MATH 253 Formula Sheet

Vectors

Lines and Planes

Limits

Limit	$\lim\limits_{\mathbf{x}\to\mathbf{a}} f(\mathbf{x}) = L$	where $\mathbf{x}\to\mathbf{a}$ in any way.
Continuity	$\lim\limits_{\mathbf{x}\to\mathbf{a}} f(\mathbf{x}) = f(\mathbf{a})$	

Showing limit DNE

- Find one way to approach $\mathbf{a}$ such that the limit DNE.
- If $\mathbf{a} = (0,0)$, we could test $\lim\limits_{t\to 0} f(t,0)$, $\lim\limits_{t\to 0} f(0,t)$, $\lim\limits_{t\to 0} f(t,t)$, etc.

Showing limit exists

- Use polar coordinates, then take $r \to 0$. θ does not matter.
- i.e. Set $x = r\cos\theta, y = r\sin\theta$, then $x^2 + y^2 = r^2$.

Partial Derivatives

First order

$$f_x = f_1 = \frac{\partial f}{\partial x} = \partial_x f = \lim_{h\to 0} \frac{f(x+h,y) - f(x,y)}{h}$$

$$f_y = f_2 = \frac{\partial f}{\partial y} = \partial_y f = \lim_{h\to 0} \frac{f(x,y+h) - f(x,y)}{h}$$

Higher order

$$f_{xx} = f_{11} = \frac{\partial^2 f}{\partial x^2}$$

$$f_{yy} = f_{22} = \frac{\partial^2 f}{\partial y^2}$$

$$f_{xy} = f_{12} = \frac{\partial f}{\partial y}\left(\frac{\partial f}{\partial x}\right)$$

$$f_{yx} = f_{21} = \frac{\partial f}{\partial x}\left(\frac{\partial f}{\partial y}\right)$$

If f_{xy} and f_{yx} both exist and are continuous, then $f_{xy} = f_{yx}$.

Chain Rule

Consider $f(x,y)$ with $x(t)$ and $y(t)$.

$$\frac{\mathrm{d}f}{\mathrm{d}t} = \frac{\partial f}{\partial x}\frac{\mathrm{d}x}{\mathrm{d}y} + \frac{\partial f}{\partial y}\frac{\mathrm{d}y}{\mathrm{d}t}$$

Consider $f(x,y)$ with $x(s,t)$ and $y(s,t)$.

$$\frac{\partial f}{\partial t} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial t}$$

Consider $f(x_1, x_2, \ldots, x_n)$ with $x_i(t_1, t_2, \ldots, t_k)$.

$$\frac{\partial f}{\partial t_i} = \frac{\partial f}{\partial x_1}\frac{\partial x_1}{\partial t_i} + \frac{\partial f}{\partial x_2}\frac{\partial x_2}{\partial t_i} + \cdots + \frac{\partial f}{\partial x_n}\frac{\partial x_n}{\partial t_i}$$

Tangent Planes

Tangent plane at $f(a,b)$:

$$-f_x(a,b)(x-a) - f_y(a,b)(y-b) + (z - f(a,b)) = 0$$

Gradient

Consider $f(x,y,z)$.

$$\nabla f = \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right\rangle$$

- $\nabla f(a,b,c)$ is the direction of normal vector at (a,b,c)
- Point-normal tangent plane:

$$\nabla f(a,b,c) \cdot \langle x-a, y-b, z-c\rangle = 0$$

- Normal line:

$$\langle x,y,z\rangle = \langle a,b,c\rangle + t \cdot \nabla f(a,b,c)$$

Linear Approximation

$$f(x,y) \approx f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$$

$$f(x+\Delta x, y+\Delta y) \approx f(a,b) + f_x(a,b)\Delta x + f_y(a,b)\Delta y$$

$$\Delta f \approx f_x(a,b)\Delta x + f_y(a,b)\Delta y$$

$$\mathrm{d}f = f_x(a,b)\mathrm{d}x + f_y(a,b)\mathrm{d}y$$