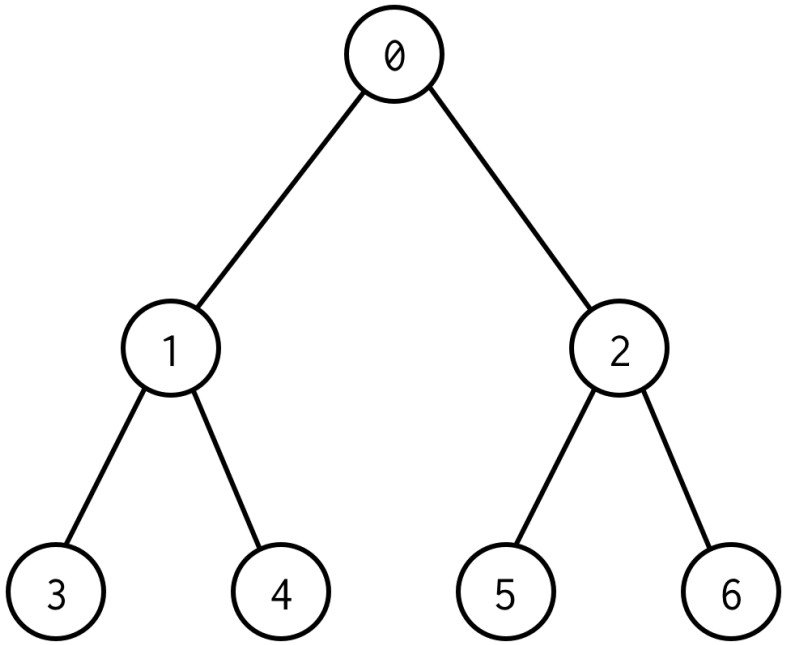
DFS on Binary Tree Array

Implementing Depth-First Search for the Binary Tree without stack and recursion.

Binary Tree Array

This is binary tree.



0 is a root node.
0 has two children: left 1 and right: 2.
Each of its children have their children and so on.
The nodes without children are *leaf* nodes (3,4,5,6).

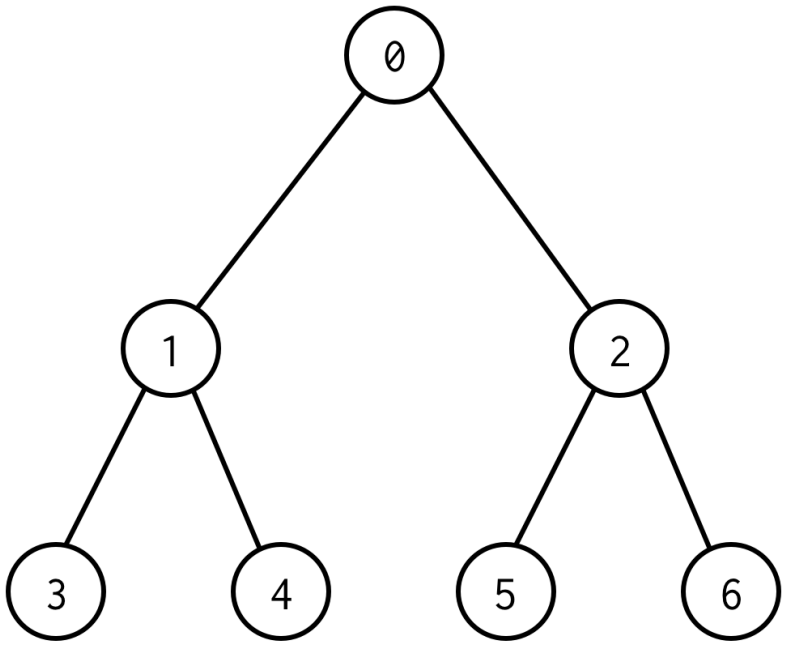
The number of edges between root and leaf nodes define tree *height*. The tree above has the height 2.

