A concise description of the complex multi-objective benchmark tasks and the optimization algorithms considered

TABLE I

COMPLEX MULTI-OBJECTIVE BENCHMARK PROBLEMS AND THEIR PROPERTIES [1]

Problem Configuration	γ	Task	Pareto Set	Properties
CIHS	0.970	$\mathcal{T}_S$	$x_1 \in [0,1]$ $x_i = 0, i = 2{:}10$	concave, unimodal, separable
		$\mathcal{T}$	$x_1 \in [0,1]$ $x_i = 0, i = 2{:}20$	concave, unimodal, separable
CIMS	0.520	$\mathcal{T}_S$	$x_1 \in [0,1]$ $x_i = 1, i = 2{:}10$	concave, multimodal, nonseparable
		$\mathcal{T}$	$x_1 \in [0,1]$ $(x_2, \dots, x_{20})^{\mathrm{T}} = s_{cm}$	concave, unimodal, nonseparable
CILS	0.070	$\mathcal{T}_S$	$x_1 \in [0,1]$, $x_i = 0, i = 2{:}10$	concave, multimodal, separable
		$\mathcal{T}$	$x_1 \in [0,1]$ $x_i = 0, i = 2{:}20$	convex, multimodal, nonseparable
PIHS	0.990	$\mathcal{T}_S$	$x_1 \in [0,1]$ $x_i = 0, i = 2{:}10$	convex, unimodal, separable
		$\mathcal{T}$	$x_1 \in [0,1]$ $(x_2, \dots, x_{20})^{\mathrm{T}} = s_{ph}$	convex, multimodal, separable
PIMS	0.550	$\mathcal{T}_S$	$x_1 \in [0,1]$, $(x_2, \dots, x_{10})^{\mathrm{T}} = s_{pm}$	concave, unimodal, nonseparable
		$\mathcal{T}$	$x_1 \in [0,1]$, $x_i = 0, i = 2{:}20$	concave, multimodal, nonseparable
PILS	0.002	$\mathcal{T}_S$	$x_1 \in [0,1]$ $x_i = 0, i = 2{:}10$	concave, multimodal, nonseparable
		$\mathcal{T}$	$x_1 \in [0,1]$ $(x_2, \dots, x_{20})^{\mathrm{T}} = s_{pl}$	concave, multimodal, nonseparable
NIHS	0.940	$\mathcal{T}_S$	$x_1 \in [0,1]$ $x_i = 0, i = 2{:}10$	concave, multimodal, nonseparable
		$\mathcal{T}$	$x_1 \in [0,1]$ $x_i = 1, i = 2{:}20$	convex, unimodal, separable
NIMS	0.510	$\mathcal{T}_S$	$x_1 \in [0,1]$, $x_2 \in [0,1]$, $x_i = 1, i = 3{:}10$	concave, multimodal, nonseparable
		$\mathcal{T}$	$x_1 \in [0,1]$, $x_2 \in [0,1]$, $x_i = 0, i = 3{:}20$	concave, unimodal, nonseparable
NILS	0.001	$\mathcal{T}_S$	$x_1 \in [0,1]$, $x_2 \in [0,1]$, $(x_3, \dots, x_{10})^{\mathrm{T}} = s_{nl}$	concave, multimodal, nonseparable
		$\mathcal{T}$	$x_1 \in [0,1]$ $x_i = 0, i = 3{:}20$	concave, multimodal, nonseparable

Table I presents the multi-objective source and target tasks (i.e., $\mathcal{T}_S$ and $\mathcal{T}$) for 9 distinct cases, along with their relevant properties. The source and target dimensionalities are set to $d_S = 10$ and $d_T = 20$, respectively. Table I also gives the similarity between the source and target fitness landscapes, denoted as γ. The four letter problem configuration represents the relationship between $\mathcal{T}_S$ and $\mathcal{T}$ in each case. For illustrative purposes: (i) CIHS: *complete intersection with high similarity*, implies that $\mathcal{T}_S$ and $\mathcal{T}$ have complete overlap between their

Pareto sets and a large γ; (ii) PIMS: *partial intersection with medium similarity*, states that $\mathcal{T}_S$ and $\mathcal{T}$ have partial overlap between their Pareto sets and a medium γ; and (iii) NILS: *no intersection with low similarity* (NILS), $\mathcal{T}_S$ and $\mathcal{T}$ have zero overlap between their Pareto sets and a small γ. Refer to [1] for the full details of these benchmark problems.

