



Introduction to Numerical Computing with Numpy

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Introduction to Numerical Computing with Numpy

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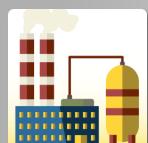
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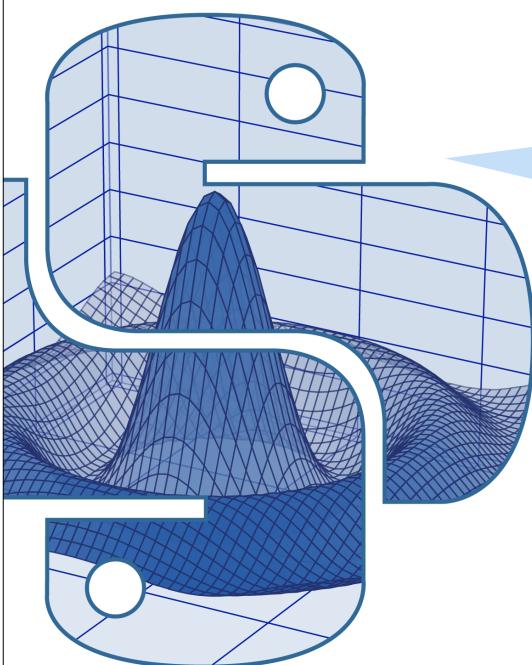
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Deployment Server

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Industry-Specific Software



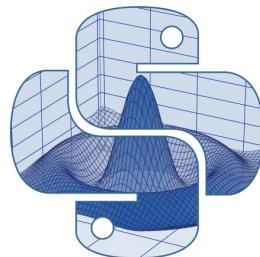
Hi there!

NumPy

The Standard Numerical Library
for Python

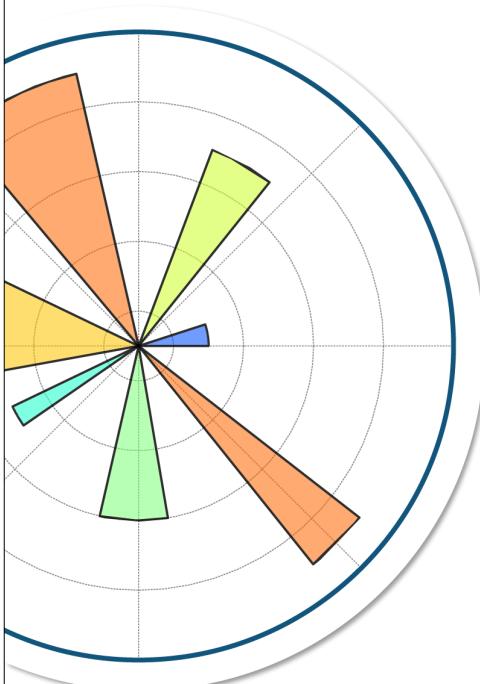
NumPy Arrays

- Defining Arrays
- Indexing and Slicing
- Creating Arrays
- Array Calculations
- The Array Data Structure
- Advanced NumPy, Overview



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matplotlib



Matplotlib

An Interlude

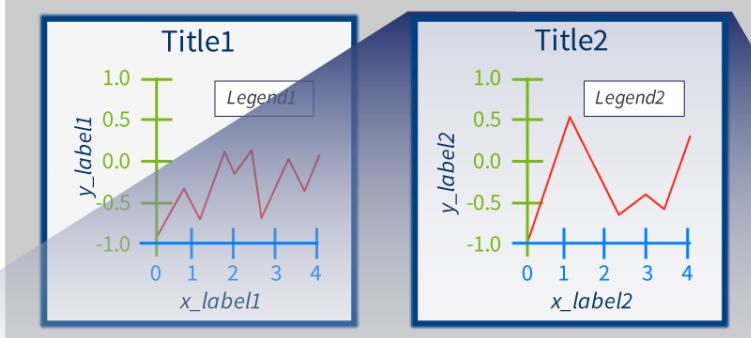
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Matplotlib Object Model

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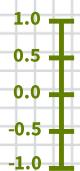
Figure



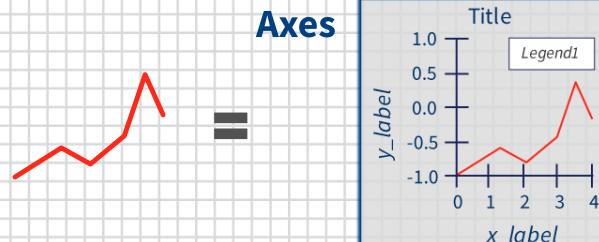
Axis



Axis



Axes



Matplotlib's "State Machine"

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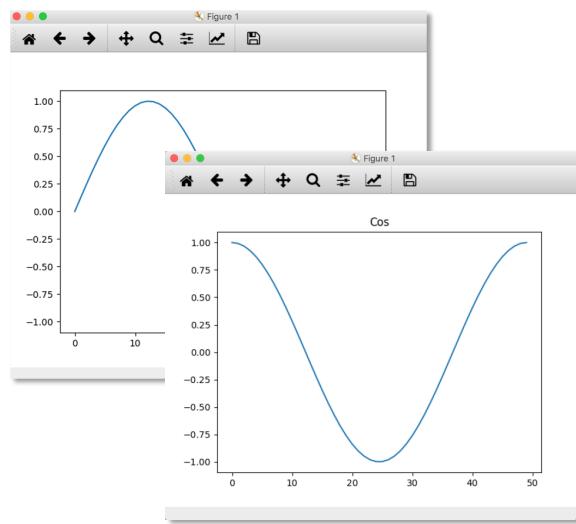
Matplotlib behaves like a state machine.

Any command is applied to the current plotting area:

```
>>> import matplotlib.pyplot as plt
>>> import numpy as np
>>> t = np.linspace(0,2*np.pi,
...                  50)
>>> x = np.sin(t)
>>> y = np.cos(t)

# Now create a figure
>>> plt.figure()
# and plot x inside it
>>> plt.plot(x)

# Now create a new figure
>>> plt.figure()
# and plot y inside it...
>>> plt.plot(y)
# ...and add a title
>>> plt.title("Cos")
```



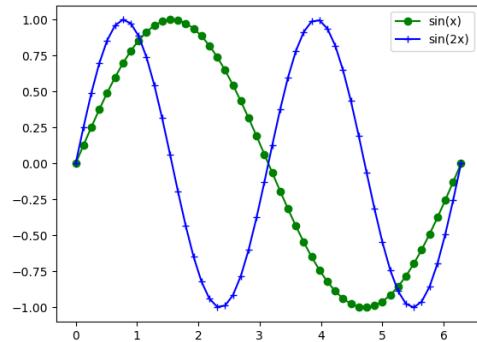
Line Plots

```
>>> x = np.linspace(0, 2*np.pi, 50)
>>> y1 = np.sin(x)
>>> y2 = np.sin(2*x)

>>> plt.figure() # Create figure
>>> plt.plot(y1)
>>> plt.plot(x, y1)

# red dot-dash circle
>>> plt.plot(x, y1, 'r-o')

# red marker only circle
>>> plt.plot(x, y1, 'ro')
>>> plt.plot(x, y1, 'g-o',
...             x, y2, 'b-+')
>>> plt.legend(['sin(x)',
...             'sin(2x)'])
```



Symbol	Color
b	Blue
g	Green
r	Red
c	Cyan
m	Magenta
y	Yellow
k	Black
w	White

Symbol	Marker
.	Point
o	Circle
< > ^ v	Triangles
8	Octagon
s	Square
*	Star
+	Plus
	...and more

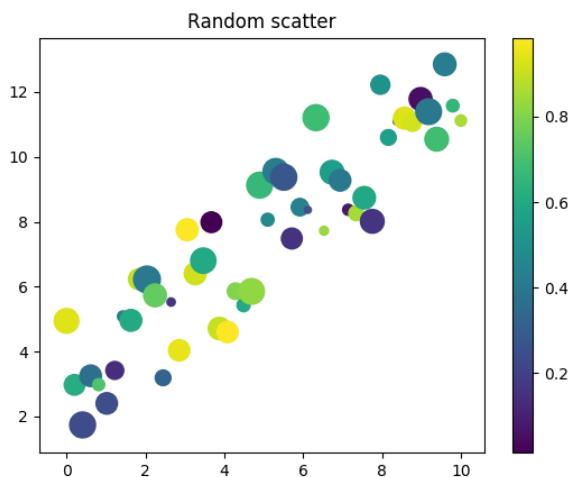
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Scatter Plots

```
>>> N = 50 # no. of points
>>> x = np.linspace(0, 10, N)
>>> from numpy.random \
...     import rand
>>> e = rand(N)*5.0 # noise
>>> y1 = x + e

>>> areas = rand(N)*300
>>> plt.scatter(x, y1, s=areas)
>>> colors = rand(N)
>>> plt.scatter(x, y1, s=areas,
...               c=colors)
>>> plt.colorbar()
>>> plt.title("Random scatter")
```



