Swaption Pricing and Hedging with Hull-White One Factor Model

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Abstract

This paper summarizes the results of our attempt to model European plain-vanilla interest rate swaptions by using the Hull-White One Factor model. There are two ways to price swaptions with this interest rate model: Jamshidian method and Monte Carlo simulation. The pricing results show Jamshidian method performs well in swaptions with long maturities and tenors (3 years or more), while Monte Carlo model is more suitable for swaptions with shorter maturities and tenors. One possible way to improve the pricing performance through recalibration is also examined, where we exclude the swaptions with high relative errors from the original calibration in the new calibration. In our swaption hedging study, portfolio PnL shows delta-hedging makes the portfolio less risky. Finally, further study shows increasing hedging frequency improves the hedging result.

Contents

1. Introduction

This thesis is concerned with the pricing of swaptions. Swaptions are financial derivatives that can cover interest rate risk or serve other purposes. The most common model to value swaptions is the Black-76 model. This model is popular because of its simplicity. The model has only one unknown parameter, namely the volatility. Hence the performance of the model depends on the reliability of the volatility parameter. If there is a liquid market on which a particular swaption is heavily traded, then the swaption volatility that is quoted in the market is likely reliable and the Black-76 model can be implemented. But what if this is not the case? How do we deal with situations in which no reliable volatility can be observed from the markets?

The answer is stochastic interest rate models. Many one-factor, two-factor and multiple factor interest rate models are at hand to value swaptions. Tools like QuantLib library in C++ can be used to achieve the goals. It should be noted that these models can also be used to value other interest rate products like caps, swaps and captions. This thesis is specially focused on Hull-White One Factor model, due to its popularity, relative simplicity and advantage of being able to fit today's term structure of interest rates. Besides the Hull-White One Factor model many more interesting interest rate models are available. However, one should always be cautious when considering more complex models. A complex model could lead to overfitting.

In addition, this thesis attempts to take a first step in the direction of modeling European swaptions with stochastic interest rate models. One should realize that the valuation of American and Bermudan swaptions with such models is more interesting but therefore also more challenging.

Because the mathematical deduction of swaption pricing is not the main focus of this thesis, mathematical equations are only explained briefly and without further deduction. For a detailed explanation of the pricing formula, the readers should refer to the references [3].

Based on the swaption price, some interesting practical applications like swaption hedging can be studied. As financial intermediaries in the majority of over-the-counter transactions, banks accumulate large amounts of interest rate derivatives in their books. These exposures can constitute a threat in case of an unexpected abrupt movement in interest rates. For this reason, financial institutions have become more concerned on how to manage and hedge these financial risks.

2. Hull-White One Factor Model

In financial mathematics, the Hull–White One Factor model is a model of future interest rates. In its most generic formulation, it belongs to the class of no-arbitrage models that are able to fit today's term structure of interest rates. It is relatively straightforward to translate the mathematical description of the evolution of future interest rates onto a tree or lattice. The model is still popular in the market today.

Hull and White (1990) assumed that the instantaneous short-rate process evolves under the risk-neutral measure according to

$$dr(t) = [\theta(t) - ar(t)]dt + \sigma dW(t). \tag{2.1}$$

Note that the Hull-White One Factor model can also be written as

$$dr(t) = a\left[\frac{\theta(t)}{a} - r(t)\right]dt + \sigma dW(t). \tag{2.2}$$

This model can be interpreted as follows: the parameter $\theta(t)$ is chosen such that the model is consistent with the current term structure of interest rates, where a and σ are usually positive constants and can be calibrated by market value of interest rate derivatives like caps and swaptions, and $W(t)$ denotes a Brownian motion under the probability measure corresponding to the money-market account. Since the Hull-White One Factor model is a no-arbitrage model, it fits the current term structure by definition. $\frac{\theta(t)}{a}$ is the mean level of reversion and the reversion speed is a. Note that the standard deviation of the short rate and the reversion speed are still constant over time. In the Hull-White One Factor model negative interest can occur as is also the case in the Vasicek model.

Assume the instantaneous forward rate that are currently observed in the market is denoted by $f^M(0,T)$. Additionally denote the zero coupon bonds that are currently observed in the market by $P^M(0,T)$. To fit the model to the current term structure of interest rate $\theta(t)$ should be defined as follows:

$$\theta(t) = \frac{\partial f^M(0,t)}{\partial T} + af^M(0,t) + \frac{\sigma^2}{2a}(1 - e^{-2at}). \tag{2.3}$$

By integrating equation (2.1), an explicit expression for $r(t)$ can be obtained:

$$r(t) = r(s)e^{-a(t-s)} + \int_s^t e^{-a(t-u)}\,\theta(u)du + \sigma\int_s^t e^{-a(t-u)}\,dW(u). \tag{2.4}$$

From this it can be seen that $r(t)$ is normally distributed.

2.1 Zero-Coupon Bond Price

Notice indeed that, due to the Gaussian distribution of $r(T)$ conditional on $\mathcal{F}(t)$, t $\leqslant$ T,

$\int_t^T r(u)du$ is itself normally distributed. So that zero-coupon bond prices can be obtained easily. The price of a zero-coupon bond in case of the Hull-White One Factor model is given by the following set of equations:

$$P(t,T) = A(t,T)e^{-B(t,T)r(t)}. \tag{2.5}$$

where $A(t,T) = \frac{P^M(0,T)}{P^M(0,t)}\exp(\mathrm{B(t,T)}f^M(0,t) - \frac{\sigma^2}{4a}(1-e^{-2at})\mathrm{B(t,T)}^2)$ and $\mathrm{B(t,T)} = \frac{1-e^{-a(T-t)}}{a}$.

2.2 Zero-Coupon Bond Option

Consider the European call with maturity T, strike X and written on a unit-principal zero coupon bond with maturity S > T leads to the pricing formula:

$$\boldsymbol{ZBC}(t,T,S,X) = E\left(e^{-\int_t^T r_u du}(P(T,S)-X)^+ \middle| \mathcal{F}(t)\right). \tag{2.6}$$

Under T-forward measure, the zero-coupon bond call price is:

$$\boldsymbol{ZBC}(t,T,S,X) = \mathrm{P(t,T)}E^T((P(T,S)-X)^+|\mathcal{F}(t)). \tag{2.7}$$

After knowing the distribution of the process $r(t)$ under the T-forward measure Q^T, the European call-option price is:

$$\boldsymbol{ZBC}(t,T,S,X) = \mathrm{P(t,S)}\Phi(h) - XP(t,T)\Phi\big(h-\sigma_p\big). \tag{2.8}$$

Where $\sigma_p = \sigma\sqrt{\frac{1-e^{-2a(T-t)}}{2a}}B(T,S)$, $h = \frac{1}{\sigma_p} ln\frac{P(t,S)}{P(t,T)X} + \frac{\sigma_p}{2}$. The price of zero coupon bond put is similar.

2.3 Coupon Bond Option

European options on coupon-bearing bonds can be explicitly priced by means of Jamshidian's (1989) decomposition. To this end, consider a European option with strike X and maturity T, written on a bond paying n coupons after the option maturity. Denote by $T_i, T_i > T$, and by c_i the payment time and value of the i-th cash flow after T. Denote by r^* the value of the spot rate at time T for which the coupon-bearing bond price equals the strike and by X_i the time-T value of a zero coupon bond maturing at T_i when the spot rate is r^*. Then the coupon bond option price at time $t < T$ is:

$$\boldsymbol{CBO}(t,T,c,X) = \textstyle\sum_{i=1}^{n} c_i \boldsymbol{ZBO}(t,T,T_i,X_i). \tag{2.8}$$

2.4 Parameters Calibration

The next section will elaborate on the valuation of swaptions under Hull-White One Factor models and we will discuss the concept of calibration. As mentioned before, there are two parameters, i.e. a and σ, in the Hull-White One Factor model. This subsection will discuss how these parameters can be determined; this process is referred to as calibration.

The purpose of the calibration is to fit the model as closely as possible to observable market data. Usually cap or swaption volatilities are taken into account but also the initial term structure can be used for this purpose. In a sense the user of the model is free to choose which data is used for the calibration. To fit the model as closely as possible to the chosen market data one has to minimize the errors. Of course, there are several possibilities. Here we decided to follow Hull and to minimize the sum of squared errors. The errors are defined as the difference between the calculated swaption value resulting from the Jamshidian method of Hull-White One Factor model and the Black-76 model. The sum runs from one to a total of calibration instruments. A calibration instrument in this setting should be interpreted as a piece of market information. For example, if the model is calibrated against five quoted swaption volatilities there are a total of five calibration instruments. The model parameters will then be set such that the sum of squared errors is minimized. Formally this can be formulated as follow:

$$min \sum_{i=1}^{n} \left(V_{Swaption_i}^{Black-76} - V_{Swaption_i}^{Jamshidian} \right)^2 . \tag{2.9}$$

Here $V_{Swaption_i}^{Black-76}$ denotes the value of swaption that has been valued according to the Black-76 method, in turn $V_{Swaption_i}^{Jamshidian}$ denotes the value of swaption that has been valued according to the Jamshidian method of Hull-White One Factor model. The parameters found by the calibration can then be used to price different swaptions. Of course, there are many possibilities to calibrate the model, there is no general rule of thumb regarding how many swaptions should be used to calibrate the parameters.

