MATH 152 Formula Sheet

Vectors

Basics

Direction Vector	$\overrightarrow{ab} = \vec{b} - \vec{a}$
Norm	$\|\vec{a}\| = \sqrt{a_1^2 + \cdots + a_n^2}$
Unit Vector	$\hat{u} = \frac{\vec{u}}{\|u\|}$

Dot Product

$\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + \cdots + a_n b_n$
$\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$
$\vec{a} \cdot \vec{b} = \|\vec{a}\| \|\vec{b}\| \cos\theta$
$\vec{a} \perp \vec{b}$ if $\vec{a} \cdot \vec{b} = 0$

Projection and Perpendicular

$\mathrm{proj}_{\vec{b}}(\vec{a}) = \frac{\vec{a}\cdot\vec{b}}{\vec{b}\cdot\vec{b}}\vec{b} = \frac{\vec{a}\cdot\vec{b}}{\|\vec{b}\|^2}\vec{b} = (\vec{a}\cdot\hat{b})\hat{b}$
$\mathrm{perp}_{\vec{b}}(\vec{a}) = \vec{a} - \mathrm{proj}_{\vec{b}}(\vec{a})$

Cross Product

$$\vec{a} \times \vec{b} = \det \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{bmatrix}$$

$\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$
$\|\vec{a} \times \vec{b}\| = \|\vec{a}\| \|\vec{b}\| \sin\theta$
$\vec{a} \parallel \vec{b}$ if $\vec{a} \times \vec{b} = \vec{0}$.

Area and Volume

$A = \|\vec{a} \times \vec{b}\|$

$$A = \left| \det \begin{bmatrix} -\vec{a}- \\ -\vec{b}- \end{bmatrix} \right|$$

$V = |\vec{a} \cdot (\vec{b} \times \vec{c})|$

$$V = \left| \det \begin{bmatrix} -\vec{a}- \\ -\vec{b}- \\ -\vec{c}- \end{bmatrix} \right|$$

Lines and Planes

Line Equations

Vector/Parametric	$\vec{x} = \vec{p} + \vec{a}t$
Two-Point	$\vec{x} = (1-t)\vec{a} + t\vec{b}$
Point-Normal Form in $\mathbb{R}^2$	$\vec{n} \cdot (\vec{x} - \vec{p}) = 0$
Standard Form in $\mathbb{R}^2$	$ax + by = c$ where $\vec{n} = \langle a, b \rangle$ and $c = \vec{n} \cdot \vec{p}$

Plane Equations in $\mathbb{R}^3$

Vector/Parametric	$\vec{x} = \vec{p} + \vec{a}s + \vec{b}t$
Point-Normal Form	$\vec{n} \cdot (\vec{x} - \vec{p}) = 0$
Standard Form in $\mathbb{R}^3$	$ax + by + cz = d$ where $\vec{n} = \langle a, b, c \rangle$ and $d = \vec{n} \cdot \vec{p}$
Two Lines/Three Points	Use $\vec{a}$, $\vec{b}$ and $\vec{n} = \vec{a} \times \vec{b}$

Hyperplanes

A hyperplane has dimension $n-1$ in $\mathbb{R}^n$

Point-Normal Form	$\vec{n} \cdot (\vec{x} - \vec{p}) = 0$
Standard Form	$a_1 x_1 + \cdots + a_n x_n = d$ where $\vec{n} = \langle a_1, \ldots a_n \rangle$ and $\vec{d} = \vec{n} \cdot \vec{p}$

Distance Between Objects

Distance between point $\vec{q}$ and line $\vec{x} = \vec{p} + \vec{a}t$:
$d = \|\mathrm{perp}_{\vec{a}}(\overrightarrow{pq})\| = \|\mathrm{proj}_{\vec{a}^\perp}(\overrightarrow{pq})\|$
Distance between point $\vec{q}$ and hyperplane with point $\vec{p}$:
$d = \|\mathrm{proj}_{\vec{n}}(\overrightarrow{pq})\|$

Intersection of Objects

Use parametric forms and see if solutions for parameters are consistent.

