Dynamic Semiparametric Portfolio Choice with Elicitable Risk Constraints

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Abstract

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1 Setup sketch

$\mathrm{D} \subseteq \mathbb{R}^p$ and $\mathcal{V} \subseteq \mathrm{D}$

Consider the myopic problem

$$\begin{aligned}
\underset{\boldsymbol{\pi} \in \mathcal{P}, \mathbf{v} \in \mathcal{V}}{\text{maximize}} \quad & \mathbb{E}_{t-1}\left[U(W_t^{\boldsymbol{\pi}})\right] && \text{(1a)}\\
\text{subject to} \quad & W_t^{\boldsymbol{\pi}} = w_0(R_t^f + \boldsymbol{\pi}^\top(\mathbf{Y}_t - R_t^f \mathbf{1}_n)), && \text{(1b)}\\
& \mathbf{v} \in \underset{\mathbf{v}'}{\arg\min}\ \mathbb{E}_{t-1}\left[L(\mathbf{v}', W_t^{\boldsymbol{\pi}})\right] && \text{(1c)}
\end{aligned}$$

2 Illustration

Find reference

Suppose we have asset returns $\mathbf{Y} \sim \mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$. Suppose we have CARA utility $U(w) = -e^{-\gamma w}$ for some risk aversion parameter $\gamma > 0$. By conventional arguments,

$$\mathbb{E}[U(W)] = -\exp\left\{-\gamma w_0 \left[R^f + \boldsymbol{\pi}^\top(\boldsymbol{\mu} - R^f \mathbf{1}_n) - \frac{\gamma w_0}{2}\boldsymbol{\pi}^\top \boldsymbol{\Sigma} \boldsymbol{\pi}\right]\right\} \tag{2}$$

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Fixing a level $\alpha \in (0,1)$, it is well known the the score function $S(x,y) = (\mathbb{1}\{x \geq y\} - \alpha)(x-y)$ will lead to the α-quantile. Hence, (1c) solves explicitly to

$$\mathrm{VaR}_\alpha(\boldsymbol{\pi}) = \arg\min_{v'} \mathbb{E}[L(v', W_t^{\boldsymbol{\pi}})] = w_0 \left(R^f + \boldsymbol{\pi}^\top(\boldsymbol{\mu} - R^f \mathbf{1}_n) + \sqrt{2}\mathrm{erf}^{-1}(2\alpha - 1)\sqrt{\boldsymbol{\pi}^\top \boldsymbol{\Sigma} \boldsymbol{\pi}} \right) \tag{3}$$

Suppose we have the constraint set $\mathcal{V} = \{v' \in \mathbb{R} : v' \leq w_0 \bar{v}\}$ for some constant $\bar{v} \leq 0$. Putting everything together, we reduce the optimization problem to

$$\begin{aligned} \underset{\boldsymbol{\pi} \in \mathbb{R}^n}{\text{maximize}} \quad & R^f + \boldsymbol{\pi}^\top(\boldsymbol{\mu} - R^f \mathbf{1}_n) - \frac{\gamma w_0}{2} \boldsymbol{\pi}^\top \boldsymbol{\Sigma} \boldsymbol{\pi} \\ \text{subject to} \quad & R^f + \boldsymbol{\pi}^\top(\boldsymbol{\mu} - R^f \mathbf{1} n) + \sqrt{2}\mathrm{erf}^{-1}(2\alpha - 1)\sqrt{\boldsymbol{\pi}^\top \boldsymbol{\Sigma} \boldsymbol{\pi}} \leq \bar{v} \end{aligned} \tag{4}$$

The constraint $v \in \mathcal{V}$ behaves like an *individual rationality* constraint in the principal-agent literature. [1]

Find reference for risk preference difference in principal-agent lit

Our highly stylized principal-agent problem here illustrates the tension between the principal ("portfolio manager") and the agent ("risk manager"). Under this stylized illustration, the principal is a mean-variance optimizer while the agent is a mean-standard deviation optimizer. Up to the VaR constraint of $\bar{v}$, the principal and the agent have the same first moment term $R^f + \pi^\top(\boldsymbol{\mu} - R^f \mathbf{1}_n)$ in their preferences. Both the principal and the agent agree to choose portfolios $\boldsymbol{\pi}$ that achieves a high mean wealth. However, the principal and the agent have different risk aversions. Let's assume no wealth effects for simplicity, so say $w_0 = 1$. Note that $\mathrm{erf}^{-1} \leq 0$ on $(-1, 0]$. Thus for conventional values of α (e.g. 0.05), it is ensured that $\mathrm{erf}^{-1}(2\alpha - 1) < 0$. Rewriting as $\sqrt{2}\mathrm{erf}^{-1}(2\alpha - 1)\sqrt{\boldsymbol{\pi}^\top \boldsymbol{\Sigma} \boldsymbol{\pi}} = \frac{1}{2} \frac{2\sqrt{2}\mathrm{erf}^{-1}(2\alpha-1)}{\sqrt{\boldsymbol{\pi}^\top \boldsymbol{\Sigma} \boldsymbol{\pi}}} \boldsymbol{\pi}^\top \boldsymbol{\Sigma} \boldsymbol{\pi}$, we can reinterpret the agent to have a risk aversion parameter of $2\sqrt{2}\mathrm{erf}^{-1}(2\alpha - 1)/\sqrt{\boldsymbol{\pi}^\top \boldsymbol{\Sigma} \boldsymbol{\pi}}$ that simultaneously depends on the level α and also on the portfolio volatility $\sqrt{\boldsymbol{\pi}^\top \boldsymbol{\Sigma} \boldsymbol{\pi}}$. Thus, our setup here illustrates one of the core conflicts between the principal and the agent whereby they have strictly different risk preferences. Of course, without the constraint $v' \in \mathcal{V}$, there will be no conflict between the principal and the agent.

3 Literature review

Primary references: [Patton et al., 2019], [Chen et al., 2016]

Secondary references: [Dimitriadis and Bayer, 2019]

Portfolio choice: [Brandt, 1999]

Mechanism design: [Rogerson, 1985], [Mirrlees, 1999] (written in 1975 but published in 1999), [Jewitt, 1988]

Score models: [Creal et al., 2013], recent surveys [Artemova et al., 2022a, Artemova et al., 2022b]

Nonlinear regression: [Huber, 1967], [White and Domowitz, 1984], [Oberhofer, 1982], [Powell, 1984], [Weiss, 1991]

Quantile regression: [Koenker, 2005, Koenker, 2017]

Financial econometrics: [Engle and Manganelli, 2004]

Elicitable risks: [Fissler and Ziegel, 2016], [Gneiting, 2011], recent survey [He et al., 2022]

[1] Strictly speaking, we should formulate the individual rationality constraint in terms of the agent's utility function. In our current setup, it would take the form $\mathbb{E}[L(v', W^{\boldsymbol{\pi}})] \geq \underline{L}$ for some constant $\underline{L}$.

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