1: Figures Made Using Tikz

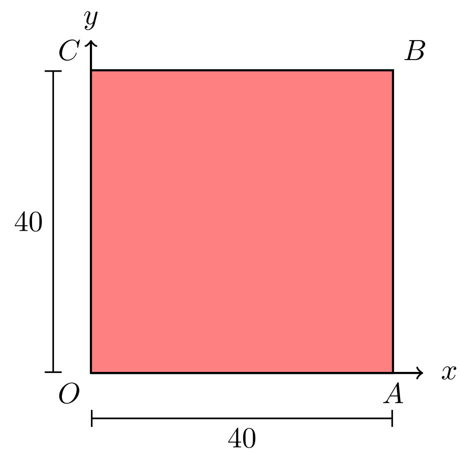

Figure 1: Square with length = 40

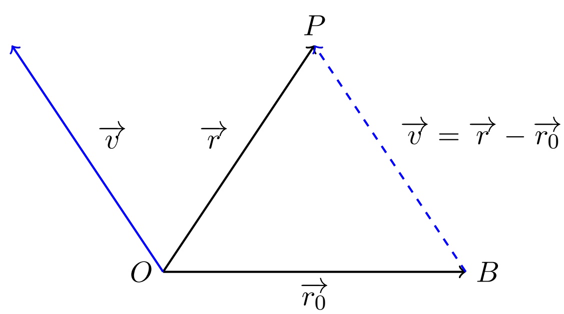

Figure 2: Displaying Vectors Using Arrows

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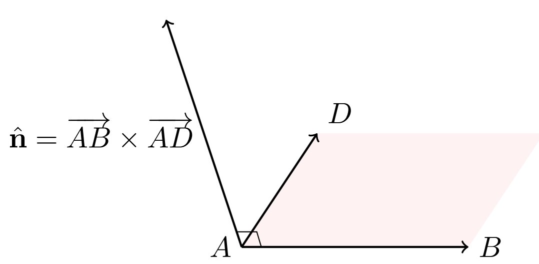

Figure 3: Cross-Product of two vectors $\overrightarrow{AB}$ and $\overrightarrow{AD}$.

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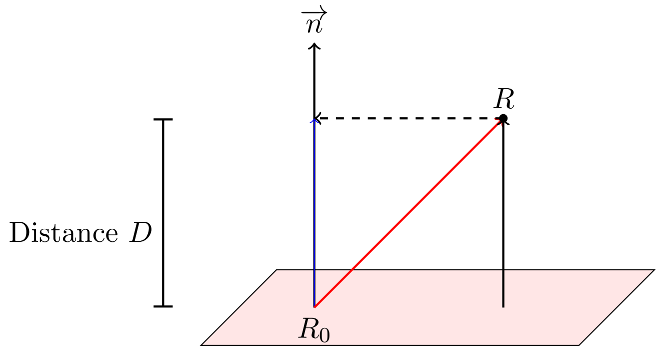

Figure 4: Projection of $\overrightarrow{RR_0}$ onto $\overrightarrow{n}$

2: Sample of Solved Math Problems

a) Given the space $\mathbb{R}^3$, the points B and P are located at $(40, 40, 0)$ and $(20, 20, 80)$ respectively, find the vector $\overrightarrow{BP}$. Hence also find the equation of the line BP

Sol: The position vectors from B and P are relative to origin $O(0, 0, 0)$

$$\overrightarrow{OP} = \overrightarrow{OB} + \overrightarrow{BP}$$
$$\overrightarrow{BP} = \overrightarrow{OP} - \overrightarrow{OB}$$
$$\overrightarrow{BP} = \begin{bmatrix} 20 \\ 20 \\ 80 \end{bmatrix} - \begin{bmatrix} 40 \\ 40 \\ 0 \end{bmatrix}$$
$$\overrightarrow{BP} = \begin{bmatrix} -20 \\ -20 \\ 80 \end{bmatrix}$$

The line is parallel to $\overrightarrow{BP}$. And it passes through the point B. The general equation of a line is given as:

$$\overrightarrow{r} = \overrightarrow{r_o} + t\overrightarrow{v}$$

here $\overrightarrow{r_o}$ is the position vector point on the line (or a point on the line) (i.e. $\overrightarrow{OB}$), and $\overrightarrow{o}$

is the direction of the line (i.e. $\overrightarrow{BP}$).