Linear Systems

Gauss-Jordan Elimination

1. Set the top left entry to 1
2. Use the first row to 'kill off' other entries in the first column
3. For column 2, use one row to 'kill off' other entries in that column
4. Repeat process until the matrix is in RREF

Solutions to Linear Systems

rank: number of leading 1s in the RREF. n: number of unknowns.

- If $\mathrm{rank}(\mathbf{A}) < \mathrm{rank}([\mathbf{A} \mid \vec{b}])$, the system is inconsistent.
- If $\mathrm{rank}(\mathbf{A}) = \mathrm{rank}([\mathbf{A} \mid \vec{b}]) = n$, there is a unique solution.
- If $\mathrm{rank}(\mathbf{A}) = \mathrm{rank}([\mathbf{A} \mid \vec{b}]) < n$, there are infinitely many solutions. k-parameter family of solutions where $k = n - \mathrm{rank}(A)$.

Polynomial Interpolation

With points $(x_1, y_1), \ldots, (x_n, y_n)$ and $p(x) = r_0 + r_1 x + \cdots + r_{n-1}x^{n-1}$, solve:

$$\left[\begin{array}{cccc|c} 1 & x_1 & \cdots & x_1^{n-1} & y_1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 1 & x_n & \cdots & x_n^{n-1} & y_n \end{array}\right]$$

Matrices

Matrix Multiplication

$\mathbf{A}_{m\times p}\mathbf{B}_{p\times n} = (\mathbf{AB})_{m\times n}$
$\mathbf{AI} = \mathbf{IA} = \mathbf{A}$

$$\begin{bmatrix} -\vec{a}_1- \\ \vdots \\ -\vec{a}_m- \end{bmatrix} \begin{bmatrix} | & & | \\ \vec{b}_1 & \cdots & \vec{b}_n \\ | & & | \end{bmatrix} = \begin{bmatrix} \vec{a}_1 \cdot \vec{b}_1 & \cdots & \vec{a}_1 \cdot \vec{b}_n \\ \vdots & \ddots & \vdots \\ \vec{a}_m \cdot \vec{b}_1 & \cdots & \vec{a}_n \cdot \vec{b}_n \end{bmatrix}$$

Transpose

$(\mathbf{A}^\mathsf{T})^\mathsf{T} = \mathbf{A}$
$(\mathbf{A} + \mathbf{B})^\mathsf{T} = \mathbf{A}^\mathsf{T} + \mathbf{B}^\mathsf{T}$
$(\mathbf{AB})^\mathsf{T} = \mathbf{B}^\mathsf{T}\mathbf{A}^\mathsf{T}$
$(k\mathbf{A})^\mathsf{T} = k\mathbf{A}^\mathsf{T}$
Symmetric: $\mathbf{A}^\mathsf{T} = \mathbf{A}$
Skew-symmetric: $\mathbf{A}^\mathsf{T} = -\mathbf{A}$

Inverse

$\mathbf{A}$ is invertible if $\det(\mathbf{A}) \neq 0$.
$\mathbf{A}^{-1}\mathbf{A} = \mathbf{A}\mathbf{A}^{-1} = \mathbf{I}$
$(\mathbf{A}^{-1})^{-1} = \mathbf{A}$
$(k\mathbf{A})^{-1} = \frac{1}{k}\mathbf{A}^{-1}$
$(\mathbf{A}^\mathsf{T})^{-1} = (\mathbf{A}^{-1})^\mathsf{T}$
$(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$
2×2 matrix inverse:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

General matrix inverse:

1. Augment $[\mathbf{A} \mid \mathbf{I}]$
2. Use Gauss-Jordan Elimination to row reduce $\mathbf{A}$ to $\mathbf{I}$, creating the matrix $[\mathbf{I} \mid \mathbf{X}]$
3. $\mathbf{A}^{-1} = \mathbf{X}$.

