PHYS 250 Formula Sheet

Constants

$1\,\text{eV} = 1.602\times10^{-19}\,\text{J}$
$q = 1.602\times10^{-19}\,\text{C}$
$c = 2.998\times10^{8}\,\text{m/s}$
$h = 6.626\times10^{-34}\,\text{J s} = 4.136\times10^{-15}\,\text{eV s}$
$hc = 1240\,\text{eV nm} = 1.986\times10^{-25}\,\text{J m}$
$\hbar = \frac{h}{2\pi} = 1.055\times10^{-34}\,\text{J s} = 6.5821\times10^{-16}\,\text{eV s}$
$k = 1.381\times10^{-23}\,\text{J/K} = 8.617\times10^{-5}\,\text{eV/K}$
$m_e = 9.11\times10^{-31}\,\text{kg} = 0.511\,\text{MeV}/c^2$
$m_p = 1.673\times10^{-27}\,\text{kg} = 938.3\,\text{MeV}/c^2$

Relativity

Lorentz Transforms

Lorentz Factor	$\beta = \frac{u}{c} = \sqrt{1-\frac{1}{\gamma^2}}$	
	$\gamma = \frac{1}{\sqrt{1-\beta^2}}$	
Approximations	$\gamma \approx 1 + \frac{1}{2}\beta^2$	$(\beta \ll 1)$
	$\beta \approx 1 - \frac{1}{2}\gamma^{-2}$	$(\gamma \gg 1)$
Transforms	$x' = \gamma(x - \beta ct)$	$ct' = \gamma(ct - \beta x)$
Inverse Transforms	$x = \gamma(x' + \beta ct')$	$ct = \gamma(ct' + \beta x')$
Time Dilation	$T_{\text{moving}} = \gamma T_{\text{rest}}$	
Length Contraction	$L_{\text{moving}} = \frac{L_{\text{rest}}}{\gamma}$	
Velocity Addition	$\beta = \frac{\beta_1+\beta_2}{1+\beta_1\beta_2}$	
Doppler Effect	$f_{\text{obs}} = f_{\text{source}}\sqrt{\frac{1-\beta}{1+\beta}}$	
	$\beta > 0 \iff$ distance increasing	

Relativistic Energy-Momentum

4-Vector	$\underrightarrow{X} = (ct, x, y, z) = (ct, \vec{x})$	
Dot Product	$\underrightarrow{X_1}\cdot\underrightarrow{X_2} = (ct_1)(ct_2) - \vec{x_1}\cdot\vec{x_2}$	
Rel Mass	$M = \gamma m$	
Kinetic	$K = (\gamma - 1)mc^2$	
Rel Momentum	$p = \gamma m v$	
Rel Energy	$E = \gamma mc^2$	
	$E^2 = (pc)^2 + (mc^2)^2$	
Transforms	$E' = \gamma(E - \beta cp)$	$p' = \gamma(p - \beta cE)$
Inverse	$E = \gamma(E' + \beta cp')$	$p = \gamma(p' + \beta cE')$
4-Momentum	$\underrightarrow{P} = \left(\frac{E_{\text{rel}}}{c}, \overrightarrow{p_{\text{rel}}}\right)$	$\underrightarrow{P}^2 = (mc)^2$
CM Energy	$E_{\text{CM}} = (\underrightarrow{P_1} + \underrightarrow{P_2})^2$	
	$= m_1^2 + 2\underrightarrow{P_1}\cdot\underrightarrow{P_2} + m_2^2$	

