CIVL 215 Formula Sheet

Basics

Weight	$W = mg$
	$g = 9.8\,\mathrm{m/s^2}$
Density	$\rho = \frac{m}{V}$
Specific weight	$\gamma = \frac{W}{V} = \frac{mg}{V} = \rho g$
Specific gravity	$s = \frac{\rho}{\rho_{\text{water}}}$ $\quad \rho_{\text{water}} = 1000\,\mathrm{kg/m^3}$

Pressure

Pressure	$P = \lim_{\Delta A \to 0} \frac{\Delta F_n}{\Delta A}$
	$1\,\mathrm{bar} = 10^5\,\mathrm{Pa}$
Shear stress	$\tau = \lim_{\Delta A \to 0} \frac{\Delta F_t}{\Delta A}$
Gage pressure	$P_{\text{gage},A} = P_A - P_{\text{atm}}$

Hydrostatics

Fluid cube	$-\frac{\partial P}{\partial x} = \rho a_x \quad -\frac{\partial P}{\partial y} = \rho a_y \quad -\frac{\partial P}{\partial z} = \rho(a_z + g)$
	$dP = \frac{\partial P}{\partial x}dx + \frac{\partial P}{\partial y}dy + \frac{\partial P}{\partial z}dz$
	$dP = -\rho a_x dx - \rho a_y dy - \rho(a_z + g)dz$
	$\nabla P = -\rho(\vec{a} - \vec{g})$
Accelerated	$a_x > 0, a_y = a_z = 0$
	$P = -\rho a_x x - \rho g z + P_{\text{atm}}$
	$z = -\frac{a_x}{g}x$
No force	$a_x = a_y = a_z = 0$
	$dP = -\rho g dz$
	$P_2 = P_1 + \rho g h$

1st law of manometry: all points on a horizontal plane have the same pressure providing they are connected by a fluid of constant density.

Atmospheric Pressure

Hydrostatic	$\frac{dP}{dz} = -\rho_a g$
Ideal gas	$P = \rho_a R_a T$
	$R_a = \frac{R}{M_a} = \frac{8.31\,\mathrm{J/(mol\,K)}}{0.029\,\mathrm{kg/mol}} = 287\,\mathrm{J/(kg\,K)}$
Temperature	$T(z) = T_0 - \alpha z$
	$\alpha = 0.0065\,\mathrm{K/m} \quad T_0 = 15\,^\circ\mathrm{C} = 288\,\mathrm{K}$
Pressure	$P = P_0\left(\frac{T_0 - \alpha z}{T_0}\right)^{\frac{g}{\alpha R_a}}$
	$P_0 = 101.3\,\mathrm{kPa} \quad \frac{g}{\alpha R_a} = 5.25$

Hydrostatic Pressure

Planar Surfaces

Average pressure	$\bar{P} = \frac{1}{2}\rho g h$
Point	$h_p = \frac{2}{3}h$
	$M = F h_p = \frac{1}{2}\rho g h^2 W h_p$

Curved Surfaces

Use F_H, F_V, F_W, and P for moment balance.

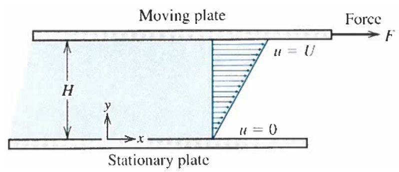
Buoyancy

Buoyancy force	$B = \rho_w g \int_0^L (h_b - h_a) W\,dx$
	$B = \rho_w g \forall$

Fluid Motion

Eulerian flow	$\vec{V} = \vec{V}(x, y, z, t)$
Steady flow	$\vec{V} = \vec{V}(x, y, z)$

Viscosity



Velocity	$u(0) = 0$
	$u(H) = U$
	$u(y) = \left(\frac{y}{H}\right)y$
Shear stress	$\tau = \mu\frac{du}{dy} = \mu\frac{U}{H}$
Force	$F = \tau A = \mu\frac{U}{H}A$
Kinematic viscosity	$\nu = \frac{\mu}{\rho}$
Reynold's number	$\mathrm{Re} = \frac{UD}{\nu} = \frac{UD\rho}{\mu}$
	$\nu_{\text{water}} = 1 \times 10^{-6}\,\mathrm{m^2/s}$
Pipes	$\mathrm{Re} \lesssim 2000$ laminar
	$\mathrm{Re} \gtrsim 2000$ turbulent