3. Swaption Pricing

Swaptions are options on an interest rate swap (IRS). There are two main types of swaptions, a payer version and a receiver version. A European payer swaption is an option giving the right (and no obligation) to enter a payer IRS at a given future time, the swaption maturity.

Usually the swaption maturity coincides with the first reset date of the underlying IRS. The underlying IRS length $(T_\beta - T_\alpha)$ is called the tenor of the swaption. Sometimes the set of reset and payment dates is called the tenor structure.

Consider the discounted payoff of a payer swaption by considering the value of the underlying payer IRS at its first reset date T_α, which is also assumed to be the swaption maturity. The price is thus:

$$ND(t, T_\alpha)(\sum_{i=\alpha+1}^{\beta} P(T_\alpha, T_i)\tau_i(F(T_\alpha; T_{i-1}, T_i) - K))^+. \tag{3.1}$$

Where N is the notional, $D(t, T_\alpha)$ is the discount factor and $F(T_\alpha; T_{i-1}, T_i)$ is the forward rate, note swaption is only exercised if the underlying payer IRS at its first reset date T_α has positive value.

As discussed in the introduction section, swaptions can be priced by different models. The most common model is the Black-76 model. Also, Hull-White One Factor model can price swaptions via Jamshidian method or Monte Carlo simulation. In this section, details of pricing methods with Black-76 model and Hull-White One Factor model will be explained.

3.1 Black-76 Model for Swaptions

The Black Scholes formula is an elegant and useful way for pricing stock options. Fisher Black extended the model such that it can cope with interest rate derivatives. One should note that the pricing of interest rate derivatives is more complex than the pricing of stock options. To be able to price interest rate derivatives, forward swap rates have to be determined. Moreover, interest rates have to be used not only to price the option but also to discount the future payoffs. Various interest rate derivatives like caps, floors and swaptions can be valued using the Black-76 model.

The Black-76 model is closely related to the Black Scholes model but makes use of one extra assumption. For the valuation of swaptions it is assumed that the underlying forward swap rate $S_{\alpha,\beta}(t)$ at the swaption maturity follows the lognormal distribution, so sometimes it is also called the Lognormal Forward-Swap model (LSM). Under such assumption, the swaption price at time 0 is:

$$\sum_{i=\alpha+1}^{\beta} P(0, T_i)N\tau_i \left(S_{\alpha,\beta}(0)\Phi(d_+) - K\Phi(d_-)\right), \tag{3.2}$$

where $d_{\pm} = \frac{ln\frac{S_{\alpha,\beta}(0)}{K} \pm \frac{\sigma_{\alpha,\beta}{}^2}{2}T_\alpha}{\sigma_{\alpha,\beta}\sqrt{T_\alpha}}$. Compared to the Black Scholes option pricing formula, there is an extra term included in equation (3.2), this term discounts the cash flows.

3.2 Jamshidian Method

The basis of Jamshidian method is that the underlying swap value of a swaption can be viewed as the difference between coupon bond and floating bond. As the value of floating bond at reset date is always its notional, swaption can also be viewed as coupon bond option with strike equals to the notional. With methods discussed in 2.3, we can price swaption by calculating the corresponding coupon bond option price.

Indeed, consider a payer swaption with strike rate X, maturity T and nominal value N, which gives the holder the right to enter at time $t_0 = T$ an interest rate swap with payment times $\{t_1, \ldots, t_n\}$, $t_1 > T$, where he pays at the fixed rate X and receives LIBOR set "in arrears". Denote by τ_i the year fraction from t_{i-1} to $t_i, i = 1, \ldots, n$ and set $c_i = 1 + X\tau_i$ for $i = 1, \ldots, n-1$. Denoting by r^* for which the value of the spot rate at time T the coupon-bearing bond price equals the strike unit-notional, that is:

$$\sum_{i=1}^{n} c_i\, A(T, t_i) e^{-B(T,t_i)r^*} = 1.$$

And setting $X_i = A(T, t_i)e^{-B(T,t_i)r^*}$, then the payer swaption price at time $t < T$ is then given by:

$$\boldsymbol{PS}(t, T, N, X) = N \sum_{i=1}^{n} c_i \boldsymbol{ZBP}(t, T, t_i, X_i). \qquad (3.3)$$

Analogously, the price of the corresponding receiver swaption is:

$$\boldsymbol{RS}(t, T, N, X) = N \sum_{i=1}^{n} c_i \boldsymbol{ZBC}(t, T, t_i, X_i). \qquad (3.4)$$

With equations in 2.2 and 2.3, an analytical solution of European swaption by Hull-White One Factor model can be deduced.

3.3 Monte Carlo Simulation

Recall the payoff function of swaption (3.1), an alternative expression for this equation, expressed in terms of the relevant forward swap rate, is at time t:

$$ND(t, T_\alpha)(S_{\alpha,\beta}(T_\alpha) - K)^+ \sum_{i=\alpha+1}^{\beta} P(T_\alpha, T_i)\tau_i, \qquad (3.5)$$

where the forward swap rate $S_{\alpha,\beta}(T_\alpha) = \frac{1-P(T_\alpha,T_\beta)}{\sum_{i=\alpha+1}^{\beta} P(T_\alpha,T_i)\tau_i}$. Notice the only uncertain part is $P(T_\alpha, T_i), i = \alpha + 1, \ldots, \beta$.

As a result, once the parameters a and σ in the Hull-White One Factor model are calibrated, forward zero-coupon bond price $P(T_\alpha, T_i), i = \alpha + 1, \ldots, \beta$ can be simulated. Then the swaption price can be obtained with Monte Carlo simulation.

4. Swaption Hedging

When one goes long/short an option in the market, there is a problem in managing the risk due to reasons such as the changing value of the option's underlying. Here, the underlying of the swaption is the swap in the option. For example, going long a payer swaption leads to the risk of losing money if its underlying swap value decreases. Since the changing swap value is the main reason of the risk of swaption, delta, which is the rate of change of the theoretical option value with respect to changes in the underlying asset's price, becomes the target Greek in this hedging study. This section will provide theoretical methods to neutralize the delta exposure.

4.1 Delta Hedging

When setting up a delta hedge, one typically wants to obtain a position that is both replicating and self-financing. The general idea of creating a portfolio that is delta neutral is that the price changes of the underlying security are compensated by the price changes of the derivative security. The delta neutral portfolio can be constructed by going long a unit of the option and going short a quantity delta (Δ) of the underlying security. Price increases of the underlying are in this way compensated by the price drops of the derivative security and vice versa and the property of self-financing is met. By setting the delta of the portfolio to zero, the risk caused by fluctuations in the underlying security should theoretically - and almost practically - be eliminated. The delta neutral portfolio is set up by

$$\Delta_{Port} = \frac{\partial V_{Port}}{\partial V_{Underlying}} = -\Delta * \frac{\partial V_{Underlying}}{\partial V_{Underlying}} + \frac{\partial V_{Derivative}}{\partial V_{Underlying}} = -\Delta * 1 + \Delta = 0 \qquad (4.1)$$

4.2 Delta Hedging of Swaptions

To set up a delta neutral portfolio with swaptions, suppose that one goes short (long) one swaption contract at time t. The procedure of delta hedging the short (long) position in the swaption contract, is to go long (short) an amount of the underlying swap contract at time t. The exact amount swap contracts needed, ω, to achieve a delta neutral portfolio is obtained by setting the delta exposure of the portfolio components that carry delta risk equal to zero as shown below.

$$-\omega * \frac{\partial V_{Swap;t}}{\partial V_{Swap;t}} + \frac{\partial V_{Swaption;t}}{\partial V_{Swap;t}} = 0$$

$$\Rightarrow \omega = \frac{\partial V_{Swaption;t}}{\partial V_{Swap;t}}. \qquad (4.2)$$

In section 3.1, the value of swaption by Black-76 model is discussed and defined as the formula:

$$\sum_{i=\alpha+1}^{\beta} P(0,T_i) N \tau_i \left(S_{\alpha,\beta}(0)\Phi(d_+) - K\Phi(d_-) \right). \qquad (4.3)$$