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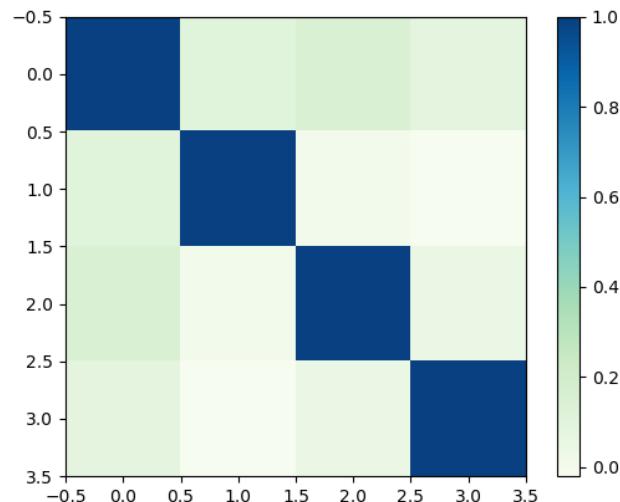
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Image "Plots"

```
>>> # Create some data
>>> e1 = rand(100)
>>> e2 = rand(100)*2
>>> e3 = rand(100)*10
>>> e4 = rand(100)*100
>>> corrmatrix =
...     np.corrcoef([e1, e2,
...                  e3, e4])
```

>>> # Plot corr matrix as image

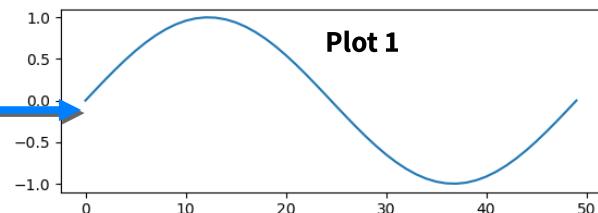
```
>>> plt.imshow(corrmatrix,
...             interpolation='none',
...             cmap='GnBu')
>>> plt.colorbar()
```



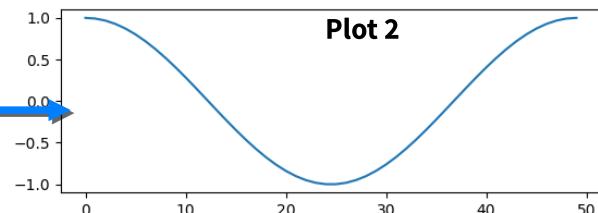
Multiple Plots Using subplot

```
>>> t = np.linspace(0, 2*np.pi)
>>> x = np.sin(t)
>>> y = np.cos(t)
```

```
# To divide the plotting area
    columns
    |
    >>> plt.subplot(2, 1, 1)
    rows      active plot
    |
    >>> plt.plot(x)
```



```
# Now activate a new plot
# area.
    >>> plt.subplot(2, 1, 2)
    >>> plt.plot(y)
```



Histogram Plots

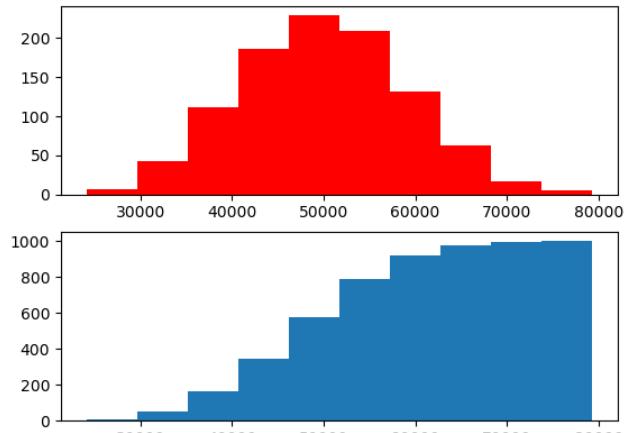
```
>>> # Create array of data
>>> from numpy.random import \
...     randint
>>> data = randint(10000,
...     size=(10,1000))

>>> # Approx norm distribution
>>> x = np.sum(data, axis=0)

>>> # Set up for stacked plots
>>> plt.subplot(2,1,1)
>>> plt.hist(x, color='r')

>>> # Plot cumulative dist
>>> plt.subplot(2,1,2)
>>> plt.hist(x, cumulative=True)

>>> # For multiple histograms:
>>> plt.hist([d1, d2, ...])
```



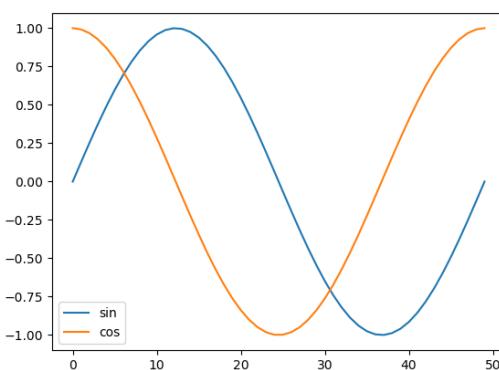
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Legends, Titles, and Axis Labels

LEGEND LABELS WITH PLOT

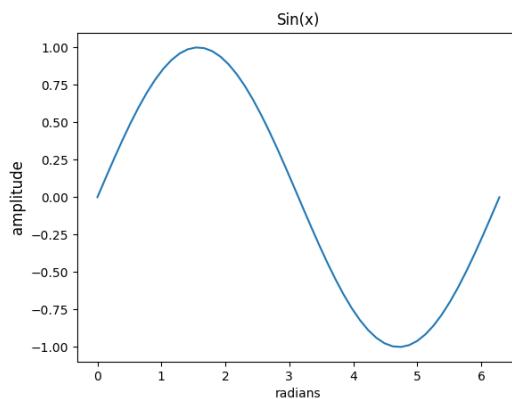
```
# Add labels in plot command.
>>> plt.plot(np.sin(t),
...           label='sin')
>>> plt.plot(np.cos(t),
...           label='cos')
>>> plt.legend()
```



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TITLES AND AXIS LABELS

```
>>> plt.plot(t, np.sin(t))
>>> plt.xlabel('radians')
# Keywords set text properties.
>>> plt.ylabel('amplitude',
...             fontsize='large')
>>> plt.title('Sin(x)')
```



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Plotting from Scripts

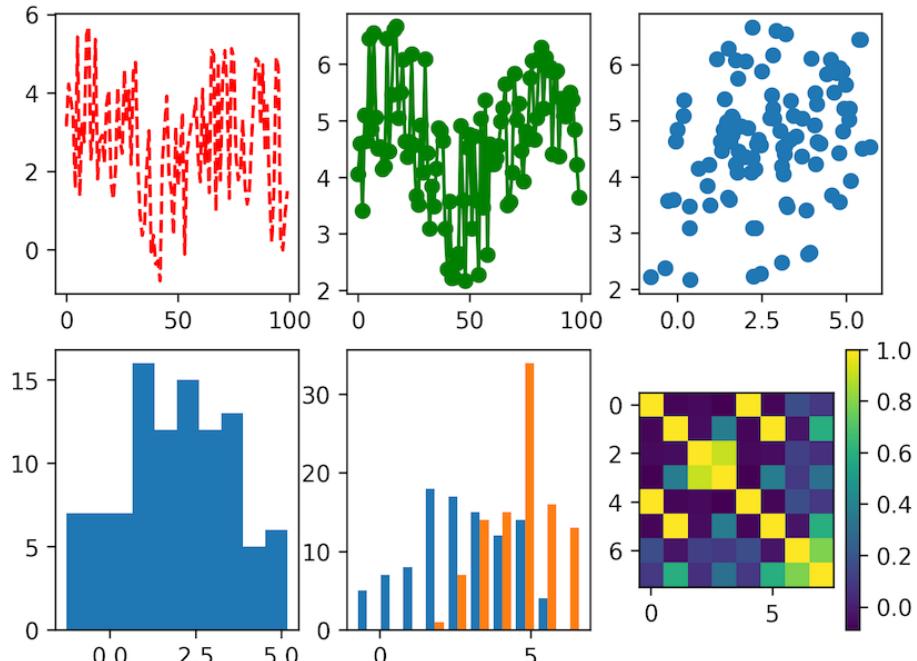
INTERACTIVE MODE

```
# In IPython, plots show up
# as soon as a plot command
# is issued:
>>> plt.figure()
>>> plt.plot(np.sin(t))
>>> plt.figure()
>>> plt.plot(np.cos(t))
```

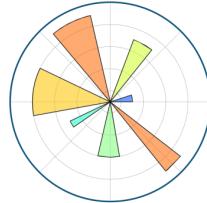
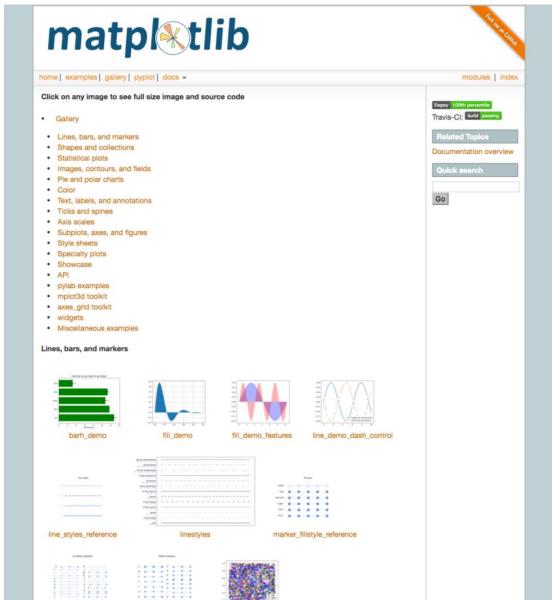
NON-INTERACTIVE MODE

```
# In a script, you must call
# the show() command to display
# plots. Call it at the end of
# all your plot commands for
# best performance.
import matplotlib.pyplot as plt
import numpy as np
t = np.linspace(0,2*np.pi,
...                           50)
plt.figure()
plt.plot(np.sin(t))
plt.figure()
plt.plot(np.cos(t))
# Plots will not appear until
# this command is run:
plt.show()
```

MPL Exercise: Desired Output



Additional Matplotlib Resources



- Simple examples with increasing difficulty
<http://matplotlib.org/examples/index.html>
- Gallery (huge)
<http://matplotlib.org/gallery.html>
- See appendix for reference materials
- Usage FAQ
http://matplotlib.org/faq/usage_faq.html

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Further Learning Online

Additional material and exercises available online:

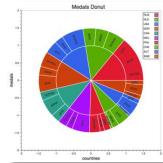
The illustration features a computer monitor in the center. On the screen, there's a presentation slide titled 'Matplotlib Basics' with the Enthought Training On Demand logo. The slide includes a circular radar chart, the word 'matplotlib', and text encouraging users to follow along with Jupyter Notebooks and work on included exercises. To the left of the monitor is a coffee cup icon with the Enthought logo. To the right are two smaller video camera icons showing two different people. The bottom of the monitor has a ribbon banner with the Enthought Training On Demand logo. The entire scene is set against a dark blue background.