Four optimizers are considered for comparison. They are: (i) the NSGA-II without knowledge transfer [2], (ii) the NSGA-II+M_S which is a recently proposed TrEO algorithm [3] that is adapted using the proposed source-to-target mapping M_S, (iii) the baseline AMTEA without solution representation learning [4], and (iv) our proposed MOTrEO+M_S with both (a) solution representation learning via the mapping M_S (for inducing positive transfers) and (b) source-target similarity capture mechanism (for mitigating negative transfers). All search populations of size $N = 50$ consist of real-coded solutions. The optimizers employ simulated binary crossover, polynomial mutation (with probability $1/d_T$) and tournament selection. The transfer interval is set to $\Delta = 2$, at which target mixture models with Gaussian components are built and sampled in the AMTEA and the MOTrEO+M_S. For the two-layer neural network in the NSGA-II+M_S^{NL} and the MOTrEO+M_S, we set the number of neurons in the hidden layer to $d_h = 2d_T$. The solution quality achieved by all the optimizers is measured using the IGD performance metric [5]. A single run of each algorithm is terminated after 50,000 function evaluations, while the average IGD values are calculated based on 20 independent runs.

We expect that a smaller extent of intersection between source-target Pareto optimal solutions (i.e., "no" < "partial" < "complete" intersection), and a lower degree of similarity in the fitness landscapes (i.e., "low" < "medium" < "high" similarity) will increase the scope for the MOTrEO+M_S to unveil useful but hidden inter-task relationships. To further affirm that the effectiveness of the MOTrEO+M_S is indeed by virtue of positive transfers induced by the source-to-target mapping M_S (and is not a spurious artifact of introducing diversity through knowledge transfers from external sources), we consider an additional MOEA that transfers the solutions sampled from a random source model (RSM), which we label as the MOTrEO(RSM). The RSM takes the form of a uniform distribution defined over the search space, which is solely intended for introducing diversity into the target population. It is worth noting that in the MOTrEO(RSM), the amount of knowledge transfers is determined by the transfer coefficients obtained in the MOTrEO+M_S, which ensures that both algorithms receive the same extent of solution transfers from external sources.

Other relevant files and information are provided as follow.

- Source data: s_data_CIHS.txt, s_data_CIMS.txt, s_data_CILS.txt, s_data_PIHS.txt, s_data_PIMS.txt, s_data_PILS.txt, s_data_NIHS.txt, s_data_NIMS.txt, s_data_NILS.txt
- Source models: s_model_CIHS.txt, s_model_CIMS.txt, s_model_CILS.txt, s_model_PIHS.txt, s_model_PIMS.txt, s_model_PILS.txt, s_model_NIHS.txt, s_model_NIMS.txt, s_model_NILS.txt
- Approximated target Pareto fronts: pf_circle.txt, pf_concave.txt, pf_convex.txt
- Rotation matrices: M_CIMS_2.txt, M_NIMS_2.txt, M_PIMS_2.txt
- Shift vectors: S_CIMS_2.txt, S_PIHS_2.txt, S_PILS_2.txt

References:

[1] Y. Yuan, Y. S. Ong, L. Feng, A. K. Qin, A. Gupta, B. Da, Q. Zhang, K. C. Tan, Y. Jin and H. Ishibuchi, "Evolutionary multitasking for multiobjective continuous optimization: Benchmark problems, performance metrics and baseline results," *Technical Report*, 2017. *arXiv preprint arXiv:1706.02766*.

[2] L. M. Pang, H. Ishibuchi, and K. Shang, "NSGA-II With Simple Modification Works Well on a Wide Variety of Many-Objective Problems," *IEEE Access*, vol. 8, pp. 190240-190250, Oct. 2020.

[3] L. Feng, Y. S. Ong, S. Jiang, and A. Gupta, "Autoencoding evolutionary search with learning across heterogeneous problems," *IEEE Trans. Evol. Comput.*, vol. 21, no. 5, 760-772, Oct. 2017.

[4] B. Da, A. Gupta, and Y. S. Ong, "Curbing negative influences online for seamless transfer evolutionary optimization," *IEEE Trans. Cybern.*, vol. 49, no. 12, pp. 4365-4378, Aug. 2018.

[5] S. Jiang, Y. S. Ong, J. Zhang, and L. Feng, "Consistencies and contradictions of performance metrics in multiobjective optimization," *IEEE Trans. Cybern.*, vol. 44, no. 12, pp. 2391-2404, Dec. 2014.