Elementary Matrices

$(\mathbf{E}_k \cdots \mathbf{E}_3\mathbf{E}_2\mathbf{E}_1)\mathbf{A} = \mathbf{I}$

$\mathbf{A} = \mathbf{E}_1^{-1}\mathbf{E}_2^{-1}\cdots\mathbf{E}_k^{-1}$

Invertible Matrix Theorem

- $\mathbf{A}$ is invertible
- $\mathbf{A}^\mathsf{T}$ is invertible
- $\operatorname{rank}(\mathbf{A}) = n$
- The RREF of $\mathbf{A}_{n\times n}$ is $\mathbf{I}_n$
- $\mathbf{A}$ is a product of elementary matrices
- The linear system $\mathbf{A}\vec{x} = \vec{b}$ has a unique solution
- The homogeneous system $\mathbf{A}\vec{x} = \vec{0}$ has only the trivial solution
- $\det(A) \neq 0$
- 0 is not an eigenvalue of $\mathbf{A}$

Matrix Equations

$\mathbf{A}\vec{x} = \vec{b} \implies \vec{x} = \mathbf{A}^{-1}\vec{b}$

Linear Transformations

$T(\vec{x}) = \mathbf{A}\vec{x}$
$T(\vec{x} + \vec{y}) = T(\vec{x}) + T(\vec{y})$
$T(s\vec{x}) = sT(\vec{x})$
$\mathbf{A} = [T(\vec{e}_1) \mid T(\vec{e}_2) \mid \cdots \mid T(\vec{e}_n)]$
$(S \circ T)(\vec{x}) = S(T(\vec{x})) = \mathbf{B}\mathbf{A}\vec{x}$
Inverse of linear transformation: $(S \circ T)(\vec{x}) = \vec{x}$. The inverse transformation T^{-1} is induced by the matrix $\mathbf{A}^{-1}$.

Rotations

$$\mathrm{Rot}_\theta = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

Reflections

$$\mathrm{Ref}_m = \frac{1}{1+m^2}\begin{bmatrix} 1-m^2 & 2m \\ 2m & m^2-1 \end{bmatrix}$$
$$\mathrm{Ref}_\theta = \begin{bmatrix} \cos(2\theta) & \sin(2\theta) \\ \sin(2\theta) & -\cos(2\theta) \end{bmatrix}$$

Compositions

$\mathrm{Rot}_\theta \circ \mathrm{Rot}_\phi = \mathrm{Rot}_{\theta+\phi}$
$\mathrm{Ref}_\theta \circ \mathrm{Ref}_\phi = \mathrm{Rot}_{2(\theta-\phi)}$
$\mathrm{Rot}_\theta \circ \mathrm{Ref}_\phi = \mathrm{Ref}_{\phi+\theta/2}$
$\mathrm{Ref}_\theta \circ \mathrm{Rot}_\phi = \mathrm{Ref}_{\phi-\theta/2}$

Projections

$$\mathrm{Proj}_m = \frac{1}{1+m^2}\begin{bmatrix} 1 & m \\ m & m^2 \end{bmatrix}$$
$$\mathrm{Proj}_\theta = \frac{1}{2}\begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}$$
$$\mathrm{Proj}_\theta = \frac{1}{2}\begin{bmatrix} 1+\cos 2\theta & \sin 2\theta \\ \sin 2\theta & 1-\cos 2\theta \end{bmatrix}$$

Simple Determinants

$\det([a]) = a$
$$\det\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$
$$\det\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

Determinant Properties

If $\mathbf{A}$ has a zero row or zero column, then $\det(\mathbf{A}) = 0$
If two rows or two columns are scalar multiples, then $\det(\mathbf{A}) = 0$
$\det(\mathbf{A}) = \det(\mathbf{A}^\mathsf{T})$
$\det(\mathbf{A}^{-1}) = \frac{1}{\det(\mathbf{A})}$
$\det(\mathbf{A}^x) = \det(\mathbf{A})^x$
$\det(\mathbf{AB}) = \det(\mathbf{A})\det(\mathbf{B})$
$\det(k\mathbf{A}) = k^n\det(\mathbf{A})$
If $\mathbf{A}$ is triangular, then $\det(\mathbf{A})$ is equal to the product of the entires on the main diagonal.

