PHYS 170 Formula Sheet

Forces and Moments

Moment definition

Scalar form $M = \pm F r \sin\theta = \pm F d$

Vector form $\vec{M} = \vec{r} \times \vec{F}$ $\quad \vec{r} = \vec{P} - \vec{O}$

$$\vec{M} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

Problems

- Moment about an axis: pick any point on axis, find moment and project it onto the axis.
- Couple moment: can move freely.
- Equivalent systems: shift $\vec{F}$, add compensating $\vec{M}$.
- Wrench reduction: parametrize point (x, y). Find $\vec{F}_R$ and $\vec{M}_R$, set them to be parallel, and solve for (x, y).

Reactions

1. Draw FBD.
2. Identify supports and introduce reaction forces and moments.
3. Solve for forces and moments using equilibrium equations.

Friction

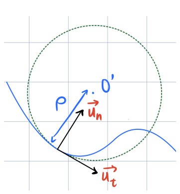
1. Draw FBD. Place N to ensure $M_R = 0$.
2. Equations vs. unknowns analysis.
3. Assume scenario. Add impending motion equations (tipping or slipping). Solve, check, and repeat if needed.

Wedge and cylinder: cannot tip.

Kinematics

$$v = \dot{s} \qquad a = \dot{v} = \ddot{s}$$

Normal-Tangential



$\vec{u}_t$ and $\vec{u}_n$

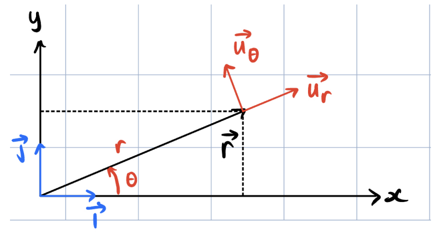
$s = s(t) \implies \vec{v} = \dot{s}\vec{u}_t$

$a_t = \dot{v} = \ddot{s}$

$a_n = \frac{v^2}{\rho}$

$a = \sqrt{a_t^2 + a_n^2}$

Polar Coordinates



$$\vec{v} = (\dot{r})\vec{u}_r + (r\dot{\theta})\vec{u}_\theta$$

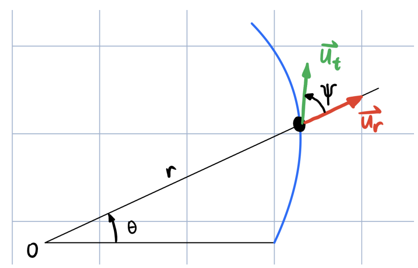
$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\vec{u}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\vec{u}_\theta$$

Constrained Motion

1. Set datums.
2. Write rope lengths and positions s.
3. Differentiate to find v.

Polar $\Leftrightarrow$ Normal-Tangential Conversion

ψ: angle from $\vec{u}_r$ to $\vec{u}_t$. Positive if CCW, negative if CW.



$$\tan\psi = \frac{r}{dr/d\theta}$$

Unify coordinate system:

1. Analyze geometry
2. Convert unit vectors
3. Convert forces

Relative motion

$$\vec{v}_{B/A} = \vec{v}_B - \vec{v}_A$$

$$\vec{a}_{B/A} = \vec{a}_B - \vec{a}_A$$

Work and Energy

Energy

Kinetic $T = \frac{mv^2}{2}$

Potential $V^{(g)} = mgh \qquad V^{(s)} = \frac{k\Delta x^2}{2}$

Work

$$U = -U_{\text{external}}$$

$$U = \int_1^2 \vec{F} \cdot d\vec{r}$$

$$U^{(g)}_{1\to2} = mg(y_1 - y_2)$$

$$U^{(s)}_{1\to2} = \frac{k}{2}(x_1^2 - x_2^2)$$

Work-Energy Principle

$$\Delta T = U_{1\to2} \quad \text{(Change in KE = external work)}$$

$$T_2 = T_1 + U_{1\to2}$$

$$T_2 + V_2 = T_1 + V_1 + U^{(\text{non-cons})}_{1\to2}$$

$$\frac{1}{2}mv_2^2 + mgy_2 + \frac{1}{2}k\Delta^2_{x_2} = \frac{1}{2}mv_1^2 + mgy_1 + \frac{1}{2}k\Delta^2_{x,1} + U^{(\text{non-cons})}_{1\to2}$$

Momentum and Impulse

$$\vec{L} = m\vec{v} \qquad \vec{I} = \int_{t_1}^{t_2} \vec{F}\,dt$$

Impulse-Momentum Principle:

$$\Delta\vec{L} = \vec{I}_{1\to2} \iff \vec{L}_2 = \vec{L}_1 + \vec{I}_{1\to2}$$

Closed system:

$$\sum m_i\vec{v}_{i,2} = \sum m_i\vec{v}_{i,1}$$