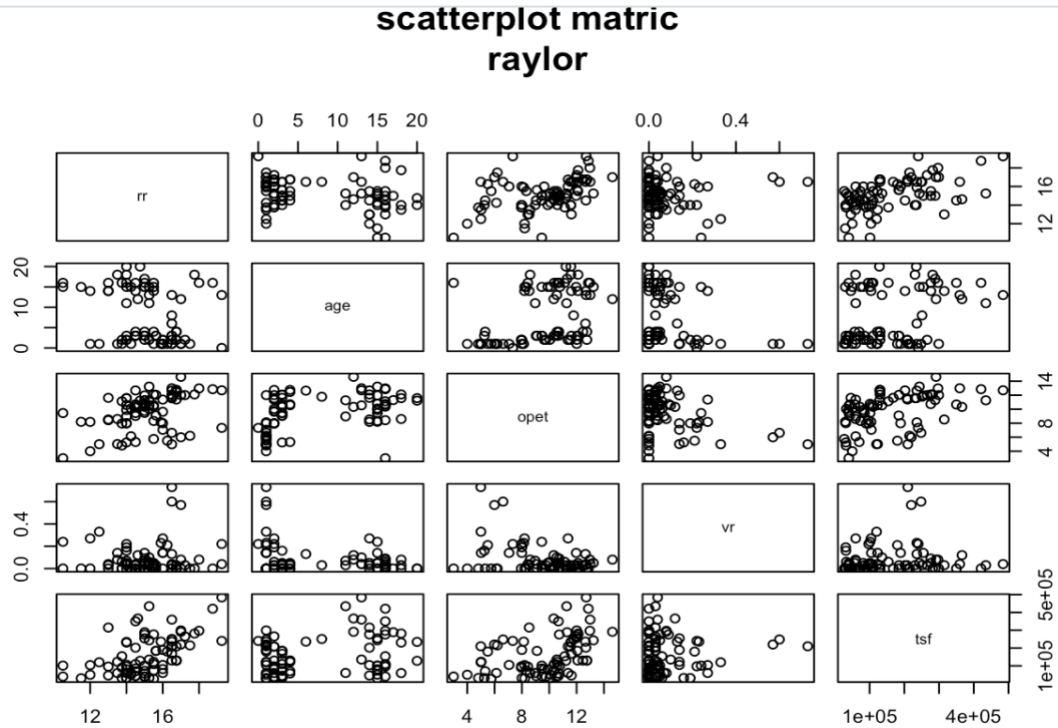


Problem1: The response variable is rental rates. The explanatory variables are age, operating expenses and taxes, vacancy rates, and total square footage.

(In this hw, I will use rr replaces rental rates, ot replaces operating expenses and taxes, vr replaces vacancy rates, and tsf replaces total square footage.)

1)



As we can see, they are all not very linearly related because the point is not like a line.

2) Find the correlation of all pairs of variables and give your comments about the association between variables.

```
> cor(pl)
```

	rr	age	opet	vr	tsf
rr	1.00000000	-0.2502846	0.4137872	0.06652647	0.53526237
age	-0.25028456	1.0000000	0.3888264	-0.25266347	0.28858350
opet	0.41378716	0.3888264	1.0000000	-0.37976174	0.44069713
vr	0.06652647	-0.2526635	-0.3797617	1.0000000	0.08061073
tsf	0.53526237	0.2885835	0.4406971	0.08061073	1.0000000

Rr and tsf have the largest correlation, and rr and tsf have second largest correlation. Then opet and age has the third largest correlation. Vr and rr have the least correlation, and vr and tsf have the second least correlation. Moreover, the correlation for rr and age, age and vr, opet and rr, vr and opet is negative. Otherwise, otis positive.

3) Run the multiple regressions with age, operating expenses and taxes, vacancy rates, and total square footage as the explanatory variables and rental rates as the response variable.

a) $Y = 1.220e1 - 1.420e-1 \cdot \text{age} + 2.82e-1 \cdot \text{ot} + 6.193e-1 \cdot \text{vr} + 7.924e-6 \cdot \text{tsf}$

b) β_0 represents the change in the mean of rental rates, per unit increase 1.22e1 when all predictor all zero.

β_1 represents the change in the mean of rental rates, per unit decrease 1.42e-1 in age when operating expenses and taxes, vacancy rates, and total square footage are held constant.

β_2 represents the change in the mean of rental rates, per unit increase 2.82e-1 in operating expenses and taxes when age, vacancy rates, and total square footage are held constant.

β_3 represents the change in the mean of rental rates, per unit increase 6.193e-1 in vacancy rates when age, operating expenses and taxes, and total square footage are held constant.

β_4 represents the change in the mean of rental rates, per unit increase 7.924e-6 in total square footage when age, operating expenses and taxes, and vacancy rates are held constant.

c) $R^2 = 0.5847$ Adj $R^2 = 0.5629$

d) $H_0: \beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$ H_A : Not all $\beta_i = 0$ (1, 2, 3 or 4) F-statistic: 26.76 on 4 and 76 DF p-value: 7.272e-14 < 0.05, so reject null hypothesis and conclude that not all $\beta_i = 0$.

e) $H_0: \beta_3 = \beta_4 = 0$ H_A : Not all $\beta_i = 0$ (3 or 4) F-statistic: 19.616 and p-value is 1.353e-07 < 0.05, so reject null hypothesis and conclude that not all $\beta_i = 0$.

f) $H_0: \beta_3 = 0$ H_A : Not $\beta_3 = 0$ T=0.57 p=0.57 > 0.05, so fail to reject null hypothesis and conclude that $\beta_3 = 0$.

g) $H_0: \beta_3 = 0$ H_A : Not $\beta_3 = 0$
f=0.3248 p=0.5704 > 0.05, so fail to reject null hypothesis and conclude that $\beta_3 = 0$.

h) Remove vr, then get the new model below $rr = 12.37 -$

```
Call:
lm(formula = rr ~ age + ot + tsf)

Residuals:
    Min       1Q   Median       3Q      Max
-3.0620 -0.6437 -0.1013  0.5672  2.9583

Coefficients:
(Intercept)  1.237e+01  4.928e-01  25.100 < 2e-16
age          -1.442e-01  2.092e-02  -6.891 1.33e-09
ot           2.672e-01  5.729e-02  4.663 1.29e-05
tsf           8.178e-06  1.305e-06   6.265 1.97e-08

(Intercept) ***
age          ***
ot           ***
tsf          ***
---
Signif. codes:
  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.132 on 77 degrees of freedom
Multiple R-squared:  0.583,    Adjusted R-squared:  0.567
F-statistic: 35.88 on 3 and 77 DF,  p-value: 1.295e-14
```

0.1442age+0.2672ot+8.17e-6 tsf

4) Confidence intervals for the regression coefficients for the final model.

- I. Give the 90% confidence intervals for each of the regression coefficients. (Where 0.90 is the "statement confidence coefficient".)

	5 %	95 %
(Intercept)	1.155005e+01	1.319112e+01
age	-1.789942e-01	-1.093351e-01
ot	1.717777e-01	3.625564e-01
tsf	6.004908e-06	1.035151e-05

Like the picture show

- II. Give Bonferroni Joint 90% confidence intervals for each of the regression coefficients. (Where 0.90 is the "family confidence coefficient".)

```
> confint(d,level=(1-0.1/4))
```

	1.25 %	98.75 %
(Intercept)	1.124390e+01	13.4972680885
age	-1.919897e-01	-0.0963396198
ot	1.361865e-01	0.3981475445
tsf	5.194017e-06	0.0000111624

Like the picture show

- III. We would like to obtain simultaneous interval estimates of the mean rental rates for four typical properties specified below. Obtain the family of estimates using a 95% family confidence coefficient.

```
> predict(d,new.data,interval="confidence",level=(1-.05/4))
```

	fit	lwr	upr
1	15.89844	15.35022	16.44666
2	15.98463	15.41526	16.55400
3	15.87816	15.32149	16.43483
4	15.91431	15.33555	16.49308

simultaneous interval is

((15.35022, 16.44666), (15.41526, 16.554), (15.32149, 16.43483) and (15.33555, 16.49308)).

- IV. Develop separate prediction intervals for the rental rates of these properties, using a 95% statement confidence coefficient in each case

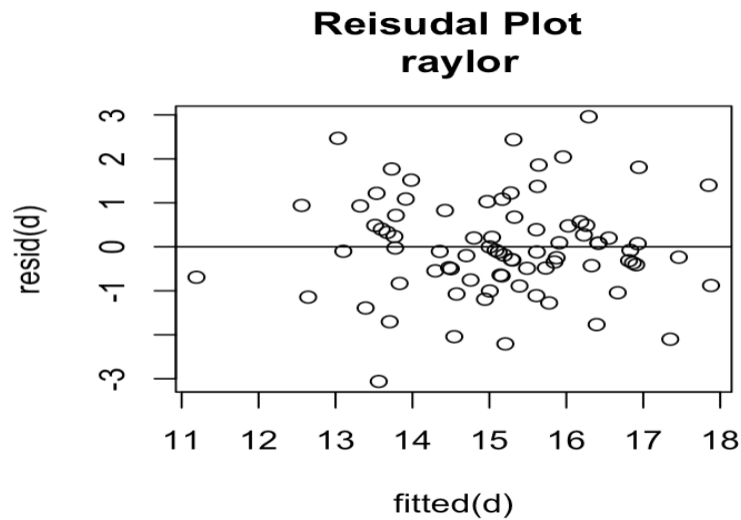
```
> predict(d,nnnew.data,interval="confidence",level=(1-.05))
```

	fit	lwr	upr
1	14.83152	14.49011	15.17293
2	14.40608	13.98934	14.82283
3	16.76079	16.26594	17.25563

Intervals are (14.75475, 15.48495), (15.17685, 15.94195) and (16.26594, 17.25563)

5) Diagnostic Plots and Tests:

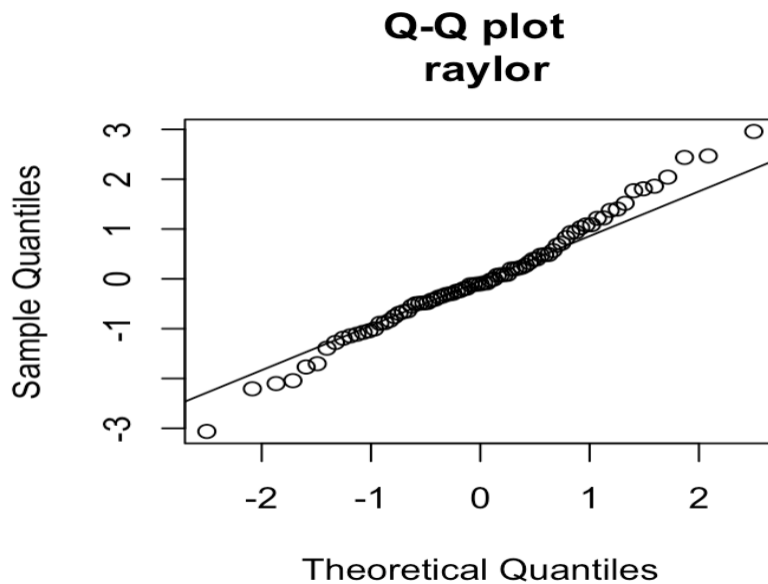
- a) Plot the residuals against the predicted (fitted) rental rate. Comment on the plot.



linear.