Photons

Types of Light	UV: $100 - 400\,\text{nm}$
	Visible: $400 - 750\,\text{nm}$
	IR: $750 - 1000\,\text{nm}$
Photon Energy	$E = hf = \frac{hc}{\lambda}$
Photon Momentum	$p = \frac{E}{c} = \frac{h}{\lambda}$
RJ Blackbody	$dI_{RJ}(f) = \frac{2kTf^2}{c^2}df$
	$dI_{RJ}(\lambda) = \frac{2kTc}{\lambda^4}d\lambda$
Wien Blackbody	$dI_W(f) = \frac{2hf^3}{c^2}\exp\left(-\frac{hf}{kT}\right)df$
	$dI_W(\lambda) = \frac{2hc^2}{\lambda^5}\exp\left(-\frac{hc}{\lambda kT}\right)d\lambda$
Planck Blackbody	$dI_P(f) = \frac{2hf^3}{c^2}\frac{1}{\exp\left(\frac{hf}{kT}\right)-1}df$
	$dI_P(\lambda) = \frac{2hc^2}{\lambda^5}\frac{1}{\exp\left(\frac{hc}{\lambda kT}\right)-1}d\lambda$
Photoelectric	$qV_{\text{stop}} = E_{k,\text{max}} = hf - \phi q$
X-ray Tube	$qV_{\text{tube}}(+q\phi) = hf_{\text{max}} = \frac{hc}{\lambda_{\text{min}}}$
Compton Scattering	$\lambda' - \lambda = \frac{h}{mc}(1 - \cos\theta)$
	$\frac{h}{mc} = 2.426\,\text{pm}$
Plane Diffraction	$2d\sin(\theta_{\text{surface}}) = n\lambda$
Slit Diffraction	$d\sin(\theta_{\text{normal}}) = n\lambda$

Atoms

Rydberg	$\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$
	$R = 1.097\times10^{7}\,\text{m}^{-1}$
Photon Energy	$E = 13.6\,\text{eV}\cdot Z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$
Orbit Energy	$E = -\frac{1}{2}\frac{q^2 Z}{4\pi\epsilon_0}\frac{1}{r} = -Z\cdot\frac{0.7202\,\text{eV nm}}{r}$
Bohr Model	$L = n\hbar$
Bohr Energy	$E = -\frac{m_e}{2}\left(\frac{q^2}{4\pi\epsilon_0\hbar}\right)^2\frac{Z^2}{n^2} = -13.6\,\text{eV}\left(\frac{Z^2}{n^2}\right)$
Bohr Radius	$r = \frac{4\pi\epsilon_0\hbar^2 n^2}{Zmq^2} = 52.97\,\text{pm}\left(\frac{n^2}{Z}\right)$
Bohr Velocity	$v = \frac{Zq^2}{4\pi\varepsilon_0 n\hbar}$ $\qquad \beta = 7.293\times10^{-3}\left(\frac{Z}{n}\right)$
Moseley's Law	$E = 13.6\,\text{eV}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)(Z - b)^2$
	Kα: $b = 1$, $n_1 = 1$, $n_2 = 2$
	Kβ: $b = 1$, $n_1 = 1$, $n_2 = 3$
	Lα: $b = 7.4$, $n_1 = 2$, $n_2 = 3$
De Broglie	$p = \frac{E}{c} = \frac{hf}{c} = \frac{h}{\lambda_{\text{photon}}}$
	$\lambda_{\text{electron}} = \frac{1.227\sqrt{\text{eV}}\cdot\text{nm}}{\sqrt{E_{k,\text{electron}}}}$
	$\lambda_{\text{matter}} = \frac{h}{\sqrt{2mE_k}}$

1-D Schrodinger

Wavefunction	$\Psi(x,t)$
Equation	$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + V(x)\Psi = H_{op}\Psi$
Hamiltonian	$H_{op} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x)$
Free Particle	$\Psi(x,t) = Ae^{i(kx-\omega t)}$
	$k = \frac{2\pi}{\lambda}$ $\quad \lambda = \frac{h}{p}$ $\quad \omega = 2\pi f$ $\quad f = \frac{E}{h}$
	$p = \hbar k$ $\qquad E = \hbar\omega$
Separation	$\Psi(x,t) = \psi(x)\phi(t)$
Time-Dependent	$\phi(t) = e^{-iEt/\hbar}$
TISE	$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V(x)\psi = E\psi$
Probability	$\rho(x) = \Psi^*\Psi$ $\qquad \int_{-\infty}^{\infty}\Psi^*\Psi\,dx = 1$
Gaussian	$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{x^2}{2\sigma^2}}$
Transform	$\sigma_x\sigma_k = 1$
Uncertainty	$\sigma_x\sigma_p \geq \frac{\hbar}{2}$

Potential Step

$$\text{Wavefunctions}\quad \begin{cases} \psi_I = e^{ikx} & \text{(Incident)} \\ \psi_R = Re^{-ikx} & \text{(Reflected)} \\ \psi_T = Te^{ik'x} & \text{(Transmitted)} \end{cases}$$