Notice, $S_{\alpha,\beta}(0)$ represents forward swap rate, but the underlying for delta hedge is swap value. Hence, a relation between forward swap rate and swap value is required. Define swap value as $W_{\alpha,\beta}$, then the swaption price becomes:

$$V_0 = \sum_{i=\alpha+1}^{\beta} P(0,T_i) N\tau_i \left(S_{\alpha,\beta}(0)\Phi(d_+) - K\Phi(d_-)\right)$$

$$= \sum_{i=\alpha+1}^{\beta} P(0,T_i) N\tau_i S_{\alpha,\beta}(0)\Phi(d_+) - K\Phi(d_-)N \sum_{i=\alpha+1}^{\beta} P(0,T_i)\tau_i$$

$$= \mathrm{N}\left[P(0,T_\alpha) - P\left(0,T_\beta\right)\right]\Phi(d_+) - K\Phi(d_-)N \sum_{i=\alpha+1}^{\beta} P(0,T_i)\tau_i$$

$$= \mathrm{N}\left[P(0,T_\alpha) - P\left(0,T_\beta\right) - K\textstyle\sum_{i=\alpha+1}^{\beta} P(0,T_i)\tau_i\right]\Phi(d_+) - K[\Phi(d_+) - \Phi(d_-)]N\textstyle\sum_{i=\alpha+1}^{\beta} P(0,T_i)\tau_i. \qquad (4.4)$$

Here,

$$W_{\alpha,\beta} = N(P(0,T_\alpha) - P\left(0,T_\beta\right) - K\sum_{i=\alpha+1}^{\beta} P(0,T_i)\tau_i)$$

$$= N(\textstyle\sum_{i=\alpha+1}^{\beta} P(0,T_i)N\tau_i\left(S_{\alpha,\beta}(0) - K\right)), \qquad (4.5)$$

is the value of the underlying swap, and

$S_{\alpha,\beta}(0) = \frac{W_{\alpha,\beta}}{\sum_{i=\alpha+1}^{\beta} P(0,T_i)N\tau_i} + K$, $d_\pm = \frac{ln\frac{S_{\alpha,\beta}(0)}{K} \pm \frac{\sigma_{\alpha,\beta}{}^2}{2}T_\alpha}{\sigma_{\alpha,\beta}\sqrt{T_\alpha}}$. Then $\frac{\partial V_0}{\partial W_{\alpha,\beta}} = \frac{\partial V_0}{\partial S_{\alpha,\beta}(0)}\frac{\partial S_{\alpha,\beta}(0)}{\partial W_{\alpha,\beta}} = \Phi(d_+)$.

This gives, that the property of a delta neutral portfolio is obtained by going long (short) an amount $\Phi(d_+)$ in swap contracts for every unit short (long) in swaptions.

5. Data

This section is devoted to the description of the data. As explained in section 2, implied volatilities and discount factors data are used to calibrate Hull-White One Factor model and price swaptions. They are also needed for calculating daily swap values in the swaption hedging section. The data that is used in this section was obtained from Bloomberg.

We chose July 1, 2008 as our settlement date for the swaption pricing, and July 1, 2008 to October 31, 2008 as our time period for swaption hedging. We thought it would be very interesting to see how the model performs when tested against such a turbulent time in the state of the economy, and whether delta-hedging is still effective in such a time period.

Considering the illiquidity of swaptions with long maturities and tenors, only 100 swaptions with maturities and tenors ranging from 1-10 years are used as the calibration. In order to see the pricing errors, those swaptions are selected as the pricing targets too, but note that the methods in this thesis are applicable to any kinds of swaptions regardless of the liquidity.

The implied volatilities data of those swaptions is shown in Table 1, in which the first column represents the maturity of the swaption and the heading represents the tenor of the swaption. For example, a swaption that will be exercised after 7 years since July 1, 2008 with a 6-year swap as underlying has the implied volatility 0.18 on July 1, 2008.

Expiry	1Yr	2Yr	3Yr	4Yr	5Yr	6Yr	7Yr	8Yr	9Yr	10Yr
1Yr	36.7	34	31.7	30	28.33	27.2	26.1	24.65	24	22.9
2Yr	28.55	27.25	26.45	25.48	24.45	23.65	22.95	22.6	22.15	21.15
3Yr	25.4	24.3	23.85	23.18	22.7	21.85	21.7	21.3	20.3	20.08
4Yr	23.3	22.55	21.95	21.45	21.05	20.63	20.18	20.05	19.7	19.1
5Yr	21.7	21.1	20.7	20.02	19.77	19.5	19.4	19	18.75	18.5
6Yr	20.3	19.5	19.45	19.15	18.8	18.55	18.2	18.3	17.85	17.5
7Yr	19.1	18.5	18.3	18.08	18.2	18	17.7	17.4	17.2	17
8Yr	18.15	17.7	17.5	17.3	17.05	16.9	16.75	16.55	16.4	16.15
9Yr	17.2	16.9	16.65	16.5	16.3	16.1	16.1	15.85	15.7	15.55
10Yr	16.5	16.2	16.4	16.3	16.1	15.68	15.8	15.55	15.4	14.95

Table 1 Swaption Volatility Surface on July 1st, 2008

Table 2 shows the discount factors data on July 1, 2008 from Bloomberg. The data is obtained by the piecewise construction and linear interpolation of the yield curve by Bloomberg. In Monte Carlo simulation section, Libor overnight rate is used as current short rate.

Term	Discount	Term	Discount
3 MO	0.992927	7 YR	0.728787
5 MO	0.986806	8 YR	0.691449
8 MO	0.978919	9 YR	0.656302
11 MO	0.970761	10 YR	0.622579
17 MO	0.962183	11 YR	0.590248
14 MO	0.953095	12 YR	0.559307
20 MO	0.943441	15 YR	0.476357
2 YR	0.931041	20 YR	0.365535
3 YR	0.88915	25 YR	0.281803
4 YR	0.847439	30 YR	0.219415
5 YR	0.806846	40 YR	0.134163
6 YR	0.76738	50 YR	0.08353

Table 2 Discount Factor on July 1st, 2008

For swaption hedging, daily swaption volatilities and discount factors data from July 1, 2008 to October 31, 2008 are used to calibrate the Hull-White One Factor model and price daily swap and swaption value.

6. Results

So far, the theoretical background and the models have been discussed. In this section, the capabilities and the limitations of the model are investigated. The delta-hedging performance based on daily swaption and swap price calculated by the model is analyzed.

6.1 Swaption Pricing

Considering swaption is usually quoted by implied volatility instead of price, both the swaption implied volatility and price are calculated. Note implied volatility can actually be converted to price by plugging into Black'76 model and vice versa.

In order to show the pricing errors, we define $\mathbf{Error} = \mathbf{Model\ Value - Market\ Value}$; $\mathbf{Relative\ Error} = \frac{(\mathbf{Model\ Value - Market\ Value})}{\mathbf{Market\ Value}}$. The 3-dimensional graphs of the results are drawn. The result by Jamshidian method is shown in Figure 1, the result of Monte Carlo is in Figure 2.

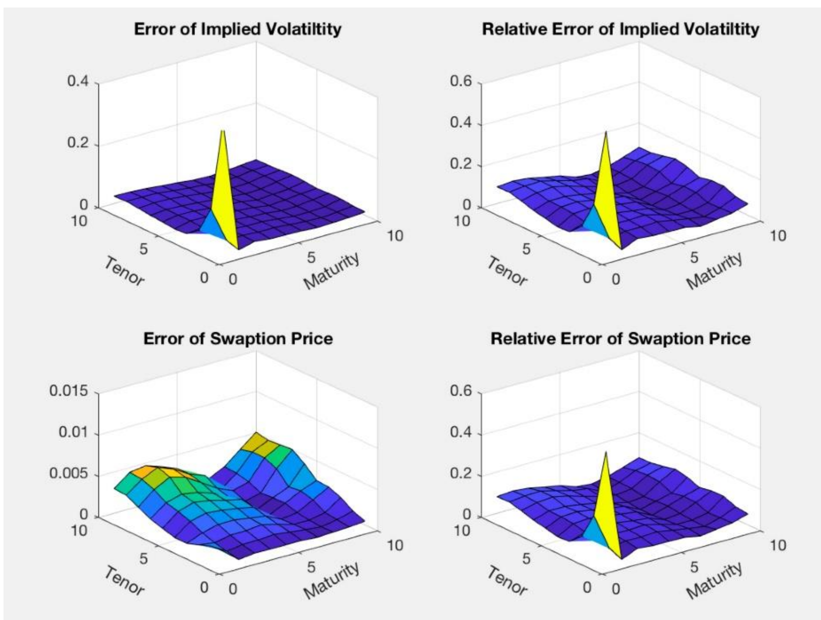

Figure 1 Pricing Result by Jamshidian Method

By Figure 1, Jamshidian method performs well for most of the swaptions, the relative errors of implied volatility and price are usually less than 10%, with median around 5%. However, for swaptions with short maturity and short tenor, the pricing errors and relative errors are pretty high. This problem is caused by the nature of Hull-White One Factor model; it cannot capture the market volatility in the short term when the market volatility is a curve with hump, which is the typical pattern for a yield curve. This problem might be solved if more advanced models are considered, such as two factors short rate (G2++) model. Also, note the error of swaption price has no specific pattern, so it's more important to focus on relative error instead of just the error in this thesis.

