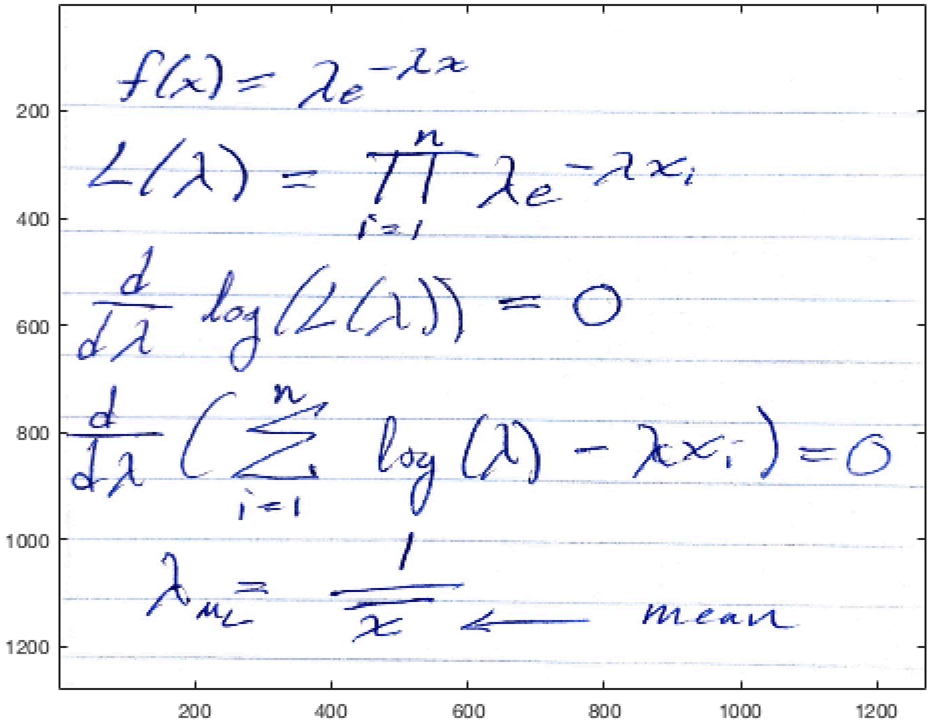
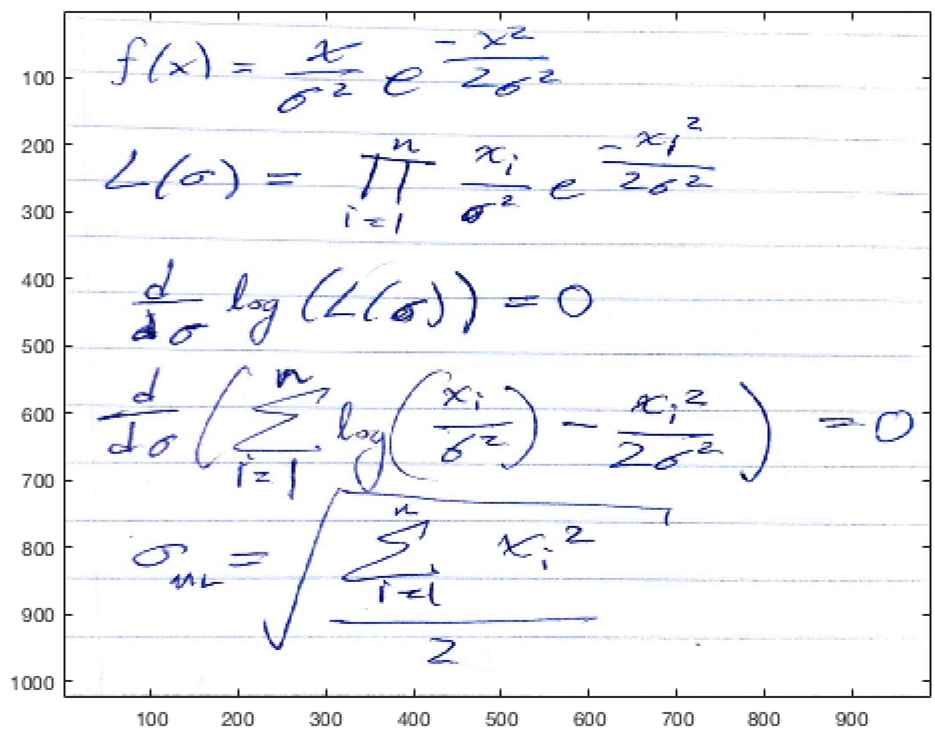
```
clc
clear
close all
```

