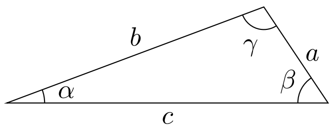
ENPH 270 Formula Sheet

Basics



Sine Law:

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

Cosine Law:

$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$

Cross Product:

$$\hat{\mathbf{i}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}} \qquad \hat{\mathbf{j}} \times \hat{\mathbf{k}} = \hat{\mathbf{i}} \qquad \hat{\mathbf{k}} \times \hat{\mathbf{i}} = \hat{\mathbf{j}}$$

$$\hat{\mathbf{i}} \times \hat{\mathbf{k}} = -\hat{\mathbf{j}} \qquad \hat{\mathbf{j}} \times \hat{\mathbf{i}} = -\hat{\mathbf{k}} \qquad \hat{\mathbf{k}} \times \hat{\mathbf{j}} = -\hat{\mathbf{i}}$$

BAC-CAB Rule:

$$\mathbf{A} \times \mathbf{B} \times \mathbf{C} = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \cdot \mathbf{B})$$

Planar Kinematics

Constant Acceleration:

$$v(t) = v_0 + at \qquad \omega(t) = \omega_0 + \alpha t$$

$$x(t) = x_0 + v_0 t + \tfrac{1}{2}at^2 \qquad \theta(t) = \theta_0 + \omega_0 + \tfrac{1}{2}\alpha t^2$$

$$v^2 = v_0^2 + 2a(x - x_0) \qquad \omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

Velocity and Acceleration:

$$v\,dv = a\,dx \qquad \omega\,d\omega = \alpha\,d\theta$$

General Planar Motion:

$$\mathbf{r}_B = \mathbf{r}_A + \mathbf{r}_{B/A}$$

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega} \times \mathbf{r}_{B/A}$$

$$\mathbf{a}_B = \mathbf{a}_A + \boldsymbol{\alpha} \times \mathbf{r}_{B/A} - \omega^2 \mathbf{r}_{B/A}$$

Instantaneous Center of Rotation (ICR):

$$\mathbf{v}_A = \boldsymbol{\omega} \times \mathbf{r}_{A/O}$$

Rotating Axes:

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega} \times \mathbf{r}_{B/A} + (\mathbf{v}_B)_{\text{Rel}}$$

$$\mathbf{a}_B = \mathbf{a}_A + \boldsymbol{\alpha} \times \mathbf{r}_{B/A} - \omega^2 \mathbf{r}_{B/A} + (\mathbf{a}_B)_{\text{Rel}} + 2\boldsymbol{\omega} \times (\mathbf{v}_B)_{\text{Rel}}$$

Planar Kinetics: Force and Acceleration

Center of Mass:

$$(x_G, y_G) = \left(\tfrac{1}{M}\textstyle\sum_i m_i x_i, \tfrac{1}{M}\textstyle\sum_i m_i y_i\right)$$

Moment of Inertia:

$$I = \int_m r^2\,dm$$

Parallel Axis Theorem:

$$I_A = I_G + md^2$$

Radius of Gyration:

$$I = mk^2$$

Planar Equations of Motion:

$$\textstyle\sum F_x = m(a_G)_x$$

$$\textstyle\sum F_y = m(a_G)_y$$

$$\textstyle\sum M_G = I_G\alpha$$

$$\textstyle\sum M_A = I_G\alpha + \left|\mathbf{r}_{G/A} \times (m\mathbf{a}_G)\right|$$

$$\textstyle\sum M_A = I_A\alpha + \left|\mathbf{r}_{G/A} \times (m\mathbf{a}_A)\right|$$

Rectilinear Translation:

$$\textstyle\sum M_G = 0$$

$$\textstyle\sum M_A = (ma_G)d$$

Curvilinear Translation:

$$\textstyle\sum F_n = m(a_G)_n = m\omega^2 r_G$$

$$\textstyle\sum F_t = m(a_G)_t = m\alpha r_G$$

Planar Kinetics: Work and Energy

Kinetic Energy:

$$T = \tfrac{1}{2}mv_G^2 + \tfrac{1}{2}I_G\omega^2$$

$$T = \tfrac{1}{2}I_O\omega^2$$

Potential Energy:

$$V_g = mgy_G$$

$$V_e = \tfrac{1}{2}ks^2$$

Work:

$$U = \int_s F\cos\theta\,ds$$

$$U_W = -W\Delta y$$

$$U_S = -\left(\tfrac{1}{2}ks_2^2 - \tfrac{1}{2}ks_1^2\right) \qquad |s_2| > |s_1|$$

Principle of Work and Energy:

$$T_1 + U_{1\to 2} = T_2$$

$$T_1 + V_1 + U_{1\to 2}^{(\text{non-cons})} = T_2 + V_2$$

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