Q1. (2 pts) Using propositional resolution, show the following propositional sentence is unsatisfiable.

`(p | q | -r) & ((-r | q | p) -> ((r | q) & -q & -p))`

To do this, convert this sentence to clausal form and derive the empty clause using resolution.

Conversion to clausal form

(p | q | -r) & ((-r | q | p) -> ((r | q) & -q & -p))

I

(p | q | -r) & (-(-r | q | p) | ((r | q) & -q & -p))

N

(p | q | -r) & ((r & -q & -p) | ((r | q) & -q & -p))

D

(p | q | -r) &
((r & -q & -p) | (r | q)) &
((r & -q & -p) | -q) &
((r & -q & -p) | -p)

(p | q | -r) &
((r | r | q) & (-q | r | q) & (-p | r | q)) & ((r | -q) & (-q | -q) & (-p | -q)) & ((r | -p) & (-q | -p) & (-p | -p))

O

{p, q, -r}
{r, q}
{-q, r, q}
{-p, r, q}
{r, -q}
{-q}
{-p, -q}
{r, -p}
{-q, -p}
{-p}

1. {p, q, -r}	Premise
2. {r, q}	Premise
3. {-p}	Premise
4. {-q}	Premise
5. {p, q}	1,2
6. {q}	3,5
7. {}	4,6

I have shown only those steps that quickly produce the empty clause.

Q2. (8 points) Every horse can outrun every dog. Some greyhounds can outrun every rabbit. Show that every horse can outrun every rabbit.

$\forall x.\forall y.(Horse(x) \wedge Dog(y) \Rightarrow Faster(x, y))$

$\exists y.(Greyhound(y) \wedge \forall z.(Rabbit(z) \Rightarrow Faster(y, z)))$

$\forall y.(Greyhound(y) \Rightarrow Dog(y))$ (*background knowledge*)

$\forall x.\forall y.\forall z.(Faster(x, y) \wedge Faster(y, z) \Rightarrow Faster(x, z))$ (*background knowledge*)

$\neg\forall x.\forall y.(Horse(x) \wedge Rabbit(y) \Rightarrow Faster(x, y))$ negated conclusion

In Prover9 format:

all x all y (Horse(x) & Dog(y) -> Faster(x, y)).

exists y (Greyhound(y) & all z (Rabbit(z)->Faster(y, z))).

all y (Greyhound(y)->Dog(y)).

all x all y all z (Faster(x, y) & Faster(y, z) -> Faster(x, z)).

Conclusion:

all x all y (Horse(x) & Rabbit(y) -> Faster(x, y)).

In clausal form they become:

$\{\neg Horse(x), \neg Dog(y), Faster(x, y)\}$

$\{Greyhound(Greg)\}$

$\{\neg Rabbit(z), Faster(Greg, z)\}$

$\{\neg Greyhound(y), Dog(y)\}$

$\{\neg Faster(x, y), \neg Faster(y, z), Faster(x, z)\}$

Let's transform the goal into clausal form:

	$\neg\forall x.\forall y.(Horse(x) \wedge Rabbit(y) \Rightarrow Faster(x, y))$
I	$\neg\forall x.\forall y.(\neg(Horse(x) \wedge Rabbit(y)) \vee Faster(x, y))$
N	$\neg\forall x.\forall y.(\neg Horse(x) \vee \neg Rabbit(y) \vee Faster(x, y))$
	$\exists x. \exists y.(Horse(x) \wedge Rabbit(y) \wedge \neg Faster(x, y))$
S	$\exists x. \exists y.(Horse(x) \wedge Rabbit(y) \wedge \neg Faster(x, y))$
E	$Horse(Jack) \wedge Rabbit(Smith) \wedge \neg Faster(Jack, Smith))$
A	$Horse(Jack) \wedge Rabbit(Smith) \wedge \neg Faster(Jack, Smith))$
D	$Horse(Jack) \wedge Rabbit(Smith) \wedge \neg Faster(Jack, Smith))$
O	$\{Horse(Jack)\}$
	$\{Rabbit(Smith)\}$
	$\{\neg Faster(Jack, Smith)\}$