$$\begin{aligned}
\overrightarrow{r} &= \overrightarrow{OB} + t\overrightarrow{BP} \\
\overrightarrow{r} &= \begin{bmatrix} 40 \\ 40 \\ 0 \end{bmatrix} + t \begin{bmatrix} -20 \\ -20 \\ 80 \end{bmatrix} \\
\overrightarrow{r} &= \begin{bmatrix} 40 - 20t \\ 40 - 20t \\ 80t \end{bmatrix} \text{ Equation of the line } BP
\end{aligned}$$

Representing this equation in parametric form:

$$x(t) = 40 - 20t \tag{1}$$
$$y(t) = 40 - 20t \tag{2}$$
$$z(t) = 80t \tag{3}$$

b) Find the cross product of the vectors:

- $\overrightarrow{A} = 40\hat{\mathbf{j}}$
- $\overrightarrow{B} = -9.5\hat{\mathbf{i}} + 30.5\hat{\mathbf{j}} + 38\hat{\mathbf{k}}$

Sol:

$$\begin{aligned}
\overrightarrow{A} \times \overrightarrow{B} &= \det \begin{bmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 0 & 40 & 0 \\ -9.5 & 30.5 & 38 \end{bmatrix} \\
&= \hat{\mathbf{i}} \begin{vmatrix} 40 & 0 \\ 30.5 & 38 \end{vmatrix} - \hat{\mathbf{j}} \begin{vmatrix} 0 & 0 \\ -9.5 & 38 \end{vmatrix} + \hat{\mathbf{k}} \begin{vmatrix} 0 & 40 \\ -9.5 & 30.5 \end{vmatrix} \\
&= (40(38) - 0)\hat{\mathbf{i}} + (0 - 0)\hat{\mathbf{j}} + (0 - (40(-9.5))\hat{\mathbf{k}} \\
\overrightarrow{A} \times \overrightarrow{B} &= 1520\hat{\mathbf{i}} + 0\hat{\mathbf{j}} + 380\hat{k}
\end{aligned}$$

c) Solve the following

i) $\int \left(\frac{\tan x}{\cos x}\right)^2 dx$ ii) $\int (2+e^x)^2$ iii) $\int_0^4 (x-5)^5$

Sol::

Ans (i)

$$= \int \left(\frac{\tan x}{\cos x}\right)^2 dx$$

$$= \int \frac{\tan^2 x}{\cos^2 x}\, dx$$

$$= \int \tan^2 x \sec^2 x \, dx \qquad \because \sec x = \frac{1}{\cos x}$$

u - substitution $\qquad (u = \tan x,\ du = \sec^2 x\, dx)$

$$= \int (\tan x)^2 (\sec^2 x\, dx)$$

$$= \int u^2\, du$$

$$= \frac{u^3}{3} + C$$

substituting back

$$= \frac{\tan^3 x}{3} + C$$

Ans(ii)

$$= \int (2+e^x)^2\, dx$$

$$= \int (2^2 + 2(2)(e^x) + (e^x)^2)\, dx \qquad \because (a+b)^2 = a^2 + 2ab + b^2$$

$$= \int (4 + 4e^x + e^{2x})\, dx$$

$$= 4\int dx + 4\int e^x\, dx + \int e^{2x}\, dx$$

$$= 4x + 4e^x + \frac{e^{2x}}{2} + C$$

Ans(iii)

$$= \int_3^4 (x-5)^5 \, dx$$

either expand, or more preferably, use u-substitution

$$= \int_3^4 u^5 \, du \qquad (u = x - 5,\ du = dx)$$

$$= \left. \frac{u^6}{6} \right|_3^4$$

substitute it back

$$= \left. \frac{(x-5)^6}{6} \right|_3^4$$

$$= \left(\frac{(4-5)^6}{6} \right) - \left(\frac{(3-5)^6}{6} \right)$$

$$= \frac{1}{6} - \frac{64}{6}$$

$$= -\frac{21}{2} = -10.5 \text{ unit}^2$$