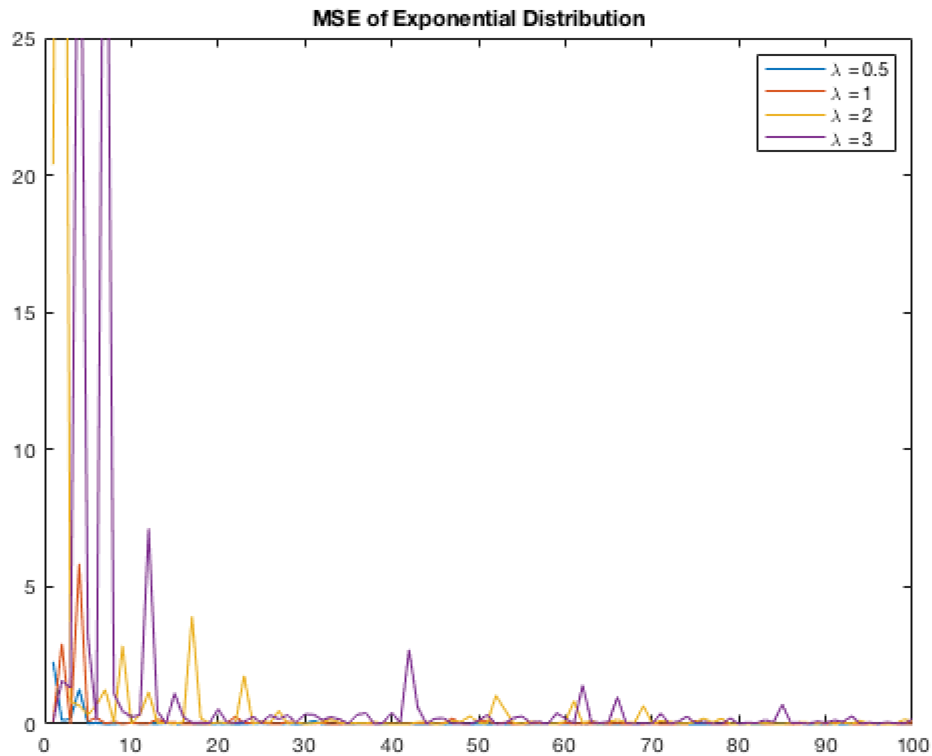
Question 1

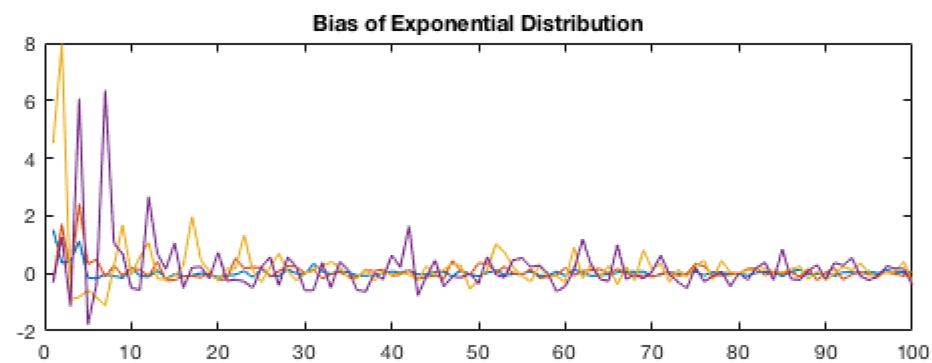
```
img1 = imread('exp.jpeg');
img2 = imread('rayl.jpeg');

%Derivations:
%Exponential:
figure;
image(img1);

%Rayleigh:
figure;
image(img2);

sample = 100;
lambda = [0.5, 1, 2, 3];
sigma = [0.5, 1, 2, 3, 4];

% preallocate exp
mse_exp = zeros(sample, length(lambda));
bias_exp = zeros(sample, length(lambda));
var_exp = zeros(sample, length(lambda));


for i = 1:length(lambda)
    for n = 1:sample
        % Generate exponential random variables
        exp_samples = exprnd(1/lambda(i), n, 1);
        lambda_ml = 1/mean(exp_samples);

        % Compute MSE, bias, and variance for exponential distribution
        mse_exp(n, i) = (lambda_ml - lambda(i))^2;
        bias_exp(n, i) = lambda_ml - lambda(i);
        var_exp(n, i) = var(exp_samples) / n;
    end
end

figure;
plot(1:sample, mse_exp);
title('MSE of Exponential Distribution');
legend(arrayfun(@(x) ['\lambda = ' num2str(x)],
 lambda, 'UniformOutput', false));
xlim([0, 100]);
ylim([0, 25]);


figure
subplot(2, 1, 1);
```