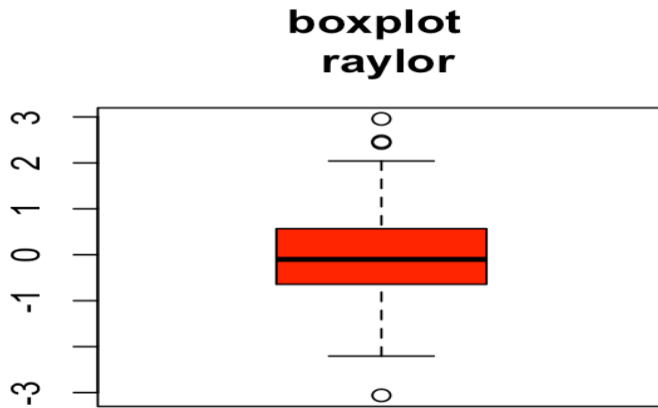
It is almost

- b) Give a QQ-plot, boxplot and histogram of the residuals with normal curve. State your conclusions from these plots.

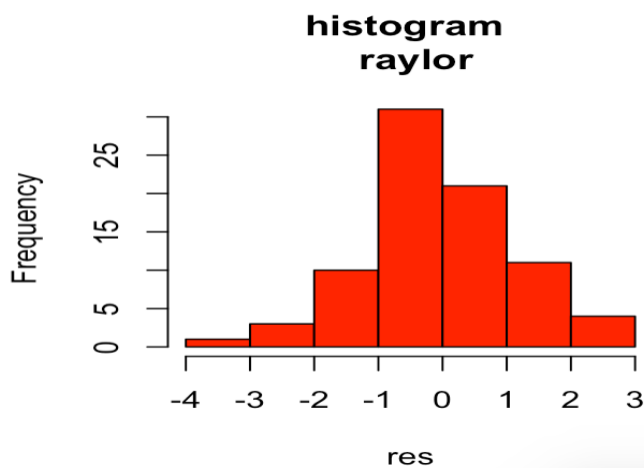


normal.

It looks like be



look normally. There are three outliers and

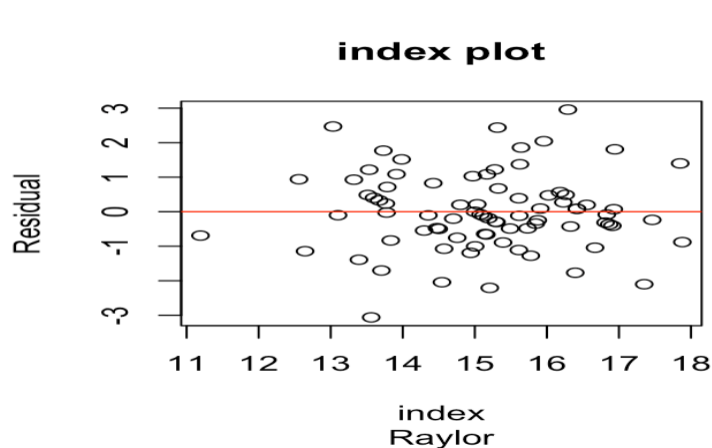


It is almost normal.

c) Conduct Breusch-Pagan Test for the constancy of the error variance.

Ho: The error variance is constant Ha: The error variance is not constant BP = 17.281, df = 3, p-value = 0.0006187 < 0.05, so reject null hypothesis and state that the error variance is not constant.

d) Index Plot to test for Independence of errors.



It looks like independent.

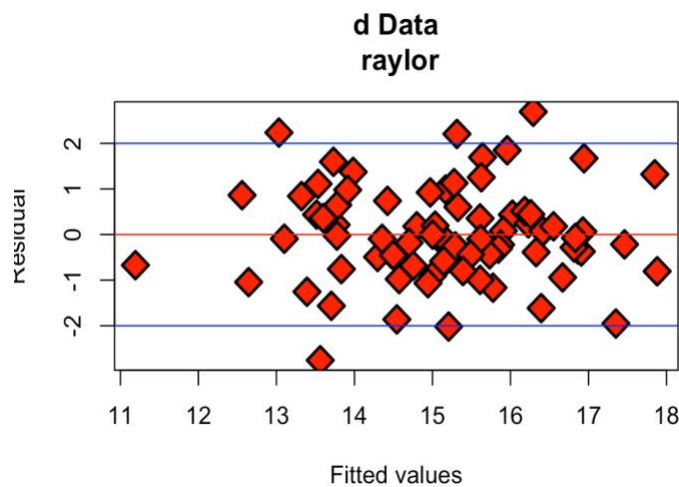
e) Conduct a Shapiro-Wilk Test on the residuals. Give the p-value for this test and explain what this means in terms of our model assumptions.

Ho: Random error comes from normal distribution Ha: Random error does not come from normal distribution. $W = 0.98776$, $p\text{-value} = 0.6406 > 0.05$, so fail to reject h_0 and state that random error comes from normal distribution

f) Conduct Durbin-Watson Test.

Ho: The errors are uncorrelated over time Ha: The errors are positively correlated $DW = 1.5867$, $p\text{-value} = 0.02463 < 0.05$, so reject null hypothesis and state that the errors are uncorrelated over time.

g) Deduct any outliers.



There are five

outliners.

6) Remove the outliers and refit the model to see how much difference in R^2 and $\text{adj } R^2$
New $R^2 = 0.6623$ and adjusted $r^2 = 0.6462$

7) Assume that a particular rental property is 5 years old, the operating expenses and taxes are 8.25, the vacancy rate is 0, and the total square footage is 250,000. Find a point estimate and 90% prediction interval for the rental rate of this property.
point estimate is 15.81573 and 90% prediction interval is (14.22162, 17.40984)

Problem2: Data from a ST430 class many years ago consist of quiz average, computer assignment average, midterm score and Final Exam score, all in percent. We seek to predict final exam score from the term work. In the data file, the quiz averages and computer averages are out of ten, but you should fix them up so they are out of 100.

Part A:

1. Start with a summary of the data frame, a correlation matrix, and a matrix of scatterplots using pairs

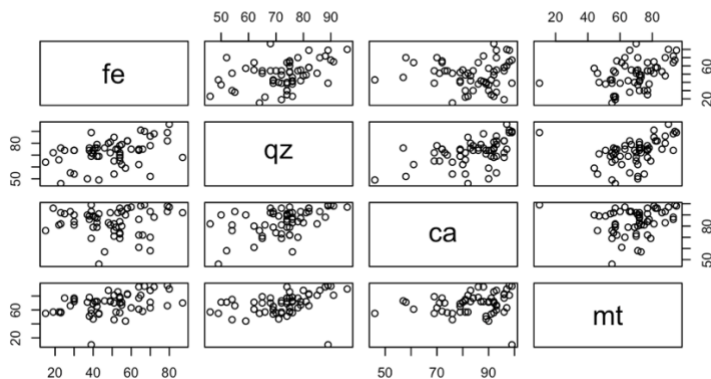
```
> summary(a[c('QuizAve', 'CompAve', 'MidTerm', 'FinalExam')])
```

QuizAve	CompAve	MidTerm	FinalExam
Min.: 4.600	Min.: 4.600	Min.: 10.00	Min.: 15.00
1st Qu.: 6.800	1st Qu.: 7.900	1st Qu.: 57.25	1st Qu.: 39.00
Median: 7.400	Median: 8.650	Median: 71.00	Median: 51.00
Mean: 7.257	Mean: 8.400	Mean: 68.88	Mean: 49.45
3rd Qu.: 7.875	3rd Qu.: 9.275	3rd Qu.: 77.00	3rd Qu.: 59.50
Max.: 9.600	Max.: 9.900	Max.: 95.00	Max.: 87.00

```
> cor(a)
```

	QuizAve	CompAve	MidTerm	FinalExam
QuizAve	1.0000000	0.49313970	0.3665234	0.39597772
CompAve	0.4931397	1.00000000	0.1600845	0.03312729
MidTerm	0.3665234	0.16008452	1.00000000	0.40363552
FinalExam	0.3959777	0.03312729	0.4036355	1.00000000

scatterplot matrix
raylor



- i. What is the median score on the computer assignments?
86.5
- ii. What is the correlation between the computer average and the final exam score?
0.03312729
- iii. Suppose you were to fit a simple regression model with quiz average as the single explanatory variable and final exam score as the response variable. Without actually fitting the model, what would R^2 be?
0.1568

2. Write the regression equation in scalar form

$$\text{Final} = 9.1368 + 0.5871 * \text{QuizAve} - 0.2934 * \text{CompAve} + 0.3246 * \text{Midterm}$$

3. What is the expected final exam score for a student with a 70% average on the quizzes, 85% on the computer assignments, and 65% on the midterm?

$$\text{Final} = 9.13675 + 0.587096 * 70 - 0.293434 * 85 + 0.32455 * 65 \\ = 46.38733$$

4. For any fixed quiz average and computer average, a score one point higher on the midterm yields an expected mark on the Final Exam that is 0.32455 higher.