Standard approach to implement binary tree is to define class called `BinaryNode`

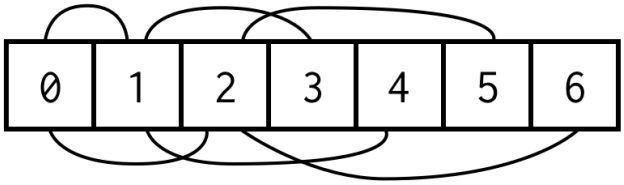
```
public class BinaryNode<T> {
    public T data;
    public BinaryNode<T> left;
```

```
    public BinaryNode<T> right;
}
```

However, there is a trick to implement binary tree using only static arrays. If you enumerate all nodes in binary tree by levels starting from zero you can pack it into array.



a[i] has children a[2*i+1] and a[2*i+2]

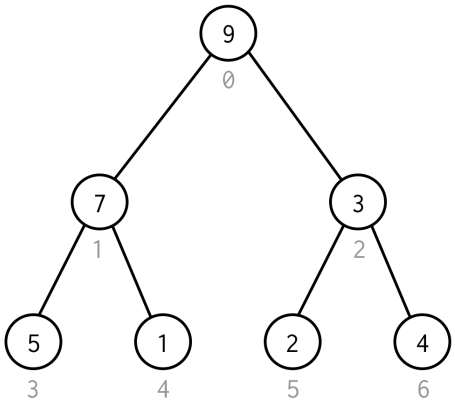


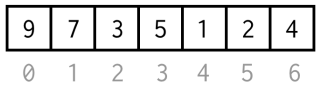
The relationship between parent index and child index, allow us to move from the parent to child, as well as from the child to the parent

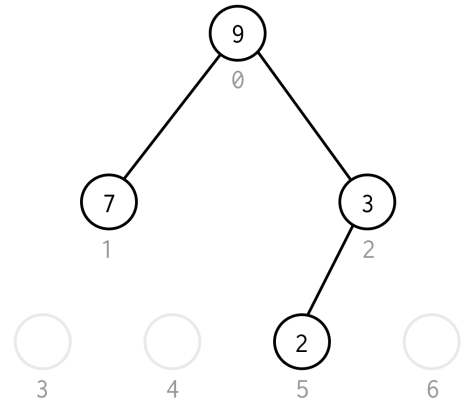
```
left_child(i)  = 2 * i + 1
right_child(i) = 2 * i + 2
parent(i)      = (i - 1) / 2
```

Important note that the tree should be *complete*, what means for the specific height it should contain exactly $2^{height+1} - 1$ elements.

You can avoid such requirement by placing `null` instead of missing elements. The enumeration still applies for empty nodes as well. This should cover any possible binary tree.





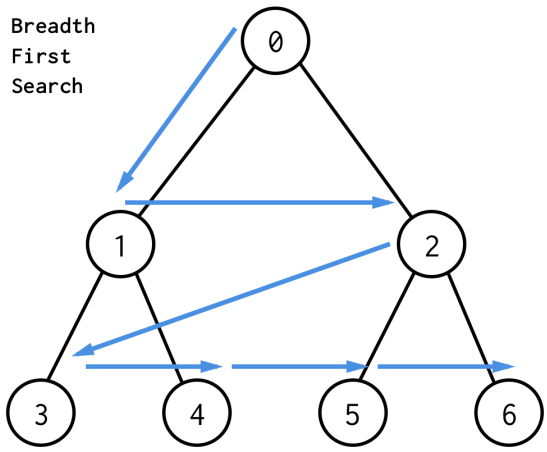


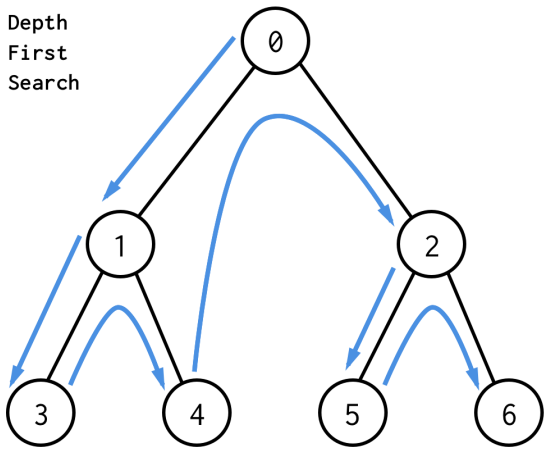
Such array representation for binary tree we will call **Binary Tree Array**. We always assume it will contain $2^n - 1$ elements.