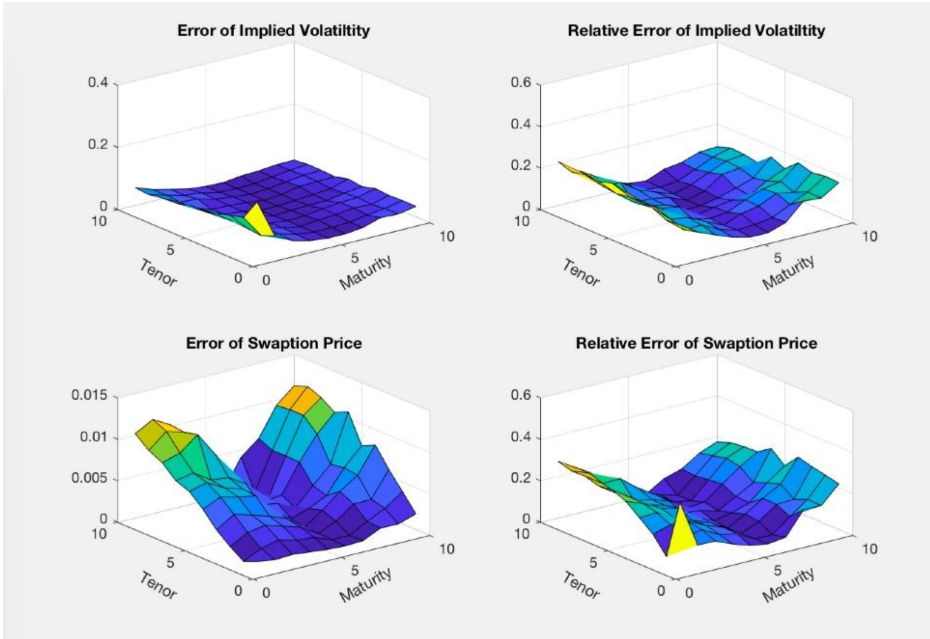

Figure 2 Pricing Result by Monte Carlo Simulation

From Figure 2, most of relative errors by Monte Carlo simulation are larger than that of Jamshidian Method. However, notice that for swaptions with short maturity and tenor, the relative errors are much less than that of Jamshidian model.

In comparison, Jamshidian method is suitable for pricing swaptions with relatively long maturity and long tenor (more than 3 years), while Monte Carlo model is more suitable for swaptions with short maturities and short tenors.

Table 3 shows the model implied volatilities, real implied volatilities, model prices, and real prices by Jamshidian method in numbers. All errors and relative errors above are calculated based on the data in Table 3.

Maturity	Tenor	Model Implied Volatility	Real Implied Volatility	Model Price	Real Price
1	1	1.1499	0.719	0.004413	0.002851
1	2	0.726947	0.598	0.008668	0.007181
1	3	0.540885	0.5	0.012732	0.01179
1	4	0.439657	0.428	0.016516	0.016085
1	5	0.378882	0.3878	0.020055	0.020522
1	6	0.339931	0.353	0.02335	0.024238
1	7	0.313522	0.3297	0.026415	0.027766
1	8	0.294273	0.3155	0.02929	0.031385
1	9	0.279632	0.3048	0.031929	0.034782
1	10	0.268112	0.2915	0.034384	0.037363
2	1	0.544053	0.5442	0.00609	0.006091
2	2	0.435353	0.4475	0.011905	0.012227
2	3	0.368863	0.3835	0.017419	0.018094
2	4	0.327623	0.3453	0.022538	0.02373
2	5	0.300354	0.3215	0.027326	0.029218
2	6	0.281565	0.3033	0.031791	0.034209
2	7	0.267714	0.2888	0.03596	0.038754
2	8	0.256848	0.281	0.039847	0.043547
2	9	0.248262	0.273	0.043434	0.047711
2	10	0.241193	0.2643	0.046771	0.051202
3	1	0.367526	0.3808	0.007217	0.007469
3	2	0.32118	0.3358	0.01409	0.014714
3	3	0.291909	0.3075	0.02049	0.021559
3	4	0.272283	0.29	0.026491	0.02818
3	5	0.258653	0.2783	0.032094	0.034487
3	6	0.24847	0.2688	0.037332	0.040334
3	7	0.240322	0.2595	0.042222	0.045537
3	8	0.233798	0.2577	0.046739	0.051442
3	9	0.228354	0.252	0.050945	0.056141
3	10	0.225227	0.2448	0.054953	0.05966
4	1	0.288447	0.2965	0.008036	0.008254
4	2	0.26725	0.2788	0.015543	0.016198
4	3	0.252844	0.2655	0.022595	0.023701
4	4	0.242786	0.2573	0.029188	0.030896
4	5	0.235143	0.25	0.035358	0.037548
4	6	0.228864	0.2442	0.041124	0.043827
4	7	0.223761	0.2385	0.046453	0.049457
4	8	0.219435	0.2375	0.051418	0.055575
4	9	0.21727	0.2348	0.056143	0.060593
4	10	0.214169	0.2288	0.06049	0.064553
5	1	0.251282	0.258	0.008512	0.008734
5	2	0.240274	0.252	0.016502	0.017287
5	3	0.232584	0.2455	0.023979	0.025279
5	4	0.226585	0.237	0.031003	0.032395
5	5	0.221527	0.233	0.037499	0.039399
5	6	0.217319	0.2285	0.043576	0.04577
5	7	0.213705	0.2242	0.049239	0.051608
5	8	0.212231	0.224	0.054603	0.05757
5	9	0.20957	0.2215	0.059574	0.062898
5	10	0.206142	0.217	0.064106	0.067419
6	1	0.232575	0.2385	0.008848	0.009067
6	2	0.226458	0.234	0.017137	0.017692
6	3	0.221487	0.229	0.024929	0.025753
6	4	0.217114	0.225	0.032142	0.03328
6	5	0.213445	0.222	0.038891	0.040412
6	6	0.210254	0.2188	0.045184	0.046978
6	7	0.209315	0.2148	0.051135	0.052445
6	8	0.206921	0.2135	0.056659	0.05842
6	9	0.203613	0.2125	0.061705	0.064339
6	10	0.204766	0.2095	0.066666	0.068175
7	1	0.222969	0.225	0.009039	0.009119
7	2	0.218525	0.2227	0.017565	0.017891
7	3	0.214471	0.217	0.025441	0.025733
7	4	0.211091	0.2125	0.032792	0.033005
7	5	0.208151	0.208	0.039673	0.039645
7	6	0.207712	0.208	0.046168	0.046231
7	7	0.205471	0.204	0.052173	0.051808
7	8	0.202119	0.2055	0.057743	0.058686
7	9	0.203688	0.205	0.063135	0.063532
7	10	0.20393	0.2008	0.068182	0.06716
8	1	0.216565	0.212	0.009153	0.008966
8	2	0.212614	0.211	0.017729	0.017599
8	3	0.209473	0.2073	0.025672	0.025414
8	4	0.206702	0.2035	0.033086	0.032588
8	5	0.206846	0.2018	0.040097	0.039146
8	6	0.20468	0.2005	0.046588	0.045662
8	7	0.201168	0.198	0.052585	0.051778
8	8	0.2033	0.198	0.058446	0.056963
8	9	0.203708	0.1977	0.063847	0.062014
8	10	0.202898	0.197	0.068881	0.066931
9	1	0.211024	0.2045	0.009185	0.00891
9	2	0.208194	0.2023	0.017695	0.017209
9	3	0.205603	0.2002	0.025639	0.024986
9	4	0.206541	0.1968	0.033142	0.031625
9	5	0.204377	0.1943	0.0401	0.03818
9	6	0.20062	0.193	0.046541	0.044823
9	7	0.203364	0.1905	0.052794	0.049547
9	8	0.204067	0.1902	0.058578	0.054709
9	9	0.203293	0.1905	0.063965	0.060052
9	10	0.201402	0.1908	0.068941	0.065412
10	1	0.207621	0.1968	0.009078	0.008621
10	2	0.205133	0.193	0.017557	0.016551
10	3	0.207198	0.1907	0.025527	0.023558
10	4	0.204771	0.1885	0.032971	0.030431
10	5	0.200604	0.1868	0.039816	0.037158
10	6	0.204104	0.186	0.046389	0.042398
10	7	0.205068	0.1848	0.052562	0.047522
10	8	0.204262	0.1865	0.058285	0.053369
10	9	0.20219	0.1865	0.063579	0.058792
10	10	0.199251	0.184	0.068476	0.063387

Table 3 Pricing Result by Jamshidian Method

6.1.1 Improvement

In order to improve the performance of the model, i.e. reduce the relative errors of implied volatility and price, calibration becomes the focus. As stated in 2.4, there is no general rule of thumb that states which calibration instruments should be used in a particular situation. Previously, the whole swaption volatility surface, which includes the full 100 swaptions, was used to do the calibration. Now, only part of those swaptions are involved in recalibration.

