Quantitative Methods: Assignment 5

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Problem 1 (30 points):
We consider the 3-period name model seen in class with the parameters

$$S_0 = 4, u = 2, d = 1/2, r = 1/5, K = 5$$

The price fo the underlying asset is denoted by S_n for $n = 0, 1, 2, 3$

1. Compute the price of V_0, at time 0, of a European call option on the underlying asset S_n with payoff $V_3 = (S_3 - 2)^+$ at time 3.

2. Give also the hedging strategy Δ_0 at time 0.

3. Compute now the price of a Lookback option with payoff

$$V_3 = \max_{0 \leq n \leq 3} (S_n - S_3)$$

   for the same set of parameters

4. Give the hedging strategy Δ_0 for the lookback option.

Problem 2 (10 points):
Consider the one-period binomial tree model seen in class with $d = 1/2, u = 1.5, r = 1, S_0 = 3$. Is there an arbitrage opportunity in this model? Can you exhibit such a strategy?

Problem 3 (10 points):
Consider the general one-period binomial tree model seen in class with payoff $V_1 = S_1$ (in other words we set the strike price to 0, K = 0)

1. Write the risk-neutral pricing formula giving the price V_0 at time 0, of this claim

2. What is the price?

Problem 4 (50 points) Asian Option:
Consider the 3-period model seen in class with $d = 1/2, u = 2, r = 1/4, S_0 = 4$ and consider the Asian option with payoff

$$V_3 = (\frac{1}{4} Y_3 - 4)^+$$

where the variable Y_n is defined as

$$Y_n = \sum_{k=0}^{n} S_k$$

1. Write a recursive formula for V_n in terms of V_{n+1} that will allow you to compute the price of the Asian option.

2. Provide a formula for the number of shares $\Delta_n = \delta_n(s, y)$ of underlying asset that should be held in the replicating portfolio at time n for $n = 0, 1, 2$

3. Use the algorithm developed in the first question to compute the price of the Asian option V_0 at time 0.

Optional Problem (0 credit): Exercise 1.9 page 22 in the textbook by Shreve.