BFS vs DFS

There are two search algorithms exist for binary tree: *breadth-first* search (BFS) and *depth-first* search (DFS).

The best way to understand them is visually





BFS search nodes level by level, starting from the root node. Then checking its children. Then children for children and so on. Until all nodes are processed or node which satisfies search condition is found.

DFS behave differently. It checks all nodes from leftmost path from the root to the leaf, then jumps up and check right node and so on.

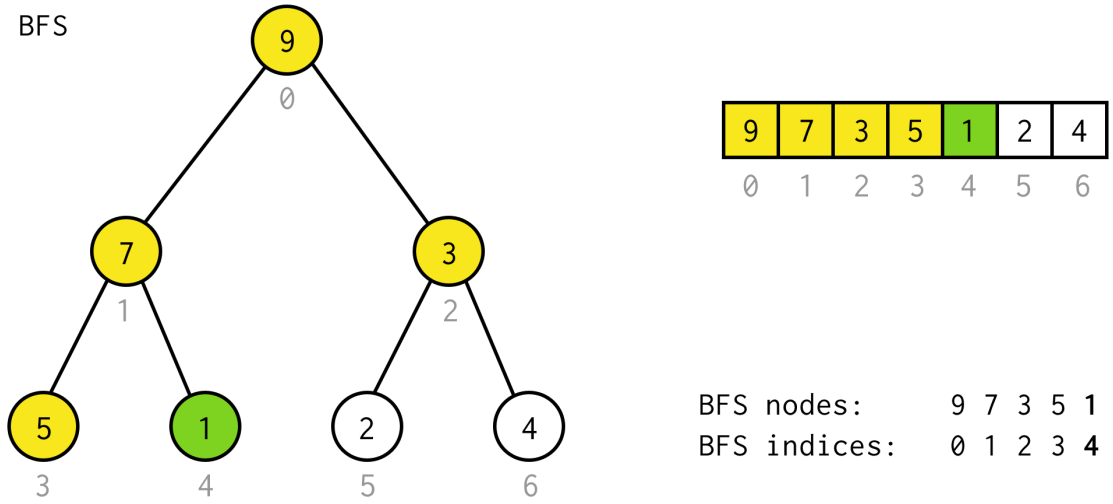
For classic binary trees BFS implemented using Queue data structure (LinkedList implements Queue)

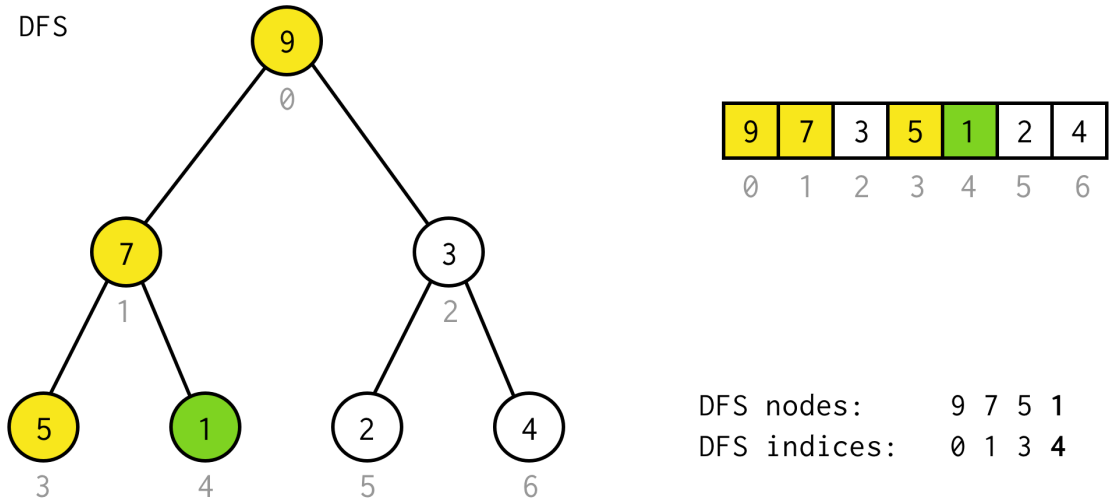
```
public BinaryNode<T> bfs(Predicate<T> predicate) {
    Queue<BinaryNode<T>> queue = new LinkedList<>();
    queue.offer(this);
    while (!queue.isEmpty()) {
        BinaryNode<T> node = queue.poll();
        if (predicate.test(node.data)) return node;
        if (node.left != null) queue.offer(node.left);
        if (node.right != null) queue.offer(node.right);
    }
    return null;
}
```

