FRE-GY 6233: Assignment 5

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Problem 1

Using the differentiation rules, find the following differentials:

(a) $d(3W^3(t) + e^{6W(t)})$

(b) $d(\frac{1}{t}\int_0^t W(u)du)$

(c) $d(t^3 cos(2W(t)))$

(d) $d(\frac{1}{t}\int_0^t e^{W(u)}du)$

(e) $d(W(t)e^{W(t)})$

Problem 2 Let $Z(T) = \int_0^T W(u)du$ be the integrated BM, and $I(T) = \int_0^T tdW(t)$

(a) Using integration by parts, find relationship bewteen $I(T)$ and $Z(T)$

(b) Using the relationship, compute the covariance between $I(T)$ and $Z(T)$

(c) Calculate the correlation between $I(T)$ and $Z(T)$

Problem 3 Let $f(t) = e^{\alpha t}$, $g(x) = cos(x)$. Using integration by parts, calculate the stochastic integral

$$\int_0^T e^{\alpha t} cos(W(t))dW(t)$$

Which choice of α makes it very simple?

Problem 4 Using the product of differentiation to calculate $d(X(t), Y(t))$, write down a general formula for integration by parts — integrate both sides of a differential of a product of two stochastic processes $X(t)Y(t)$—

$$\int_0^T Y(t)dX(t) = \cdots$$

Using the integration by parts, calculate 2 stochastic integrals:

1. $\int_0^T W(t)dW(t)$
2. $\int_0^T W^2(t)dW(t)$