We want a hypothesis test to answer this question:

5. Are any of the term work variables useful in predicting final exam score?

i. State the null and alternate hypothesis in terms of scalar β_j values.

Ho: $\beta_1 = \beta_2 = \beta_3 = 0$ Ha: At least one β_i does not equal to 0, $i=1, 2, 3$ F-statistic: 6.528 on 3 and 54 DF, p-value: 0.000755 < 0.05, so reject null hypothesis and conclude that at least one β_i does not equal to 0, $i=1, 2, 3$.

ii. The null hypothesis could be tested using the full-reduced model approach. Give the regression equation for the reduced model.

$$Y = \beta_0 = 49.448$$

6. Controlling for computer assignment average and midterm score, is quiz average related to Final Exam score?

i. State the null and alternate hypothesis in terms of scalar β_j values.

Ho: $\beta_1 = 0$ Ha: β_1 does not equal to 0 F = 7.5217. p-value = 0.008253 < 0.05, so reject H_0 and conclude that β_1 does not equal to 0.

ii. The null hypothesis could be tested using the full-reduced model approach. Give the regression equation for the reduced model.

$$\text{Final} = 22.19359 - 0.04598 * xi_2 + 0.45176 * xi_3$$

7. Allowing for quiz average and computer assignment average, is midterm score a predictor of Final Exam score?

i. State the null and alternate hypothesis in terms of scalar β_j values.

Ho: $\beta_3 = 0$ Ha: β_3 does not equal to 0 F = 5.4909 p-value = 0.02283 < 0.05, so reject H_0 and conclude that β_3 does not equal to 0.

ii. The null hypothesis could be tested using the full-reduced model approach. Give the regression equation for the reduced model.

$$\text{Final} = 20.2595 + 0.7551 * xi_1 - 0.3048 * xi_2$$

8. Holding for quiz average and midterm score fixed, is computer assignment average connected to Final Exam score?

i. State the null and alternate hypothesis in terms of scalar β_j values.

Ho: $\beta_2 = 0$ Ha: β_2 does not equal to 0 F = 2.367 p-value = 0.1298 > 0.05, so fail to reject H_0 and conclude that β_2 is equal to 0.

ii. The null hypothesis could be tested using the full-reduced model approach. Give the regression equation for the reduced model.

$$\text{Final} = -4.5802 + 0.4313 * xi_1 + 0.33 * xi_3$$

9. Controlling for computer assignment average, is quiz average or midterm score (or both) related to Final Exam score?

- i. State the null and alternate hypothesis in terms of scalar β_j values.

Ho: $\beta_1 = \beta_3 = 0$ Ha: one of $\beta_1, 3$ does not equal to 0 F = 9.7522 p-value = 0.

0002422 < 0.05, so reject H_0 and conclude that one of $\beta_1, 3$ does not equal to 0.

- ii. The null hypothesis could be tested using the full-reduced model approach. Give the regression equation for the reduced model.

Final = 45.4889 + 0.04714 * xi2

10. The professor thinks that the quizzes and midterm should have equal weight, and should be worth twice as much as the computer assignments. If this idea is correct, it should be reflected in the relationship of the term marks to the final exam. Also, it makes sense that if a student got zero on all three components of the term mark, he or she should also expect a zero on the final exam - even though this extreme case is outside the range of the data. Taken together, these ideas represent an unusual but testable null hypothesis. If it is rejected, we could say that the professor's ideas are not supported by the data.

- i. State the null and alternate hypothesis in terms of scalar β_j values.

Ho: $\beta_0 = 0$ Ha: β_0 does not equal to 0

- ii. The null hypothesis could be tested using the full-reduced model approach. Give the regression equation for the reduced model.

Final = 0.4xi1 + 0.2xi2 + 0.4xi3

Part B:

1. Write the regression equation in matrix form

$$\begin{array}{ccccccc} \mathbf{Y1} & & \mathbf{1} & & \mathbf{x1\ 1.} & & \mathbf{e1} \\ \mathbf{Y2} & & \mathbf{1} & & \mathbf{x2\ 1} & & \mathbf{\beta0} \\ \mathbf{\cdot} & = & \mathbf{\cdot} & & \mathbf{\cdot} & * & \mathbf{\beta1} \\ \mathbf{\cdot} & & \mathbf{\cdot} & & \mathbf{\cdot} & & \mathbf{\beta2} \\ \mathbf{\cdot} & & \mathbf{\cdot} & & \mathbf{\cdot} & & \mathbf{\beta3} \\ \mathbf{Y58} & & \mathbf{1} & & \mathbf{x58\ 1} & & \mathbf{e58} \end{array}$$

- i. What are the dimensions of the X matrix?
58 rows and 4 columns 58*4
- ii. What are the dimensions of $\hat{\beta} = b$?
4 rows and 1 column 4*1
- iii. What are the dimensions of $\hat{e} = e$?
58 rows and 1 column 58*1
- iv. What are the dimensions of $e'e$?
1 row and 1 column 1*1
- v. What are the dimensions of the \hat{y} matrix?
58 rows and 1 column 58*1

vi. What are the dimensions of the hat matrix H?

58 rows and 58 columns 58*58

vii. What is $e'e$

Sse

2. Calculate:

$$\hat{\beta} = (X'X)^{-1}X'Y$$

> b

[,1]

[1,] 9.1367509

[2,] 0.5870963

[3,] -0.2934339

[4,] 0.3245532

$$\hat{Y} = Xb = X(X'X)^{-1}X'y$$

[,1]