If you replace queue with the stack you will get DFS implementation.

```
public BinaryNode<T> dfs(Predicate<T> predicate) {
    Stack<BinaryNode<T>> stack = new Stack<>();
    stack.push(this);
    while (!stack.isEmpty()) {
        BinaryNode<T> node = stack.pop();
        if (predicate.test(node.data)) return node;
        if (node.right != null) stack.push(node.right);
        if (node.left != null) stack.push(node.left);
    }
    return null;
}
```

For example, assume you need to find node with value 1 in the following tree.





Yellow cell are cells which are tested in search algorithm before needed node found. Take into account that for some cases DFS require less nodes for processing. For some cases it requires more.

BFS and DFS for Binary Tree Array

Breadth-first search for binary tree array is trivial. Since elements in binary tree array are placed level by level and bfs is also checking element level by level, bfs for binary tree array is just *linear search*

```
public int bfs(Predicate<T> predicate) {
    for (int i = 0; i < array.length; i++) {
        if (array[i] != null && predicate.test(array[i])) return i;
    }
    return -1; // not found
}
```

Depth-first search is a bit harder.

Using stack data structure it could be implemented the same way as for classic binary tree, just put indices into the stack.

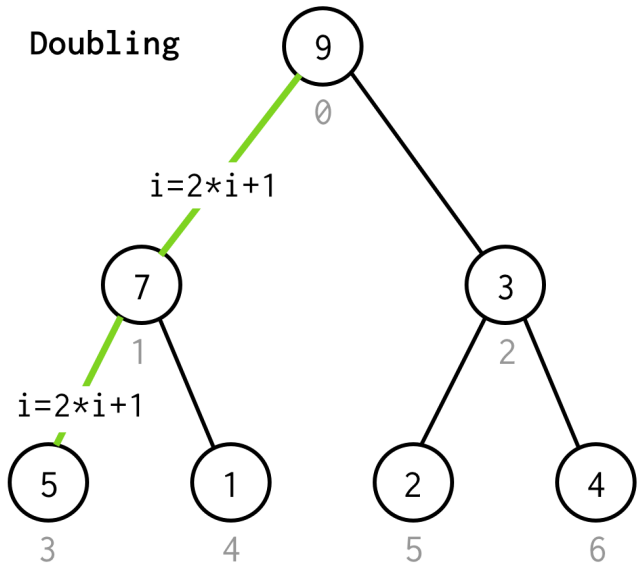
From this point recursion is not different at all, you just use implicit method call stack instead of data structure stack.

What if we could implement DFS without stack and recursion. In the end is just a selecting correct index on each step?

Well, let's try.

DFS for Binary Tree Array (without stack and recursion)

As in BFS we start from the root index `0`.
Then we need to inspect its left child, which index could be calculated by formula `2*i+1`.
Let's call process of going to next left child **doubling**.



We continue doubling the index until we don't stop at the leaf node.

So how can we understand it's a leaf node?

If next left child produce index out of array bounds exception we need to stop doubling and *jump* to the right child.

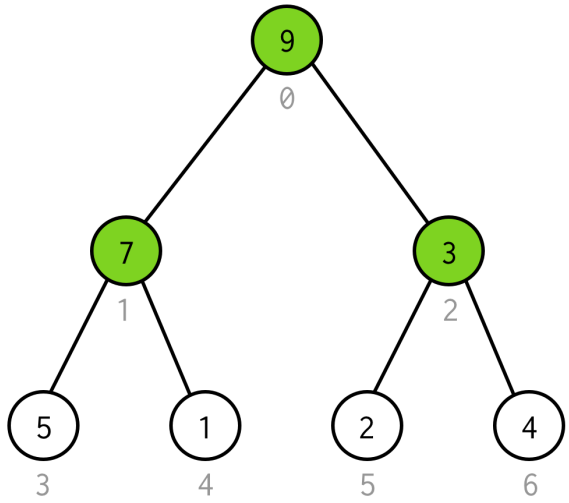
That's a valid solution, but we can avoid unnesessary doubling. The observation here is the number of elements in the tree of some `height` is two times larger (*and plus one*) than the tree with `height - 1`.