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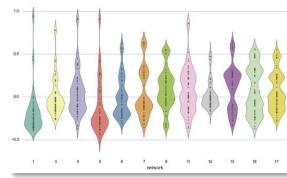
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Other Visualization Libraries

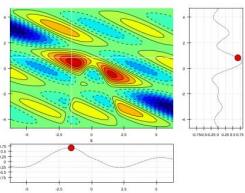
Seaborn: Better looking, high-level plots based on matplotlib



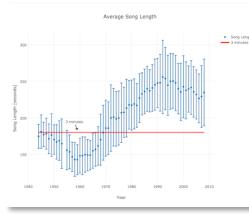
bokeh: D3-like visualization in web browsers (HTML file, or inline in IPython notebooks)



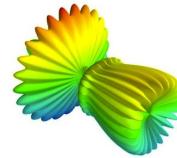
plot.ly: D3-based visualization with support for multiple languages (R, Python, Matlab, Julia and more.)



chaco: For interactive, custom plots embedded in user applications

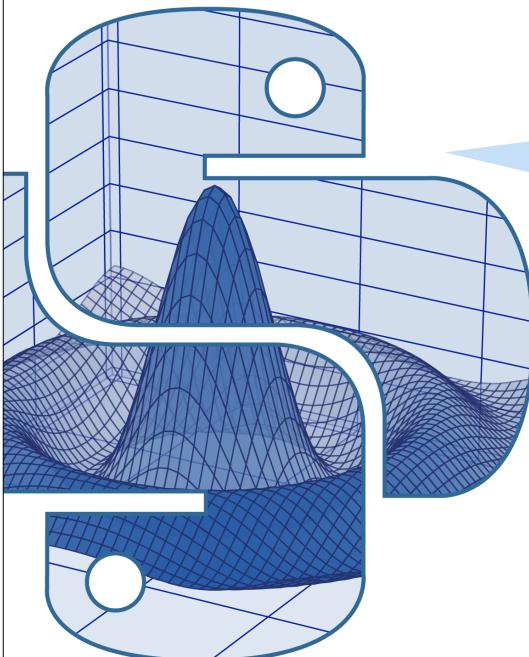


mayavi: 3D visualization, based on VTK



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a



NumPy
Introducing Arrays

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Introducing NumPy Arrays

SIMPLE ARRAY CREATION

```
>>> a = np.array([0, 1, 2, 3])
>>> a
array([0, 1, 2, 3])
```

CHECKING THE TYPE

```
>>> type(a)
numpy.ndarray
```

NUMERIC "TYPE" OF ELEMENTS

```
>>> a.dtype
dtype('int32')
```

NUMBER OF DIMENSIONS

```
>>> a.ndim
1
```

ARRAY SHAPE

```
# Shape returns a tuple
# listing the length of the
# array along each dimension.
>>> a.shape
(4,)
```

BYTES PER ELEMENT

```
>>> a.itemsize
4
```

BYTES OF MEMORY USED

```
# Return the number of bytes
# used by the data portion of
# the array.
>>> a.nbytes
16
```

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Array Operations

SIMPLE ARRAY MATH

```
>>> a = np.array([1, 2, 3, 4])
>>> b = np.array([2, 3, 4, 5])
>>> a + b
array([3, 5, 7, 9])

>>> a * b
array([ 2,  6, 12, 20])

>>> a ** b
array([ 1,  8,  81, 1024])
```



NumPy defines these constants:
 $\pi = 3.14159265359$
 $e = 2.71828182846$

MATH FUNCTIONS

```
# create array from 0. to 10.
>>> x = np.arange(11.)

# multiply entire array by
# scalar value
>>> c = (2 * np.pi) / 10.
>>> c
0.62831853071795862
>>> c * x
array([ 0., 0.628, ..., 6.283])

# in-place operations
>>> x *= c
>>> x
array([ 0., 0.628, ..., 6.283])

# apply functions to array
>>> y = np.sin(x)
```

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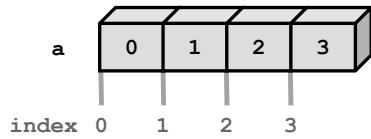
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Setting Array Elements

ARRAY INDEXING

```
>>> a[0]
```

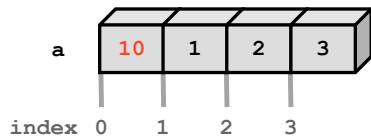
0



```
>>> a[0] = 10
```

```
>>> a
```

array([10, 1, 2, 3])



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BEWARE OF TYPE COERCION

```
>>> a.dtype
```

dtype('int32')

assigning a float into
an int32 array truncates
the decimal part

```
>>> a[0] = 10.6
```

```
>>> a
```

array([10, 1, 2, 3])

fill has the same behavior

```
>>> a.fill(-4.8)
```

```
>>> a
```

array([-4, -4, -4, -4])

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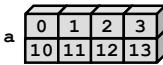
Multi-Dimensional Arrays

MULTI-DIMENSIONAL ARRAYS

```
>>> a = np.array([[ 0,  1,  2,  3],  
...                 [10, 11, 12, 13]])
```

```
>>> a  
array([[ 0,  1,  2,  3],  
       [10, 11, 12, 13]])
```

2D Array



SHAPE = (ROWS, COLUMNS)

```
>>> a.shape
```

dim 1 → dim 0 ↓

(2, 4)

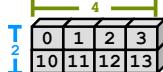


ELEMENT COUNT

```
>>> a.size
```

2 × 4 = 8 a

8

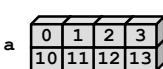


NUMBER OF DIMENSIONS

```
>>> a.ndim
```

dim 1 → dim 0 ↓

2



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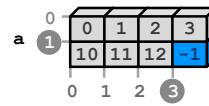
GET / SET ELEMENTS

```
>>> a[1, 3]
```

↑ ↑
 column
 row

```
>>> a[1, 3] = -1
```

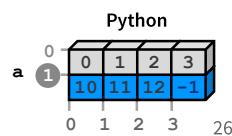
```
>>> a  
array([[ 0,  1,  2,  3],  
       [10, 11, 12, -1]])
```



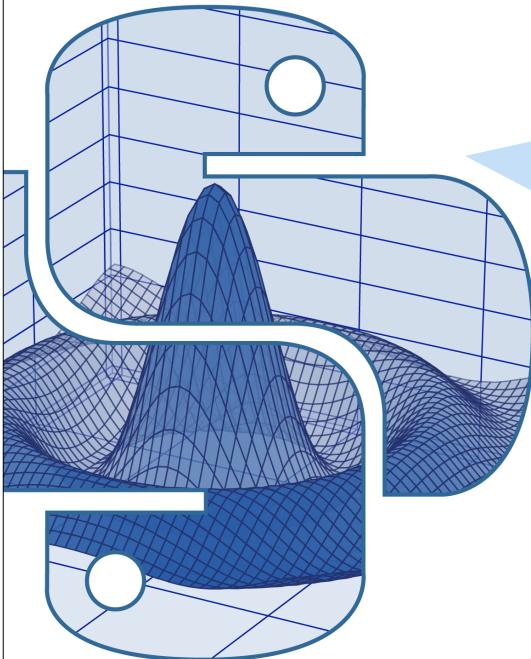
ADDRESS SECOND (ONETH) ROW USING SINGLE INDEX

```
>>> a[1]
```

array([10, 11, 12, -1])



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0	1	2	3	4	5
10	11	12	13	14	15
20	21	22	23	24	25
30	31	32	33	34	35
40	41	42	43	44	45
50	51	52	53	54	55

NumPy

Indexing and Slicing

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Slicing

var[lower:upper:step]

Extracts a portion of a sequence by specifying a lower and upper bound.

The lower-bound element is included, but the upper-bound element is **not** included.

Mathematically: [lower, upper). The step value specifies the stride between elements.

