples are equal if and only if their corresponding elements are equal

2 tuples
↳ ordered pairs
(a, b) and (c, d) are equal if & only if $a = c$ and $b = d$

Cartesian Product "$A \times B$"

→ combo of all elements in given 2 sets
→ set of ordered pairs (a, b) where $a \in A$ and $b \in B$

Example:
$A = \{a, b\}$ $B = \{1, 2, 3\}$

$A \times B = \{(a,1), (a,2), (a,3),$

$A \times B = \{(a,b) \mid a \in A \wedge b \in B\}$

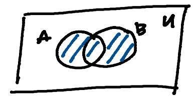
* subset R of $A \times B$ is called a relation

Set Operations

- Union! "$\cup$"

  $\rightarrow \{x \mid x \in A \vee x \in B\}$

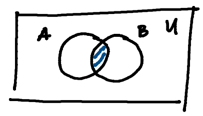
$A \cup B$



- Intersection

  $\rightarrow \{x \mid x \in A \wedge x \in B\}$

$A \cap B$

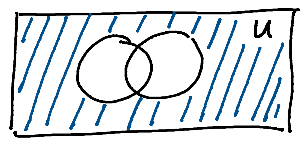


- Complement

  $\rightarrow U - A$

  $\rightarrow \{x \in U \mid x \notin A\}$

A' or $\bar{A}$



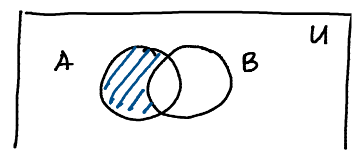
- Difference

  $\{1,2,3\} - \{3,4,5\} = \{1,2\}$

  $\{3,4,5\} - \{1,2,3\} = \{3,4,5\}$

  $\rightarrow A - B = \{x \mid x \in A \wedge x \notin B\} = A \cap B^c$

$A - B$ or $A \cap B^c$



Cardinality of the Union of Two Sets

- Inclusion - Exclusion

  $|A \cup B| = |A| + |B| - |A \cap B|$ .. ?

Set Identities!

1) Identity Laws

   $A \cup \emptyset = A$

   $A \cap U = A$

2) Domination Laws

   $A \cup U = U$

   $A \cap \emptyset = \emptyset$

3) Idempotent Laws

   $A \cup A = A$

   $A \cap A = A$

4) Complementation Laws

   $\overline{(\bar{A})} = A$

5) Commutative Laws

   $A \cup B = B \cup A$

   $A \cap B = B \cap A$

6) Associative Laws

   $A \cup (B \cup C) = (A \cup B) \cup C$

   $A \cap (B \cap C) = (A \cap B) \cap C$

7)