Simplifying Determinants

Swap Rows	$\det(\mathbf{B}) = -\det(\mathbf{A})$
Multiply Row by k	$\det(\mathbf{B}) = k\det(\mathbf{A})$
Add Factor of a Row	$\det(\mathbf{B}) = \det(\mathbf{A})$

These rules also apply if operations are performed on columns.

Calculating Determinants

Minor: M_{ij} is the determinant of the submatrix that remains after removing row i and column j
Cofactor: $C_{ij} = (-1)^{i+j}M_{ij}$
$\det(\mathbf{A})$ is equal to the products of entries and cofactors along any row or column

Determinants and Inverse

$\mathbf{C}_\mathbf{A}$: matrix of cofactors.
Adjoint matrix: $\operatorname{adj}(\mathbf{A}) = (\mathbf{C}_\mathbf{A})^\mathsf{T}$
$\mathbf{A}^{-1} = \frac{1}{\det(A)}\operatorname{adj}(\mathbf{A})$

Cramer's Rule

Let $\mathbf{A}_{n\times n}$ be an invertible matrix, and let $\vec{b} \in \mathbb{R}^n$ be a constant vector.
Define $\mathbf{A}_i$ to be matrix $\mathbf{A}$ with column i replaced by $\vec{b}$.
Then, $x_i = \frac{\det(\mathbf{A}_i)}{\det(\mathbf{A})}$

Complex Numbers

Definitions

Imaginary Number	$i^2 = -1$
Complex Number	$z = a + bi$
Conjugate	$\overline{z} = a - bi$
Real Part	$\Re(z) = \frac{z+\overline{z}}{2}$
Imaginary Part	$\Im z = \frac{z-\overline{z}}{2}$
Norm	$\lvert z\rvert = \sqrt{a^2+b^2}$

Operations and Identities

$\overline{(z \pm u)} = \overline{z} \pm \overline{u}$
$\overline{zu} = \overline{z}\cdot\overline{u}$
$\overline{\frac{z}{u}} = \frac{\overline{z}}{\overline{u}}$
$z \cdot \overline{z} = |z|^2$
$|zu| = |z||u|$
$\frac{u}{z} = \frac{u\overline{z}}{|z|^2}$

Polar Form

$z = r(\cos\theta + i\sin\theta) = re^{i\theta}$
$\arg(z) = \theta = \arctan(\frac{b}{a})$
$z \cdot w = (re^{i\theta})\cdot(se^{i\phi}) = (rs)e^{i(\theta+\phi)}$

Powers and Roots

$z^n = r^n(\cos(n\theta) + i\sin(n\theta))$
Solving $z^n = w$, where $w = se^{i\phi}$:
$$\begin{aligned} z_1 &= \sqrt[n]{s}e^{i(\phi)/n} \\ z_2 &= \sqrt[n]{s}e^{i(\phi+2\pi\cdot 1)/n} \\ &\vdots \\ z_n &= \sqrt[n]{s}e^{i(\phi+2\pi\cdot(n-1))/n} \end{aligned}$$

Vector Spaces

Vector Space Axioms

1. $\vec{u} + \vec{v}$ must be in V (Closure)
2. $k \cdot \vec{v}$ must be in V (Closure)
3. $(\vec{u} + \vec{v}) + \vec{w} = \vec{u} + (\vec{v} + \vec{w})$ (Associativity)
4. $k \cdot (m \cdot \vec{v}) = (km) \cdot \vec{v}$ (Associativity)
5. $k \cdot (\vec{u} + \vec{v}) = k \cdot \vec{u} + k \cdot \vec{v}$ (Distributivity)
6. $(k + m) \cdot \vec{v} = k \cdot \vec{v} + m \cdot \vec{v}$ (Distributivity)
7. $\vec{u} + \vec{v} = \vec{v} + \vec{u}$ (Commutativity)
8. $\vec{v} + \vec{0} = \vec{0} + \vec{v} = \vec{v}$ (Additive Identity)
9. $\vec{v} + (-\vec{v}) = \vec{0}$ (Additive Inverse)
10. $1 \cdot \vec{v} = \vec{v}$ (Multiplicative Identity)