The new and improved calibration only uses those swaptions with relative errors of price less than the median of the 100 relative errors from the original calibration. Recalibration leads to the new parameters, new implied volatilities and new swaption prices. The comparison between the result by improved calibration and the one by original calibration is shown in Figure 3.

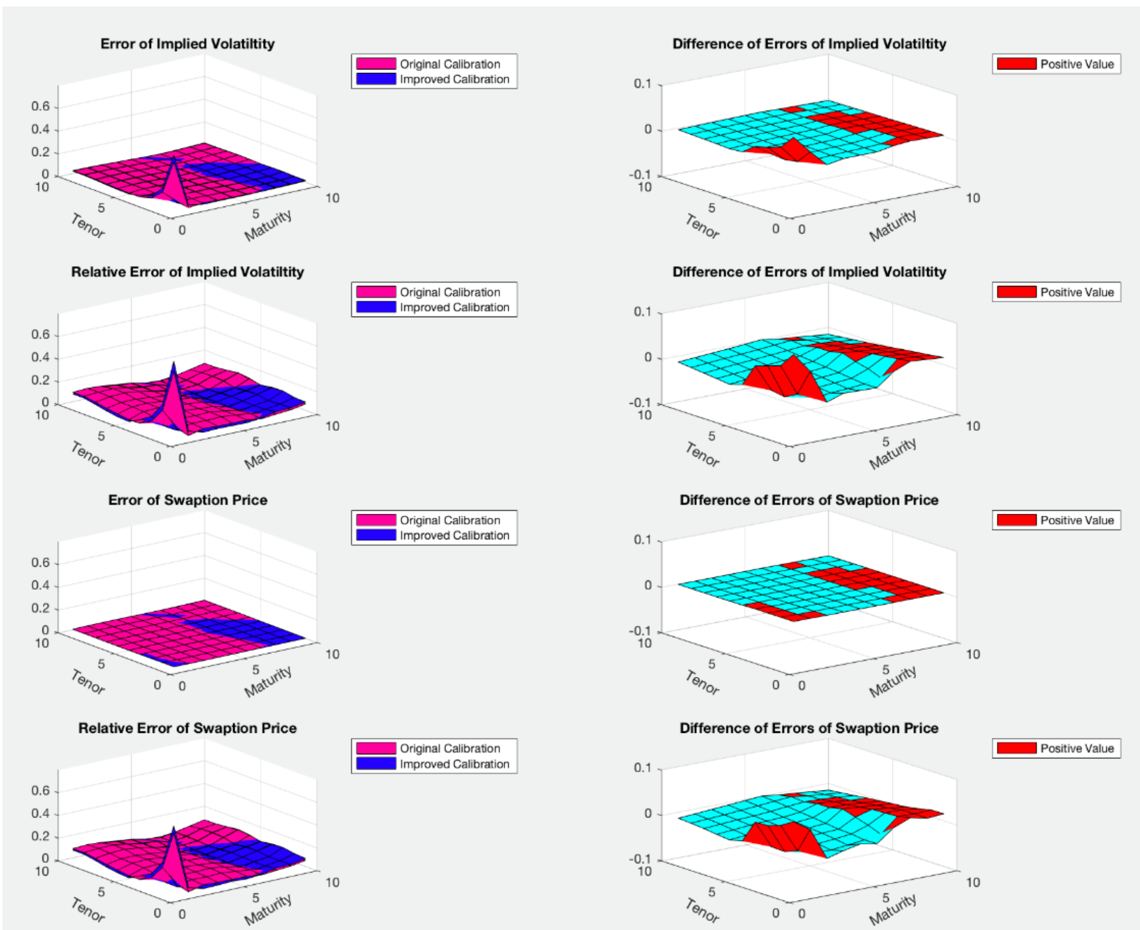

Figure 3 Pricing Results by Original Calibration and Improved Calibration

From the first column in Figure 3, both the errors and relative errors of improved calibration are less than original calibration for most swaptions. The second column in Figure 3 shows the difference between the result by improved calibration and the result by original calibration. It demonstrates that most of them are negative values (light blue parts), meaning the new improved calibration does reduce the errors and relative errors for most swaptions.

However, notice that, for swaption with long maturities and short tenors, as well as short maturities and short tenors, the original calibration performs better. The reason is that these parts are the deleted swaptions in the improved swaption. It makes sense that improved calibration performs worse in those areas, because it doesn't try to fit those swaptions at all. Nevertheless, even though half of the 100 swaptions are deleted, the improved calibration performs better for more than half of the swaptions. That means the new calibration does improve the result of the pricing of the swaption surface. Table 4 shows the result by improved calibration in numbers.

So, if one needs to price a specific swaption, it is recommended that after the basic calibration based on the whole swaption surface, a recalibration should be done. The swaptions with high relative errors from the original calibration should be excluded in order to get a better result.