[1,] 42.25694

[2,] 51.40981

[3,] 63.43346

[4,] 60.09480

[5,] 48.48723

[6,] 57.03233

[7,] 47.56108

[8,] 35.12579

[9,] 52.69078

[10,] 60.72272

[11,] 31.64672

[12,] 53.06346

[13,] 60.89836

[14,] 52.92403

[15,] 44.78210

[16,] 45.10529

[17,] 53.83590

[18,] 35.58390

[19,] 55.72236

[20,] 60.31012

[21,] 46.69436

[22,] 49.54066

[23,] 41.41330

[24,] 65.16249

[25,] 51.79477

[26,] 42.61649

[27,] 40.46978

[28,] 35.55821

[29,] 41.83024

[30,] 61.26717

[31,] 47.43500

[32,] 51.30172

[33,] 56.04691

[33,] 56.04691

[34,] 50.86582

[35,] 52.76889

[36,] 45.68987

[37,] 63.46435

[38,] 66.24470

[39,] 54.08463

[40,] 45.15274

[41,] 49.18293

[42,] 30.58113

[43,] 49.01117

[44,] 57.53001

[45,] 54.83674

[46,] 55.27241

[47,] 33.46842

[48,] 42.26036

[49,] 50.86999

[50,] 41.73756

[51,] 45.88024

[52,] 51.33192

[53,] 52.70574

[54,] 53.35644

[55,] 39.95897

[56,] 43.01847

[57,] 44.93513

[58,] 49.97307

$$H = X(X'X)^{-1}X'$$

[1,]	[2,]	[3,]	[4,]
[1,]	0.212689843	-0.031154549	-7.329939e-02
[2,]	-0.031154549	0.030800180	3.410654e-02
[3,]	-0.073299382	0.031065399	9.261622e-02
[4,]	-0.066239423	0.043572594	7.315297e-02
[5,]	0.040257285	0.025180604	-3.048066e-03
[6,]	0.050851056	0.006916066	1.965547e-02
[7,]	0.043981858	0.011625667	1.654318e-03
[8,]	0.012315953	-0.010770813	-8.735204e-04
[9,]	0.088410797	-0.010049865	7.179239e-03
[10,]	0.1323943193	0.000804899	-1.191990e-03
[11,]	0.0108375277	0.196939275	-3.056986e-02
[12,]	-0.0273797701	0.032066714	0.086523e-02
[13,]	-0.0411216220	0.028660825	7.222925e-02
[14,]	0.0136071171	0.013696179	3.125747e-02
[15,]	-0.0140174963	0.015704821	2.187276e-02
[16,]	0.0440868529	-0.001256932	3.179046e-03
[17,]	0.0024978579	0.017092559	3.736284e-02
[5,]	[6,]	[7,]	[8,]
[1,]	0.040257285	0.05805106	0.043981858
[2,]	0.025180606	0.006916066	0.011625667
[3,]	-0.0030488061	0.019655468	0.001654318
[4,]	0.0153278250	0.015558697	0.004672376
[5,]	0.0409545352	0.042626915	0.020289866
[6,]	0.0246269148	0.052157143	0.021849600
[7,]	0.0220829662	0.021849600	0.02134357
[8,]	-0.026801712	-0.023134538	0.016574902
[9,]	0.0101472855	0.047687988	0.024436305
[10,]	0.0478182503	0.004994151	0.003072020
[11,]	0.0196574488	-0.027739555	0.020774031
[12,]	0.0175067894	0.011444158	0.011048200
[13,]	0.0047590373	0.024750303	0.006677375
[14,]	0.0007868259	-0.023767914	0.015420903
[15,]	-0.0001283099	-0.005878190	0.013256488
[16,]	0.0078501781	0.014308065	0.020477266
[17,]	0.0061903180		
[1,]	[2,]	[3,]	[4,]
[1,]	0.0028591781	0.014308065	0.020477266
[17,]	0.0092676384	0.022313256	0.013857212
[9,]	[10,]	[11,]	[12,]
[1,]	8.418008e-02	0.1233943193	0.0108375277
[2,]	-1.004086e-02	0.0008004899	0.016992849
[3,]	7.179239e-03	-0.001191900	-0.030650895
[4,]	-7.654024e-03	0.005128072	-0.0195299405
[5,]	1.014729e-02	0.0478182503	0.0196574488
[6,]	4.768799e-02	0.004994151	-0.027739555
[7,]	2.443630e-02	0.030720200	0.027740307
[8,]	2.552899e-02	0.008072406	0.0794144134
[9,]	6.317565e-02	0.0099809191	-0.0143043314
[10,]	6.990892e-02	0.161737853	-0.0565619135
[11,]	-1.430436e-02	-0.0565619135	0.1017694101
[12,]	-8.574677e-05	0.003495468	0.0102155400
[13,]	1.395264e-02	0.015665014	-0.0248320654
[14,]	2.613515e-02	0.025989450	0.0009091208
[15,]	7.154954e-03	-0.035994994	0.043606327
[16,]	3.515975e-02	0.005402874	0.0296105632
[17,]	2.153404e-02	0.0188560522	-0.009492762
[13,]	[14,]	[15,]	[16,]
[1,]	-0.0411216220	0.0136071171	-0.0140174963
[2,]	0.0286608249	0.0136961786	0.0157048214
[3,]	0.0722292454	0.0312574670	0.021877632
[4,]	0.058302942	0.0252808070	0.0138536662
[5,]	0.0047590373	0.0096321110	0.021823099
[6,]	0.0427503831	0.0237679144	-0.0058781902
[7,]	0.0066773754	0.015240926	0.0132564877
[8,]	-0.0049524603	0.016410049	0.0688135637
[9,]	0.0139526403	0.0261351467	0.0071584950
[10,]	0.015665014	0.025989450	-0.035994994
[11,]	-0.0248320654	0.0039912208	0.043606327
[12,]	0.038875862	0.017730347	0.0044535316
[13,]	0.0583649910	0.0182156825	0.016291394
[1,]	[2,]	[3,]	[4,]
[1,]	0.0028591781	0.014308065	0.020477266
[17,]	0.0092676384	0.022313256	0.013857212
[9,]	[10,]	[11,]	[12,]
[1,]	8.418008e-02	0.1233943193	0.0108375277
[2,]	-1.004086e-02	0.0008004899	0.016992849
[3,]	7.179239e-03	-0.001191900	-0.030650895
[4,]	-7.654024e-03	0.005128072	-0.0195299405
[5,]	1.014729e-02	0.0478182503	0.0196574488
[6,]	4.768799e-02	0.004994151	-0.027739555
[7,]	2.443630e-02	0.030720200	0.027740307
[8,]	2.552899e-02	0.008072406	0.0794144134
[9,]	6.317565e-02	0.0099809191	-0.0143043314
[10,]	6.990892e-02	0.161737853	-0.0565619135
[11,]	-1.430436e-02	-0.0565619135	0.1017694101
[12,]	-8.574677e-05	0.003495468	0.0102155400
[13,]	1.395264e-02	0.015665014	-0.0248320654
[14,]	2.613515e-02	0.025989450	0.0009091208
[15,]	7.154954e-03	-0.035994994	0.043606327
[16,]	3.515975e-02	0.005402874	0.0296105632
[17,]	2.153404e-02	0.0188560522	-0.009492762
[13,]	[14,]	[15,]	[16,]
[1,]	-0.0411216220	0.0136071171	-0.0140174963
[2,]	0.0286608249	0.0136961786	0.0157048214
[3,]	0.0722292454	0.0312574670	0.021877632
[4,]	0.058302942	0.0252808070	0.0138536662
[5,]	0.0047590373	0.0096321110	0.021823099
[6,]	0.0427503831	0.0237679144	-0.0058781902
[7,]	0.0066773754	0.015240926	0.0132564877
[8,]	-0.0049524603	0.016410049	0.0688135637
[9,]	0.0139526403	0.0261351467	0.0071584950
[10,]	0.015665014	0.025989450	-0.035994994
[11,]	-0.0248320654	0.0039912208	0.043606327
[12,]	0.038875862	0.017730347	0.0044535316
[13,]	0.0583649910	0.0182156825	0.016291394
[1,]	[2,]	[3,]	[4,]
[1,]	0.0028591781	0.014308065	0.020477266
[17,]	0.0092676384	0.022313256	0.013857212
[9,]	[10,]	[11,]	[12,]
[1,]	8.418008e-02	0.1233943193	0.0108375277
[2,]	-1.004086e-02	0.0008004899	0.016992849
[3,]	7.179239e-03	-0.001191900	-0.030650895
[4,]	-7.654024e-03	0.005128072	-0.0195299405
[5,]	1.014729e-02	0.0478182503	0.0196574488
[6,]	4.768799e-02	0.004994151	-0.027739555
[7,]	2.443630e-02	0.030720200	0.027740307
[8,]	2.552899e-02	0.008072406	0.0794144134
[9,]	6.317565e-02	0.0099809191	-0.0143043314
[10,]	6.990892e-02	0.161737853	-0.0565619135
[11,]	-1.430436e-02	-0.0565619135	0.1017694101
[12,]	-8.574677e-05	0.003495468	0.0102155400
[13,]	1.395264e-02	0.015665014	-0.0248320654
[14,]	2.613515e-02	0.025989450	0.0009091208
[15,]	7.154954e-03	-0.035994994	0.043606327
[16,]	3.515975e-02	0.005402874	0.0296105632
[17,]	2.153404e-02	0.0188560522	-0.009492762
[13,]	[14,]	[15,]	[16,]
[1,]	-0.0411216220	0.0136071171	-0.0140174963
[2,]	0.0286608249	0.0136961786	0.0157048214
[3,]	0.0722292454	0.0312574670	0.021877632
[4,]	0.058302942	0.0252808070	0.0138536662
[5,]	0.0047590373	0.0096321110	0.021823099
[6,]	0.0427503831	0.0237679144	-0.0058781902
[7,]	0.0066773754	0.015240926	0.0132564877
[8,]	-0.0049524603	0.016410049	0.0688135637
[9,]	0.0139526403	0.0261351467	0.0071584950
[10,]	0.015665014	0.025989450	-0.035994994
[11,]	-0.0248320654	0.0039912208	0.043606327
[12,]	0.038875862	0.017730347	0.0044535316
[13,]	0.0583649910	0.0182156825	0.016291394
[1,]	[2,]	[3,]	[4,]
[1,]	0.0028591781	0.014308065	0.020477266
[17,]	0.0092676384	0.022313256	0.013857212
[9,]	[10,]	[11,]	[12,]
[1,]	8.418008e-02	0.1233943193	0.0108375277
[2,]	-1.004086e-02	0.0008004899	0.016992849
[3,]	7.179239e-03	-0.001191900	-0.030650895
[4,]	-7.654024e-03	0.005128072	-0.0195299405
[5,]	1.014729e-02	0.0478182503	0.0196574488
[6,]	4.768799e-02	0.004994151	-0.027739555
[7,]	2.443630e-02	0.030720200	0.027740307
[8,]	2.552899e-02	0.008072406	0.0794144134
[9,]	6.317565e-02	0.0099809191	-0.0143043314
[10,]	6.990892e-02	0.161737853	-0.0565619135
[11,]	-1.430436e-02	-0.0565619135	0.1017694101
[12,]	-8.574677e-05	0.003495468	0.0102155400
[13,]	1.395264e-02	0.015665014	-0.0248320654
[14,]	2.613515e-02	0.025989450	0.0009091208
[15,]	7.154954e-03	-0.035994994	0.043606327
[16,]	3.515975e-02	0.005402874	0.0296105632
[17,]	2.153404e-02	0.0188560522	-0.009492762
[13,]	[14,]	[15,]	[16,]
[1,]	-0.0411216220	0.0136071171	-0.0140174963
[2,]	0.0286608249	0.0136961786	0.0157048214
[3,]	0.0722292454	0.0312574670	0.021877632
[4,]	0.058302942	0.0252808070	0.0138536662
[5,]	0.0047590373	0.0096321110	0.021823099
[6,]	0.0427503831	0.0237679144	-0.0058781902
[7,]	0.0066773754	0.015240926	0.0132564877
[8,]	-0.0049524603	0.016410049	0.0688135637
[9,]	0.0139526403	0.0261351467	0.0071584950
[10,]	0.015665014	0.025989450	-0.035994994
[11,]	-0.0248320654	0.0039912208	0.043606327
[12,]	0.038875862	0.017730347	0.0044535316
[13,]	0.0583649910	0.0182156825	0.016291394
[1,]	[2,]	[3,]	[4,]
[1,]	0.0028591781	0.014308065	0.020477266
[17,]	0.0092676384	0.022313256	0.013857212
[9,]	[10,]	[11,]	[12,]
[1,]	8.418008e-02	0.1233943193	0.0108375277
[2,]	-1.004086e-02	0.0008004899	0.016992849
[3,]	7.179239e-03	-0.001191900	-0.030650895
[4,]	-7.654024e-03	0.005128072	-0.0195299405
[5,]	1.014729e-02	0.0478182503	0.0196574488
[6,]	4.768799e-02	0.004994151	-0.027739555
[7,]	2.443630e-02	0.030720200	0.027740307
[8,]	2.552899e-02	0.008072406	0.0794144134
[9,]	6.317565e-02	0.0099809191	-0.0143043314
[10,]	6.990892e-02	0.161737853	-0.0565619135
[11,]	-1.430436e-02	-0.0565619135	0.1017694101
[12,]	-8.574677e-05	0.003495468	0.0102155400
[13,]	1.395264e-02	0.015665014	-0.0248320654
[14,]	2.613515e-02	0.025989450	0.0009091208
[15,]	7.154954e-03	-0.035994994	0.043606327
[16,]	3.515975e-02	0.005402874	0.0296105632
[17,]	2.153404e-02	0.0188560522	-0.009492762
[13,]	[14,]	[15,]	[16,]
[1,]	-0.0411216220	0.0136071171	-0.0140174963
[2,]	0.0286608249	0.0136961786	0.0157048214
[3,]	0.0722292454	0.0312574670	0.021877632
[4,]	0.058302942	0.0252808070	0.0138536662
[5,]	0.0047590373	0.0096321110	0.021823099
[6,]	0.0427503831	0.0237679144	-0.0058781902
[7,]	0.0066773754	0.015240926	0.0132564877
[8,]			

$$\hat{Y} = X(X'X)^{-1}X'y = Hy$$

> Yhat

```

      [,1] [28,] 35.55821
[1,] 42.25694 [29,] 41.83024
[2,] 51.40981 [30,] 61.26717
[3,] 63.43346 [31,] 47.43500
[4,] 60.09480 [32,] 51.30172
[5,] 48.48723 [33,] 56.04691
[6,] 57.03233 [34,] 50.86582
[7,] 47.56108 [35,] 52.76889
[8,] 35.12579 [36,] 45.68987
[9,] 52.69078 [37,] 63.46435
[10,] 60.72272 [38,] 66.24470
[11,] 31.64672 [39,] 54.08463
[12,] 53.06346 [40,] 45.15274
[13,] 60.89836 [41,] 49.18293
[14,] 52.92403 [42,] 30.58113
[15,] 44.78210 [43,] 49.01117
[16,] 45.10529 [44,] 57.53001
[17,] 53.83590 [45,] 54.83674
[18,] 35.58390 [46,] 55.27241
[19,] 55.72236 [47,] 33.46842
[20,] 60.31012 [48,] 42.26036
[21,] 46.69436 [49,] 50.86999
[22,] 49.54066 [50,] 41.73756
[23,] 41.41330 [51,] 45.88024
[24,] 65.16249 [52,] 51.33192
[25,] 51.79477 [53,] 52.70574
[26,] 42.61649 [54,] 53.35644
[27,] 40.46978 [55,] 39.95897
      [,1] [56,] 43.01847
      [,1] [57,] 44.93513
      [,1] [58,] 49.97307

