MECH 260 Formula Sheet

General Loading

Stress and Strain:

$$\sigma = \frac{F}{A} \qquad \varepsilon = \frac{\delta}{L_0}$$

Young's Modulus:

$$E = \frac{\sigma}{\varepsilon} \qquad \delta = \frac{FL_0}{AE}$$

Poisson's Ratio:

$$\varepsilon_{y,z} = \frac{-\nu}{E}\sigma_x$$

Hooke's Law:

$$\varepsilon = \frac{1}{E}\left[\sigma_\parallel - \nu(\sigma_{\perp 1} + \sigma_{\perp 2})\right] + \alpha_L \Delta T$$

Rearranged Hooke's Law:

$$\sigma = \left(\frac{E}{(1+\nu)(1-2\nu)}\right)\left[(1-\nu)\varepsilon_\parallel - \nu(\varepsilon_{\perp 1} + \varepsilon_{\perp 2})\right] - \left(\frac{E}{1-2\nu}\right)\alpha_L \Delta T$$

Volumetric Strain:

$$\varepsilon_v \approx \varepsilon_x + \varepsilon_y + \varepsilon_z$$

Bulk Modulus:

$$\left(\frac{\sigma_x + \sigma_y + \sigma_z}{3}\right) = \underbrace{\frac{E}{3(1-2\nu)}}_{K}\left[(\varepsilon_x + \varepsilon_y + \varepsilon_z) - 3\alpha_L \Delta T\right]$$

Shear Stress and Strain:

$$\tau = \frac{V}{A} \qquad \gamma = \frac{\delta}{L}$$

Shear Stress:

τ_{xy} means stress on x plane in y direction

Shear Strain:

γ_{xy} is angle by which $(+x, +y)$ scissors close

Shear Modulus:

$$\tau_{xy} = G\gamma_{xy}$$

Table of Elastic Constants:

	Find			
Without	E	ν	G	K
E		$\frac{3K-2G}{6K+2G}$	$\frac{3K(1-2\nu)}{2(1+\nu)}$	$\frac{2G(1+\nu)}{3(1-2\nu)}$
ν	$\frac{9KG}{G+3K}$		$\frac{3EK}{9K-E}$	$\frac{EG}{9G-3E}$
G	$3K(1-2\nu)$	$\frac{1}{2} - \frac{E}{6K}$		$\frac{E}{3(1-2\nu)}$
K	$2G(1+\nu)$	$\frac{E}{2G} - 1$	$\frac{E}{2(1+\nu)}$	

Stress Concentration

Nominal Stress:

$$\sigma_{\text{nom}} = \frac{F}{A_{\text{min}}}$$

Stress Concentration:

$$K = \frac{\sigma_{\text{max}}}{\sigma_{\text{nom}}}$$

Factor of Safety (Ductile):

$$|\sigma_{\text{nom}}| = \frac{\sigma_{\text{Y}}}{f_{\text{s}}}$$

Factor of Safety (Brittle):

$$|\sigma_{\text{max}}| = \begin{cases} \frac{\sigma_{\text{UT}}}{f_{\text{s}}} & \text{Tension} \\ \frac{\sigma_{\text{UC}}}{f_{\text{s}}} & \text{Compression} \end{cases}$$

Failure:

$$f\text{s} \le 1$$

Torsion

Torsion Compatibility:

$$\gamma_{\text{surf}} = \frac{r_{\text{surf}}}{L}\phi \qquad \gamma(r) = \frac{r}{L}\phi$$

Torsion Hooke's Law:

$$\tau_{\text{surf}} = \frac{G r_{\text{surf}}}{L}\phi \qquad \tau(r) = \frac{Gr}{L}\phi$$

Second Polar Moment of Area:

$$T = \int_{A_\perp} r \cdot \tau(r)\,\mathrm{d}A = \frac{G\phi}{L}\underbrace{\int_{A_\perp} r^2\,\mathrm{d}A}_{J}$$

$$J = \begin{cases} \frac{\pi}{2} r_{\text{surf}}^4 & \text{Solid Shaft} \\ \frac{\pi}{2}\left(r_{\text{surf}}^4 - r_{\text{in}}^4\right) & \text{Hollow Shaft} \\ 2\pi t r_{\text{surf}}^3 & \text{Thin Tube } (t/r_{\text{surf}} \lesssim 0.01) \end{cases}$$