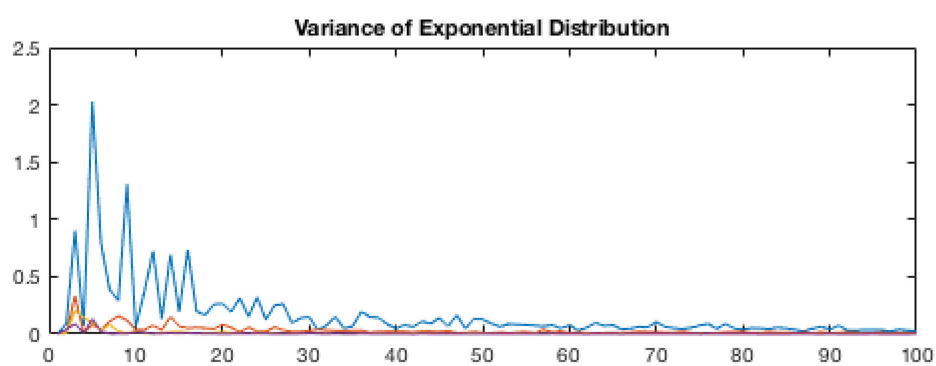
```
plot(1:sample, bias_exp);
title('Bias of Exponential Distribution');


subplot(2, 1, 2);
plot(1:sample, var_exp);
title('Variance of Exponential Distribution');


% preallocate ray
mse_ray = zeros(sample, length(sigma));
bias_ray = zeros(sample, length(sigma));
var_ray = zeros(sample, length(sigma));


for i = 1:length(sigma)
    for n = 1:sample
        % Generate Rayleigh random variables
        ray_samples = raylrnd(sigma(i), n, 1);
        sigma_ml = sqrt(mean(ray_samples.^2) / 2);

        % Compute MSE, bias, and variance for Rayleigh distribution
        mse_ray(n, i) = (sigma_ml - sigma(i))^2;
        bias_ray(n, i) = sigma_ml - sigma(i);
        var_ray(n, i) = var(ray_samples) / (2*n);
    end
end

figure;
plot(1:sample, mse_ray);
title('MSE of Rayleigh Distribution');
legend(arrayfun(@(x) ['\sigma = ' num2str(x)], sigma, 'UniformOutput',
 false));
xlim([0, 100]);
ylim([0, 5]);


figure
subplot(2, 1, 1);
plot(1:sample, bias_ray);
title('Bias of Rayleigh Distribution');

subplot(2, 1, 2);
plot(1:sample, var_ray);
title('Variance of Rayleigh Distribution');
```

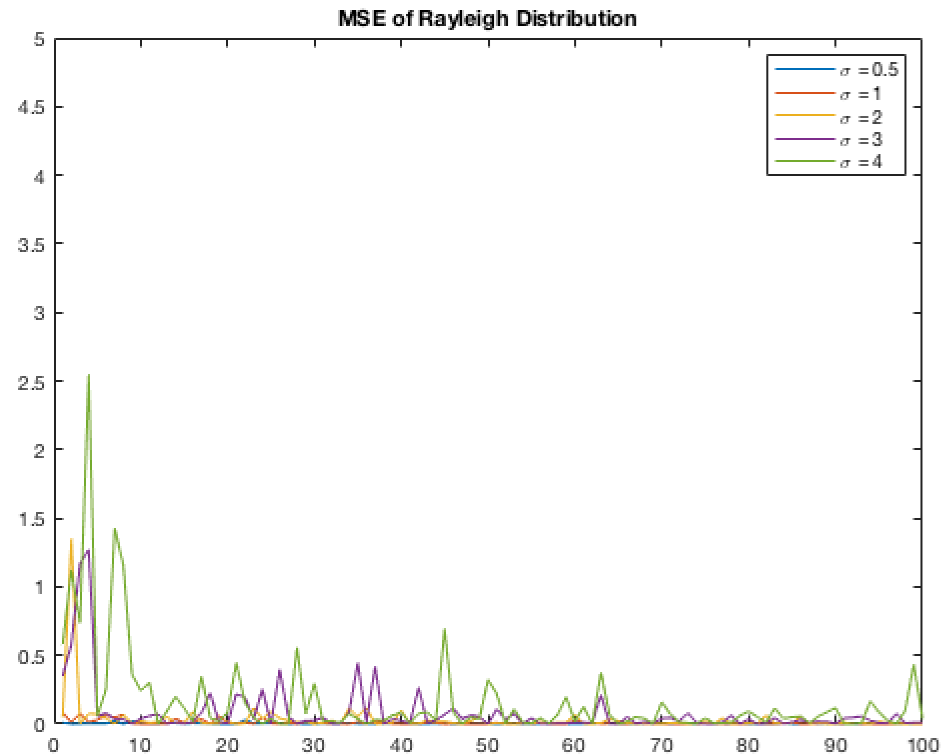

$f(x) = \lambda e^{-\lambda x}$
$L(\lambda) = \prod_{i=1}^{n} \lambda e^{-\lambda x_i}$
$\frac{d}{d\lambda} \log(L(\lambda)) = 0$
$\frac{d}{d\lambda} \left(\sum_{i=1}^{n} \log(\lambda) - \lambda x_i \right) = 0$
$\lambda_{ML} = \frac{1}{\bar{x}} \leftarrow$ mean
200
400
600
800
1000
1200

