Precalculus

Compound Interest

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This work would not have been possible without the hard work and dedication of many individuals to make many technical things possible. This document was produced in LaTeX, making use of KOMA-script, as well as TikZ and PGF.

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In this chapter we will learn about compound interest.

9.1 Introduction

Recall: when we want to compute a percentage of a number, we multiply the number by the percentage in **decimal form.** For example, 15% of 20 is $(0.15)(20) = 3$. To convert from percent to decimal, we divide by 100.

Let x be a number. If we say x has increased by 4%, then what is the new value? We convert 4% to decimal: 0.04. Then we add 4% of x to x:

$$x + 0.04x.$$

We can combine these terms, though, and write $(1 + 0.04)x = 1.04x$.

Example 9.1: Exponential Growth

1. Sandy's salary increases by 15% from \$15,000. What is her current salary?

   Here our *rate* is 15%, which we convert to 0.15. Our principal, or starting value, is \$15,000. To get the new value, we compute

   $$\$15,000 + (0.15)(\$15,000) = (1.15)(\$15,000) = \$17,250.$$

   Notice here we kept track of our units. Our final answer is in **dollars.**

2. Suppose Jane borrows \$3,000 at an interest rate of 3% compounded yearly. Assume no payments are made on the loan.

   a) Find the amount owed after 1 year.

      Here again we have a rate of 3%, or in decimal 0.03. The interest is compounded yearly, and we are asked to find the amount owed after 1 year. Our principal is \$3,000. Then $(1.03)(\$3000) = \$3090$.

   b) Find the amount owed after 2 years.

      To find the amount owed after 2 years, we must take the amount owed after 1 year and repeat the computation. Notice that we don't just add \$90. That is, the answer **is not** $\$3000 + (0.03)(\$3000) + (0.03)(\$3000)$.

We compute:

$$(1.03)(\$3090) = \$3182.70$$

c) Write a general formula for the amount Jane owes after t years, assuming no payments are made.

Notice, with each passing year, we take the last years value and multiply by 1.03 to compute a 3% increase. In (a) we saw after one year she owed $(1.03)(\$3000)$. After two years she owed $(1.03)(\$3090)$, or more generally, $(1.03)[(1.03)(\$3000)] = (1.03)^2(\$3000)$. In general, after t years Jane will owe $(1.03)^t(\$3000)$.

9.2 Compound Interest Formula - Discrete

To calculate the final amount in a compound interest problem where things grow discretely (the interest accrues a defined number of times per year), we use the following formula.

$$A = P\left(1 + \frac{r}{n}\right)^{nt}.$$

- A is the final amount.
- P is the initial amount or initial principal.
- r is the interest rate in decimal form. For example, 15% is 0.15, or 4% is 0.04.
- n is the number of times interest is applied per time period.
- t is the number of time periods elapsed.

For our purposes, t is the **number of years** and n is the **number of times interest is applied per year.** We have special keywords for n. For example

- Compounded
  - yearly: $n = 1$
  - semiannually: $n = 2$.
  - quarterly: $n = 4$
  - monthly: $n = 12$
  - daily: $n = 365$

In compound interest word problems, one is tasked with identifying **given values** and substituting the given values into the formula properly.

Example 9.2: Compound Interest
To be added.