SLICING ARRAYS

```
#           -5 -4 -3 -2 -1
# indices:      0  1  2  3  4
>>> a = np.array([10,11,12,13,14])

# [10, 11, 12, 13, 14]
>>> a[1:3]
array([11, 12])

# negative indices work also
>>> a[1:-2]
array([11, 12])
>>> a[-4:3]
array([11, 12])
```

OMMITTING INDICES

```
# omitted boundaries are
# assumed to be the beginning
# (or end) of the list

# grab first three elements
>>> a[:3]
array([10, 11, 12])

# grab last two elements
>>> a[-2:]
array([13, 14])

# every other element
>>> a[::2]
array([10, 12, 14])
```

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Array Slicing

SLICING WORKS MUCH LIKE
STANDARD PYTHON SLICING

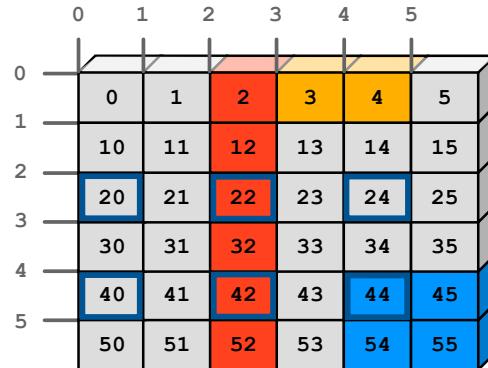
```
>>> a[0, 3:5]
array([3, 4])

>>> a[4:, 4:]
array([[44, 45],
       [54, 55]])

>>> a[:, 2]
array([2, 12, 22, 32, 42, 52])
```

STRIDED ARE ALSO POSSIBLE

```
>>> a[2::2, ::2]
array([[20, 22, 24],
       [40, 42, 44]])
```



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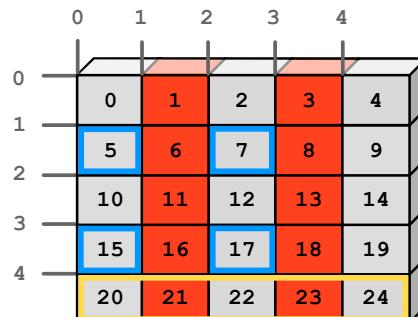
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Give it a try!

Create the array below with the command

```
a = np.arange(25).reshape(5, 5)
```

and extract the slices as indicated.



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Slices are References

Slices are references to memory in the original array.
Changing values in a slice also changes the original array.

```
>>> a = np.array((0, 1, 2, 3, 4))
# create a slice containing only the last
# element of a
>>> b = a[2:4]
>>> b
array([2, 3])
>>> b[0] = 10

# changing b changed a!
>>> a
array([ 0,  1, 10,  3,  4])
```

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Fancy Indexing

INDEXING BY POSITION

```
>>> a = np.arange(0, 80, 10)

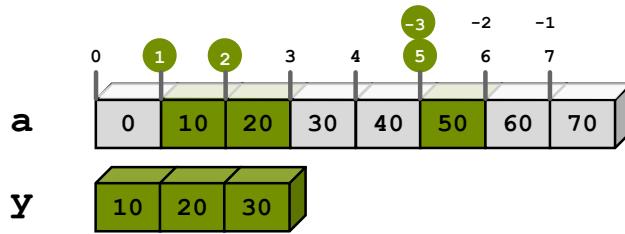
# fancy indexing
>>> indices = [1, 2, -3]
>>> y = a[indices]
>>> print(y)
[10 20 50]
```

INDEXING WITH BOOLEANS

```
# manual creation of masks
>>> mask = np.array(
...     [0, 1, 1, 0, 0, 1, 0, 0],
...     dtype=bool)

# conditional creation of masks
>>> mask2 = a < 30

# fancy indexing
>>> y = a[mask]
>>> print(y)
[10 20 50]
```



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Fancy Indexing in 2-D

```
>>> a[[0, 1, 2, 3, 4],  
...     [1, 2, 3, 4, 5]]  
array([ 1, 12, 23, 34, 45])  
  
>>> a[3:, [0, 2, 5]]  
array([[30, 32, 35],  
      [40, 42, 45],  
      [50, 52, 55]])  
  
>>> mask = np.array(  
...     [1, 0, 1, 0, 0, 1],  
...     dtype=bool)  
>>> a[mask, 2]  
array([2, 22, 52])
```

0	1	2	3	4	5
10	11	12	13	14	15
20	21	22	23	24	25
30	31	32	33	34	35
40	41	42	43	44	45
50	51	52	53	54	55



Unlike slicing, fancy indexing creates copies instead of a view into original array.

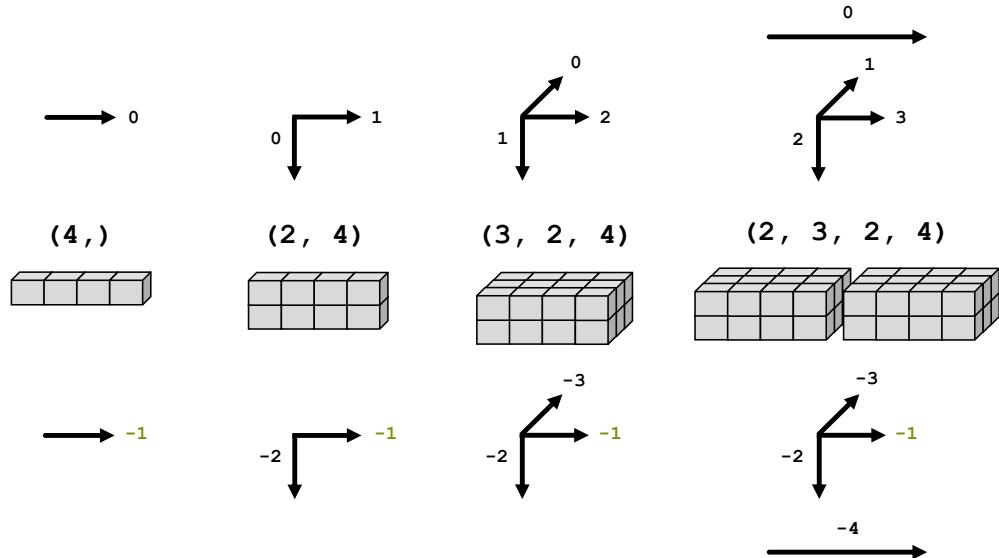
Give it a try!

1. Create the array below with
 $a = np.arange(25).reshape(5, 5)$
 and extract the elements indicated in blue.
2. Extract all the numbers divisible by 3 using a boolean mask.

0	1	2	3	4
0	1	2	3	4
5	6	7	8	9
10	11	12	13	14
15	16	17	18	19
20	21	22	23	24

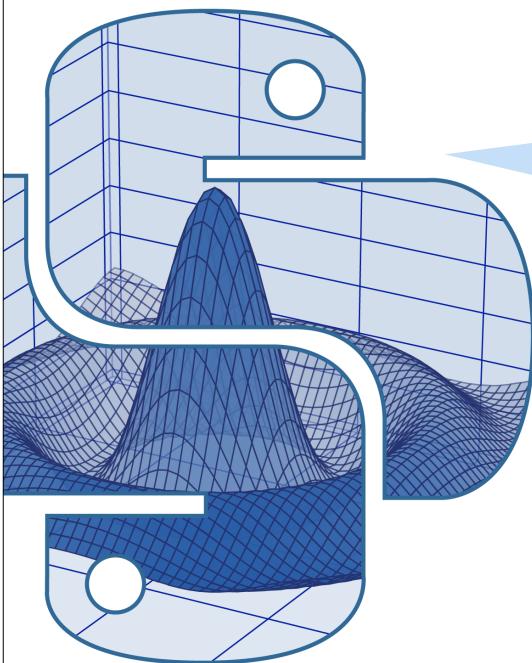
Multi-Dimensional Arrays

VISUALIZING MULTI-DIMENSIONAL ARRAYS



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`arange()`
`linspace()`
`array()`
`zeros()`
`ones()`

NumPy
 Creating Arrays

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Array Constructor Examples

FLOATING POINT ARRAYS

```
# Default to double precision
>>> a = np.array([0,1.0,2,3])
>>> a.dtype
dtype('float64')
>>> a.nbytes
32
```

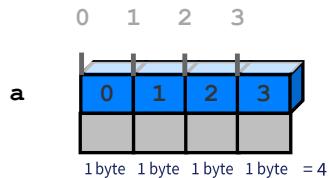
REDUCING PRECISION

```
>>> a = np.array([0,1.,2,3],
...               dtype='float32')
>>> a.dtype
dtype('float32')
>>> a.nbytes
16
```

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UNSIGNED INTEGER BYTE

```
>>> a = np.array([0,1,2,3],
...               dtype='uint8')
>>> a.dtype
dtype('uint8')
>>> a.nbytes
4
```



Base 2	Base 10
00000000	$\rightarrow 0 = 0 \cdot 2^{**0} + 0 \cdot 2^{**1} + \dots + 0 \cdot 2^{**7}$
00000001	$\rightarrow 1 = 1 \cdot 2^{**0} + 0 \cdot 2^{**1} + \dots + 0 \cdot 2^{**7}$
00000010	$\rightarrow 2 = 0 \cdot 2^{**0} + 1 \cdot 2^{**1} + \dots + 0 \cdot 2^{**7}$
...	...
11111111	$\rightarrow 255 = 1 \cdot 2^{**0} + 1 \cdot 2^{**1} + \dots + 1 \cdot 2^{**7}$