Let's try to infer the {} using resolution:

1.	{¬Horse(x), ¬Dog(y), Faster(x, y)}	
2.	{Greyhound(Greg)}	
3.	{¬Rabbit(z), Faster(Greg, z)}	
4.	{¬Greyhound(y), Dog(y)}	
5.	{¬Faster(x, y), ¬Faster(y, z), Faster(x, z)}	
6.	{Horse(Jack)}	
7.	{Rabbit(Smith)}	
8.	{¬Faster(Jack, Smith)}	
9.	{¬Dog(y), Faster(Jack, y)}	1,6
10.	{Faster(Jack, y), ¬Greyhound(y)}	4,9
11.	{Faster(Jack, Greg)}	2,10
12.	{Faster(Greg, Smith)}	3,7
13.	{¬Faster(Greg, z), Faster(x, z)}	5,11
14.	{Faster(x, Smith)}	12,13
15.	{}	8,14

Q3. (2 points) All hummingbirds are richly colored. No large birds live on honey. Birds that do not live on honey are dull in color. Conclusion: All hummingbirds are small.

```
all x (hummingbird(x) -> richly_colored(x)).
-(exists x (bird(x) & large(x) & lives_on_honey(x))).
 all x (bird(x) & -lives_on_honey(x) -> -richly_colored(x)).

%background knowledge
 all x (hummingbird(x) -> bird(x)).

Theorem
all x (hummingbird(x) -> -large(x)).
```

Q4. (2 points) My gardener is well worth listening to on military subjects; No one can remember the battle of Waterloo, unless he is very old; Nobody is really worth listening to on military subjects, unless he can remember the battle of Waterloo. Conclusion: My gardener is very old.

```
all x (gardener(Me, x) -> worthListening(x)).
 all x (remember(x, Waterloo) -> old(x)).
-(  exists x (worthListening(x) & -remember(x, Waterloo)) ).

Theorem
all x (gardener(Me, x) -> old(x)).
```

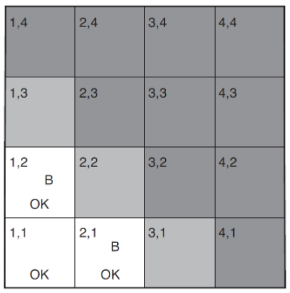
Q5. (3 pt) Redo the probability calculation for pits in [1,3], [2,2], [3,1], assuming that each square contains a pit with probability 0.01, independent of the other squares. What can you say about the relative performance of a logical versus a probabilistic agent in this case?

We need to find

$P(p_{13}|b_{12}, b_{21})$

$P(p_{31}|b_{12}, b_{21})$

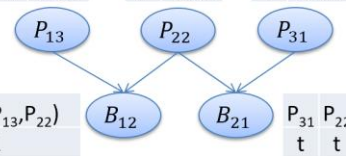
$P(p_{22}|b_{12}, b_{21})$



P_{13}	$P(P_{13})$
t	0.01

P_{22}	$P(P_{22})$
t	0.01

P_{31}	$P(P_{31})$
t	0.01



P_{13}	P_{22}	$P(B_{12}|P_{13},P_{22})$
t	t	1
t	f	1
f	t	1
f	f	0

P_{31}	P_{22}	$P(B_{21}|P_{31},P_{22})$
t	t	1
t	f	1
f	t	1
f	f	0

$$P(p_{13}|b_{12}, b_{21}) = \alpha \sum_{p_{22}} \sum_{p_{31}} P(b_{12}|p_{13}, p_{22}) \cdot P(b_{21}|p_{31}, p_{22}) \cdot P(p_{13}) \cdot P(p_{22}) \cdot P(p_{31}) =$$

$\alpha[P(b_{12}|p_{13}, p_{22}) \cdot P(b_{21}|p_{31}, p_{22}) \cdot P(p_{13}) \cdot P(p_{22}) \cdot P(p_{31}) +$

