FRE6233, assignment week 2

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Please use the updated slides posted after the lecture.

Problem 1

i Suppose that $(\Omega, \mathcal{F}, \mathcal{P})$ is a probability space where $\Omega = \{a, b, c, d, e, f\}$, $\mathcal{F}$ is σ-algebra, and $\mathcal{P}$ is uniform (so $\mathcal{P}(a) = \mathcal{P}(b)... = \frac{1}{6}$).

ii Let X, Y, Z be r.v. given by

$$X(a) = 1, X(b) = X(c) = 3, X(d) = X(e) = 5, X(f) = 7$$

$$Y(a) = Y(b) = 2, Y(c) = Y(d) = 1, Y(e) = Y(f) = 7$$

$$Z(a) = Z(b) = Z(c) = Z(d) = 3, Z(e) = Z(f) = 2$$

Solve the following questions:

1. Write down $\sigma(X), \sigma(Y), \sigma(Z), \sigma(Z)$. Are there any relationships in between them?

2. Define a r.v. $\mathbb{E}[X|\sigma(Y)]$

3. Check directly the averaging property

$$\mathbb{E}[\mathbb{E}[X|\sigma(Y)]] = \mathbb{E}[X]$$

4. Show directly (by calculating) that

$$\mathbb{E}[\mathbb{E}[X|\sigma(Y)]|\sigma(Z)] = \mathbb{E}[X|\sigma(Z)]$$

5. Check if X and Y, or Y and Z are independent under given probability. Use the definition from the last slide 16.

Problem 2 Prove Markov and Tchebyshev inequalities.

Problem 3 Let X be a r.v. and $\lambda > 0$. Prove that the following bound holds:

$$P(X \geq \lambda) \leq \frac{\mathbb{E}[e^{tX}]}{e^{\lambda t}}, \forall t > 0$$

Use Markov inequality.