PHYS 158 Formula Sheet

Mechanical Waves

Periodic Waves

$$T = \frac{1}{f} \qquad \omega = 2\pi f \qquad \omega = \frac{2\pi}{T}$$

$$v = \lambda f$$

Sinusoidal Wave

Motion in $+x$:

$$y(x,t) = A\cos(kx - \omega t) \qquad k = \frac{2\pi}{\lambda} \qquad \omega = 2\pi f$$

Speed of Wave on String

$$v = \sqrt{\frac{F}{\mu}} \qquad \mu = \frac{m}{L}$$

Standing Wave on String

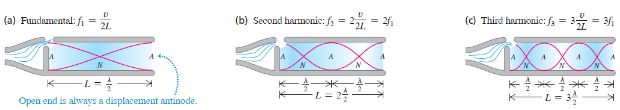
$$\lambda_n = \frac{2L}{n} \qquad f_n = \frac{nv}{2L} = nf_1 \qquad (n = 1, 2, 3, \ldots)$$

Sound Waves

Pipes

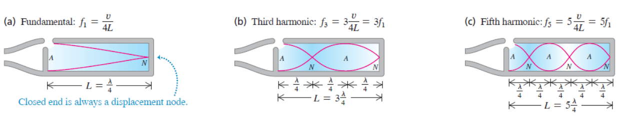
Open end: pressure node, displacement antinode
Closed end: pressure antinode, displacement node
Open Pipe

$$f_1 = \frac{v}{2L} \qquad f_n = \frac{nv}{2L} = nf_1 \qquad (n = 1, 2, 3, \ldots)$$



Closed Pipe

$$f_1 = \frac{v}{4L} \qquad f_n = \frac{nv}{4L} = nf_1 \qquad (n = 1, 3, 5, \ldots)$$



Beats

$$f_{\text{beat}} = f_a - f_b$$

Interference

Coherent: same f and constant ϕ relationship

Sources in Phase

Constructive interference:

$$r_2 - r_1 = m\lambda \qquad (m = 0, \pm 1, \pm 2, \ldots)$$

Destructive interference:

$$r_2 - r_1 = (m + \tfrac{1}{2})\lambda \qquad (m = 0, \pm 1, \pm 2, \ldots)$$

Two Source Interference

Bright regions (constructive):

$$d\sin\theta = m\lambda \qquad (m = 0, \pm 1, \pm 2, \ldots)$$

Dark regions (destructive):

$$d\sin\theta = (m + \tfrac{1}{2})\lambda \qquad (m = 0, \pm 1, \pm 2, \ldots)$$

Position of the mth bright band:

$$y_m = R\tan\theta_m$$

For small angles:

$$y_m \approx R\sin\theta_m = R\frac{m\lambda}{d} \qquad (m = 0, \pm 1, \pm 2, \ldots)$$

Thin Film Interference

Refractive index

$$n = \frac{c}{v} \qquad n_1 v_1 = n_2 v_2$$

$$\lambda_{\text{medium}} = \frac{\lambda_{\text{air}}}{n}$$

For thin film: $\Delta r = 2t$, $t =$ thickness
Half-cycle phase shift occurs if $n_2 > n_1$ (reflected off heavier medium)
No phase shift:
Constructive: $2t = m\lambda \qquad (m = 0, 1, 2 \ldots)$
Destructive: $2t = (m + \frac{1}{2})\lambda \qquad (m = 0, 1, 2, \ldots)$
Half-cycle phase shift:
Constructive: $2t = (m + \frac{1}{2})\lambda \qquad (m = 0, 1, 2, \ldots)$
Destructive: $2t = m\lambda \qquad (m = 0, 1, 2 \ldots)$

Capacitance

$$C = \frac{Q}{V_{ab}} \qquad Q = CV$$

In series: $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots$
In parallel: $C_{eq} = C_1 + C_2 + \cdots$
Kirchoff's law: positive V if coming out of the $+$ plate
Energy in capacitor:

$$E = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

Current, Resistance, and EMF

Current and Resistance:

$$I = \frac{dQ}{dt} \qquad R = \frac{\rho L}{A}$$

Ohm's Law:

$$V = IR$$

$$V_{ab} = \mathcal{E} - Ir$$

Power (any circuit element):

$$P = VI$$

Power for resistor:

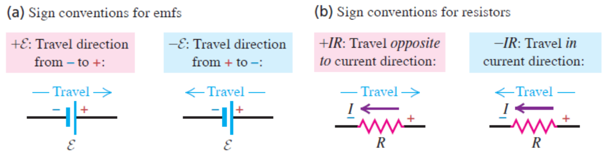
$$P = VI = I^2 R = \frac{V^2}{R}$$

DC Circuits

Resistors in series: $R_{eq} = R_1 + R_2 + \cdots$
Resistors in parallel: $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots$

Kirchoff's Laws

$$\sum I_{\text{junction}} = 0 \qquad \sum V_{\text{loop}} = 0$$



RC Circuits

Charging:

$$q = C\mathcal{E}(1 - e^{-t/RC}) \qquad i = \frac{dq}{dt} = \frac{\mathcal{E}}{R} e^{-t/RC}$$