$P(b_{12}|p_{13}, \neg p_{22}) \cdot P(b_{21}|p_{31}, \neg p_{22}) \cdot P(p_{13}) \cdot P(\neg p_{22}) \cdot P(p_{31})$

$P(b_{12}|p_{13}, p_{22}) \cdot P(b_{21}|\neg p_{31}, p_{22}) \cdot P(p_{13}) \cdot P(p_{22}) \cdot P(\neg p_{31}) +$

$P(b_{12}|p_{13}, \neg p_{22}) \cdot P(b_{21}|\neg p_{31}, \neg p_{22}) \cdot P(p_{13}) \cdot P(\neg p_{22}) \cdot P(\neg p_{31})]$

=α(1*1*.01*.01*.01 + 1*1*.01*.99*.01 + 1*1*.01*.01*.99 + 0) = α*.000199

$$P(\neg p_{13}|b_{12}, b_{21}) = \alpha \sum_{p_{22}} \sum_{p_{31}} P(b_{12}|\neg p_{13}, p_{22}) \cdot P(b_{21}|p_{31}, p_{22}) \cdot P(\neg p_{13}) \cdot P(p_{22}) \cdot P(p_{31}) =$$

$\alpha[P(b_{12}|\neg p_{13}, p_{22}) \cdot P(b_{21}|p_{31}, p_{22}) \cdot P(\neg p_{13}) \cdot P(p_{22}) \cdot P(p_{31}) +$

$P(b_{12}|\neg p_{13}, \neg p_{22}) \cdot P(b_{21}|p_{31}, \neg p_{22}) \cdot P(\neg p_{13}) \cdot P(\neg p_{22}) \cdot P(p_{31}) +$

$P(b_{12}|\neg p_{13}, p_{22}) \cdot P(b_{21}|\neg p_{31}, p_{22}) \cdot P(\neg p_{13}) \cdot P(p_{22}) \cdot P(\neg p_{31}) +$

$P(b_{12}|\neg p_{13}, \neg p_{22}) \cdot P(b_{21}|\neg p_{31}, \neg p_{22}) \cdot P(\neg p_{13}) \cdot P(\neg p_{22}) \cdot P(\neg p_{31})]$

=α(1*1*.99*.01*.01 + 0 + 1*1*.99*.01*.99 + 0) = α*.0099

$$P(p_{13}|b_{12}, b_{21}) = \alpha * .000199$$

$$P(\neg p_{13}|b_{12}, b_{21}) = \alpha * .0099$$

$$\alpha = \frac{1}{.000199 + .0099}$$

$$P(p_{13}|b_{12}, b_{21}) = \frac{.000199}{.000199 + .0099} = 1.97\%$$

$$P(\neg p_{13}|b_{12}, b_{21}) = \frac{.0099}{.000199 + .0099} = 98.03\%$$

The case of p_{31} is symmetric (same).

$$P(p_{22}|b_{12},b_{21}) = \alpha \sum_{p_{13}} \sum_{p_{31}} P(b_{12}|p_{13},p_{22}) \cdot P(b_{21}|p_{31},p_{22}) \cdot P(p_{13}) \cdot P(p_{22}) \cdot P(p_{31}) =$$

$\alpha[P(b_{12}|p_{13},p_{22}) \cdot P(b_{21}|p_{31},p_{22}) \cdot P(p_{13}) \cdot P(p_{22}) \cdot P(p_{31}) +$
$P(b_{12}|\neg p_{13},p_{22}) \cdot P(b_{21}|p_{31},p_{22}) \cdot P(\neg p_{13}) \cdot P(p_{22}) \cdot P(p_{31}) +$
$P(b_{12}|p_{13},p_{22}) \cdot P(b_{21}|\neg p_{31},p_{22}) \cdot P(p_{13}) \cdot P(p_{22}) \cdot P(\neg p_{31}) +$
$P(b_{12}|\neg p_{13},p_{22}) \cdot P(b_{21}|\neg p_{31},p_{22}) \cdot P(\neg p_{13}) \cdot P(p_{22}) \cdot P(\neg p_{31})]$
=α(1*1*.01*.01*.01 + 1*1*.99*.01*.01 + 1*1*.01*.01*.99 + 1*1*.99*.01*.99) = α*.01

$$P(\neg p_{22}|b_{12},b_{21}) = \alpha \sum_{p_{13}} \sum_{p_{31}} P(b_{12}|p_{13},p_{22}) \cdot P(b_{21}|p_{31},p_{22}) \cdot P(p_{13}) \cdot P(p_{22}) \cdot P(p_{31}) =$$