Bernoulli's Equation

Bernoulli	$\frac{p_1}{\gamma} + y_1 + \frac{v_1^2}{2g} = \frac{p_2}{\gamma} + y_2 + \frac{v_2^2}{2g}$
Torricelli	$v_2 = \sqrt{2gh}$
	$T = \sqrt{\frac{2H}{g}}\frac{1}{A_R}, \quad A_R = \frac{A_2}{A_1}$
Corrected	$V = C_v\sqrt{2gh}$
	$Q = AV = C_c A_o V$
	$Q = C_c C_v A_o \sqrt{2gh} = C_d A_o \sqrt{2gh}$

Conservation

Mass	$\frac{d}{dt}m_{\text{sys}} = 0$	$\frac{d}{dt}m_{\text{cv}} = \dot{m}_{\text{in}} - \dot{m}_{\text{out}}$
Momentum	$\sum \vec{F} = \frac{d}{dt}(m_{\text{sys}}\vec{u}) = m_{\text{sys}}\vec{a}$	

Reynold's Transport Theorem

Extensive	B
Intensive	$\beta = \frac{dB}{dm}$
RTT	$\frac{dB_{\text{sys}}}{dt} = \frac{d}{dt}\left[\int_{\text{cv}} \beta\rho\,d\forall\right] + \beta\dot{m}_{\text{out}} - \beta\dot{m}_{\text{in}}$
Mtm eqn	$\sum \vec{F} = \sum \dot{m}_{\text{out}}\vec{U}_{\text{out}} - \sum \dot{m}_{\text{in}}\vec{U}_{\text{in}}$
	$\sum \vec{F} = \rho Q(\vec{U}_2 - \vec{U}_1)$
	$\sum \vec{F} = P_1 A_1 - P_2 A_2 - F_{\text{pipe}}$

Pipe Flow

Head loss	$h_L = h_f + h_m$
Friction loss	$h_f = f\left(\frac{L}{D}\right)\frac{v^2}{2g}$
Minor loss	$h_m = \sum K_i \frac{v^2}{2g}$
Head	$H = \left(f\frac{L}{D} + \sum K_i + 1\right)\frac{v^2}{2g}$
Speed	$v = \frac{\sqrt{2gH}}{\sqrt{1 + f\frac{L}{D} + \sum K_i}}$
EGL	$Z_{\text{EGL}} = \frac{p}{\gamma} + z + \frac{v^2}{2g}$
HGL	$Z_{\text{HGL}} = \frac{p}{\gamma} + z$

Compressibility

Bulk modulus	$B = \rho\frac{\partial P}{\partial \rho}\Big\|_T$
Sound speed	$c = \sqrt{\frac{B}{\rho}}$
Capillary	$h = \frac{2\sigma\cos\theta}{r\gamma}$

Dimensional Analysis

n variables, m basic dimensions $\Longrightarrow$ $(n - m)$ dimensionless π-groups:

$$\pi_1 = f(\pi_2, \pi_3, \ldots, \pi_{n-m})$$

Similarity

Length scale	$L_R = \frac{L_P}{L_M} = \frac{L_{\text{prototype}}}{L_{\text{model}}}$
Area scale	$A_R = L_R^2$
Volume scale	$V_R = L_R^3$
Reynolds	$\mathrm{Re} = \frac{VL}{\nu}$ (viscosity)
Froude	$\mathrm{Fr} = \frac{V}{\sqrt{gH}}$ (free surface)
Euler	$\mathrm{Eu} = \frac{\Delta P}{\rho V^2}$ (falling objects, pipe flow)
Weber	$\mathrm{W} = \frac{rhoV^2 l}{\sigma}$ (surface tension)

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