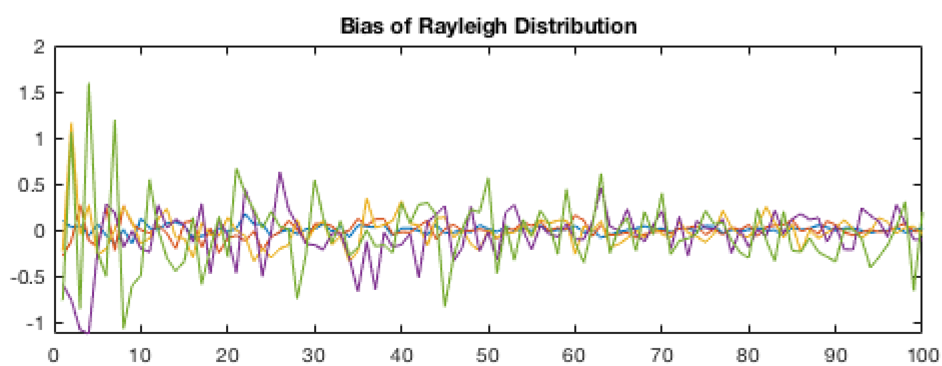

$f(x) = \frac{x}{\sigma^2} e^{\frac{-x^2}{2\sigma^2}}$
$L(\sigma) = \prod_{i=1}^{n} \frac{x_i}{\sigma^2} e^{\frac{-x_i^2}{2\sigma^2}}$
$\frac{d}{d\sigma} \log(L(\sigma)) = 0$
$\frac{d}{d\sigma} \left(\sum_{i=1}^{n} \log\left(\frac{x_i}{\sigma^2}\right) - \frac{x_i^2}{2\sigma^2} \right) = 0$
$\sigma_{ML} = \sqrt{\frac{\sum_{i=1}^{n} x_i^2}{2}}$
100
200
300
400
500
600
700
800
900
1000

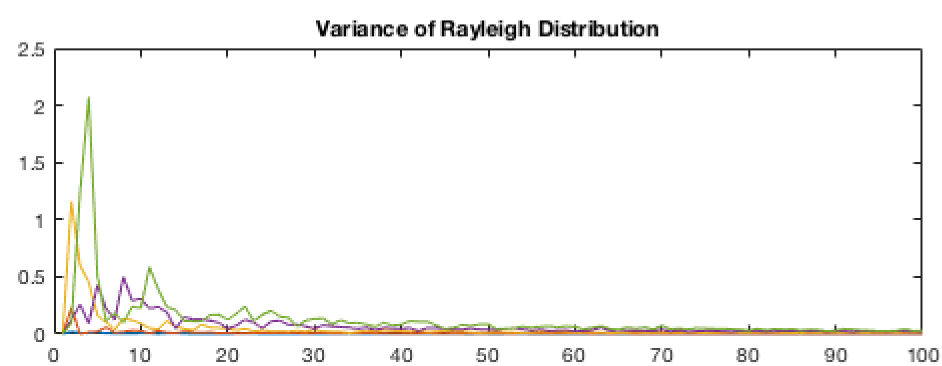

MSE of Exponential Distribution
λ = 0.5
λ = 1
λ = 2
λ = 3


Bias of Exponential Distribution


Variance of Exponential Distribution


MSE of Rayleigh Distribution
σ = 0.5
σ = 1
σ = 2
σ = 3
σ = 4


Bias of Rayleigh Distribution


Variance of Rayleigh Distribution

Question 2

```
load('data.mat');

% Compute MLE for the exponential distribution parameter (lambda)
lambda_ml = 1/mean(data)

% Compute MLE for the Rayleigh distribution parameter (sigma)
sigma_ml = sqrt(mean(data.^2)/2)

% Compute the log-likelihoods for both distributions
log_likelihood_exp = sum(log(lambda_ml * exp(-lambda_ml * data)))
log_likelihood_ray = sum(log((data/sigma_ml^2) .* exp(-data.^2/
(2*sigma_ml^2))))

% Compare the log-likelihoods
 disp('The distribution with the higher log-likelihood value is
 considered a better fit for the data, so the data is more likely
 drawn from a Rayleigh distribution.');
```

```
lambda_ml =

    7.7948


sigma_ml =

    0.1016


log_likelihood_exp =

   1.0535e+03


log_likelihood_ray =

   1.3655e+03

The distribution with the higher log-likelihood value is considered
 a better fit for the data, so the data is more likely drawn from a
 Rayleigh distribution.
```

Published with MATLAB® R2020a