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Array Creation Functions

ARANGE

```
arange([start[, stop[, step]],  
       dtype=None])
```

Nearly identical to Python's `range()`. Creates an array of values in the range [start,stop) with the specified step value. Allows non-integer values for start, stop, and step. Default `dtype` is derived from the start, stop, and step values.

```
>>> np.arange(4)
array([0, 1, 2, 3])
>>> np.arange(0, 2*pi, pi/4)
array([ 0.000,  0.785,  1.571,
2.356,  3.142,  3.927,  4.712,
5.497])

# Be careful...
>>> np.arange(1.5, 2.1, 0.3)
array([ 1.5,  1.8,  2.1])
```

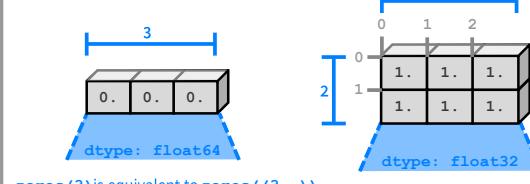
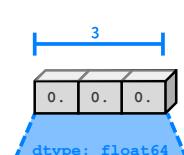
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ONES, ZEROS

```
ones(shape, dtype='float64')
zeros(shape, dtype='float64')
```

`shape` is a number or sequence specifying the dimensions of the array. If `dtype` is not specified, it defaults to `float64`.

```
>>> np.ones((2, 3),  
           dtype='float32')  
array([[ 1.,  1.,  1.],  
      [ 1.,  1.,  1.]],  
      dtype=float32)  
>>> np.zeros(3)  
array([ 0.,  0.,  0.])
```



`zeros(3)` is equivalent to `zeros((3,))`

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Array Creation Functions (cont'd)

IDENTITY

```
# Generate an n by n identity
# array. The default dtype is
# float64.
>>> a = np.identity(4)
>>> a
array([[ 1.,  0.,  0.,  0.],
       [ 0.,  1.,  0.,  0.],
       [ 0.,  0.,  1.,  0.],
       [ 0.,  0.,  0.,  1.]])
>>> a.dtype
dtype('float64')
>>> np.identity(4, dtype=int)
array([[ 1,  0,  0,  0],
       [ 0,  1,  0,  0],
       [ 0,  0,  1,  0],
       [ 0,  0,  0,  1]])
```

EMPTY AND FILL

```
# empty(shape, dtype=float64,
#       order='C')
>>> a = np.empty(2)
>>> a
array([1.78021120e-306,
       6.95357225e-308])

# fill array with 5.0
>>> a.fill(5.0)
array([5.,  5.])

# alternative approach
# (slightly slower)
>>> a[:] = 4.0
array([4.,  4.])
```

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Array Creation Functions (cont'd)

LINSPACE

```
# Generate N evenly spaced
# elements between (and including)
# start and stop values.
>>> np.linspace(0, 1, 5)
array([0., 0.25, 0.5, 0.75, 1.0])
```

ARRAYS FROM/TO TXT FILES

Data.txt

```
-- BEGINNING OF THE FILE
% Day, Month, Year, Skip, Avg Power
01, 01, 2000, x876, 13 % crazy day!
% we don't have Jan 03rd
04, 01, 2000, xfed, 55
```

LOGSPACE

```
# Generate N evenly spaced
# elements on a log scale
# between base**start and
# base**stop (default base=10.
>>> np.logspace(0, 1, 5)
array([1., 1.77, 3.16, 5.62, 10.])
```

```
# loadtxt() automatically
# generates an array from the
# txt file
arr = np.loadtxt('Data.txt',
...                 skiprows=1,
...                 dtype=int, delimiter=",",
...                 usecols = (0,1,2,4),
...                 comments = "%")

# Save an array into a txt file
np.savetxt('filename', arr)
```

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“Structured” Arrays

```
>>> import numpy as np
# "Data structure" (dtype) that describes the fields and
# type of the items in each array element.
>>> particle_dtype = np.dtype([('mass','float32'),
...                           ('velocity', 'float32')])
# This must be a list of tuples.
>>> particles = np.array([(1,1), (1,2), (2,1), (1,3)],
...                      dtype=particle_dtype)
>>> print(particles)
[(1.0, 1.0) (1.0, 2.0) (2.0, 1.0) (1.0, 3.0)]
# Retrieve the mass for all particles through indexing.
>>> print(particles['mass'])
[ 1.  1.  2.  1.]
# Retrieve particle 0 through indexing.
>>> particles[0]
(1.0, 1.0)
# Sort particles in place, with velocity as the primary field and
# mass as the secondary field.
>>> particles.sort(order=('velocity','mass'))
>>> print(particles)
[(1.0, 1.0) (2.0, 1.0) (1.0, 2.0) (1.0, 3.0)]
# See demo/multitype_array/particle.py.
```

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“Structured” Arrays

Elements of an array can be any fixed-size data structure!

EXAMPLE

```
name  char[10]
age   int
weight double
```

Brad	Jane	John	Fred
33	25	47	54
135.0	105.0	225.0	140.0
Henry	George	Brian	Amy
29	61	32	27
154.0	202.0	137.0	187.0
Ron	Susan	Jennifer	Jill
19	33	18	54
188.0	135.0	88.0	145.0

```
# structured data format
>>> fmt = np.dtype([('name', 'S10'),
...                  ('age', int),
...                  ('weight', float)
... ])
>>> a = np.empty((3,4), dtype=fmt)
>>> a.itemsize
22
>>> a['name'] = [['Brad', ... , 'Jill']]
>>> a['age'] = [[33, ... , 54]]
>>> a['weight'] = [[135, ... , 145]]
>>> print(a)
[[('Brad', 33, 135.0)
...
('Jill', 54, 145.0)]]
```

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Nested Datatype

nested.dat

Time	Size	Position			Gain	Samples (2048) ...				
		Az	EI	Type						
1172581077060	4108	0.715594	-0.148407	1	4	40	561	1467	997	-30
1172581077091	4108	0.706876	-0.148407	1	4	40	7	591	423	
1172581077123	4108	0.698157	-0.148407	1	4	40	49	-367	-565	-35
1172581077153	4108	0.689423	-0.148407	1	4	40	-55	-953	-1151	-30
1172581077184	4108	0.680683	-0.148407	1	4	40	-719	-1149	-491	38
1172581077215	4108	0.671956	-0.148407	1	4	40	-1503	-683	661	148
1172581077245	4108	0.663232	-0.148407	1	4	40	-2731	-281	2327	291
1172581077276	4108	0.654511	-0.148407	1	4	40	-3493	-159	3277	380
1172581077306	4108	0.645787	-0.148407	1	4	40	-3255	-247	3145	385
1172581077339	4108	0.637058	-0.148407	1	4	40	-2303	-101	2079	247
1172581077370	4108	0.628321	-0.148407	1	4	40	-1495	-553	571	107
1172581077402	4108	0.619599	-0.148407	1	4	40	-955	-1491	-1207	-25
1172581077432	4108	0.61087	-0.148407	1	4	40	-875	-3009	-2987	-93
1172581077463	4108	0.602148	-0.148407	1	4	40	-491	-3681	-4193	-175
1172581077497	4108	0.593438	-0.148407	1	4	40	167	-3501	-4573	-250
1172581077547	4108	0.584696	-0.148407	1	4	40	1007	-2613	-4463	-303
1172581077599	4108	0.575972	-0.148407	1	4	40	1261	-2155	-4299	-339
1172581077650	4108	0.567244	-0.148407	1	4	40	1537	-2633	-4945	-367
1172581077700	4108	0.559511	-0.148407	1	4	40	1105	-2701	-6120	420

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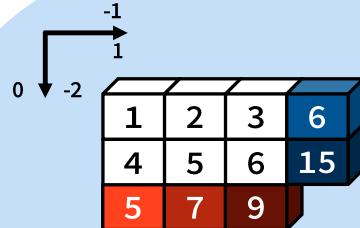
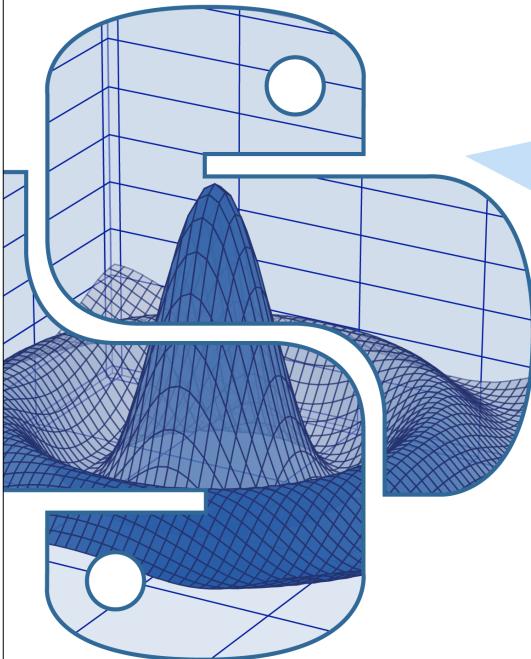
Nested Datatype (cont'd)

The data file can be extracted with the following code:

```
>>> dt = np.dtype([('time', 'uint64'),
...                 ('size', 'uint32'),
...                 ('position', [('az', 'float32'),
...                               ('el', 'float32'),
...                               ('region_type', 'uint8'),
...                               ('region_ID', 'uint16')]),
...                 ('gain', 'uint8'),
...                 ('samples', 'int16', 2048)])

>>> data = np.loadtxt('nested.dat', dtype=dt, skiprows=2)
>>> data['position']['az']
array([ 0.71559399,  0.70687598,  0.69815701,  0.68942302,
       0.68068302, ...], dtype=float32)
```

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NumPy

Array Calculation Methods

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Computations with Arrays

- Rule 1:** Operations between multiple array objects are first checked for proper shape match.
- Rule 2:** Mathematical operators (+ - * / exp, log, ...) apply element by element, on the values.
- Rule 3:** Reduction operations (mean, std, skew, kurt, sum, prod, ...) apply to the whole array, unless an axis is specified.
- Rule 4:** Missing values propagate unless explicitly ignored (nanmean, nansum, ...).