Maturity	Tenor	Model Implied Volatility	Real Implied Volatility	Model Price	Real Price
1	1	1.21168	0.719	0.00458809	0.002851
1	2	0.758218	0.598	0.00898693	0.007181
1	3	0.562136	0.5	0.0131414	0.01179
1	4	0.453898	0.428	0.0170481	0.016085
1	5	0.390054	0.3878	0.0206282	0.020522
1	6	0.348945	0.353	0.023938	0.024238
1	7	0.320841	0.3297	0.0270049	0.027766
1	8	0.30032	0.3155	0.029843	0.031385
1	9	0.284509	0.3048	0.0324537	0.034782
1	10	0.271933	0.2915	0.0348746	0.037363
2	1	0.567004	0.5442	0.00631881	0.006091
2	2	0.451725	0.4475	0.0123176	0.012227
2	3	0.381443	0.3835	0.0179554	0.018094
2	4	0.337286	0.3453	0.0232111	0.02373
2	5	0.308323	0.3215	0.0280411	0.029218
2	6	0.288187	0.3033	0.0325126	0.034209
2	7	0.273163	0.2888	0.0366689	0.038754
2	8	0.261355	0.281	0.0404977	0.043547
2	9	0.251864	0.273	0.0440341	0.047711
2	10	0.243955	0.2643	0.0473171	0.051202
3	1	0.381469	0.3808	0.00747229	0.007469
3	2	0.332237	0.3358	0.014526	0.014714
3	3	0.300653	0.3075	0.0211229	0.021559
3	4	0.279596	0.29	0.0271992	0.02818
3	5	0.264788	0.2783	0.0328332	0.034487
3	6	0.253554	0.2688	0.0380767	0.040334
3	7	0.244533	0.2595	0.042913	0.045537
3	8	0.23717	0.2577	0.0473845	0.051442
3	9	0.23094	0.252	0.0515394	0.056141
3	10	0.227145	0.2448	0.0554145	0.05966
4	1	0.298505	0.2965	0.00827953	0.008254
4	2	0.275383	0.2788	0.0160461	0.016198
4	3	0.259724	0.2655	0.0232145	0.023701
4	4	0.248597	0.2573	0.0298697	0.030896
4	5	0.23998	0.25	0.0360706	0.037548
4	6	0.232871	0.2442	0.0417955	0.043827
4	7	0.226969	0.2385	0.0470926	0.049457
4	8	0.221888	0.2375	0.0520181	0.055575
4	9	0.219065	0.2348	0.0566043	0.060593
4	10	0.215342	0.2288	0.0608212	0.064553
5	1	0.259144	0.258	0.0088184	0.008734
5	2	0.246971	0.252	0.0169728	0.017287
5	3	0.238259	0.2455	0.0245524	0.025279
5	4	0.231332	0.237	0.0316213	0.032395
5	5	0.225426	0.233	0.0381528	0.039399
5	6	0.220442	0.2285	0.0441996	0.04577
5	7	0.21608	0.2242	0.0498252	0.051608
5	8	0.213949	0.224	0.0550546	0.05757
5	9	0.210686	0.2215	0.0598717	0.062898
5	10	0.206602	0.217	0.0642657	0.067419
6	1	0.239048	0.2385	0.00908764	0.009067
6	2	0.231942	0.234	0.0175427	0.017692
6	3	0.22607	0.229	0.0254126	0.025753
6	4	0.220852	0.225	0.0327278	0.03328
6	5	0.216426	0.222	0.0394476	0.040412
6	6	0.212516	0.2188	0.0456703	0.046978
6	7	0.210903	0.2148	0.051525	0.052445
6	8	0.207894	0.2135	0.0568964	0.05842
6	9	0.203946	0.2125	0.0618081	0.064339
6	10	0.204514	0.2095	0.066629	0.068175
7	1	0.228454	0.225	0.0092562	0.009119
7	2	0.223111	0.2227	0.0178792	0.017891
7	3	0.21823	0.217	0.0258574	0.025733
7	4	0.214069	0.2125	0.0332903	0.033005
7	5	0.210386	0.208	0.040117	0.039645
7	6	0.209256	0.208	0.0465136	0.046231
7	7	0.206378	0.204	0.0524077	0.051808
7	8	0.202376	0.2055	0.0578064	0.058686
7	9	0.203369	0.205	0.0630437	0.063532
7	10	0.202989	0.2008	0.0679074	0.06716
8	1	0.221188	0.212	0.00934371	0.008966
8	2	0.216426	0.211	0.0179943	0.017599
8	3	0.212487	0.2073	0.0260145	0.025414
8	4	0.208951	0.2035	0.0334847	0.032588
8	5	0.20838	0.2018	0.0403964	0.039146
8	6	0.205562	0.2005	0.0467929	0.045662
8	7	0.201399	0.198	0.0526618	0.051778
8	8	0.202884	0.198	0.0583197	0.056963
8	9	0.202722	0.1977	0.0635943	0.062014
8	10	0.201348	0.197	0.0684018	0.066931
9	1	0.214888	0.2045	0.00930077	0.00891
9	2	0.21126	0.2023	0.0179278	0.017209
9	3	0.207901	0.2002	0.0259665	0.024986
9	4	0.208101	0.1968	0.0333938	0.031625
9	5	0.20526	0.1943	0.0402774	0.03818
9	6	0.200822	0.193	0.0466047	0.044823
9	7	0.202911	0.1905	0.0526671	0.049547
9	8	0.202983	0.1902	0.0583256	0.054709
9	9	0.201623	0.1905	0.0634895	0.060052
9	10	0.199199	0.1908	0.0682267	0.065412
10	1	0.210767	0.1968	0.0092156	0.008621
10	2	0.207458	0.193	0.0178064	0.016551
10	3	0.208811	0.1907	0.0257333	0.023558
10	4	0.205722	0.1885	0.0330906	0.030431
10	5	0.200768	0.1868	0.0398652	0.037158
10	6	0.203625	0.186	0.046318	0.042398
10	7	0.203935	0.1848	0.0523482	0.047522
10	8	0.202532	0.1865	0.0578581	0.053369
10	9	0.199916	0.1865	0.0629205	0.058792
10	10	0.196413	0.184	0.0675556	0.063387

Table 4 Pricing Result by Improved Calibration

6.2 Swaption Hedging

Since the swaption prices on July 1, 2008 are obtained in the previous section, this thesis chooses a 7x6 swaption as the hedging target, due to its small relative error of price. Daily swaption price and its underlying swap value from July 1, 2008 to October 31, 2008 are calculated by Jamshidian method and swap analytical pricing formula respectively. Applying the method discussed in section 4.2, delta value of the swaption by Black-76 model, i.e. $\Phi(d_+)$, on July 1, 2008 can be also calculated.

The purpose here is to construct a delta-neutral portfolio by going long one-unit swaption and shorting delta amount of the underlying swap, then comparing the Profit and Loss (PnL) of the delta-hedged portfolio and the unhedged swaption. Figure 4 shows the performance of the unhedged portfolio (single swaption) and hedged portfolio (one-unit swaption and delta amount of swap) in four months starting from July 1, 2008 via their PnLs.

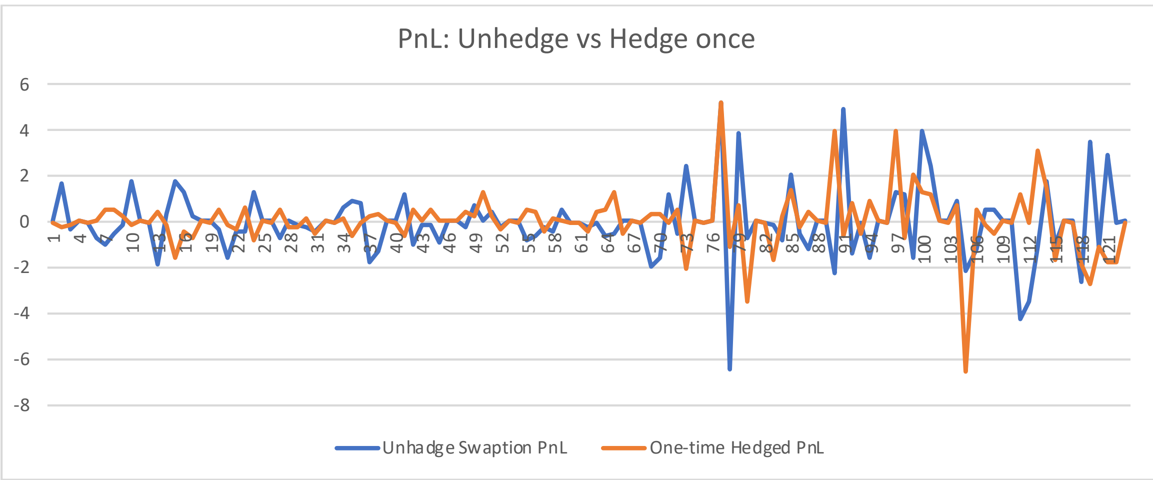

Figure 4 PnL Result for Unhedged Swaption and Hedged Once Swaption

In Figure 4, the blue line represents PnL of the unhedged swaption while the orange line stands for PnL of the hedged swaption. It can be seen that in the first two and half months (75 days), the hedged swaption preforms better because its daily PnLs are closer to zero, which means its value is more stable and with less risk. However, for the rest of the time, the hedged swaption becomes rather volatile.

6.2.1 Hedging Frequency

When performing delta hedging, the hedging frequency can be changed for different strategies. Previously, one-time hedging was set at the beginning and the delta remained constant in the four months. In order to show the effect of hedging frequency, monthly hedging and ten-day hedging are studied for comparison, the result is shown in Table 5.

To see how delta hedge with different hedging frequencies helps to improve the portfolio PnL clearly, density graphs are drawn in Figure 5. For a certain point at x-axis, if the mapping y-axis value (density) is higher, this x-axis point occupies larger percentage in the whole data set. As shown in the figure, higher frequency implied better hedging results and more stable PnLs for the portfolio. Ten-day hedging has the highest density around zero, monthly hedging and one-time hedging are sequentially lower, and the unhedged swaption is the lowest. This comparison implies that the hedging frequency affects the hedging result: the higher frequency, the better hedging result.