Wavenumbers	$k = \frac{\sqrt{2mE}}{\hbar}$	$k' = \frac{\sqrt{2m(E-V)}}{\hbar}$
Amplitudes	$R = \frac{k-k'}{k+k'}$	$T = \frac{2k}{k+k'}$
	$R = \frac{\sqrt{E}-\sqrt{E-V}}{\sqrt{E}+\sqrt{E-V}}$	$T = \frac{2\sqrt{E}}{\sqrt{E}+\sqrt{E-V}}$
Flux	$\Phi = \rho\cdot v$	
	$\Phi_I = \frac{\hbar k}{m}$	$\Phi_I = \Phi_R + \Phi_T$
	$\Phi_R = \frac{\hbar k}{m}R^2$	$\Phi_T = \frac{\hbar k'}{m}T^2$
Probabilities	$P(R) = \frac{\Phi_R}{\Phi_I} = R^2$	$P(T) = \frac{\Phi_T}{\Phi_I} = \frac{k'}{k}T^2$

Potential Barrier

$$\text{Wavefunctions}\quad \begin{cases} \psi_I = e^{ikx} & \text{(Incident)} \\ \psi_R = Re^{-ikx} & \text{(Reflected)} \\ \psi_T = Te^{ikx} & \text{(Transmitted)} \\ \psi_F = Fe^{ik'x} & \text{(Forward)} \\ \psi_B = Be^{-ik'x} & \text{(Backward)} \end{cases}$$

$$\begin{bmatrix} -1 & 1 & 1 & 0 \\ k & k' & -k' & 0 \\ 0 & e^{ik'w} & e^{-ik'w} & -e^{ikw} \\ 0 & k'e^{ik'w} & -k'e^{-ik'w} & -ke^{ikw} \end{bmatrix}\begin{bmatrix} R \\ F \\ B \\ T \end{bmatrix} = \begin{bmatrix} 1 \\ k \\ 0 \\ 0 \end{bmatrix}$$

Tunneling $\quad P(T) \approx 16\frac{E}{V}\left(1 - \frac{E}{V}\right)e^{-2\kappa w}$

$$\kappa = \frac{\sqrt{2m(V-E)}}{\hbar}$$

Infinite Square Well

Energy $E_n = \frac{\hbar^2\pi^2}{2m}\frac{n^2}{w^2}$

$\frac{\hbar^2\pi^2}{2m_e} = 0.376\,03\,\mathrm{eV\,nm}$ (for electron)

Inside $\psi_n(x) = \sqrt{\frac{2}{w}}\sin(k_n x)$ $\quad k_n = \frac{n\pi}{w}$

Outside $\psi_n(x) = e^{\pm k' x}$ $\quad k'_n = \frac{n\pi}{w}$

Finite Potential Wells

Energy $E_n \approx \frac{\hbar^2\pi^2 n^2}{2mw^2} - V, \quad E < 0$

Inside $\psi_n(x) = A\sin(kx) + B\cos(kx)$ $\quad k = \frac{\sqrt{2mE}}{\hbar}$

Outside $\psi_n(x) = Ce^{k'x} + De^{-k'x}$ $\quad k' = \frac{\sqrt{2m(V-E)}}{\hbar}$

Quantum Harmonic Oscillator

Potential $V(x) = \frac{1}{2}k'x^2$

Energy $E_n = \left(n + \frac{1}{2}\right)\hbar\omega$ $\quad \omega = \sqrt{\frac{k'}{m}}$

Wavefunction $\psi_n(x) = H_n(\frac{x}{b})e^{-\frac{x^2}{2b^2}}$ $\quad b^2 = \frac{\hbar}{\sqrt{k'm}}$

Hermite $H_0(x) = 1$ $\quad H_1(x) = x$

$H_2(x) = x^2 - \frac{1}{2}$ $\quad H_3(x) = x^3 - \frac{3}{2}x$

3-D Schrodinger

Wavefunction $\Psi(x,y,z,t) = \Psi(\vec{x},t)$

Laplacian $\nabla^2\Psi(x,y,z) = \frac{\partial^2\Psi}{\partial x^2} + \frac{\partial^2\Psi}{\partial y^2} + \frac{\partial^2\Psi}{\partial z^2}$

Equation $i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\Psi + V(\vec{x})\Psi$