```

$$Var(\hat{\beta}) = MSE(X'X)^{-1}$$

> var

```

      [,1] [,2] [,3] [,4]
[1,] 260.8936587 -1.000599069 -1.6695697647 -0.6454847451
[2,] -1.0005991 0.044991612 -0.0189632577 -0.0097486757
[3,] -1.6695698 -0.018963258 0.0357155755 0.0006621061
[4,] -0.6454847 -0.009748676 0.0006621061 0.0188346709

```

$$SSTO = Y'Y - \left(\frac{1}{n}\right)Y'JY$$

15548.34

$$SSE = Y'Y - b'X'Y$$

11410.04

$$SSR = b'X'Y - \left(\frac{1}{n}\right)Y'JY$$

4138.305

$$E(Y_h) = X_h'\beta$$

49.44828

$$\hat{Y}_h = X_h'b \text{ where } X_h' = (1, X_h)$$

49.44828

$$\sigma^2(\hat{Y}_h) = \sigma^2 X_h'(X'X)^{-1}X_h$$

3.576815

$$S^2(\hat{Y}_h) = MSE X_h'(X'X)^{-1}X_h$$

211.0321

3. What is the predicted final exam score for a student with a 70% average on the quizzes, 85% on the computer assignments, and 65% on the midterm?

46.38757 and confident interval is (42.29514, 50.48)

4. For any fixed quiz average and computer average, a score one point higher on the midterm yields a predicted mark on the Final Exam that is 0.3246 higher.

5. What is the largest e_i in absolute value?

42.2179

6. For each of the following null hypotheses, give the value of the test statistic and the p-value. The answers are numbers that appear in the output from summary table. Also state whether you reject H_0 at $\alpha = 0.05$.

H0	Test Statistic	p-value	Reject H0?
$\beta_1 = \beta_2 = \beta_3 = 0$	6.528	0.000755	Reject
$\beta_0 = 0$	0.56	0.57746	Not reject
$\beta_1 = 0$	2.743	0.00825	Reject
$\beta_2 = 0$	-1.538	0.12977	Not reject
$\beta_3 = 0$	2.343	0.02283	reject

7. What proportion of the variation (sum of squares) in final exam mark is explained by the term work?

26.62%

8. Controlling for computer assignment average and midterm score, is quiz average related to Final Exam score?

i. State the null and alternate hypothesis in terms of scalar β_j values.

Ho: $\beta_1 = 0$ Ha: β_1 does not equal to 0

ii. A nice 2-sided t-test is part of the default output. Give the value of the t statistic, the degrees of freedom, and the p-value.

t=2.743 df= 54 p-value = 0.00825 < 0.05

iii. Do you reject the null hypothesis at $\alpha = 0.05$? Answer Yes or No.

reject h0 and conclude that β_1 is equal to 0.

iv. Are the results statistically significant at the $\alpha = 0.05$ level? Answer Yes or No.

Yes

v. Carry out the same test using the full-reduced model approach.

F=7.5217 p=0.008253 < 0.05, so reject h0 and conclude that β_1 is equal to 0.

9. Give a 95% confidence interval for β_1 . Why does this confidence interval provide one more way of testing $H_0 : \beta_1 = 0$?

(0.15791733, 1.01627535), yes because it can look if β_1 is included or not. This CI doesn't include 0 and reject H_0 and conclude that β_1 is not equal to 0.

10. Consider a student who is "average" on all three explanatory variables. The expected final exam score for such a student would be

$$E(y/x_1 = \bar{x}_1, x_2 = \bar{x}_2, x_3 = \bar{x}_3) = \beta_0 + \beta_1\bar{x}_1 + \beta_2\bar{x}_2 + \beta_3\bar{x}_3$$

- i. Give a point estimate of the expected value.
49.44828
- ii. Give a 95% confidence interval for the expected value.
(45.62161, 53.27495)

11. For each of the following questions, give the null and alternate hypothesis, test statistic and p-value.

i. Controlling for quiz average and computer average, is mark on the midterm test related to mark on the final exam?

$H_0: \beta_3 = 0$ $H_a: \beta_3 \neq 0$ $F = 5.4909$ $p\text{-value} = 0.02283 < 0.05$, so we reject null hypothesis and conclude that $\beta_3 \neq 0$.

ii. Allowing for mark on the midterm test and quiz average, is computer average a useful predictor of mark on the final exam?

$H_0: \beta_2 = 0$ $H_a: \beta_2 \neq 0$ $F = 2.367$ $p\text{-value} = 0.1298 > 0.05$, so we fail to reject null hypothesis and conclude that $\beta_2 = 0$.

iii. Considering mark on the midterm test and computer average, is quiz average related to mark on the final exam?

$H_0: \beta_1 = 0$ $H_a: \beta_1 \neq 0$ $F = 7.5217$ $p\text{-value} = 0.008253 < 0.05$, so we reject null hypothesis and conclude that $\beta_1 \neq 0$.

iv. Are any of the predictor variables useful?

Controlling for mark on the midterm test, are the other two variables (either or both) related to mark on the Final exam?

$H_0: \beta_1 = \beta_2 = 0$ $H_a: \beta_1 \text{ and } \beta_2 \neq 0$ $F = 3.7983$ $p\text{-value} = 0.02862 < 0.05$, so we reject null hypothesis and conclude that β_1 or $\beta_2 \neq 0$.

Q1

```
> p1 = read.table (file.choose(), header = TRUE)
```

```
> p1
```

	rr	age	opet	vr	tsf
1	13.500	1	5.02	0.14	123000
2	12.000	14	8.19	0.27	104079
3	10.500	16	3.00	0.00	39998
4	15.000	4	10.70	0.05	57112
5	14.000	11	8.97	0.07	60000
6	10.500	15	9.45	0.24	101385
7	14.000	2	8.00	0.19	31300
8	16.500	1	6.62	0.60	248172
9	17.500	1	6.20	0.00	215000
10	16.500	8	11.78	0.03	251015
11	17.000	12	14.62	0.08	291264
12	16.500	2	11.55	0.03	207549
13	16.000	2	9.63	0.00	82000
14	16.500	13	12.99	0.04	359665
15	17.225	2	12.01	0.03	265500
16	17.000	1	12.01	0.00	299000
17	16.000	1	7.99	0.14	189258
18	14.625	12	10.33	0.12	366013
19	14.500	16	10.67	0.00	349930
20	14.500	3	9.45	0.03	85335
21	16.500	6	12.65	0.13	235932
22	16.500	3	12.08	0.00	130000
23	15.000	3	10.52	0.05	40500
24	15.000	3	9.47	0.00	40500
25	13.000	14	11.62	0.00	45959
26	12.500	1	5.00	0.33	120000
27	14.000	15	9.89	0.05	81243
28	13.750	16	11.13	0.06	153947
29	14.000	2	7.96	0.22	97321
30	15.000	16	10.73	0.09	276099
31	13.750	2	7.95	0.00	90000
32	15.625	3	9.10	0.00	184000
33	15.625	3	12.05	0.03	184718
34	13.000	16	8.43	0.04	96000
35	14.000	16	10.60	0.04	106350
36	15.250	13	10.55	0.10	135512
37	16.250	1	5.50	0.21	180000
38	13.000	14	8.53	0.03	315000
39	14.500	3	9.04	0.04	42500
40	11.500	15	8.20	0.00	30005
41	14.250	1	6.13	0.00	60000
42	15.500	15	8.32	0.00	73521

```

43 12.000 1 4.00 0.00 50000
44 14.250 15 10.10 0.00 50724
45 14.000 3 5.25 0.16 31750
46 16.500 3 11.62 0.00 168000
47 14.500 4 5.31 0.00 70000
48 15.500 1 5.75 0.00 27000
49 16.750 4 12.46 0.03 129614
50 16.750 4 12.75 0.00 129614
51 16.750 2 12.75 0.00 130000
52 16.750 2 11.38 0.00 209000
53 17.000 1 5.99 0.57 220000
54 16.000 2 11.37 0.27 60000
55 14.500 3 10.38 0.00 110000
56 15.000 15 10.77 0.05 101206
57 15.000 17 11.30 0.00 288847
58 16.000 1 7.06 0.14 105000
59 15.500 14 12.10 0.05 276425
60 15.250 2 10.04 0.06 33000
61 16.500 1 4.99 0.73 210000
62 19.250 0 7.33 0.22 240000
63 17.750 18 12.11 0.00 281552
64 18.750 16 12.86 0.00 421000
65 19.250 13 12.70 0.04 484290
66 14.000 20 11.58 0.00 234493
67 14.000 18 11.58 0.03 230675
68 18.000 16 12.97 0.08 296966
69 13.750 1 4.82 0.00 32000
70 15.000 2 9.75 0.03 38533
71 15.500 16 10.36 0.02 109912
72 15.900 1 8.13 0.23 236000
73 15.250 15 13.23 0.05 243338
74 15.500 4 10.57 0.04 122183
75 14.750 20 11.22 0.00 128268
76 15.000 3 10.34 0.00 72000
77 14.500 3 10.67 0.00 43404
78 13.500 18 8.60 0.08 59443
79 15.000 15 11.97 0.14 254700
80 15.250 11 11.27 0.03 434746
81 14.500 14 12.68 0.03 201930