For tree with `height = 2` we have `7` elements, for tree with `height = 3` we have `7*2+1 = 15` elements.

Using this observation we build simple condition

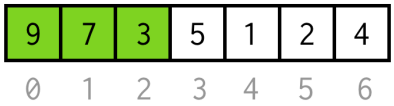
```
i < array.length / 2
```

You can check this condition is valid for any non-leaf node in the tree.



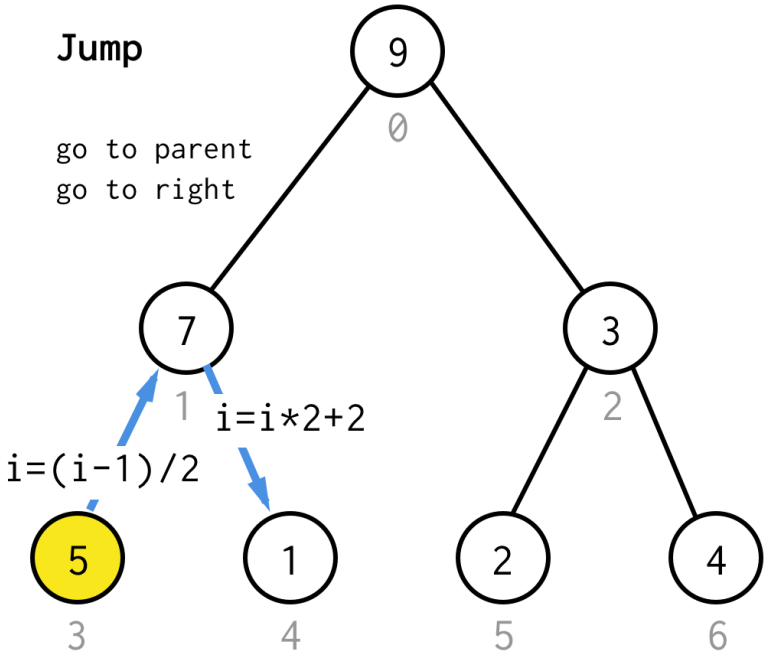

```
i < array.length / 2
i < 7 / 2
i < 3

Condition holds
for all non-leaf nodes
```

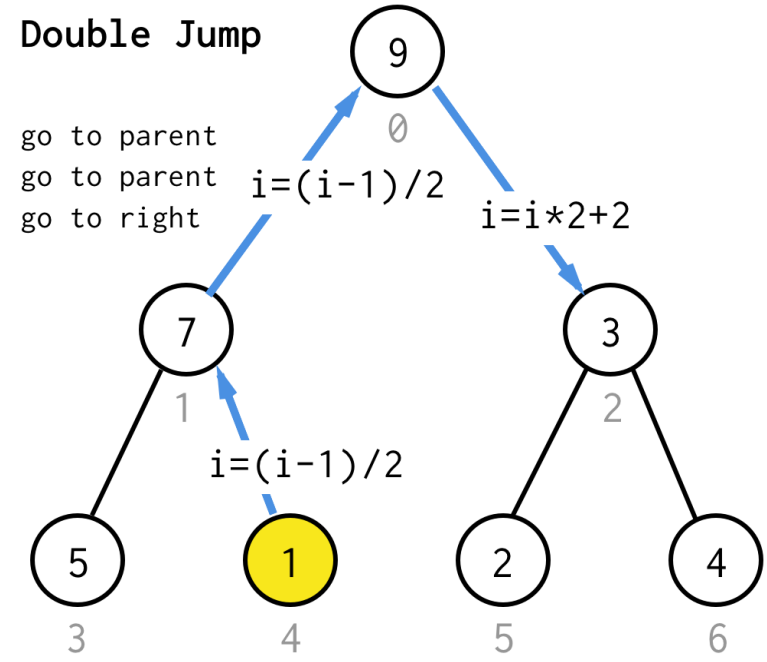


Nice so far. We can run doubling, and we even know when to stop. But this way we can check only left children of the nodes.