Free Particle $\Psi(\vec{x},t) = Ae^{i(\vec{k}\cdot\vec{x}-\omega t)}$

$\vec{p} = \hbar\vec{k}$ $\quad E = \hbar\omega$

Separation $\Psi(\vec{x},t) = \psi(\vec{x})\phi(t)$

Time-Dependent $\phi(t) = e^{-iEt/\hbar}$

TISE $-\frac{\hbar^2}{2m}\nabla^2\psi + V(\vec{x})\psi = E\psi$

Infinite Box Well

Energy $E_{n_x,n_y,n_z} = \left(\frac{n_x^2}{a^2} + \frac{n_y^2}{b^2} + \frac{n_z^2}{c^2}\right)\frac{\hbar^2\pi^2}{2m}$

Cube $E_{n_x,n_y,n_z} = (n_x^2 + n_y^2 + n_z^2)\frac{\hbar^2\pi^2}{2m}\frac{1}{w^2}$

Wavefunction $\psi_{\vec{n}}(x,y,z) = \sin(\frac{n_x\pi x}{a})\sin(\frac{n_y\pi y}{b})\sin(\frac{n_z\pi z}{c})$

Spherical Coordinates

$$\nabla^2\Psi = \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\frac{\partial\Psi}{\partial r}\right] + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left[\sin\theta\frac{\partial\Psi}{\partial\theta}\right] + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\Psi}{\partial\phi^2}$$

Separation $\psi(r,\theta,\phi) = F(r)G(\theta)H(\phi)$

Equations
$$\begin{cases} \frac{\partial^2 H(\phi)}{\partial\phi^2} = -\mu H(\phi) \\ \sin\theta\frac{\partial}{\partial\theta}\left[\sin\theta\frac{\partial G(\theta)}{\partial\theta}\right] = (\mu - \lambda\sin^2\theta)G(\theta) \\ -\frac{\hbar^2}{2M}\frac{\partial^2 U(r)}{\partial r^2} + \left[V(r) + \frac{\hbar^2\lambda}{2Mr^2}U(r)\right] = EU(r) \end{cases}$$

Solutions $H(\phi) = e^{im\phi}, m \in \mathbb{Z}$

$G(\theta) = P_l^m(\theta)$ (Legendre functions)

$F(r) = \frac{1}{r}U_{kl}(r)$

Harmonics $Y_l^m(\theta,\phi) = P_l^m(\theta)e^{im\phi}$

General $\psi_{klm}(r,\theta,\phi) = \frac{1}{r}U_{kl}(r)Y_l^m(\theta,\phi)$

	$\ell=0$	$\ell=1$	$\ell=2$	$\ell=3$	$\ell=4$
$m=4$					$+\sqrt{\frac{315}{512\pi}}\sin^4\theta\, e^{4i\phi}$
$m=3$				$-\sqrt{\frac{35}{64\pi}}\sin^3\theta\, e^{3i\phi}$	$-\sqrt{\frac{315}{64\pi}}\sin^3\theta\cos\theta\, e^{3i\phi}$
$m=2$			$+\sqrt{\frac{15}{32\pi}}\sin^2\theta\, e^{2i\phi}$	$+\sqrt{\frac{35}{64\pi}}\sin^2\theta\cos\theta\, e^{2i\phi}$	$+\sqrt{\frac{45}{128\pi}}\sin^2\theta\left(7\cos^2\theta-1\right)e^{2i\phi}$
$m=1$		$-\sqrt{\frac{3}{4\pi}}\sin\theta\, e^{i\phi}$	$-\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta\, e^{i\phi}$	$-\sqrt{\frac{21}{64\pi}}\sin\theta\left(5\cos^2\theta-1\right)e^{i\phi}$	$-\sqrt{\frac{45}{64\pi}}\sin\theta\left(7\cos^3\theta-3\cos\theta\right)e^{i\phi}$
$m=0$	$\sqrt{\frac{1}{4\pi}}$	$+\sqrt{\frac{3}{4\pi}}\cos\theta$	$+\sqrt{\frac{5}{16\pi}}\left(3\cos^2\theta-1\right)$	$+\sqrt{\frac{7}{16\pi}}\left(5\cos^3\theta-3\cos\theta\right)$	$+\sqrt{\frac{9}{256\pi}}\left(35\cos^4\theta-30\cos^2\theta+3\right)$
$m=-1$		$+\sqrt{\frac{3}{4\pi}}\sin\theta\, e^{-i\phi}$	$+\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta\, e^{-i\phi}$	$+\sqrt{\frac{21}{64\pi}}\sin\theta\left(5\cos^2\theta-1\right)e^{-i\phi}$	$+\sqrt{\frac{45}{64\pi}}\sin\theta\left(7\cos^3\theta-3\cos\theta\right)e^{-i\phi}$
$m=-2$			$+\sqrt{\frac{15}{32\pi}}\sin^2\theta\, e^{-2i\phi}$	$+\sqrt{\frac{35}{64\pi}}\sin^2\theta\cos\theta\, e^{-2i\phi}$	$+\sqrt{\frac{45}{128\pi}}\sin^2\theta\left(7\cos^2\theta-1\right)e^{-2i\phi}$
$m=-3$				$+\sqrt{\frac{35}{64\pi}}\sin^3\theta\, e^{-3i\phi}$	$+\sqrt{\frac{315}{64\pi}}\sin^3\theta\cos\theta\, e^{-3i\phi}$
$m=-4$					$+\sqrt{\frac{315}{512\pi}}\sin^4\theta\, e^{-4i\phi}$