$\alpha[P(b_{12}|p_{13},\neg p_{22}) \cdot P(b_{21}|p_{31},\neg p_{22}) \cdot P(p_{13}) \cdot P(\neg p_{22}) \cdot P(p_{31}) +$
$P(b_{12}|\neg p_{13},\neg p_{22}) \cdot P(b_{21}|p_{31},\neg p_{22}) \cdot P(\neg p_{13}) \cdot P(\neg p_{22}) \cdot P(p_{31}) +$
$P(b_{12}|p_{13},\neg p_{22}) \cdot P(b_{21}|\neg p_{31},\neg p_{22}) \cdot P(p_{13}) \cdot P(\neg p_{22}) \cdot P(\neg p_{31}) +$
$P(b_{12}|\neg p_{13},\neg p_{22}) \cdot P(b_{21}|\neg p_{31},\neg p_{22}) \cdot P(\neg p_{13}) \cdot P(\neg p_{22}) \cdot P(\neg p_{31})]$
=α(1*1*.01*.99*.01 + 0 + 0 + 0) = α* 0.000099

$$P(p_{22}|b_{12},b_{21}) = \alpha * .01$$
$$P(\neg p_{22}|b_{12},b_{21}) = \alpha * .000099$$
$$\alpha = \frac{1}{.01 + .000099}$$
$$P(p_{22}|b_{12},b_{21}) = \frac{.01}{.01 + .000099} = 99\%$$
$$P(\neg p_{22}|b_{12},b_{21}) = \frac{.000099}{.01 + .000099} = 1\%$$

Conclusion.
Going to [2,2] is almost certain death. So, a probabilistic agent will never choose to go to [2,2]. On the other hand, to a logical agent, squares [1,3], [2,2], [3,1] look the same. So, the logical agent would choose either one with equal chance (1/3). By doing that, the agent will die with a chance of about 1/3.

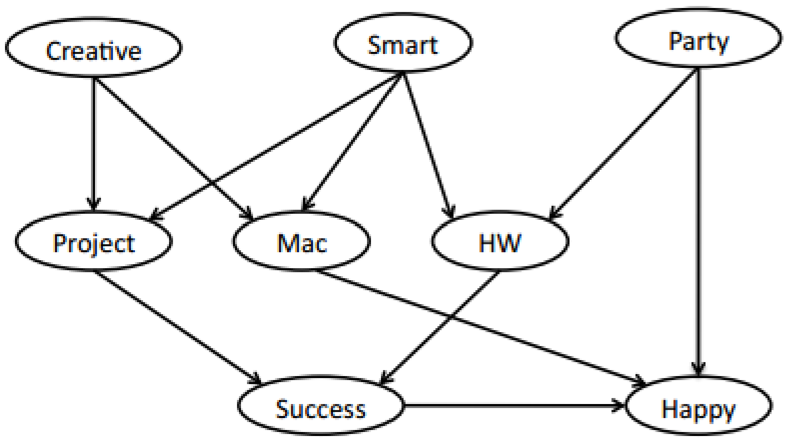
Q6. (9 pts) [Adapted from a CMU machine learning assignment]
As part of a comprehensive study of the role of CMU 10-601 (Machine Learning) on people's happiness, CMU has been collecting data from graduating students. In an optional survey, the following questions were asked:
• Do you party frequently [Party: Yes/No]?
• Are you wicked smart [Smart: Yes/No]?
• Are you very creative [Creative: Yes/No]? (Please only answer Yes or No)
• Did you do well on all your homework assignments? [HW: Yes/No]
• Do you use a Mac? [Mac: Yes/No]
• Did your course project succeed? [Project: Yes/No]
• Did you succeed in your most important class (which is 10-601)? [Success: Yes/No]
• Are you currently Happy? [Happy: Yes/No]