When we stuck at the leaf node and can not apply doubling anymore, we perform **jump** operation. Basically, if doubling is go to left child, jump is go to parent and go to right child.



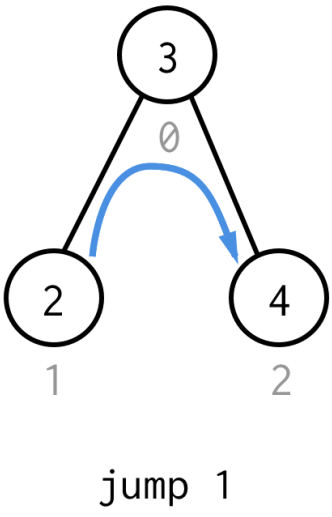
Such jump operation can be implemented by just incrementing the leaf index. But when we stuck at the next leaf node, increment doesn't move node to the next node, required for DFS. So we need to do **double jump**



Make it more generic, for our algorithm we could define operation **jump N**, which specifies how many jumps to the parent it requires and always finishes with one jump to the right child.

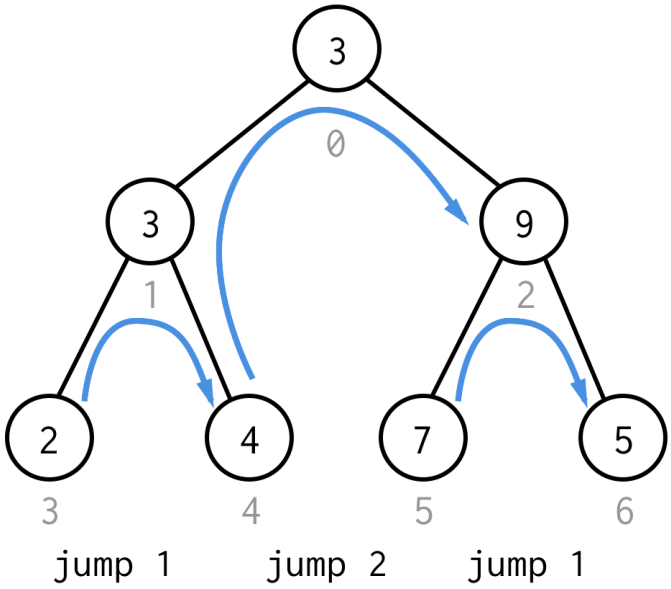
For the tree with height 1, we need one jump, jump 1

Tree with height = 1



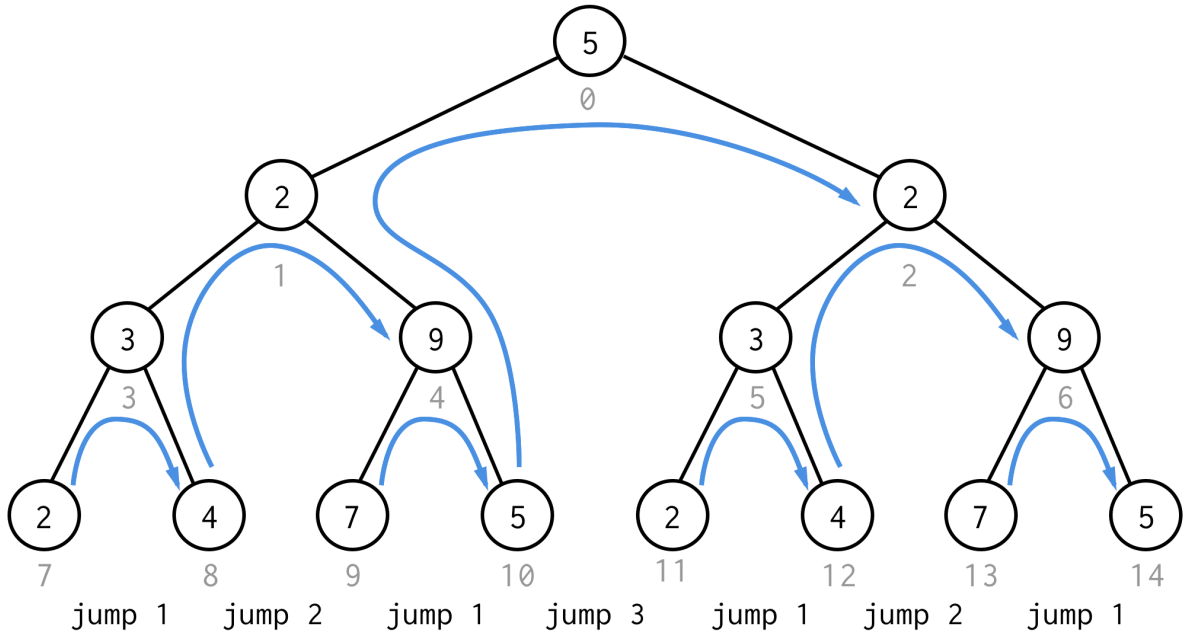
For the tree with height 2, we need one jump, one double jump, and than one jump again