Time constant: $\tau = RC$
Discharging:

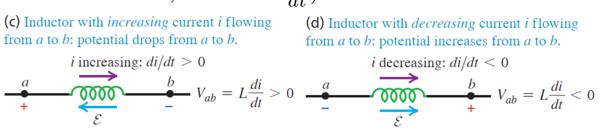
$$q = Q_0 e^{-t/RC} \qquad i = \frac{dq}{dt} = -\frac{Q_0}{RC} e^{-t/RC}$$

Inductance

$$\mathcal{E} = -L\frac{di}{dt} \qquad \text{(Opposes change in current)}$$

$$V_{ab} = L\frac{di}{dt}$$

Kirchoff's law: same convention as resistor (if same direction as current, then $-L\frac{di}{dt}$)



Energy in inductor:

$$E = \frac{1}{2} L I^2$$

RL Circuits

Current growth:

$$i = \frac{\mathcal{E}}{R}(1 - e^{-(R/L)t}) \qquad \frac{di}{dt} = \frac{\mathcal{E}}{L} e^{-(R/L)t}$$

Time constant: $\tau = \frac{L}{R}$

Current decay:

$$i = I_0 e^{-(R/L)t}$$

LC Circuit

Start with capacitor fully charged:

$$\omega = \frac{1}{\sqrt{LC}}$$

Capacitor: $V(t) = V_0 \cos(\frac{t}{\sqrt{LC}}) = V_0 \cos(\omega t)$
Inductor: $I(t) = -\frac{V_0}{\sqrt{L/C}} \sin(\frac{t}{\sqrt{LC}}) = -\frac{V_0}{\sqrt{L/C}} \sin(\omega t)$

RLC Circuits

LC with damping.

$$\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} \qquad \tau = \frac{2L}{R}$$

$$q = Q_0 e^{-(R/2L)t} \cos\left(\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} t + \phi\right)$$

$$q = Q_0 e^{-t/\tau} \cos(\omega t + \phi)$$

$$i = -\frac{dq}{dt} = Q_0 e^{-t/\tau}(\omega \sin(\omega t + \phi) + \frac{1}{\tau}\cos(\omega t + \phi))$$

Limiting Behaviour

$t \to 0^+$: $C \approx$ wire, $L \approx$ broken circuit (no change in i)
$t \to \infty$: $C \approx$ broken circuit, $L \approx$ wire

AC Circuits

Sinusoid voltage/current:

$$v = V\cos\omega t \qquad i = I\cos\omega t$$

RMS values

$$I_{\text{rms}} = \frac{I}{\sqrt{2}} \qquad V_{\text{rms}} = \frac{V}{\sqrt{2}}$$

Resistors

$$V_R = IR \qquad v_R = V_R \cos\omega t$$

v_r is in phase with i.

Inductors

$$V_L = IX_L \qquad X_L = \omega L \qquad v_L = V_L\cos(\omega t + 90°)$$

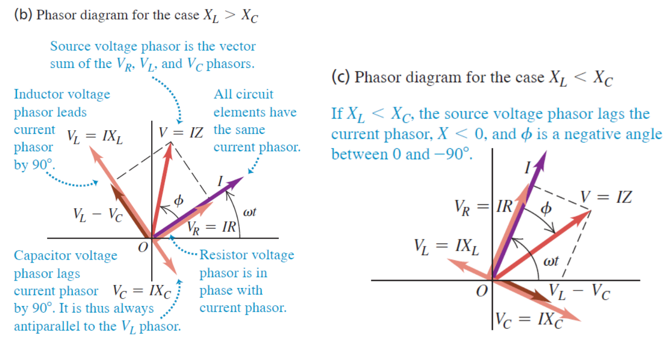
v_L leads i by $90°$.
Inductors block high frequencies and permit low frequencies + DC to pass through (low-pass filter)

Capacitors

$$V_C = IX_C \qquad X_C = \frac{1}{\omega C} \qquad v_C = V_C\cos(\omega t - 90°)$$

v_C lags i by $90°$.
Capacitors block low frequencies + DC and permit high frequencies to pass through (high-pass filter)

LRC AC Circuit



Impedance

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$V = IZ \qquad \text{(amplitudes)}$$

$$\tan\phi = \frac{X_L - X_C}{R} = \frac{\omega L - 1/\omega C}{R}$$

If $i = I\cos\omega t$, then source voltage $v = V\cos(\omega t + \phi)$

Power

Instantaneous: $p = vi$
Resistor: $P_{\text{av}} = \frac{1}{2}VI = V_{\text{rms}} I_{\text{rms}}$
Capacitor and inductor: $P_{\text{av}} = 0$
General AC circuit:

$$P_{av} = \frac{1}{2}VI\cos\phi = V_{\text{rms}} I_{\text{rms}} \cos\phi$$

Power factor: $\cos\phi$