Torsion Summary:

$$\frac{\tau(r)}{r} = \frac{T}{J} = \frac{G}{L}\phi$$

Horsepower:

$$\text{HP} = \frac{\text{RPM} \cdot T}{5252}$$

Gears:

$$\frac{T_1}{r_1} = \frac{T_2}{r_2} \qquad r_1\phi_1 = -r_2\phi_2$$

Pulleys:

$$T = (F_1 - F_2)r \qquad \delta_1 = \delta_2 = r\phi$$

Bending

Shear (downward):

$$V(z) = \int_0^z w_{\text{upward}}(z)\,\mathrm{d}z + \sum_0^z F_{\text{upward}}(z)$$

Moment (counterclockwise):

$$M(z) = \int_0^z V(z)\,\mathrm{d}z + \sum_0^z T_{\text{CW}}(z)$$

Neutral Axis:

$$y^*_{\text{NA}} = \frac{1}{A_\perp}\int_{A_\perp} y^*\,\mathrm{d}A$$

$$y^*_{\text{NA}} = \frac{1}{A_{\text{total}}}\sum_{i=1}^n y^*_{\text{NA},i} \cdot A_i$$

Second Rectangular Moment of Area:

$$M = \int_{A_\perp} (-y \cdot \sigma_z(y))\,\mathrm{d}A = \frac{E}{\rho}\underbrace{\int_{A_\perp} y^2\,\mathrm{d}A}_{I_x}$$

$$I_x = \int_{y^*_{\text{min}}}^{y^*_{\text{max}}} (y^* - y^*_{\text{NA}})^2 \cdot w(y^*)\,\mathrm{d}y^*$$

$$I_x = \int_{y^*_{\text{min}}}^{y^*_{\text{max}}} (y^*)^2 \cdot w(y^*)\,\mathrm{d}y^* - A_\perp (y^*_{\text{NA}})^2$$

$$I_x = \sum_{i=1}^n I_{B_i,i} + \sum_{i=1}^n A_i\left(y^*_{\text{NA}} - y^*_{B_i}\right)^2$$

$$I_x = \begin{cases} \frac{bh^3}{12} & \text{Rectangle} \\ \frac{bh^3}{36} & \text{Triangle} \\ \frac{\pi}{4} r^4 & \text{Circle} \\ \left(\frac{\pi}{8} - \frac{8}{9\pi}\right) r^4 & \text{Semi-Circle} \end{cases}$$

Pure Bending Summary:

$$\frac{\sigma_z(y)}{-y} = \frac{M}{I_x} = \frac{E}{\rho}$$

Composite Bending:

$$w_{\text{scaled}} = w \cdot \frac{E}{E_{\text{ref}}} \qquad \sigma = \sigma_{\text{scaled}} \cdot \frac{E}{E_{\text{ref}}}$$

Stress Transformation and Failure Criteria

Plane Stress Transformation:

$$\sigma_{x'} = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\sigma_{y'} = \left(\frac{\sigma_x + \sigma_y}{2}\right) - \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

Mohr's Circle:

$$x : (\sigma_x, -\tau_{xy}) \qquad y : (\sigma_y, +\tau_{xy}) \qquad k : (\sigma_k, \tau_k)$$

Principal Stresses:

$$\tan 2\theta_i = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

$$\sigma_i = \left(\frac{\sigma_x + \sigma_y}{2}\right) \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Tresca Criterion:

$$f_{\mathrm{s}}^{\mathrm{Tresca}} = \frac{\sigma_{\mathrm{Y}}/2}{\tau_{\max}}$$

$$\max(\sigma_1, \sigma_2, \sigma_3) - \min(\sigma_1, \sigma_2, \sigma_3) = \frac{\sigma_{\mathrm{Y}}}{f_{\mathrm{s}}^{\mathrm{Tresca}}}$$

Von Mises Criterion:

$$\frac{1}{2}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right] = \left(\frac{\sigma_{\mathrm{Y}}}{f_{\mathrm{s}}^{\mathrm{vM}}}\right)^2$$

Comparison of Tresca and von Mises:

$$f_{\mathrm{s}}^{\mathrm{vM}} \geq f_{\mathrm{s}}^{\mathrm{Tresca}} \qquad f_{\mathrm{s}}^{\mathrm{vM}} \leq \frac{2}{\sqrt{3}} f_{\mathrm{s}}^{\mathrm{Tresca}}$$

Mohr Criterion:

$$\frac{\max(\sigma_1, \sigma_2, \sigma_3)}{\sigma_{\mathrm{UT}}} - \frac{\min(\sigma_1, \sigma_2, \sigma_3)}{|\sigma_{\mathrm{UC}}|} = \frac{1}{f_s^{\mathrm{Mohr}}}$$

Gauge Pressure:

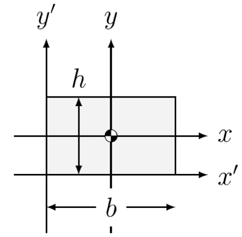
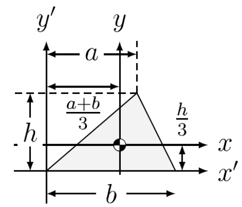
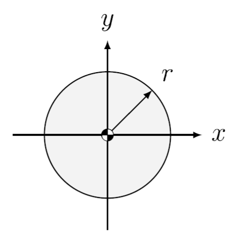
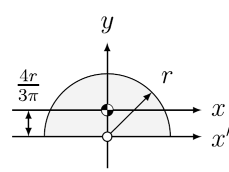
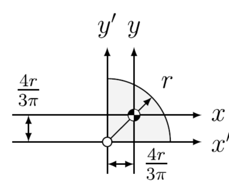
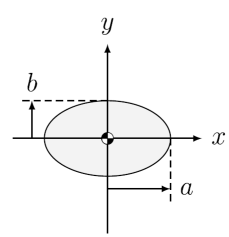
$$P = P_{\mathrm{inside}} - P_{\mathrm{outside}} \qquad P = \rho g h$$

Thin-Walled Vessels ($t/r \lesssim 0.1$):

$$\sigma_{\mathrm{ax}} = \frac{Pr}{2t} \qquad \sigma_\theta^{\mathrm{cyl}} = \frac{Pr}{t} \qquad \sigma_\theta^{\mathrm{sph}} = \frac{Pr}{2t}$$

Tips

1. Sign convention for σ (compressive is negative)
2. Gears: $r_1\phi_1 = -r_2\phi_2$ (not $\phi_1 = -\phi_2$)
3. Sign convention for $V(z)$ and $M(z)$
4. σ_{nom} vs. $\sigma_{\max}$
5. Radius vs. diameter for torsion and bending
6. Check $t/r \lesssim 0.1$ for thin-walled vessels

		Second Moment of Area for...		
Description	**Diagram**	**Bending about axis $\parallel x$**	**Bending about axis $\parallel y$**	**Torsion about axis $\parallel z$**
Rectangle		$I_x = \frac{bh^3}{12}$ $I_{x'} = \frac{bh^3}{3}$	$I_y = \frac{b^3h}{12}$ $I_{y'} = \frac{b^3h}{3}$	$J_{\odot} = \frac{1}{12}bh(b^2+h^2)$
Triangle		$I_x = \frac{bh^3}{36}$ $I_{x'} = \frac{bh^3}{12}$	$I_y = \frac{b^3h - b^2ha + hba^2}{36}$ $I_{y'} = \frac{b^3h - b^2ha + hba^2}{12}$	—
Circle		$I_x = \frac{\pi}{4}r^4$	$I_y = \frac{\pi}{4}r^4$	$J_{\odot} = \frac{\pi}{2}r^4$
Semi-Circle		$I_x = \left(\frac{\pi}{8} - \frac{8}{9\pi}\right)r^4$ $I_{x'} = \frac{\pi}{8}r^4$	$I_y = \frac{\pi}{8}r^4$	$J_{\circ} = \frac{\pi}{4}r^4$
Quarter-Circle		$I_x = \left(\frac{\pi}{16} - \frac{4}{9\pi}\right)r^4$ $I_{x'} = \frac{\pi}{16}r^4$	$I_y = \left(\frac{\pi}{16} - \frac{4}{9\pi}\right)r^4$ $I_{y'} = \frac{\pi}{16}r^4$	$J_{\circ} = \frac{\pi}{8}r^4$
Ellipse		$I_x = \frac{\pi}{4}ab^3$	$I_y = \frac{\pi}{4}a^3b$	$J_{\odot} = \frac{\pi}{4}ab\left(a^2+b^2\right)$