```

```
> summary(pl[c('rr','age','opet','vr','tsf')])
```

```

      rr      age      opet      vr
Min. :10.50 Min. :0.000 Min. :3.000 Min. :0.00000
1st Qu.:14.00 1st Qu.: 2.000 1st Qu.: 8.130 1st Qu.:0.00000
Median :15.00 Median : 4.000 Median :10.360 Median :0.03000
Mean   :15.14 Mean   : 7.864 Mean   : 9.688 Mean   :0.08099
3rd Qu.:16.50 3rd Qu.:15.000 3rd Qu.:11.620 3rd Qu.:0.09000

```


Max. :19.25 Max. :20.000 Max. :14.620 Max. :0.73000

tsf

Min. : 27000

1st Qu.: 70000

Median :129614

Mean :160633

3rd Qu.:236000

Max. :484290

```
> pairs(~rr+age+opet+vr+tsf, data=p1,main="scatterplot matrix \n raylor")
```

```
> rr=p1$rr
```

```
> age=p1$age
```

```
> ot=p1$opet
```

```
> vr=p1$vr
```

```
> tsf=p1$tsf
```

```
> cor(p1)
```

	rr	age	opet	vr	tsf
rr	1.00000000	-0.2502846	0.4137872	0.06652647	0.53526237
age	-0.25028456	1.00000000	0.3888264	-0.25266347	0.28858350
opet	0.41378716	0.3888264	1.00000000	-0.37976174	0.44069713
vr	0.06652647	-0.2526635	-0.3797617	1.00000000	0.08061073
tsf	0.53526237	0.2885835	0.4406971	0.08061073	1.00000000

```
> model <- lm(rr~age+ot+vr+tsf)
```

```
> summary(model)
```

Call:

```
lm(formula = rr ~ age + ot + vr + tsf)
```

Residuals:

Min	1Q	Median	3Q	Max
-3.1872	-0.5911	-0.0910	0.5579	2.9441

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	1.220e+01	5.780e-01	21.110	< 2e-16 ***
age	-1.420e-01	2.134e-02	-6.655	3.89e-09 ***
ot	2.820e-01	6.317e-02	4.464	2.75e-05 ***
vr	6.193e-01	1.087e+00	0.570	0.57
tsf	7.924e-06	1.385e-06	5.722	1.98e-07 ***

Signif. codes:

0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.137 on 76 degrees of freedom

Multiple R-squared: 0.5847, Adjusted R-squared: 0.5629

F-statistic: 26.76 on 4 and 76 DF, p-value: 7.272e-14

```
> anova(model)
```

Analysis of Variance Table

Response: rr

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
age	1	14.819	14.819	11.4649	0.001125 **
ot	1	72.802	72.802	56.3262	9.699e-11 ***
vr	1	8.381	8.381	6.4846	0.012904 *
tsf	1	42.325	42.325	32.7464	1.976e-07 ***
Residuals	76	98.231	1.293		

Signif. codes:

0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
> model1 <- lm(rr~vr+tsf)
> summary(model1)

Call:

lm(formula = rr ~ vr + tsf)

Residuals:

Min	1Q	Median	3Q	Max
-4.1886	-0.7879	0.3140	0.9820	3.4021

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	1.376e+01	3.027e-01	45.469	< 2e-16 ***
vr	3.007e-01	1.226e+00	0.245	0.807
tsf	8.407e-06	1.512e-06	5.561	3.63e-07 ***

Signif. codes:

0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.47 on 78 degrees of freedom

Multiple R-squared: 0.2871, Adjusted R-squared: 0.2688

F-statistic: 15.7 on 2 and 78 DF, p-value: 1.859e-06

> a=lm(rr~age+ot)

> anova(a,model)

Analysis of Variance Table

Model 1: rr ~ age + ot

Model 2: rr ~ age + ot + vr + tsf

	Res.Df	RSS	Df	Sum of Sq	F	Pr(>F)
1	78	148.937				
2	76	98.231	2	50.706	19.616	1.353e-07 ***

Signif. codes:

0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```
> d=lm(rr~age+ot+tsf)
> summary(d)
```

Call:

```
lm(formula = rr ~ age + ot + tsf)
```

Residuals:

```
    Min      1Q  Median      3Q     Max
-3.0620 -0.6437 -0.1013  0.5672  2.9583
```

Coefficients:

```
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 1.237e+01  4.928e-01  25.100 < 2e-16
age         -1.442e-01  2.092e-02  -6.891 1.33e-09
ot           2.672e-01  5.729e-02   4.663 1.29e-05
tsf          8.178e-06  1.305e-06   6.265 1.97e-08
```

(Intercept) ***

age ***

ot ***

tsf ***

Signif. codes:

0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.132 on 77 degrees of freedom

Multiple R-squared: 0.583, Adjusted R-squared: 0.5667

F-statistic: 35.88 on 3 and 77 DF, p-value: 1.295e-14

```
> confint(d,level=0.9)
```

```
            5 %      95 %
(Intercept) 1.155005e+01 1.319112e+01
age         -1.789942e-01 -1.093351e-01
ot           1.717777e-01  3.625564e-01
tsf          6.004908e-06  1.035151e-05
```

```
> confint(d,level=(1-0.1/4))
```

```
            1.25 %      98.75 %
(Intercept) 1.124390e+01 13.4972680885
age         -1.919897e-01 -0.0963396198
ot           1.361865e-01  0.3981475445
tsf          5.194017e-06  0.0000111624
```

```
>
```

```
new.data=data.frame(age=c(5,6,14,12),ot=c(8.25,8.5,11.5,10.25),tsf=c(250000,270000,300000,310000))
```

```
> predict(d,new.data,interval="confidence",level=(1-.05/4))
```

```
      fit      lwr      upr
1 15.89844 15.35022 16.44666
```

```

2 15.98463 15.41526 16.55400
3 15.87816 15.32149 16.43483
4 15.91431 15.33555 16.49308
>
>
nnew.data=data.frame(age=c(6,14,12),ot=c(10,11.5,12.5),tsf=c(80000,120000,340000))
> predict(d,nnew.data,interval="confidence",level=(1-.05/4))
      fit      lwr      upr
1 14.83152 14.39298 15.27006
2 14.40608 13.87078 14.94139
3 16.76079 16.12517 17.39640
> plot(fitted(d), resid(d),main="Residual Plot \n raylor")
> abline(h=0)
>
> qqnorm(resid(d),main= "Q-Q plot \n raylor")
> qqline(resid(d))
> plot (d$fitted.values, d$residuals, main="Index Plot", sub= "raylor",xlab="Fitted
values",ylab="Residual")
> abline(h=0, col="red")
> bptest(d, studentize = FALSE)

```

Breusch-Pagan test

```

data: d
BP = 17.281, df = 3, p-value = 0.0006187

> plot(d$fitted.values,d$residuals,main="index plot", sub=
"Raylor",xlab="index",ylab="Residual")
> abline(h=0,col="red")
> shapiro.test(resid(d))

```

Shapiro-Wilk normality test

```

data: resid(d)
W = 0.98776, p-value = 0.6406
> dwtest(d)

```

Durbin-Watson test

```

data: d
DW = 1.5867, p-value = 0.02463
alternative hypothesis: true autocorrelation is greater than 0

> rstandard(d)
      1      2      3      4      5
-0.97755761 -1.25401611 -0.67191935 -0.10784521  0.29435399

```

6	7	8	9	10
-2.75734430	-0.42746706	0.43454717	1.69420847	0.07399060
11	12	13	14	15
0.06506769	-0.32948755	0.60461995	-0.37163043	-0.21543236
16	17	18	19	20
-0.80591580	0.08189521	-1.61432640	-1.16507694	-0.59099078
21	22	23	24	25
-0.28349894	0.24536238	-0.07221777	0.18038226	-0.75846965
26	27	28	29	30
-1.86090930	0.43783689	-0.49137300	-0.89871942	-0.17007734
31	32	33	34	35
-1.06666638	-0.22250001	-0.94070474	-0.09179945	0.21147055
36	37	38	39	40
0.73903240	0.98700409	-2.01833988	-0.18030785	-1.04344579
41	42	43	44	45
-0.09457136	2.23542440	-1.56396092	0.84233923	0.36262270
46	47	48	49	50
0.07508971	0.64753002	1.37328967	0.51342789	0.44454389
51	52	53	54	55
0.18085669	-0.07374863	1.25591023	0.35371952	-0.99497514
56	57	58	59	60
0.97916007	-0.27212934	0.92338213	-0.31056246	0.19479994
61	62	63	64	65
1.12771528	2.69207186	2.20586445	1.67146835	1.32044244
66	67	68	69	70
-0.45321947	-0.68237999	1.84718053	-0.02356378	-0.00203972
71	72	73	74	75
1.59571850	-0.38729516	-0.43540397	-0.10473058	1.11051846
76	77	78	79	80
-0.26011939	-0.58210287	0.86254288	-0.43922831	-1.95155044
81				
-0.80086111				

```
> shapiro.test(resid(d))
```

Shapiro-Wilk normality test

data: resid(d)

W = 0.98776, p-value = 0.6406

```
> plot(d$fitted.values, rstandard(d), main="d Data\n raylor", xlab="Fitted
values", ylab="Residual", pch=23, bg="red", cex=2, lwd=2)
> abline(h=0, col="red")
> abline(h=2, col="blue")
> abline(h=-2, col="blue")
> (1:length(rstandard(d)))[rstandard(d)>2]
> (1:length(rstandard(d)))[rstandard(d)<-2]
```

```
> p2 = read.table (file.choose(), header = TRUE)
> rr=p2$rr
> age=p2$age
> ot=p2$opet
> vr=p2$vr
> tsf=p2$tsf
> e=lm(rr~age+ot+tsf)
> summary(e)
```

Call:

```
lm(formula = rr ~ age + ot + tsf)
```

Residuals:

```
      Min       1Q   Median       3Q      Max
-2.00369 -0.58142 -0.06623  0.54297  2.06131
```

Coefficients:

