ELEC 204 Formula Sheet

Basics

Current, Voltage, Power

$$e = 1.602 \times 10^{-19}\ \mathrm{C}$$

$$I = \frac{\mathrm{d}q}{\mathrm{d}t} \qquad V = \frac{\mathrm{d}W}{\mathrm{d}q} \qquad P = VI$$

Resistors

$$R = \rho \frac{l}{A}$$

$$V = IR \qquad P = I^2 R$$

$$R = R_1 + R_2 + \cdots + R_n \qquad \text{(series)}$$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n} \qquad \text{(parallel)}$$

KCL

$$\sum I_{in} = \sum I_{out}$$

KVL

$$\sum_{loop} \Delta V = 0$$

Current-Voltage Division

$$I_1 = I_2 \qquad V_1 = \frac{R_1}{R_1 + R_2} V \qquad \text{(series)}$$

$$V_1 = V_2 \qquad I_1 = \frac{R_2}{R_1 + R_2} V \qquad \text{(parallel)}$$

Wye-Delta

$\Delta \to$ Y:

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

Y $\to \Delta$:

$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}$$

Circuit Analysis

Nodal Analysis

1. Set reference node voltage to 0.
2. Assign variable voltages to other nodes.
3. Write KCL at all unknown nodes, then solve.
4. Use supernode if necessary.

Mesh Analysis

1. Assign loop currents.
2. Write KVL for each loop, then solve.
3. Use supermesh if necessary.

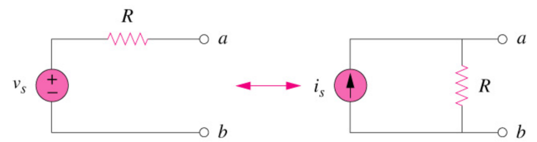
Linearity

We can assume a value, then scale linearly in the end.

Superposition

1. Set all independent sources to 0, except one. Repeat for each source, then sum.
2. 0 voltage source = wire.
3. 0 current source = broken circuit.

Source Transform



$$V = IR$$

Thevenin

1. $V_{th} = V_{oc}$, turn off independent sources to get R_{th}
2. $V_{th} = V_{oc}$, find I_{sc} then $R_{th} = V_{oc}/I_{sc}$
3. Apply test current/test voltage
4. Source transform

Norton

$$I_n = V_{th}/R_{th} \qquad R_n = R_{th}$$

Maximum Power

$$R_L = R_{th} \qquad P_{max} = \frac{V^2}{4R_{th}}$$

Capacitors and Inductors

Capacitors

$$q = CV \qquad i = C\frac{\mathrm{d}v}{\mathrm{d}t} \qquad W = \frac{1}{2}Cv^2$$

Inductors

$$v = L\frac{\mathrm{d}i}{\mathrm{d}t} \qquad W = \frac{1}{2}Li^2$$

First/Second Order Circuits

First Order

$$\tau = RC \qquad \tau = \frac{L}{R} \qquad \text{(use } R = R_{th}\text{)}$$

$$x(t) = \left[x(t_0^+) - x(\infty)\right] e^{-\frac{t-t_0}{\tau}} + x(\infty)$$

Second Order

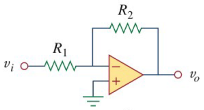
Solve second order constant coefficient DE.

OpAmps

- Saturate at $+V_{cc}$ and $-V_{cc}$
- Amplifier: $V_{out} = AV_{in}$
- Negative feedback: $i_+ = i_- = 0 \qquad V_+ = V_-$

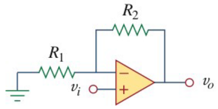
- Inverting amplifier

$$v_o = -\frac{R_2}{R_1} v_i$$



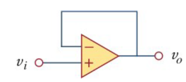
- Non-inverting amplifier

$$v_o = \left(1 + \frac{R_2}{R_1}\right) v_i$$



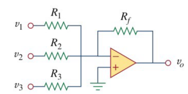
- Buffer (voltage follower)

$$v_o = v_i$$



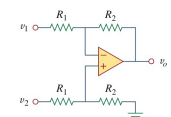
- Summer (adder)

$$v_o = -\left(\frac{R_f}{R_1}v_1 + \frac{R_f}{R_2}v_2 + \frac{R_f}{R_3}v_3\right)$$