Vector Subspace

1. W contains the zero vector of V: $\vec{0} \in W$
2. W is closed under vector addition: $\vec{w}_1 + \vec{w}_2 \in W$
3. W is closed under scalar multiplication: $k \cdot \vec{w}_1 \in W$

It suffices to show that $a\vec{w}_1 + b\vec{w}_2 \in W$

Linear Independence

$\vec{v}_1, \vec{v}_2, \dots, \vec{v}_n$ are linearly independent

If $\left[\begin{array}{cccc|c} | & | & & | & | \\ \vec{v}_1 & \vec{v}_2 & \cdots & \vec{v}_n & \vec{0} \\ | & | & & | & | \end{array}\right]$ has only the trivial solution

Or if $\det \begin{bmatrix} | & | & & | \\ \vec{v}_1 & \vec{v}_2 & \cdots & \vec{v}_n \\ | & | & & | \end{bmatrix} \neq 0$

Span

If $S = \{\vec{v}_1, \vec{v}_2, \dots, \vec{v}_n\}$, then $\text{span}(S)$ is the set of all linear combinations of the vectors in S.
Checking if $\vec{b} \in \text{span}(S)$: solve

$$\left[\begin{array}{cccc|c} | & | & & | & | \\ \vec{v}_1 & \vec{v}_2 & \cdots & \vec{v}_n & \vec{b} \\ | & | & & | & | \end{array}\right]$$

If V is a vector space and $\text{span}(S) = V$, then S is a generating set of V

Basis

If B is linearly independent and a generating set of V, then B is a basis for V

Dimension

The dimension of a vector space V is the number of vectors in its basis, denoted as $\dim(V)$

Coordinates

If $B = \{\vec{v}_1, \vec{v}_2, \dots, \vec{v}_n\}$ is a basis of V, and $\vec{v} = c_1\vec{v}_1 + c_2\vec{v}_2 + \cdots + c_n\vec{v}_n$, then

$\begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix}$ are the coordinates of $\vec{v}$ relative to the basis B.

Column and Null Space

Given a matrix $\mathbf{A}_{n\times n}$:
Column space: $\text{Col}(\mathbf{A}) = \{\vec{b} \in \mathbb{R}^n \mid \mathbf{A}\vec{x} = \vec{b}\}$ where $\vec{x} \in \mathbb{R}^n$
Column space: $\text{Col}(\mathbf{A}) = \text{span}(\{\vec{a}_1, \dots, \vec{a}_n\})$ (column vectors)
Null space: $\text{Null}(\mathbf{A}) = \{\vec{x} \in \mathbb{R}^n \mid \mathbf{A}\vec{x} = \vec{0}\}$
$\text{rank}(\mathbf{A}) + \dim(\text{Null}(\mathbf{A})) = n$

Eigen-Analysis

Eigenvectors and Eigenvalues

Consider $\mathbf{A}_{n\times n}$. $\vec{x} \in \mathbb{R}^n$ is an eigenvector of $\mathbf{A}$ with associated eigenvalue $\lambda \in \mathbb{R}$ if:
$\mathbf{A}\vec{x} = \lambda\vec{x}$

Characteristic Polynomial

$p_A(\lambda) = \det(\mathbf{A} - \lambda\mathbf{I})$
Eigenvalues: λ such that $p_A(\lambda) = 0$

Solving Method

1. Use $\det(\mathbf{A} - \lambda\mathbf{I}) = 0$ to solve for λ
2. Use $(\mathbf{A} - \lambda_i\mathbf{I})\vec{x}_i = \vec{0}$ to solve for $\vec{x}_i$. Each eigenvector is not unique, scalar multiples are valid

Algebraic multiplicity: number of times the eigenvalue appears as a root
Geometric multiplicty: number of corresponding eigenvectors for the eigenvalue
2×2 matrices:
$\mathbf{A} - \lambda\mathbf{I} = \begin{bmatrix} a & b \\ ? & ? \end{bmatrix} \implies \vec{x} = \begin{bmatrix} -b \\ a \end{bmatrix}$

Properties

Triangular Matrix Diagonal entries are eigenvalues
$\mathbf{A}^m\vec{x} = \lambda^m\vec{x}$
$\det(\mathbf{A}) = \lambda_1 \times \lambda_2 \times \cdots \times \lambda_n$
$\text{tr}(\mathbf{A}) = a_11 + a_22 + \cdots + a_nn = \lambda_1 + \lambda_2 + \cdots + \lambda_n$
If $\mathbf{P}$ is an invertible matrix, then $\mathbf{A}$ is similar to $\mathbf{P}^{-1}\mathbf{AP}$ and have the same eigenvalues

Diagonalization

$\mathbf{A}$ is diagonalizable if $\mathbf{A}$ is similar to a diagonal matrix $\mathbf{D}$, where $\mathbf{D} = \mathbf{P}^{-1}\mathbf{AP}$

$\mathbf{P} = \begin{bmatrix} | & | & & | \\ \vec{x}_1 & \vec{x}_2 & \cdots & \vec{x}_n \\ | & | & & | \end{bmatrix}$ (matrix of eigenvectors)

$\mathbf{D} = \begin{bmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & \lambda_n \end{bmatrix}$ (eigenvalues in same order)

Diagonalization Theorem

1. $\mathbf{A}_{n\times n}$ is diagonalizable if and only if $\mathbf{A}$ has n linearly independent eigenvectors
2. Or if algebraic multiplicty matches geometric multiplicity for all eigenvalues
3. Or if $\mathbf{A}$ has n distinct eigenvalues

$\mathbf{A}^k = \mathbf{PD}^k\mathbf{P}^{-1}$. For diagonal matrices,

$$\mathbf{D}^k = \begin{bmatrix} d_1^k & 0 & \cdots & 0 \\ 0 & d_2^k & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & d_n^k \end{bmatrix}$$

Markov Chains

A matrix whose columns are numbers between $[0, 1]$, which sum to 1
a_{ij}: probability of moving from j to i

State vector: $\vec{x}_n = \begin{bmatrix} x_{1n} \\ \vdots \\ x_{nn} \end{bmatrix}$ (each amount at time n)

Markov Chain equation: $\vec{x}_{n+1} = \mathbf{A}\vec{x}_n$
Steady state vector: $\vec{x}_s = \mathbf{A}\vec{x}_s$.
$\vec{x}_n$ approaches $\vec{x}_s$ for large n
$(\mathbf{I} - \mathbf{A})\vec{x}_s = \vec{0}$, solve using Gauss-Jordan elimination

Linear Dynamical Systems

$\vec{v}_{n+1} = \mathbf{A}\vec{v}_n$
$\vec{v}_n = \mathbf{A}^n\vec{v}_0 = \mathbf{PD}^n\mathbf{P}^{-1}\vec{v}_0$

Recurrence Relations

$x_{n+2} = cx_n + dx_{n+1}$, $x_0 = a$, $x_1 = b$. Let $y_n = x_{n+1}$

$$\begin{bmatrix} x_{n+1} \\ y_{n+1} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ c & d \end{bmatrix} \begin{bmatrix} x_n \\ y_n \end{bmatrix}$$

Differential Equations

Simple Differential Equation

Solution to $y' = ay$ is $y(x) = ce^{ax}$

System of Linear Differential Equations

$$\begin{bmatrix} y_1' \\ y_2' \\ \vdots \\ y_n' \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$$

1. Set $\vec{y}' = \mathbf{A}y$
2. Find the eigenvalues and eigenvectors of $\mathbf{A}$
3. $\vec{x}(t) = (c_1e^{\lambda_1 x})\vec{x}_1 + (c_2e^{\lambda_2 x})\vec{x}_2 + \cdots + (c_ne^{\lambda_n x})\vec{x}_n$

Higher Order Differential Equation

Substitute $y_1 = y, y_2 = y', \dots$
Convert to system of linear differential equations with single derivatives on the left hand side