```
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 1.232e+01  4.170e-01  29.536 < 2e-16 ***
age         -1.381e-01  1.823e-02  -7.578 9.38e-11 ***
ot           2.697e-01  4.862e-02   5.547 4.54e-07 ***
tsf          7.860e-06  1.128e-06   6.970 1.26e-09 ***
---

```

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.9383 on 72 degrees of freedom

Multiple R-squared: 0.6603, Adjusted R-squared: 0.6462

F-statistic: 46.66 on 3 and 72 DF, p-value: < 2.2e-16

```
nnnew.data=data.frame(age=c(6,14,12),ot=c(10,11.5,12.5),tsf=c(80000,120000,340000))
```

```
> predict(d,nnnew.data,interval="confidence",level=(1-.05))
```

```
      fit      lwr      upr
1 14.83152 14.49011 15.17293
2 14.40608 13.98934 14.82283
3 16.76079 16.26594 17.25563
```

Q2

```
> a = read.table (file.choose(), header = TRUE)
> summary(a[c('QuizAve','CompAve','MidTerm', 'FinalExam')])
      QuizAve      CompAve      MidTerm
Min.   :4.600  Min.   :4.600  Min.   :10.00
1st Qu.:6.800  1st Qu.:7.900  1st Qu.:57.25
Median :7.400  Median :8.650  Median :71.00
Mean   :7.257  Mean   :8.400  Mean   :68.88
```

```

3rd Qu.:7.875  3rd Qu.:9.275  3rd Qu.:77.00
Max.   :9.600  Max.   :9.900  Max.   :95.00
FinalExam
Min.   :15.00
1st Qu.:39.00
Median :51.00
Mean   :49.45
3rd Qu.:59.50
Max.   :87.00
> qz= a$QuizAve*10
> ca= a$CompAve*10
> mt= a$MidTerm
> fe= a$FinalExam
> pairs(~fe+qz+ca+mt, data=a,main="scatterplot matric \n raylor")
> cor(a)
      QuizAve  CompAve  MidTerm  FinalExam
QuizAve  1.0000000  0.4931397  0.3665234  0.39597772
CompAve   0.4931397  1.0000000  0.1600845  0.03312729
MidTerm   0.3665234  0.1600845  1.0000000  0.40363552
FinalExam 0.3959777  0.0331272  0.4036355  1.00000000
> median(ca)
[1] 86.5
> b=lm(fe~qz+ca+mt)
> summary(b)

```

Call:

```
lm(formula = fe ~ qz + ca + mt)
```

Residuals:

Min	1Q	Median	3Q	Max
-27.260	-10.293	1.302	7.221	42.218

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	9.1368	16.3011	0.560	0.57746
qz	0.5871	0.2141	2.743	0.00825 **
ca	-0.2934	0.1907	-1.538	0.12977
mt	0.3246	0.1385	2.343	0.02283 *

Signif. codes:

0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 14.54 on 54 degrees of freedom

Multiple R-squared: 0.2662, Adjusted R-squared: 0.2254

F-statistic: 6.528 on 3 and 54 DF, p-value: 0.000755

```
> c=lm(fe~qz)
```

```
> summary(c)
```

Call:

```
lm(formula = fe ~ qz)
```

Residuals:

Min	1Q	Median	3Q	Max
-30.109	-10.253	1.209	11.164	40.275

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	6.1938	13.5537	0.457	0.64945
qz	0.5960	0.1847	3.227	0.00209 **

Signif. codes:

0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 15.3 on 56 degrees of freedom

Multiple R-squared: 0.1568, Adjusted R-squared: 0.1417

F-statistic: 10.41 on 1 and 56 DF, p-value: 0.002092

```
> d=lm(fe~ca+mt)
```

```
> summary(d)
```

Call:

```
lm(formula = fe ~ ca + mt)
```

Residuals:

Min	1Q	Median	3Q	Max
-28.546	-9.482	0.522	10.623	37.413

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	22.19359	16.48882	1.346	0.18383
ca	-0.04598	0.17772	-0.259	0.79681
mt	0.45176	0.13803	3.273	0.00184 **

Signif. codes:

0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 15.37 on 55 degrees of freedom

Multiple R-squared: 0.1639, Adjusted R-squared: 0.1335

F-statistic: 5.392 on 2 and 55 DF, p-value: 0.00727

```
> e= lm(fe~1)
```

```
> anova(e, b)
```

Analysis of Variance Table

Model 1: fe ~ 1

Model 2: fe ~ qz + ca + mt

	Res.Df	RSS	Df	Sum of Sq	F	Pr(>F)
1	57	15548				
2	54	11410	3	4138.3	6.5284	0.000755 ***

Signif. codes:

0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

> summary(e)

Call:

lm(formula = fe ~ 1)

Residuals:

Min	1Q	Median	3Q	Max
-34.448	-10.448	1.552	10.052	37.552

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	49.448	2.169	22.8	<2e-16 ***

Signif. codes:

0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 16.52 on 57 degrees of freedom

> f= lm(fe~ca+mt)

> anova(f, b)

Analysis of Variance Table

Model 1: fe ~ ca + mt

Model 2: fe ~ qz + ca + mt

	Res.Df	RSS	Df	Sum of Sq	F	Pr(>F)
1	55	12999				
2	54	11410	1	1589.3	7.5217	0.008253 **

Signif. codes:

0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

> summary(f)

Call:

lm(formula = fe ~ ca + mt)

Residuals:

Min	1Q	Median	3Q	Max
-28.546	-9.482	0.522	10.623	37.413

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	22.19359	16.48882	1.346	0.18383
ca	-0.04598	0.17772	-0.259	0.79681
mt	0.45176	0.13803	3.273	0.00184 **

Signif. codes:

0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 15.37 on 55 degrees of freedom
Multiple R-squared: 0.1639, Adjusted R-squared: 0.1335
F-statistic: 5.392 on 2 and 55 DF, p-value: 0.00727

```
> g=lm(fe~qz+ca)
> summary(g)
```

Call:

lm(formula = fe ~ qz + ca)

Residuals:

Min	1Q	Median	3Q	Max
-30.417	-10.213	0.108	11.509	43.440

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	20.2595	16.2189	1.249	0.216906
qz	0.7551	0.2098	3.599	0.000685 ***
ca	-0.3048	0.1983	-1.537	0.129952

Signif. codes:

0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 15.12 on 55 degrees of freedom
Multiple R-squared: 0.1915, Adjusted R-squared: 0.1621
F-statistic: 6.515 on 2 and 55 DF, p-value: 0.002888

```
> anova(g,b)
```

Analysis of Variance Table

Model 1: fe ~ qz + ca

Model 2: fe ~ qz + ca + mt

	Res.Df	RSS	Df	Sum of Sq	F	Pr(>F)
1	55	12570				
2	54	11410	1	1160.2	5.4909	0.02283 *

Signif. codes:

0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```
> h=lm(fe~qz+mt)
```

```
> summary(h)
```

Call:

```
lm(formula = fe ~ qz + mt)
```

Residuals:

Min	1Q	Median	3Q	Max
-27.745	-10.068	-0.087	8.411	39.152

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	-4.5802	13.8153	-0.332	0.7415
qz	0.4313	0.1909	2.259	0.0279 *
mt	0.3300	0.1402	2.354	0.0222 *

Signif. codes:

0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 14.72 on 55 degrees of freedom

Multiple R-squared: 0.234, Adjusted R-squared: 0.2061

F-statistic: 8.4 on 2 and 55 DF, p-value: 0.0006553

```
> anova(h,b)
```

Analysis of Variance Table

Model 1: fe ~ qz + mt

Model 2: fe ~ qz + ca + mt

	Res.Df	RSS	Df	Sum of Sq	F	Pr(>F)
1	55	11910				
2	54	11410	1	500.14	2.367	0.1298

```
> i=lm(fe~ca)
```

```
> summary(i)
```

Call:

```
lm(formula = fe ~ ca)
```

Residuals:

Min	1Q	Median	3Q	Max
-34.071	-10.590	1.787	9.451	37.175

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	45.48890	16.11186	2.823	0.00657 **

```
ca      0.04714  0.19003  0.248  0.80501
```

```
---
```

```
Signif. codes:
```

```
0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
Residual standard error: 16.65 on 56 degrees of freedom
```

```
Multiple R-squared:  0.001097,    Adjusted R-squared:  -0.01674
```

```
F-statistic: 0.06152 on 1 and 56 DF,  p-value: 0.805
```

```
> anova(i,b)
```

```
Analysis of Variance Table
```

```
Model 1: fe ~ ca
```

```
Model 2: fe ~ qz + ca + mt
```

```
Res.Df  RSS Df Sum of Sq    F  Pr(>F)
```

```
1    56 15531
```

```
2    54 11410  2   4121.2 9.7522 0.0002422 ***
```

```
---
```

```
Signif. codes:
```

```
0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
> data= read.table (file.choose(), header = TRUE)
```

```
> quiz<- data[, 1]
```

```
> comp<- data[, 2]
```

```
> mid<- data[, 3]
```

```
> final<- data[, 4]
```

```
> X=matrix(c( rep(1, 58), data[, 1],data[, 2], data[, 3]), ncol=4)
```

```
> Y=matrix(data[,4])
```

```
> b=solve(t(X)%*%X)%*%(X)%*% Y
```

```
b
```

```
      [,1]
```

```
[1,]  9.1367509
```

```
[2,]  0.5870963
```

```
[3,] -0.2934339
```

```
[4,]  0.3245532
```

```
> yhat=X%*%b
```

```
> yhat
```

```
      [,1]
```

```
[1,] 42.25694
```

```
[2,] 51.40981
```

```
[3,] 63.43346
```

```
[4,] 60.09480
```

```
[5,] 48.48723
```

```
[6,] 57.03233
```

```
[7,] 47.56108
```

```
[8,] 35.12579
```

```
[9,] 52.69078
```