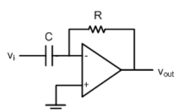
- Difference Amplifier

$$v_o = \frac{R_2}{R_1}(v_2 - v_1)$$



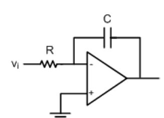
- Differentiator

$$v_o = -RC\frac{dv_i}{dt}$$



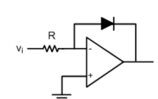
- Integrator

$$v_o = -\frac{1}{RC}\int v_i dt$$



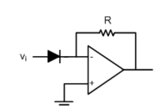
- Taking natural logarithm

$$v_o = -V_T \ln\left(\frac{v_i}{R \cdot I_s}\right)$$



- Raising to the power of e

$$v_o = -R \cdot I_s e^{\frac{v_i}{V_T}}$$



Sinusoidal Analysis and Power

Sinusoidal Analysis

$$v(t) = V_m \cos(\omega t + \theta) = \mathrm{Re}\left[V_m e^{j\theta} e^{j\omega t}\right]$$

$$v(t) \equiv \mathrm{Re}\left[V_m e^{j\theta}\right] = \mathrm{Re}\left[V_m \angle\theta\right] = \mathrm{Re}\left[\boldsymbol{V}\right]$$

$$\boldsymbol{Z} = R + jX$$

$$X_L = \omega L \qquad X_C = -\frac{1}{\omega C}$$

- $\boldsymbol{V}, \boldsymbol{I}, \boldsymbol{Z}$ can be treated as V, I, R.
- Thevenin, Nodal, and Mesh analysis ✓

Real Power

$$p = \underbrace{\frac{V_m I_m}{2}\cos(\theta_v - \theta_i)}_{P\text{: active power}} + \frac{V_m I_m}{2}\cos(\theta_v - \theta_i)\cos 2\omega t$$

$$-\underbrace{\frac{V_m I_m}{2}\sin(\theta_v - \theta_i)}_{Q\text{: reactive power}}\sin 2\omega t$$

Power factor: $pf = \cos(\theta_v - \theta_i)$.
RMS:

$$I_{rms} = I_{eff} = \frac{I}{\sqrt{2}} \qquad V_{rms} = V_{eff} = \frac{V}{\sqrt{2}}$$

Complex Power

$$\boldsymbol{S} = P + jQ = \boldsymbol{V}_{rms}\boldsymbol{I}^*_{rms} = \frac{1}{2}\boldsymbol{V}\boldsymbol{I}^*$$

$$\boldsymbol{S} = \boldsymbol{Z}\boldsymbol{I}_{eff}\boldsymbol{I}^*_{eff} = \boldsymbol{Z}\left|\boldsymbol{I}_{eff}\right|^2$$

$$P = \left|\boldsymbol{I}_{eff}\right|^2 R = \frac{1}{2}\left|\boldsymbol{I}\right|^2 R$$

$$Q = \left|\boldsymbol{I}_{eff}\right|^2 X = \frac{1}{2}\left|\boldsymbol{I}\right|^2 X$$

Maximum Power

$$\boldsymbol{Z}_L = \boldsymbol{Z}^*_{th} \qquad P_{max} = \frac{1}{4}\frac{\left|\boldsymbol{V}_{th}\right|^2}{R_{th}}$$

Restricted R_L and X_L:

- Adjust X_L to be close to $-X_{th}$
- Adjust R_L to be close to $\sqrt{R_{th}^2 + (X_L + X_{th})^2}$

Fixed angle of $\boldsymbol{Z}_L$:

- Set $|\boldsymbol{Z}_L| = |\boldsymbol{Z}_{th}|$

Diodes

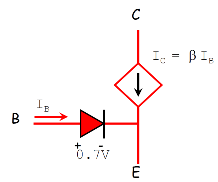
$$i_D = I_0\left(e^{\frac{v_D}{nV_T}} - 1\right) \qquad V_T \equiv \frac{kT}{q_e}$$

$$V_D = 0.7\,\mathrm{V}$$

1. Assume whether each diode is on or off, solve for currents/voltage, and check if consistent with assumption
2. If inconsistent, try another combination

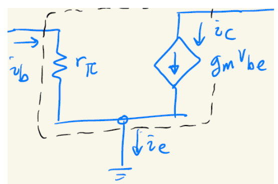
BJTs - NPN

Linear zone: BE forward biased and BC reverse biased



1. DC operating point: turn off AC sources, solve
2. AC small-signal: turn off DC sources, use hybrid-pi small signal equivalent

$$g_m = \frac{I_C}{V_T} \qquad r_\pi = \frac{\beta}{g_m} = \frac{\beta V_T}{I_C} = \frac{V_T}{I_B}$$



MOSFETs - NMOS

Activate when $V_{GS} > V_{Th}$

- $V_{DS} \ll V_{GS} - V_{Th}$ (deep triode):

$$I_D = \mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{Th}) \cdot V_{DS}$$

- $V_{DS} \leq V_{GS} - V_{Th}$ (triode):

$$I_D = \mu_n C_{ox}\frac{W}{L}\left[(V_{GS} - V_{Th}) \cdot V_{DS} - \frac{1}{2}V_{DS}^2\right]$$

$$g_m = \mu_n C_{ox}\frac{W}{L}V_{DS}$$

- $V_{DS} > V_{GS} - V_{Th}$ (saturation):

$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{Th})^2$$

$$g_m = \mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{Th})$$

Small signal equivalent: