A summary of the vehicle routing problems and the optimization algorithms considered

The Truck Delivery (Source) Problem

We define the source task on an undirected graph $G = (V, E)$, where $V = \{v_i\}$ is the set of vertices (i.e., customer nodes) and $E = \{e_{ij}\}$ is the set of edges connecting vertices v_i and v_j, for $i, j = 1, \dots, d_S$, while d_S is the number of customers. The single depot is represented by vertex v_0 which stations K number of trucks. The vehicles are allowed to do multiple trips that start and end at the depot, and they are constrained by delivery capacity Q and maximum travel distance limit L per trip. Each customer i has a known deterministic demand $c_{S,i}$ and a distance $z_{S,i}$ from the depot. They are served by vehicle k in route r, where $k = 1, \dots, K$ and $r = 1, \dots, R$. Let $D_{k,r}$ be the distance travelled and $W_{k,r}$ be the weight capacity served by vehicle k in route r. The objective and constraint functions are then formulated as follows:

$$\min \;\; f_S = \sum_{r=1}^{R} . \sum_{k=1}^{K} D_{k,r},$$
$$s.t., D_{k,r} \leq L \; and \; W_{k,r} \leq Q, \forall k, \forall r, \tag{1}$$

where f_S is the source objective function to minimize the total truck distance travelled after serving all customer demands.

The Multi-Objective Drone Delivery (Target) Problem

We define the target task on an undirected graph $H = (U, A)$, where $U = \{u_i\}$ is the set of vertices and $A = \{a_{ij}\}$ is the set of edges connecting vertices u_i and u_j, for $i, j = 1, \dots, d_T$, such that d_T is the number of customer nodes. The single depot is represented by vertex u_0, at which each drone starts and ends a route b. All the drones have a delivery capacity q, a maximum travel distance l per route and a maximum flight range from the depot h. Any customer j has a known deterministic demand $c_{T,j}$ and a distance $z_{T,j}$ from the depot. They can only be served once by a drone y in route b, where $y, b = 1, \dots, B$. We assume that the drones are not allowed to do multiple trips due to their limited power capacity that requires time-consuming battery recharging. As such, a total of B number of drones is needed to complete B number of routes. Further, let $D_{y,b}$ be the distance travelled and $W_{y,b}$ be the payload served by drone y in route b. We formulate the objective and constraint functions as follows:

$$\min_{\boldsymbol{x}_T} \;\; \boldsymbol{F}(\boldsymbol{x}_T) = \left(f_T^1(\boldsymbol{x}_T), f_T^2(\boldsymbol{x}_T)\right),$$
$$s.t., f_T^1 = B, \; f_T^2 = \sum_{y,b}^{B} D_{y,b}, \tag{2}$$
$$q \ll Q, l \ll L, W_{y,b} \leq q, D_{y,b} \leq l \; and \; z_{T,j} \leq h, \forall y, \forall b, \forall j,$$

where $\boldsymbol{x}_T$ is a target candidate solution, f_T^1 and f_T^2 are the target objective functions to minimize the number of drones and the total flight distance, respectively. Eq. (2) accounts for the essential operating constraints of real commercial drones including smaller payload, shorter flight distance per route and limited flight range from the depot.

Table I shows the vehicle routing problem instances that are taken from various datasets [1], [2], [3]. For the truck delivery (source) instances, we either consider the original datasets or modify the datasets by removing an arbitrary number of customer nodes. The dimensionalities of the source and target problems, denoted as d_S and d_T, respectively, are moderately large with up to a few hundred customers. In our implementation, we generate new multi-objective drone delivery (target) instances according to the following procedure: (i) create a new target instance using only the Cartesian coordinates of customer nodes from the original dataset; (ii) arbitrarily transform the geometric distribution of customer nodes and randomly assign an index number to each node; (iii) set the drone travel distance limit $l = 50$, and randomly generate the flight range h between $[5,10]$ in L_2-norm distances, the payload limit q between $[20,200]$ in kilograms, as well as the customer demands $c_{T,j}$ between $[1,5]$ in kilograms, for $j = 1, \ldots, d_T$; (iv) remove customer nodes located beyond the range h. The drone constraints and customer demands values are motivated by the specifications of real commercial drones.

TABLE I

VEHICLE ROUTE OPTIMIZATION PROBLEM INSTANCES AND THEIR PROPERTIES.

Problem Instance	Source Sizes, d_S	Target Size, d_T
M-n151 [1]	135, 140	144
X-n214 [2]	205, 213	200
Golden_17 [3]	210, 240	222
Golden_18 [3]	290, 300	282
CMT4 [1]	130, 140, 150	133
X-n162 [2]	140, 150, 161	143

Four optimizers are considered for the performance comparison of the target problem instances in Table I. They are: (i) the NSGA-II without knowledge transfer [4], (ii) a multi-objective variant of the edge histogram-based sampling algorithm (EHBSA) which is a probabilistic model-based optimizer for combinatorial problems [5], (iii) the AMTEA without solution representation learning [6], and (iv) our proposed MOTrEO+M_S with both (a) solution representation learning via the mapping M_S (for inducing positive transfers) and (b) source-target similarity capture mechanism (for mitigating negative transfers). All search populations of size $N = 100$ consist of permutation-coded solutions. Excluding EHBSA, the other algorithms execute optimized crossover (with probability 0.75) along with inversion or swap sequence mutation (with probability 0.2) [7]. For EHBSA, the sampling without template strategy is used with its default parameter settings.

The transfer interval is set to $\Delta = 5$, at which target probabilistic mixture models with edge histogram-based source and target components [5] are built in the AMTEA and the MOTrEO+M_S. During non-transfer search iterations, both AMTEA and MOTrEO+M_S execute NSGA-II as the base MOEA. The solution quality achieved by all the optimizers is measured using the IGD performance metric [8]. A single-run of each optimizer is terminated after 10,000 function evaluations, whereas the target Pareto front is approximated at the end of 100,000 function evaluations of the NSGA-II.

Other relevant files and information are provided as follow.

- Source instances: M_151_source1_instance.txt, M_151_source2_instance.txt, X_214_source1_instance.txt, X_214_source2_instance.txt, Golden_17_source1_instance.txt, Golden_17_source2_instance.txt, Golden_18_source1_instance.txt, Golden_18_source2_instance.txt, vrpnc4_source1_instance.txt, vrpnc4_source2_instance.txt, vrpnc4_source3_instance.txt, X_162_source1_instance.txt, X_162_source2_instance.txt, X_162_source3_instance.txt
- Target instances: M_151_target_instance.txt, X_214_target_instance.txt, Golden_17_target_instance.txt, Golden_18_target_instance.txt, vrpnc4_target_instance.txt, X_162_target_instance.txt
- Source data: M_151_s1_data.xlsx, M_151_s2_data.xlsx, X_214_s1_data.xlsx, X_214_s2_data.xlsx, Golden_17_s1_data.xlsx, Golden_17_s2_data.xlsx, Golden_18_s1_data.xlsx, Golden_18_s2_data.xlsx,

vrpnc4_s1_data.xlsx, vrpnc4_s2_data.xlsx, vrpnc4_s3_data.xlsx, X_162_s1_data.xlsx, X_162_s2_data.xlsx, X_162_s3_data.xlsx

- Approximated target Pareto fronts: pf_M_151.txt, pf_X_214.txt, pf_Golden_17.txt, pf_Golden_18.txt, pf_vrpnc4.txt, pf_X_162.txt

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