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Array Calculation Methods

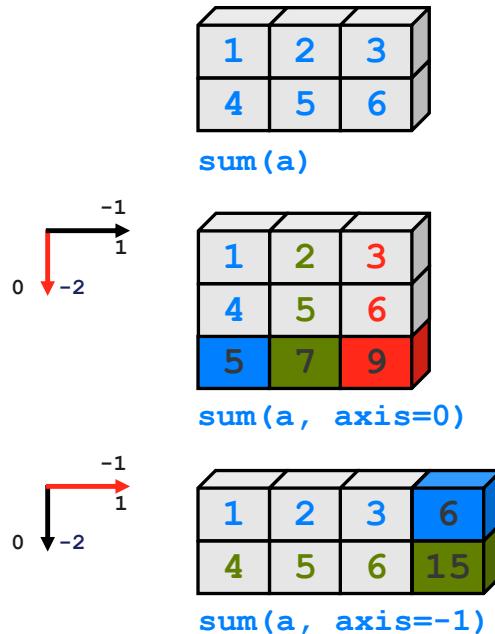
SUM METHOD

```
# Methods act on data stored
# in the array
>>> a = np.array([[1,2,3],
   [4,5,6]])

# .sum() defaults to adding up
all the values in an array.
>>> a.sum()
21

# supply the keyword axis to
# sum along the 0th axis
>>> a.sum(axis=0)
array([5, 7, 9])

# supply the keyword axis to
# sum along the last axis
>>> a.sum(axis=-1)
array([ 6, 15])
```



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Other Operations on Arrays

SUM FUNCTION

```
# Functions work on data
# passed to it
>>> a = np.array([[1,2,3],
   [4,5,6]])

# sum() defaults to adding
# up all values in an array.
>>> np.sum(a)
21

# supply an axis argument to
# sum along a specific axis
>>> np.sum(a, axis=0)
array([5, 7, 9])
```

OTHER METHODS AND FUNCTIONS

Mathematical functions

- `sum`, `prod`
- `min`, `max`, `argmin`, `argmax`
- `ptp` (`max` - `min`)

Statistics

- `mean`, `std`, `var`

Truth value testing

- `any`, `all`

See the Numpy appendix for more.

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Min/Max

MIN

```
>>> a = np.array([2.,3.,0.,1.])
>>> a.min(axis=0)
0.0
# Use NumPy's min() instead
# of Python's built-in min()
# for speedy operations on
# multi-dimensional arrays.
>>> np.min(a, axis=0)
0.0
```

ARGMIN

```
# Find index of minimum value.
>>> a.argmin(axis=0)
2
# as a function
>>> np.argmax(a, axis=0)
2
```

MAX

```
>>> a = np.array([2.,3.,0.,1.])
>>> a.max(axis=0)
3.0
# as a function
>>> np.max(a, axis=0)
3.0
```

ARGMAX

```
# Find index of maximum value.
>>> a.argmax(axis=0)
1
# as a function
>>> np.argmax(a, axis=0)
1
```

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Statistics Array Methods

MEAN

```
>>> a = np.array([[1,2,3],
...                 [4,5,6]])

# mean value of each column
>>> a.mean(axis=0)
array([ 2.5,  3.5,  4.5])
>>> np.mean(a, axis=0)
array([ 2.5,  3.5,  4.5])
```

STANDARD DEV./VARIANCE

```
# Standard Deviation
>>> a.std(axis=0)
array([ 1.5,  1.5,  1.5])
# For sample, set ddof=1
>>> a.std(axis=0, ddof=1)
array([ 1.22,  1.22,  1.22])

# variance
>>> a.var(axis=0)
array([2.25, 2.25, 2.25])
>>> np.var(a, axis=0)
array([2.25, 2.25, 2.25])
```

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Give it a try!

Create the array below with

```
a = np.arange(-15, 15).reshape(5, 6) ** 2
```

and compute:

1. The maximum of each row (one max per row)
2. The mean of each column (one mean per column)
3. The position of the overall minimum (requires 2-3 steps)

	0	1	2	3	4	5
0	225	196	169	144	121	100
1	81	64	49	36	25	16
2	9	4	1	0	1	4
3	9	16	25	36	49	64
4	81	100	121	144	169	196

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Array Broadcasting

NumPy arrays of different dimensionality can be combined in the same expression. Arrays with smaller dimension are **broadcasted** to match the larger arrays, *without copying data*. Broadcasting has **two rules**.

RULE 1: PREPEND ONES TO SMALLER ARRAYS' SHAPE

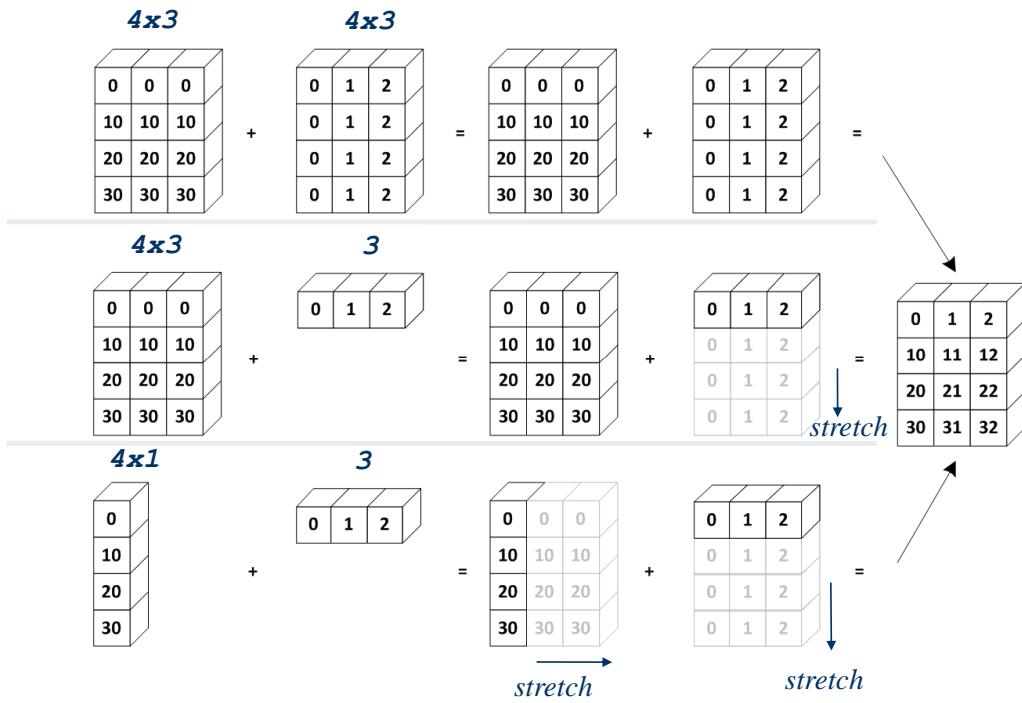
```
>>> import numpy as np
>>> a = np.ones((3, 5)) # a.shape == (3, 5)
>>> b = np.ones((5,)) # b.shape == (5,)
>>> b.reshape(1, 5) # result is a (1,5)-shaped array.
>>> b[np.newaxis, :] # equivalent, more concise.
```

RULE 2: DIMENSIONS OF SIZE 1 ARE REPEATED WITHOUT COPYING

```
>>> c = a + b # c.shape == (3, 5)
# is logically equivalent to...
>>> tmp_b = b.reshape(1, 5)
>>> tmp_b_repeat = tmp_b.repeat(3, axis=0)
>>> c = a + tmp_b_repeat
# But broadcasting makes no copies of "b"s data!
```

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Array Broadcasting

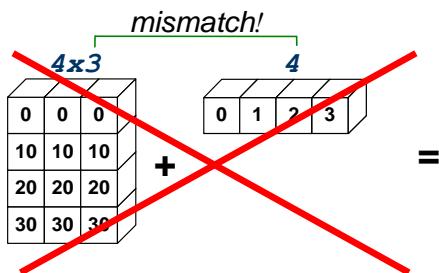


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Broadcasting Rules

The *trailing axes* of either arrays must be 1 or both must have the same size for broadcasting to occur. Otherwise,

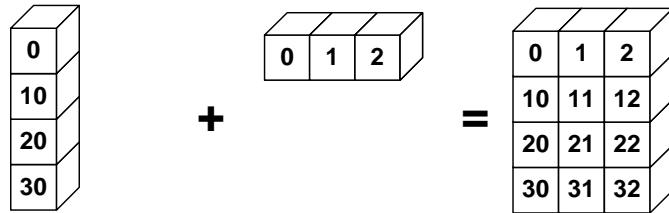
"`ValueError: shape mismatch: objects cannot be broadcast to a single shape`" exception is thrown.