Angular $l = \{0,1,2,\ldots\}$

Magnetic $m = \{\ldots,-1,0,1,\ldots\}, \quad |m| \le l$

Radial $k \in \mathbb{Z}$, starting from 0 or 1

Principal $n = k + l$ (for $\frac{1}{r}$ potential)

$\lambda = l(l+1)$

Infinite Spherical Well

Energy $E_{k0} = \frac{\hbar^2\pi^2}{2M}\frac{k^2}{R^2}$

Wavefunction $U_{k0}(r) = \sin\left(\frac{k\pi r}{R}\right)$

$\psi_{k00} = \frac{1}{r}\sin\left(\frac{k\pi r}{R}\right)Y_0^0(\theta,\phi) = \frac{1}{r}\sin\left(\frac{k\pi r}{R}\right)$

Infinite Shell

Energy $E_{kl} = \frac{\hbar^2\pi^2}{2M}\left[\frac{k^2}{\Delta R^2} + \frac{l(l+1)}{\pi^2(R+\Delta R/2)^2}\right]$

Centrifugal Term $\frac{\hbar^2 l(l+1)}{2M(R+\Delta R/2)^2} \approx \frac{\hbar^2 l(l+1)}{2MR^2} \quad (\Delta R \ll R)$

Wavefunction $U_{k0}(r) = \sin\left(\frac{k\pi(r-R)}{\Delta R}\right)$

$\psi_{k00} = \frac{1}{r}\sin\left(\frac{k\pi(r-R)}{\Delta R}\right)$

Coulomb Potential and Hydrogen

Potential $V(r) = -\frac{q^2}{4\pi\varepsilon_0 r}$

Wavefunction $\psi_{nlm}(r,\theta,\phi) = e^{-\frac{\rho}{2}}\rho^l L_{n-l-1}^{2l+1}(\rho)Y_l^m(\theta,\phi)$

$\rho = \frac{2r}{na_0}$ $\quad a_0 = 52.97\,\mathrm{pm}$

$\rho = \frac{r}{a_0}$

$U_{n\ell}(\rho)$	$\ell=0, m=0$	$\ell=1, m=0,\pm1$	$\ell=2, m=0,\pm1,\pm2$
$n=3$	$\left[\rho^3 - 9\rho^2 + \frac{27}{2}\rho\right]\cdot e^{\frac{-\rho}{3}}$	$\left[\rho^3 - 6\rho^2\right]\cdot e^{\frac{-\rho}{3}}$	$\rho^3\cdot e^{\frac{-\rho}{3}}$
$n=2$	$\left[\rho^2 - 2\rho\right]\cdot e^{\frac{-\rho}{2}}$	$\rho^2\cdot e^{\frac{-\rho}{2}}$	
$n=1$	$\rho\cdot e^{-\rho}$		

Principal $\exp\left(-\frac{r}{na_0}\right) \implies n$

Angular sin power + max cos power $\implies l$

Magnetic $\exp(im\phi) \implies m$

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