Tree with height = 2



For the tree with height 3, you can apply the same rules and get 7 jumps.

```
Tree with height = 3
```



See the pattern?

$$Jumps_{height} = Jumps_{height-1}\ Height\ Jumps_{height-1}$$

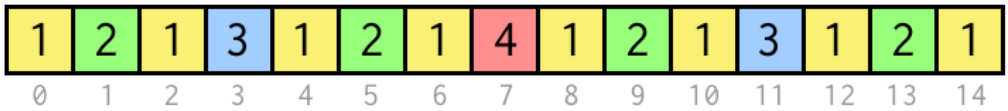
If you apply the same rule to the tree of height 4 and write down only jump values you will get next sequence number

```
1 2 1 3 1 2 1 4 1 2 1 3 1 2 1
```

This sequence known as Zimin words and it has recursive definiton, which is not great, because we not allowed to use recursion.

Let's find some another way to calculate it.

If you look carefully at the jump sequence, you can see non recursive pattern.



Every 2nd element starting from 0 is 1
Every 4th element starting from 1 is 2
Every 8th element starting from 3 is 3
Every 16th element starting from 7 is 4

$$\frac{2^k}{2^{k-1}-1} \quad k$$

We can track index for current leaf `j`, which corresponds to the index in sequence array above.

To verify statement like "Every 8th element starting from 3" we just check `j % 8 == 3`, if it's true three jumps to the parent node required.

Iterative jumping process will be following

- Start with `k = 1`
- Jump once to the parent by applying `i = (i - 1)/2`
- If `leaf % 2*k == k - 1` all jumps performed, exit
- Otherwise, set k to `2*k` and repeat.

If you putting all together what we've discussed

```
public int dfs(Predicate<T> predicate) {
    int i = 0;
    int leaf = 0;
    while (i < array.length) {
        if (array[i] != null && predicate.test(array[i])) return i; // node found
        if (i < array.length / 2) { // not leaf node, can be advanced
            i = 2 * i + 1; // try left child
        } else { // leaf node, jump
            int k = 1;
            while (true) {
                i = (i - 1) / 2; // jump to the parent
                int p = k * 2;
                if (leaf % p == k - 1) break; // correct number of jumps found
                k = p;
            }
            // after we jumped to the parent, go to the right child
            i = 2 * i + 2;
            leaf++; // next leaf, please
```

```
            }
        }
        return -1;
    }
```

Done. DFS for binary tree array without stack and recursion

Complexity

The complexity looks like quadratic because we have loop in a loop, but let's see what reality is.

- For half of the total nodes (`n/2`) we use doubling strategy, which is constant `O(1)`
- For other half of the total nodes (`n/2`) we use leaf jumping strategy.
- Half of the leaf jumping nodes (`n/4`) require only one parent jump, `1` iteration. `O(1)`
- Half from this half (`n/8`) require two jumps, or `2` iterations. `O(1)`
- ...
- Only *one* leaf jumping node require full jump to the root, which is `O(logN)`
- So, if all cases has `O(1)` except one which is `O(logN)`, we can say the average for jump is `O(1)`
- Total complexity is `O(n)`

Performance

Although theory is good, let's see how this algorithm works in practice, comparing to others.