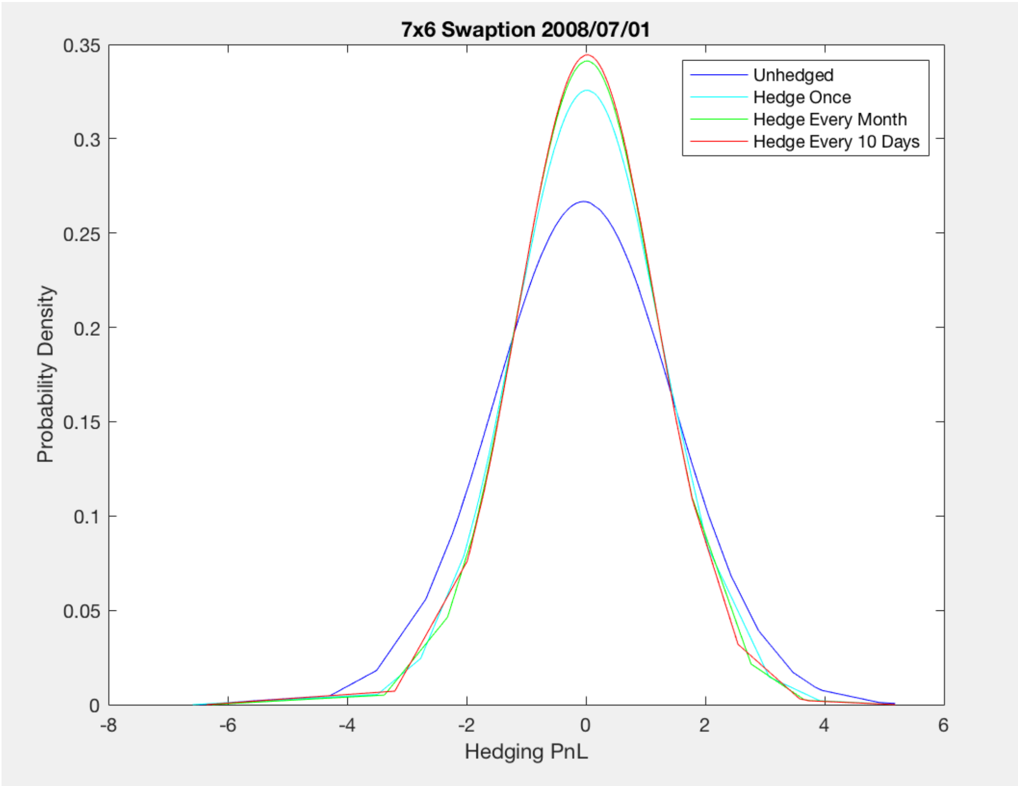

Figure 5 PnL Density Graphs for Different Hedging Frequency

Date	Swap Value	Swaption Value	Swap PnL	Unhadge Swaption PnL	One-time Hedged PnL	Monthly Hedged PnL	Tendays Hedged PnL
20080701	9.07979	42.1707	0.26885	0.0765	-0.093903851	-0.093903851	-0.093903851
20080702	9.34864	42.2472	3.06956	1.6753	-0.270263867	-0.270263867	-0.270263867
20080703	12.4182	43.9225	-0.287	-0.3828	-0.200892225	-0.200892225	-0.200892225
20080704	12.1312	43.5397	0.0032	-0.0016	-0.00362824	-0.00362824	-0.00362824
20080705	12.1344	43.5381	0.0032	-0.0106	-0.01262824	-0.01262824	-0.01262824
20080706	12.1376	43.5275	-1.2767	-0.7471	0.062104377	0.062104377	0.062104377
20080707	10.8609	42.7804	-2.45375	-1.0406	0.514648094	0.514648094	0.514648094
20080708	8.40715	41.7398	-1.78634	-0.5774	0.55482695	0.55482695	0.55482695
20080709	6.62081	41.1624	-0.55945	-0.1658	0.188793396	0.188793396	0.188793396
20080710	6.06136	40.9966	2.92286	1.7226	-0.12998174	-0.12998174	-0.129569616
20080711	8.98422	42.7192	0.00188	-0.0026	-0.003791591	-0.003791591	-0.003791326
20080712	8.9861	42.7166	0.00188	-0.0113	-0.012491591	-0.012491591	-0.012491326
20080713	8.98798	42.7053	-3.7269	-1.9105	0.451702393	0.451702393	0.4511769
20080714	5.26108	40.7948	0.86337	0.3648	-0.18242549	-0.18242549	-0.182303755
20080715	6.12445	41.1596	5.26745	1.7883	-1.550341496	-1.550341496	-1.549598786
20080716	11.3919	42.9479	2.608	1.2495	-0.4035156	-0.4035156	-0.403147872
20080717	13.9999	44.1974	1.5508	0.2378	-0.74513581	-0.74513581	-0.744917147
20080718	15.5507	44.4352	0.0048	-0.0021	-0.00514236	-0.00514236	-0.005141683
20080719	15.5555	44.4331	0.0048	-0.0094	-0.01244236	-0.01244236	-0.012441683
20080720	15.5603	44.4237	-1.3217	-0.3632	0.474526503	0.474526503	0.499226432
20080721	14.2386	44.0605	-2.1307	-1.547	-0.196509072	-0.196509072	-0.156690551
20080722	12.1079	42.5135	-0.0752	-0.4017	-0.35403636	-0.35403636	-0.352631022
20080723	12.0327	42.1118	-1.5853	-0.4218	0.583002772	0.583002772	0.612628859
20080724	10.4474	41.69	3.4037	1.3192	-0.838150152	-0.838150152	-0.901758498
20080725	13.8511	43.0092	0.0033	-0.002	-0.004091623	-0.004091623	-0.004153293
20080726	13.8544	43.0072	0.0032	-0.0085	-0.01052824	-0.01052824	-0.010588042
20080727	13.8576	42.9987	-2.0573	-0.7683	0.535668173	0.535668173	0.574114995
20080728	11.8003	42.2304	0.4121	-0.0033	-0.264499283	-0.264499283	-0.272200607
20080729	12.2124	42.2271	0.1536	-0.1905	-0.28785552	-0.28785552	-0.290725997
20080730	12.366	42.0366	-0.6473	-0.2461	0.164174923	0.164174923	0.176271665
20080731	11.7187	41.7905	0.2295	-0.398	-0.543462838	-0.545124188	-0.545124188
20080801	11.9482	41.3925	0.0027	-0.0033	-0.005011327	-0.005030873	-0.005030873
20080802	11.9509	41.3892	0.0028	-0.0106	-0.01237471	-0.012394979	-0.012394979
20080803	11.9537	41.3786	0.7286	0.6031	0.141295105	0.13602077	0.13602077
20080804	12.6823	41.9817	2.3358	0.8684	-0.612088435	-0.628997291	-0.628997291
20080805	15.0181	42.8501	1.3705	0.7733	-0.095357163	-0.105278212	-0.105278212
20080806	16.3886	43.6234	-3.1235	-1.736	0.243752388	0.266363404	0.266363404
20080807	13.2651	41.8874	-2.6667	-1.3454	0.344821128	0.364125369	0.364125369
20080808	10.5984	40.542	0.0024	-0.003	-0.00452118	-0.004538554	-0.004538554
20080809	10.6008	40.539	0.0025	-0.0095	-0.011084563	-0.01110266	-0.01110266
20080810	10.6033	40.5295	2.9307	1.2019	-0.655650927	-0.676866265	-0.695748765
20080811	13.534	41.7314	-2.5493	-1.0561	0.559710072	0.578164455	0.594589595
20080812	10.9847	40.6753	-0.2521	-0.129	0.030787283	0.032612234	0.034236515
20080813	10.7326	40.5463	-1.06045	-0.1705	0.501639721	0.509316319	0.516148798
20080814	9.67215	40.3758	-1.4698	-0.9022	0.029395985	0.040035867	0.049505789
20080815	8.20235	39.4736	0.00072	-0.0079	-0.008356354	-0.008361566	-0.008366205
20080816	8.20307	39.4657	0.00072	-0.0078	-0.008256354	-0.008261566	-0.008266205
20080817	8.20379	39.4579	-1.09174	-0.2828	0.409172105	0.417075211	0.424109292
20080818	7.11205	39.1751	0.78359	0.7174	0.220741068	0.21506866	0.21001999
20080819	7.89564	39.8925	-1.91983	0.0302	1.24703625	1.260933899	1.273303364
20080820	5.97581	39.9227	0.35446	0.4567	0.23203439	0.229468455	0.236684197
20080821	6.33027	40.3794	0.07995	-0.2631	-0.313774309	-0.314353067	-0.312725525
20080822	6.41022	40.1163	0.00268	-0.0027	-0.004398651	-0.004418052	-0.004363495
20080823	6.4129	40.1136	0.00268	-0.0098	-0.011498651	-0.011518052	-0.011463495
20080824	6.41558	40.1038	-2.21615	-0.8453	0.559351274	0.575393984	0.530279818
20080825	4.19943	39.2585	-1.64999	-0.6167	0.429104912	0.441049189	0.407460343
20080826	2.54944	38.6418	0.3088	-0.2278	-0.42352516	-0.425760563	-0.419474322
20080827	2.85824	38.414	-0.84627	-0.4038	0.132587083	0.138713231	0.121485713
20080828	2.01197	38.0102	0.7853	0.5364	0.038657228	0.032972441	0.048958793
20080829	2.79727	38.5466	0.00152	-0.0136	-0.014563414	-0.014574417	-0.014543475
20080830	2.79879	38.533	0.00151	-0.0108	-0.011757076	-0.011768007	-0.011737268
20080831	2.8003	38.5222	0.30624	-0.2222	-0.416302568	-0.407986008	-0.407986008
20080901	3.10654	38.3	-0.83238	-0.0702	0.457383254	0.43477831	0.43477831
20080902	2.27416	38.2298	-1.823731	-0.6128	0.543126301	0.493599238	0.493599238
20080903	0.450429	37.617	-2.786279	-0.4909	1.275113287	1.199446308	1.199446308
20080904	-2.33585	37.1261	0.85864	0.0005	-0.543727498	-0.520409412	-0.520409412
20080905	-1.47721	37.1266	-0.00034	-0.0058	-0.0055845	-0.005593733	-0.005593733
20080906	-1.47755	37.1208	-0.00034	-0.0118	-0.0115845	-0.011593733	-0.011593733
20080907	-1.47789	37.109	-3.6057	-1.9663	0.319082802	0.221162808	0.221162808
20080908	-5.08359	35.1427	-2.97496	-1.5857	0.299904022	0.219113033	0.219113033
20080909	-8.05855	33.557	2.00498	1.2031	-0.067706449	-0.013257207	-0.013257207
20080910	-6.05357	34.7601	-1.63402	-0.5104	0.525282726	0.480907645	0.414406299
20080911	-7.68759	34.2497	7.06327	2.4343	-2.042577108	-1.850759884	-1.563298922
20080912	-0.62432	36.684	-0.000171	-0.0058	-0.005691616	-0.00569626	-0.005703219
20080913	-0.624491	36.6782	-0.000172	-0.0114	-0.011290982	-0.011295653	-0.011302653
20080914	-0.624663	36.6668	-0.000172	0.0058	0.005909018	0.005904347	0.005897347
20080915	-0.624835	36.6726	-0.000171	5.1827	5.182808384	5.18280374	5.182796781
20080916	-0.625006	41.8553	-8.425804	-6.4807	-1.14021478	-1.369034339	-1.71194771
20080917	-9.05081	35.3746	4.93758	3.8432	0.713638356	0.847728217	1.048677847
20080918	-4.11323	39.2178	4.29895	-0.767	-3.491781984	-3.375035399	-3.200076732
20080919	0.18572	38.4508	0.000841	-0.0061	-0.006633047	-0.006610208	-0.006575981
20080920	0.186561	38.4447	0.000841	-0.0109	-0.011433047	-0.011410208	-0.011403806
20080921	0.187402	38.4338	2.420538	-0.1276	-1.661797498	-1.596062947	-1.577637812
20080922	2.60794	38.3062	-1.704707	-0.8642	0.216285914	0.169991186	0.157014957
20080923	0.903233	37.442	1.094707	2.051	1.357147336	1.386876294	1.395209203
20080924	1.99794	39.493	-0.48623	-0.5378	-0.22961527	-0.242819818	-0.246521001
20080925	1.51171	38.9552	-2.425416	-1.1613	0.375989296	0.310122274	0.291660007
20080926	-0.913706	37.7939	0.002419	-0.0027	-0.004233223	-0.00416753	-0.004149116
20080927	-0.911287	37.7912	0.002419	-0.0104	-0.011933223	-0.01186753	-0.011849116
20080928	-0.908868	37.7808	-9.710232	-2.2373	3.917287797	3.653587027	3.579672741
20080929	-10.6191	35.5435	8.81364	4.9134	-0.672905373	-0.433553352	-0.366463924
20080930	-1.80546	40.4569	-3.43877	-1.4212	0.758378395	0.600604189	0.600604189
20081001	-5.24423	39.0357	0.79789	-0.0338	-0.539522629	-0.502914638	-0.502914638
20081002	-4.44634	39.0019	-3.8631	-1.5879	0.860629358	0.683386466	0.683386466
20081003	-8.30944	37.414	0.00283	-0.0056	-0.007393725	-0.007263882	-0.007263882
20081004	-8.30661	37.4084	0.00283	-0.0109	-0.012693725	-0.012563882	-0.012563882
20081005	-8.30378	37.3975	-4.12022	1.3155	3.926998442	3.737958628	3.737958628
20081006	-12.424	38.713	2.99035	1.1996	-0.695758589	-0.55855834	-0.55855834
20081007	-9.43365	39.9126	-5.68295	-1.5607	2.041295784	1.780556355	1.780556355
20081008	-15.1166	38.3519	4.1576	3.96	1.32480918	1.515564026	1.515564026
20081009	-10.959	42.3119	1.94489	2.4284	1.195680096	1.284913594	1.284913594
20081010	-9.01411	44.7403	0.01641	0.0041	-0.006301068	-0.005548161	-0.005756831
20081011	-8.9977	44.7444	0.01641	-0.0083	-0.018701068	-0.017948161	-0.018156831
20081012	-8.98129	44.7361	0.32816	0.8756	0.667603988	0.682660297	0.678487414
20081013	-8.65313	45.6117	7.0209	-2.1366	-6.586621943	-6.26449603	-6.353773794
20081014	-1.63223	43.4751	-2.66216	-1.1808	0.506543562	0.384400999	0.418253026
20081015	-4.29439	42.2943	1.08603	0.5432	-0.145152965	-0.095324822	-0.10913478
20081016	-3.20836	42.8375	1.69548	0.5378	-0.536837611	-0.459047293	-0.480607017
20081017	-1.51288	43.3753	0.01231	0.0033	-0.004502386	-0.003937591	-0.004094125
20081018	-1.50057	43.3786	0.01232	-0.0072	-0.015008724	-0.01444347	-0.014600131
20081019	-1.48825	43.3714	-8.67365	-4.2861	1.211476211	0.813520476	0.923814609
20081020	-10.1619	39.0853	-5.3889	-3.5055	-0.089880457	-0.337128578	-0.515533502
20081021	-15.5508	35.5798	-6.5961	-1.1095	3.071273083	2.768637418	2.550266932
20081022	-22.1469	34.4703	0.5655	1.8007	1.442271963	1.468217668	1.486939111
20081023	-21.5814	36.271	1.0357	-1.0226	-1.679052553	-1.631533601	-1.597245717
20081024	-20.5457	35.2484	0.0128	-0.0004	-0.00851296	-0.007925683	-0.007501926
20081025	-20.5329	35.248	0.0127	-0.0073	-0.015349578	-0.014766889	-0.014346443
20081026	-20.5202	35.2407	-1.3361	-2.681	-1.834146418	-1.895448022	-1.939680948
20081027	-21.8563	32.5597	9.843	3.4709	-2.767839475	-2.316232792	-1.990370434
20081028	-12.0133	36.0306	0.0191	-1.1248	-1.136906058	-1.13602973	-1.135397406
20081029	-11.9942	34.9058	7.41738	2.8929	-1.808420879	-1.468104067	-1.222544284
20081030	-4.57682	37.7987	2.69169	-0.0907	-1.796760414	-1.673262985	-1.584151896
20081031	-1.88513	37.708	0.01935	-0.0046	-0.016864514	-0.016045331	-0.016045331