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Broadcasting in Action

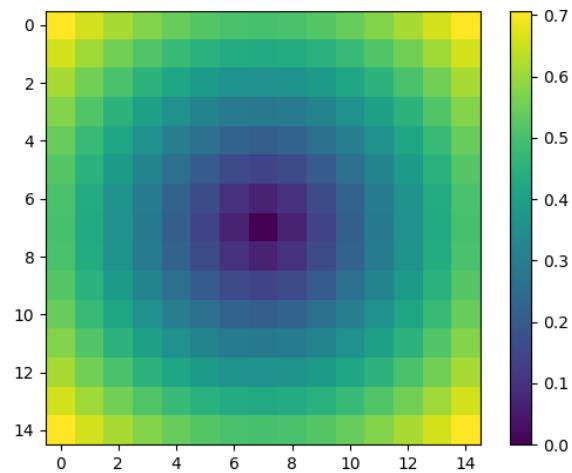
```
>>> a = np.array((0, 10, 20, 30))
>>> b = np.array((0, 1, 2))
>>> y = a[:, np.newaxis] + b
```



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Application: Distance from Center

```
>>> import matplotlib.pyplot as plt
>>> a = np.linspace(0, 1, 15) - 0.5
>>> b = a[:, np.newaxis] # b.shape == (15, 1)
>>> dist2 = a**2 + b**2 # broadcasting sum.
>>> dist = np.sqrt(dist2)
>>> plt.imshow(dist)
>>> plt.colorbar()
```



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Broadcasting's Usefulness

Broadcasting can often be used to replace needless data replication inside a NumPy array expression.

`np.meshgrid()` – use `newaxis` appropriately in broadcasting expressions.

`np.repeat()` – broadcasting makes repeating an array along a dimension of size 1 unnecessary.

MESHGRID: COPIES DATA

```
>>> x, y = \
...     np.meshgrid([1,2], \
...                 [3,4,5]) \
>>> z = x + y
```

BROADCASTING: NO COPIES

```
>>> x = np.array([1, 2])
>>> y = np.array([3, 4, 5])
>>> z = x[:, np.newaxis] + y[:, np.newaxis]
```

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Broadcasting Indices

Broadcasting can also be used to slice elements from different “depths” in a 3-D (or any other shape) array. This is a *very* powerful feature of indexing.

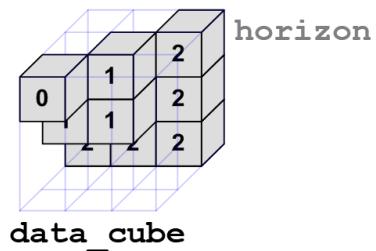
```
>>> yi, xi = np.meshgrid(np.arange(3), np.arange(3),
...                         sparse=True)
>>> zi = np.array([[0, 1, 2],
...                  [1, 1, 2],
...                  [2, 2, 2]])
>>> horizon = data_cube[xi, yi, zi]
```

Indices

Selected Data

	yi	0	1	2
xi	0	0	1	2
	1	1	1	2
	2	2	2	2

zi



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Universal Function Methods

The mathematical, comparative, logical, and bitwise operators *op* that take two arguments (binary operators) have special methods that operate on arrays:

```
op.reduce(a, axis=0)
op.accumulate(a, axis=0)
op.outer(a, b)
op.reduceat(a, indices)
```

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op.reduce()

op.reduce(a) applies *op* to all the elements in a 1-D array **a** reducing it to a single value.

For example:

$$\begin{aligned} y &= \text{add.reduce}(a) \\ &= \sum_{n=0}^{N-1} a[n] \\ &= a[0] + a[1] + \dots + a[N-1] \end{aligned}$$

ADD EXAMPLE

```
>>> a = np.array([1, 2, 3, 4])
>>> np.add.reduce(a)
10
```

STRING LIST EXAMPLE

```
>>> a = np.array(
    ['ab', 'cd', 'ef'],
    dtype='object')
>>> np.add.reduce(a)
'abcdef'
```

LOGICAL OP EXAMPLES

```
>>> a = np.array([1, 1, 0, 1])
>>> np.logical_and.reduce(a)
False
>>> np.logical_or.reduce(a)
True
```

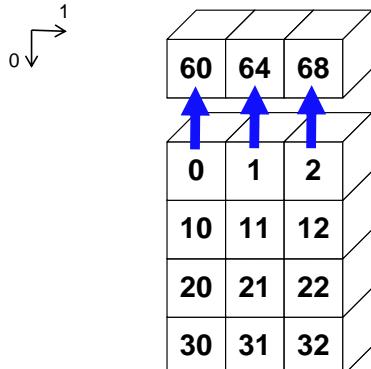
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op.reduce()

For multidimensional arrays, `op.reduce(a, axis)` applies `op` to the elements of `a` along the specified `axis`. The resulting array has dimensionality one less than `a`. The default value for `axis` is 0.

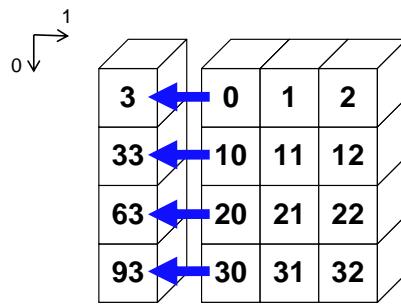
SUM COLUMNS BY DEFAULT

```
>>> np.add.reduce(a)
array([60, 64, 68])
```



SUMMING UP EACH ROW

```
>>> np.add.reduce(a, 1)
array([ 3, 33, 63, 93])
```



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op.accumulate()

`op.accumulate(a)` creates a new array containing the intermediate results of the `reduce` operation at each element in `a`.

For example:

$$y = \text{add.accumulate}(a) \\ = \left[\sum_{n=0}^0 a[n], \sum_{n=0}^1 a[n], \dots, \sum_{n=0}^{N-1} a[n] \right]$$

ADD EXAMPLE

```
>>> a = np.array([1,2,3,4])
>>> np.add.accumulate(a)
array([ 1,  3,  6, 10])
```

STRING LIST EXAMPLE

```
>>> a = np.array(
    ['ab', 'cd', 'ef'],
    dtype='object')
>>> np.add.accumulate(a)
array(['ab', 'abcd', 'abcdef'],
      dtype=object)
```

LOGICAL OP EXAMPLES

```
>>> a = np.array([True, False, True])
>>> np.logical_and.accumulate(a)
array([True, False, False])
>>> np.logical_or.accumulate(a)
array([True, True, True])
```

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op.reduceat()

`op.reduceat(a, indices)`

applies `op` to ranges in the 1-D array `a` defined by the values in `indices`. The resulting array has the same length as `indices`.

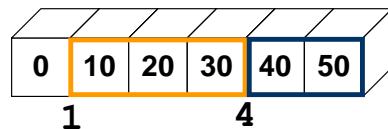
For example:

`y = add.reduceat(a, indices)`

$$y[i] = \sum_{n=indices[i]}^{indices[i+1]} a[n]$$

EXAMPLE

```
>>> a = np.array([0,10,20,30,40,50])
>>> indices = np.array([1,4])
>>> np.add.reduceat(a, indices)
array([60, 90])
```

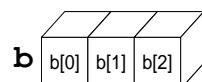
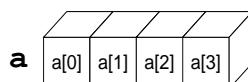


For multidimensional arrays, `reduceat()` is always applied along the *last axis* (sum of rows for 2-D arrays). This is different from the default for `reduce()` and `accumulate()`.

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op.outer()

`op.outer(a,b)` forms all possible combinations of elements between `a` and `b` using `op`. The shape of the resulting array results from concatenating the shapes of `a` and `b`. (Order matters.)



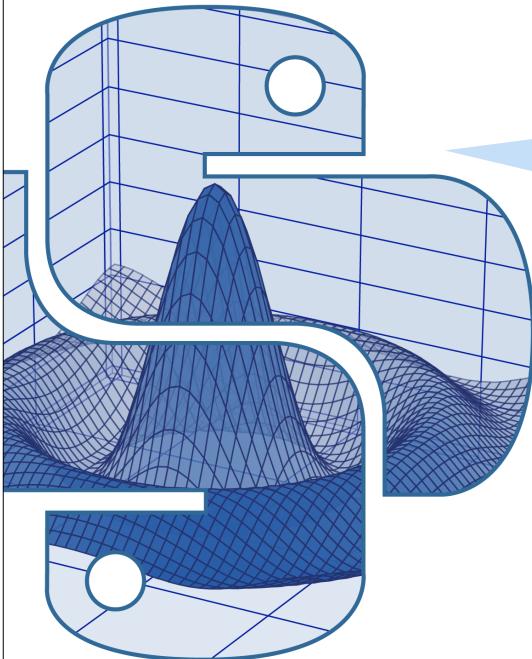
```
>>> np.add.outer(a,b)
```

a[0]+b[0]	a[0]+b[1]	a[0]+b[2]
a[1]+b[0]	a[1]+b[1]	a[1]+b[2]
a[2]+b[0]	a[2]+b[1]	a[2]+b[2]
a[3]+b[0]	a[3]+b[1]	a[3]+b[2]