You can obtain the comma-separated survey results from the accompanying file.
Each row in *students.csv* corresponds to the responses of a separate student.
The columns in *students.csv* correspond to each question (random variable) in the order Party, Smart, Creative, HW, Mac, Project, Success, and Happy. The entries are either zero,

corresponding to a No response, or one, corresponding to a Yes response. After consulting a behavioral psychologist, they obtained the following complete set of conditional relationships:

- HW depends only on Party and Smart
- Mac depends only on Smart and Creative
- Project depends only on Smart and Creative
- Success depends only on HW and Project
- Happy depends only on Party, Mac, and Success

1. (1 pt) Draw the Bayesian network.



2. (2 pt) Estimate the probabilities of the conditional probability tables using the data provided (you can use Excel pivot tables for counting).

$$\mathbf{P}(\text{creative} = T) = 0.69932$$
$$\mathbf{P}(\text{smart} = T) = 0.70472$$
$$\mathbf{P}(\text{party} = T) = 0.60216$$

creative	smart	$\mathbf{P}(\text{project} = T \mid \text{creative}, \text{smart})$
T	T	0.90484
T	F	0.40307
F	T	0.79326
F	F	0.10731

creative	smart	$\mathbf{P}(\text{mac} = T \mid \text{creative}, \text{smart})$
T	T	0.68564
T	F	0.89635
F	T	0.41347
F	F	0.12329

smart	party	$\mathbf{P}(\text{hw} = T \mid \text{smart}, \text{party})$
T	T	0.80252
T	F	0.89790
F	T	0.09447
F	F	0.30556

project	hw	$\mathbf{P}(\text{success} = T \mid \text{project}, \text{hw})$
T	T	0.89633
T	F	0.20737
F	T	0.30714
F	F	0.05066

success	mac	party	$\mathbf{P}(\text{happy} = T \mid \text{success}, \text{mac}, \text{party})$
T	T	T	0.95842
T	T	F	0.35837
T	F	T	0.72082
T	F	F	0.30769
F	T	T	0.49234
F	T	F	0.20619
F	F	T	0.42043
F	F	F	0.09646

3. (2 pts) What is the probability of being happy given that you party often, are wicked smart, but not very creative?
Show details of computation.

P (happy = T | party = T, smart = T, creative = F) = 0.6922

P (h | p, s, -c)
= alpha*$\Sigma_{hw,m,pr,su}$(P(-c)P(s)P(p)P(pr|-c,s)P(m|-c,s)P(hw|s,p)P(su|pr,hw)P(h|p,m,su))
= alpha*(…)

P (-h | p, s, -c)
= alpha*$\Sigma_{hw,m,pr,su}$(P(-c)P(s)P(p)P(pr|-c,s)P(m|-c,s)P(hw|s,p)P(su|pr,hw)P(-h|p,m,su))
= alpha*(…)

alpha=1/(P (h | p, s, -c) + P (-h | p, s, -c))

4. (2 pts) What is the probability of being happy given that you are wicked smart and very creative?
No details required. Use the AIspace tool.

P (happy = T |smart = T, creative = T) = 0.58132

5. (0.5 pts) What is the probability of being happy given you do not party, and do well on all your homework and class project?
No details required. Use the AIspace tool.

P (happy = T | party = F, hw = T, project = T) = 0.32108

6. (0.5 pts) What is the probability of being happy given you own a mac?
No details required. Use the AIspace tool.

P (happy = T | mac = T) = 0.56269

7. (0.5 pts) What is the probability that you party often given you are wicked smart?
No details required. Use the AIspace tool.

P (party = T |smart = T) = 0.60216

8. (0.5 pts) What is the probability that you party often given you are wicked smart and happy?
No details required. Use the AIspace tool.

P (party = T |smart = T, happy = T) = 0.79204