Table 5 Swaption Hedging Result for Different Hedging Frequency

7. Conclusions and Recommendation

This paper examined how swaption can be priced and delta-hedged with Hull-White One Factor model. There are two ways to price swaption with this interest rate model: Jamshidian method provides an analytical solution of swaption price, while in Monte Carlo simulation, forward swap rate is simulated and swaption price can be obtained naturally.

In this paper, 100 ATM swaptions on July 1, 2008 were chosen as the pricing targets. The pricing results show that Jamshidian method performs well in swaptions with long maturities and tenors (3 years or more), while Monte Carlo model is relatively more suitable for swaptions with shorter maturities and tenors, even though there still are pricing errors. The problem can be solved with more advanced interest rate models such as G2++.

Section 6.1.1 shows one possible way to improve the pricing performance through recalibration. If one needs to price a specific swaption, it is recommended that after the basic calibration based on the whole swaption surface, a recalibration should be done. The swaptions with high relative errors from the original calibration should be excluded in order to get a better result. Note the standard of choosing financial instruments for recalibration is not unique.

Based on the swaption pricing methods, swaption hedging is studied next. Delta hedging is chosen as the study target in this thesis, because the change of underlying value is the main factor driven the portfolio value change. However, one should know that it can be very useful of using delta-gamma hedge or vega hedge even though they are not studied.

For swaption hedging, this paper studies a 7x6 swaption in four months starting from July 1, 2008. The comparison between the PnLs of the unhedged portfolio (single swaption) and the hedged portfolio (one-unit swaption and delta amounts of swap) shows that delta-hedging makes the portfolio more stable and less risky. Further study, which includes one-time hedging, monthly hedging and ten-day hedging, shows that increasing hedging frequency improves the hedging result.

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