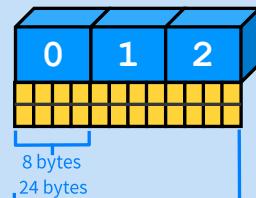
```
>>> np.add.outer(b,a)
```

b[0]+a[0]	b[0]+a[1]	b[0]+a[2]	b[0]+a[3]
b[1]+a[0]	b[1]+a[1]	b[1]+a[2]	b[1]+a[3]
b[2]+a[0]	b[2]+a[1]	b[2]+a[2]	b[2]+a[3]

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Memory Block



NumPy

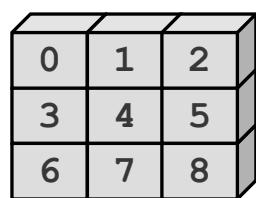
The Array Data Structure

Array Data Structure

Memory Block

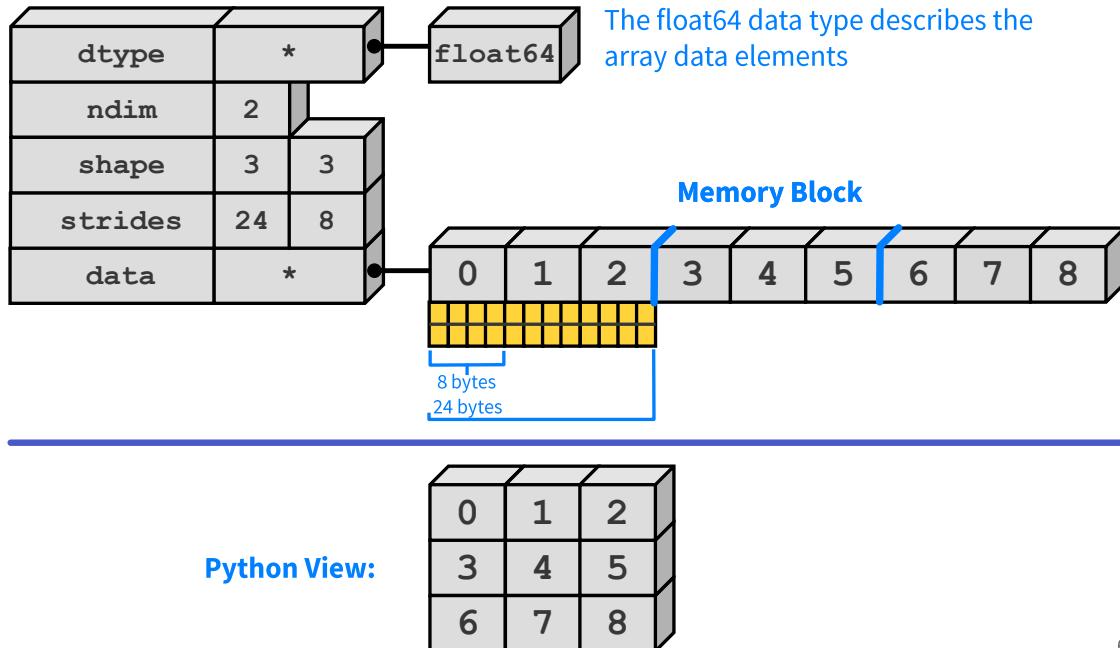


Python View:



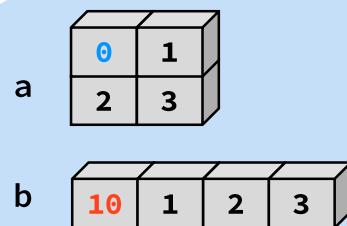
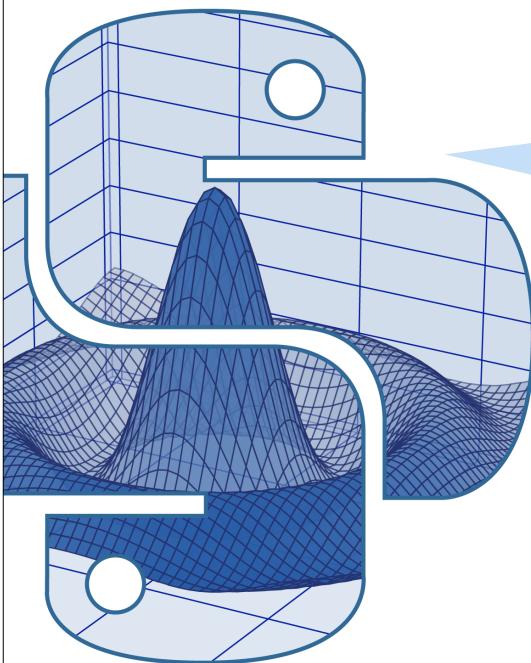
Array Data Structure

NDArray Data Structure



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NumPy
Structure Operations

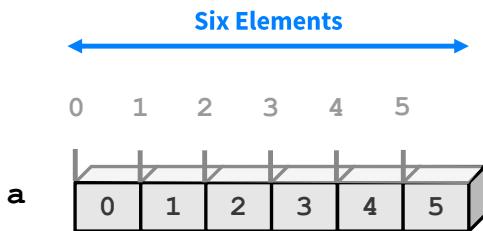
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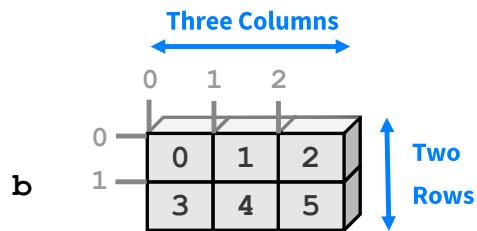
Operations on the Array Structure

Operations that only affect the array structure, not the data, can be executed without copying memory.

```
>>> a = np.arange(6)
>>> a
```



```
>>> b = a.reshape(2, 3)
>>> b
```



This is not a new array.

The original data does not get reordered.

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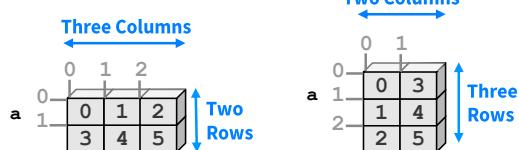
Transpose

TRANSPOSE

```
>>> a = np.array([[0,1,2],
...                 [3,4,5]])
>>> a.shape
(2, 3)

# Transpose swaps the order
# of axes.

>>> a.T
array([[0, 3],
       [1, 4],
       [2, 5]])
>>> a.T.shape
(3, 2)
```

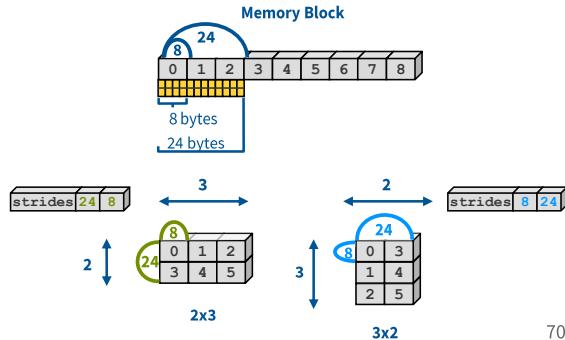


TRANSPOSE RETURNS VIEWS

```
# Transpose does not move
# values around in memory.
# It only changes the order
# of "strides" in the array

>>> a.strides
(24, 8)

>>> a.T.strides
(8, 24)
```



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Reshaping Arrays

RESHAPE

```
>>> a = np.array([[0,1,2],
...                 [3,4,5]])

# Return a new array with a
# different shape (a view
# where possible)
>>> a.reshape(3,2)
array([[0, 1],
       [2, 3],
       [4, 5]])

# Reshape cannot change the
# number of elements in an
# array
>>> a.reshape(4,2)
ValueError: total size of new
array must be unchanged
```

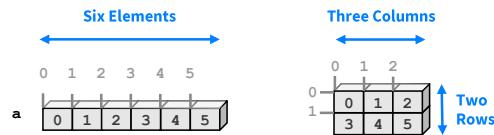
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SHAPE

```
>>> a = np.arange(6)
>>> a
array([0, 1, 2, 3, 4, 5])
>>> a.shape
(6,)

# Reshape array in-place to
# 2x3

>>> a.shape = (2,3)
>>> a
array([[0, 1, 2],
       [3, 4, 5]])
```



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Flattening Arrays

FLATTEN (SAFE)

`a.flatten()` converts a multi-dimensional array into a 1-D array. The new array is a copy of the original data.

```
# Create a 2D array
>>> a = np.array([[0,1,
...                 [2,3]])

# Flatten out elements to 1D
>>> b = a.flatten()
>>> b
array([0,1,2,3])

# Changing b does not change a
>>> b[0] = 10
>>> b
array([10,1,2,3])
>>> a
array([[0, 1],
       [2, 3]])
```

no change

b	10	1	2	3
---	----	---	---	---

a	0	1
	2	3

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RAVEL (EFFICIENT)

`a.ravel()` is the same as `a.flatten()`, but returns a reference (or view) of the array if possible (i.e., the memory is contiguous). Otherwise the new array copies the data.

```
# Flatten out elements to 1-D
>>> b = a.ravel()
>>> b
array([0,1,2,3])

# Changing b does change a
>>> b[0] = 10
>>> b
array([10,1,2,3])
>>> a
array([[10, 1],
       [2, 3]])
```

changed!

b	10	1	2	3
---	----	---	---	---

a	2	3
---	---	---

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