We test 5 algorithms, using JMH.

- BFS on Binary Tree (linked list as queue)
- BFS on Binary Tree Array (linear search)
- DFS on Binary Tree (stack)
- DFS on Binary Tree Array (stack)
- Iterative DFS on Binary Tree Array

We've precreated a tree and using predicate to match the last node, to make sure we always hit the same number of nodes for all algorithms.

So, results

```
Benchmark                                  Mode  Cnt         Score        Error  Units
-----------------------------------------------------------------------------------------
BinaryTreeBenchmarks.bfsOnArray           thrpt  200  91378159.672 ± 625885.079  ops/s
BinaryTreeBenchmarks.bfsOnBinaryTree      thrpt  200   7892439.856 ±  84124.694  ops/s
```

```
BinaryTreeBenchmarks.dfsOnArray          thrpt   200  19980835.346 ± 209143.115  ops/s
BinaryTreeBenchmarks.dfsOnBinaryTree     thrpt   200  13615106.113 ±  91863.833  ops/s
BinaryTreeBenchmarks.dfsOnStackOnArray   thrpt   200  11643533.145 ±  87018.132  ops/s
```

Obviously, BFS on array wins. It's just a linear search, so if you can represent binary tree as array, do it.

Iterative DFS, which we just described in the article, took 2nd place and `4.5` times slower then linear search (BFS on array)

The rest three algorithms even slower then iterative DFS, probably, because they involve additional data structures, extra space and memory allocation.

And the worst case is BFS on Binary Tree. I suspect here the linked list is a problem.

Conclusion

Binary Tree Array is a nice trick to represent hierarchical data structure as a static array (contiguous region in the memory). It defines a simple index relationship for moving from parent to child, as well as moving from child to parent, which is not possible with classic binary array representation. Some drawbacks, like slow insertion/deletion exists, but I'm sure there are plenty applications for binary tree array.

As we have shown, search algorithms, BFS and DFS implemented on arrays are much faster then their brothers on a reference based structure. For real world problems it's not so important, and, probably, I still will use the reference based binary array.

Anyway, it was a nice excercise. Expect question *"DFS on binary array without recursion and stack"* on your next interview.

algorithms datastructures programming

MISHADOFF 10 SEPTEMBER 2017

← Clojure Euler: Problem 026 Evolville 0.0.1: The Beginning →

Frank Rose • 9 months ago

I just got to your blog via the Clojure Design Patterns post - I loved it.

Considering your code here, what if you just wanted to visit depth first vs search? Say you get `op` instead of `predicate`. I think you'd do the following in the leaf clause:

```
        } else {
            op(array[i]); // <--- Perform operation on leaf
            int k = 1;
            while (true) {
                i = (i - 1) / 2;
                int p = k * 2;
                if (leaf % p == k - 1) break;
                op(array[i]); // <--- Perform operation on parent as you pa
                k = p;
            }
            i = 2 * i + 2;
            leaf++;
        }
```

Thinking about this caused me to realize that your current algo is not perfectly depth first. Specifically, if a parent node matches `predicate`, it will be returned rather than a left child which also matches the predicate.

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mishadoff Mod ➚ Frank Rose • 9 months ago

I think you may be wrong, classic DFS traversal checks parent node first, then it goes to the left child.

stack.push(root);
while (!stack.isEmpty()) {
Node node = stack.pop() // <-- check item
if (found(node)) return;
else {
stack.push(node.right);
stack.push(node.left);
}
}

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Frank Rose ➚ mishadoff • 9 months ago

You are correct.

My expectation was that a search would follow vertex ordering "postordering" (thanks Wikipedia). I still think that makes some sense - that a Depth First search would find the matching node at the

greatest Depth First. But I realize I'm adding my own constraint there with "greatest". Depth First really just means Child First (vs Sibling First for Breadth). One might more aptly call my expectation "Descendent First", which wasn't even on the table.

Thanks!

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