[10,] 60.72272
[11,] 31.64672
[12,] 53.06346
[13,] 60.89836
[14,] 52.92403
[15,] 44.78210
[16,] 45.10529
[17,] 53.83590
[18,] 35.58390
[19,] 55.72236
[20,] 60.31012
[21,] 46.69436
[22,] 49.54066
[23,] 41.41330
[24,] 65.16249
[25,] 51.79477
[26,] 42.61649
[27,] 40.46978
[28,] 35.55821
[29,] 41.83024
[30,] 61.26717
[31,] 47.43500
[32,] 51.30172
[33,] 56.04691
[34,] 50.86582
[35,] 52.76889
[36,] 45.68987
[37,] 63.46435
[38,] 66.24470
[39,] 54.08463
[40,] 45.15274
[41,] 49.18293
[42,] 30.58113
[43,] 49.01117
[44,] 57.53001
[45,] 54.83674
[46,] 55.27241
[47,] 33.46842
[48,] 42.26036
[49,] 50.86999
[50,] 41.73756
[51,] 45.88024
[52,] 51.33192
[53,] 52.70574
[54,] 53.35644
[55,] 39.95897

[56,] 43.01847

[57,] 44.93513

[58,] 49.97307

> H=X%%solve(t(X)%%X)%%t(X)

> H

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[5,] 2.733943e-02 0.0200264549 0.035662273 0.0354549452
[6,] 4.207420e-02 0.0202702235 -0.008812779 -0.0027462598
[7,] 2.310979e-02 0.0141089085 0.019103541 0.0178977348
[8,] -1.230930e-02 -0.0051256122 0.013615690 0.0020764689
[9,] 3.948108e-02 0.0106973532 -0.017717721 -0.0150490041
[10,] 7.010282e-02 0.0222024894 -0.015010932 -0.0054485763
[11,] -7.365477e-03 0.0038902980 0.066904529 0.0535303075
[12,] 9.820392e-03 0.0249315957 0.020743607 0.0232459388
[13,] 1.362343e-02 0.0295905345 -0.007515290 0.0005937533
[14,] 1.918661e-02 0.0185594202 0.002169068 0.0046354073
[15,] -1.149093e-03 0.0122901622 0.021536198 0.0175796572
[16,] 1.602578e-02 0.0071747118 0.007041797 0.0036690352
[17,] 1.726251e-02 0.0203749879 0.002602800 0.0056937089
      [,57]      [,58]
[1,] -0.0180698955 -0.0284711502
[2,] 0.0360306910 0.0333482398
[3,] 0.0105704524 0.0319297708
[4,] 0.0271309298 0.0374078604
[5,] 0.0291336079 0.0193443924
[6,] -0.0041242122 0.0037732107
[7,] 0.0149853162 0.0118294475
[8,] 0.0066604165 0.0071296021
[9,] -0.0159823706 -0.0065853158
[10,] -0.0136244295 -0.0081239003
[11,] 0.0474578278 0.0250635498
[12,] 0.0265600998 0.0290676813
[13,] 0.0097929335 0.0262194144
[14,] 0.0075283458 0.0145799550
[15,] 0.0211354337 0.0213712733
[16,] 0.0038515206 0.0054979435
[17,] 0.0095045510 0.0173677457
[ reached getOption("max.print") -- omitted 41 rows ]

```

```
> Yhat= H%*% Y
```

```
> Yhat
```

```
    [,1]
```

```
[1,] 42.25694  
[2,] 51.40981  
[3,] 63.43346  
[4,] 60.09480  
[5,] 48.48723  
[6,] 57.03233  
[7,] 47.56108  
[8,] 35.12579  
[9,] 52.69078  
[10,] 60.72272  
[11,] 31.64672  
[12,] 53.06346  
[13,] 60.89836  
[14,] 52.92403  
[15,] 44.78210  
[16,] 45.10529  
[17,] 53.83590  
[18,] 35.58390  
[19,] 55.72236  
[20,] 60.31012  
[21,] 46.69436  
[22,] 49.54066  
[23,] 41.41330  
[24,] 65.16249  
[25,] 51.79477  
[26,] 42.61649  
[27,] 40.46978  
[28,] 35.55821  
[29,] 41.83024  
[30,] 61.26717  
[31,] 47.43500  
[32,] 51.30172  
[33,] 56.04691  
[34,] 50.86582  
[35,] 52.76889  
[36,] 45.68987  
[37,] 63.46435  
[38,] 66.24470  
[39,] 54.08463  
[40,] 45.15274  
[41,] 49.18293  
[42,] 30.58113  
[43,] 49.01117
```

```

[44,] 57.53001
[45,] 54.83674
[46,] 55.27241
[47,] 33.46842
[48,] 42.26036
[49,] 50.86999
[50,] 41.73756
[51,] 45.88024
[52,] 51.33192
[53,] 52.70574
[54,] 53.35644
[55,] 39.95897
[56,] 43.01847
[57,] 44.93513
[58,] 49.97307
> sse=(t(Y)%*%Y) - t(b)%*%t(X)%*%Y
> n = nrow(Y)
> mse = as.vector(sse/(n-3))
>
> var = mse*solve(t(X)%*%X)
> var
      [,1]      [,2]      [,3]      [,4]
[1,] 260.8936587 -1.000599069 -1.6695697647 -0.6454847451
[2,] -1.0005991  0.044991612 -0.0189632577 -0.0097486757
[3,] -1.6695698 -0.018963258  0.0357155755  0.0006621061
[4,] -0.6454847 -0.009748676  0.0006621061  0.0188346709
>
> J = matrix(1, n, n)
> SSTO = t(Y)%*%Y-(1/n)*t(Y)%*%J%*%Y
> SSTO
      [,1]
[1,] 15548.34
>
> sse
      [,1]
[1,] 11410.04
> SSR = t(b)%*%t(X)%*%Y-(1/n)*t(Y)%*%J%*%Y
> SSR
      [,1]
[1,] 4138.305
<xh=matrix(c(1,50))
<s_sqyh=mse*t(xh)%*%xtrxin%*%xh
<s_sqyh
<s_yh=sqrt(s_sqyh)
<s_yh
<s_sqp=mse*(1+t(xh)%*%xtrxin%*%xh)

```

```

<s_sqp
<s_p = sqrt(s_sqp)
<s_p
> newmodel=lm(final~quiz+comp+mid)
> new.data=data.frame(quiz=70, comp= 85, mid= 65)
> predict(newmodel,new.data,interval="confidence",level=(1-.05), se.fit=TRUE)
$fit
      fit      lwr      upr
1 46.38757 42.29514 50.48

```

```

$se.fit
[1] 2.041236

```

```

$df
[1] 54

```

```

$residual.scale
[1] 14.53606
> summary(newmodel)

```

```

Call:
lm(formula = final ~ quiz + comp + mid)

```

```

Residuals:
    Min     1Q   Median     3Q    Max
-27.260 -10.293  1.302   7.221  42.218

```

```

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  9.1368    16.3011   0.560 0.57746
quiz         0.5871     0.2141   2.743 0.00825 **
comp        -0.2934     0.1907  -1.538 0.12977
mid          0.3246     0.1385   2.343 0.02283 *
---

```

```

Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```

```

Residual standard error: 14.54 on 54 degrees of freedom
Multiple R-squared:  0.2662, Adjusted R-squared:  0.2254
F-statistic: 6.528 on 3 and 54 DF, p-value: 0.000755

```

```

> e=Y-yhat
> max(e)
[1] 42.2179
> n=lm(final~quiz+comp+mid)
> summary(n)

```

```

Call:

```

```
lm(formula = final ~ quiz + comp + mid)
```

Residuals:

Min	1Q	Median	3Q	Max
-27.260	-10.293	1.302	7.221	42.218

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	9.1368	16.3011	0.560	0.57746
quiz	0.5871	0.2141	2.743	0.00825 **
comp	-0.2934	0.1907	-1.538	0.12977
mid	0.3246	0.1385	2.343	0.02283 *

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 14.54 on 54 degrees of freedom
Multiple R-squared: 0.2662, Adjusted R-squared: 0.2254
F-statistic: 6.528 on 3 and 54 DF, p-value: 0.000755

```
> o= lm(final~comp+mid)
```

```
> anova(n, o)
```

Analysis of Variance Table

Model 1: final ~ quiz + comp + mid

Model 2: final ~ comp + mid

	Res.Df	RSS	Df	Sum of Sq	F	Pr(>F)
1	54	11410				
2	55	12999	-1	-1589.3	7.5217	0.008253 **

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```
> confint (n, level = 0.95)
```

	2.5 %	97.5 %
--	-------	--------

(Intercept)	-23.54493534	41.81843706
-------------	--------------	-------------

quiz	0.15791733	1.01627535
------	------------	------------

comp	-0.67581953	0.08895166
------	-------------	------------

mid	0.04686867	0.60223776
-----	------------	------------

```
> mean (quiz)
```

```
[1] 72.56897
```

```
> mean (comp)
```

```
[1] 84
```

```
> mean (mid)
```

```
[1] 68.87931
```

```
> average= data.frame(quiz=72.56897, comp=84, mid=68.87931)
```

```
> predict(n, average, interval="confidence", level=(1-.05))
```

	fit	lwr	upr
--	-----	-----	-----

1	49.44828	45.62161	53.27495
---	----------	----------	----------

```
>
```

```
> k= lm(final~ mid)
> anova(k, n)
Analysis of Variance Table
```

Model 1: final ~ mid

Model 2: final ~ quiz + comp + mid

	Res.Df	RSS	Df	Sum of Sq	F	Pr(>F)
1	56	13015				
2	54	11410	2	1605.1	3.7983	0.02862 *

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```
> v= lm(final~quiz+mid)
```

```
> anova(v, n)
```

Analysis of Variance Table

Model 1: final ~ quiz + mid

Model 2: final ~ quiz + comp + mid

	Res.Df	RSS	Df	Sum of Sq	F	Pr(>F)
1	55	11910				
2	54	11410	1	500.14	2.367	0.1298

```
> h= lm(final~quiz+comp)
```

```
> anova(h, n)
```

Analysis of Variance Table

Model 1: final ~ quiz + comp

Model 2: final ~ quiz + comp + mid

	Res.Df	RSS	Df	Sum of Sq	F	Pr(>F)
1	55	12570				
2	54	11410	1	1160.2	5.4909	0.02283 *

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```
> s= lm(final~comp+mid)
```

```
> anova(s, n)
```

Analysis of Variance Table

Model 1: final ~ comp + mid

Model 2: final ~ quiz + comp + mid

	Res.Df	RSS	Df	Sum of Sq	F	Pr(>F)
1	55	12999				
2	54	11410	1	1589.3	7.5217	0.008253 **

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1