# INTRO TO DATA SCIENCE REGRESSION & REGULARIZATION

INTRO TO DATA SCIENCE, REGRESSION & REGULARIZATION

### DATA SCIENCE IN THE NEWS

WHY SECURITY DATA SCIENCE MATTERS AND HOW ITS DIFFERENT: PITFALLS AND PROMISES OF DATA SCIENCE BASED BREACH DETECTION AND THREAT INTELLIGENCE



FROM FALSE POSITIVES TO ACTIONABLE ANALYSIS: BEHAVIORAL INTRUSION DETECTION MACHINE LEARNING AND THE SOC

THE APPLICATIONS OF DEEP LEARNING ON TRAFFIC IDENTIFICATION

Why security data science matters and how it's different: pitfalls and promises of data science based breach detection and threat intelligence

Joshua Saxe, Invincea Labs Work presented in this talk contains significant contributions from Alex Long, David Slater, Giacomo Bergamo, Konstantin Berlin, and Robert Gove



### CLASSIFICATION OF ENCRYPTED WEB TRAFFIC USING MACHINE LEARNING ALGORITHMS

THESIS

William Charles Barto

AFIT-ENG-13-J-11

### Cleaning MoMA's Artwork Collection Using Python

09 AUG 2015 on python, art, and cleaning

### Use Python to Clean the Museum of Modern Art's Collection

Art is a messy business. Over centuries, artists have created everything from simple paintings to complex sculptures, and art historians have been cataloging everything they can along the way. The Museum of Modern Art, or MoMA for short, is considered one of the most influential museums in the world and recently released a dataset of all the artworks they've cataloged in their collection. This dataset contains basic information on metadata for each artwork and is part of MoMA's push to make art more accessible to everyone.

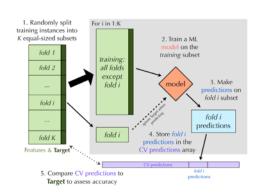
### **LAST TIME:**

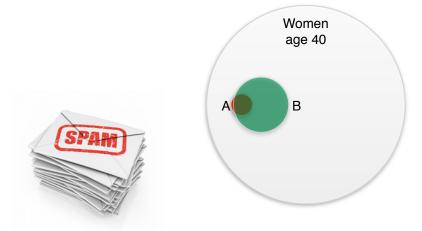
- I. CROSS VALIDATION
- II. INTRO TO PROBABILITY
- III. NAÏVE BAYESIAN CLASSIFICATION

**EXERCISES:** 

IV. NAÏVE BAYES CLASSIFICATION IN PYTHON

**QUESTIONS?** 





INTRO TO DATA SCIENCE

### QUESTIONS?

WHAT WAS THE MOST INTERESTING THING YOU LEARNT?

WHAT WAS THE HARDEST TO GRASP?

### **REVIEW GAME**



I. LINEAR REGRESSION (INCL. MULTIPLE REGRESSION)
II. POLYNOMIAL REGRESSION
III. REGULARIZATION

LAB:

IV. IMPLEMENTING MULTIPLE REGRESSION & POLYNOMIAL REGRESSION IN PYTHON

#### **KEY OBJECTIVES**

- WHAT ARE LINEAR AND POLYNOMIAL REGRESSION
- WHICH PROBLEMS CAN BE TACKLED WITH REGRESSION TECHNIQUES
- HOW TO IMPLEMENT LINEAR AND POLYNOMIAL REGRESSION IN PYTHON
- WHAT IS REGULARIZATION
- HOW REGULARIZATION CAN HELP WHEN DATA IS NOISY
- HOW TO IMPLEMENT REGULARIZATION IN PYTHON

### L LINEAR REGRESSION

	Continuous	Categorical	
Supervised	???	???	
Unsupervised	???	???	

	Continuous	Categorical
Supervised (	regression	classification
Unsupervised	dimension reduction	clustering

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#### INTRO TO REGRESSION

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 $\varepsilon$  = residual (the prediction error)

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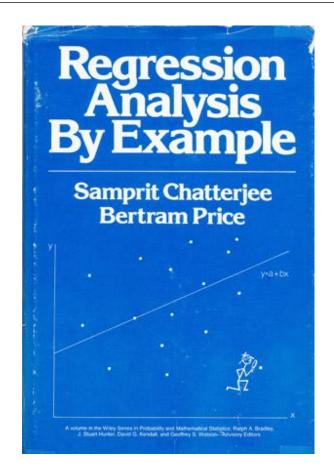
We can extend this model to several input variables, giving us the multiple linear regression model:

$$y = \alpha + \beta_1 x_1 + \dots + \beta_n x_n + \varepsilon$$

$$y = \alpha + \begin{bmatrix} \beta_1 \beta_2 \dots \beta_n \end{bmatrix} * \begin{bmatrix} x_1 \\ x_2 \\ \dots \\ x_n \end{bmatrix} + \varepsilon$$

Linear regression involves several technical assumptions and is often presented with lots of mathematical formality.

The math is not very important for our purposes, but you should check it out if you get serious about solving regression problems.



## Statistical Models Theory and Practice REVISED EDITION David A. Freedman

### INTRO TO REGRESSION

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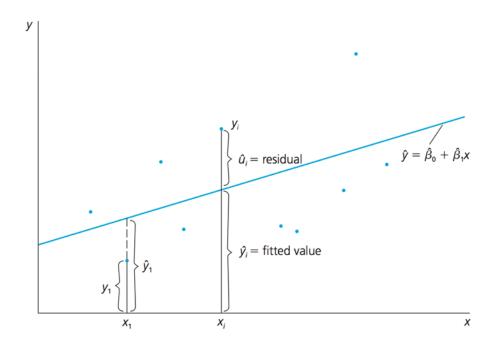
But again, if you get serious about regression, you should learn how this works!

A: In theory, minimize the sum of the squared residuals (OLS).

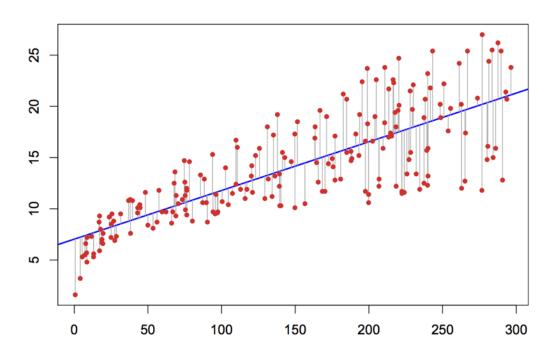
$$\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i.$$

$$\hat{u}_i = y_i - \hat{y}_i = y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i.$$

$$\sum_{i=1}^{n} \hat{u}_i^2 = \sum_{i=1}^{n} (y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i)^2,$$



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### II: POLYNOMIAL REGRESSION

#### Consider the following polynomial regression model: $y = \alpha + \beta_1 x + \beta_2 x^2 + \epsilon$

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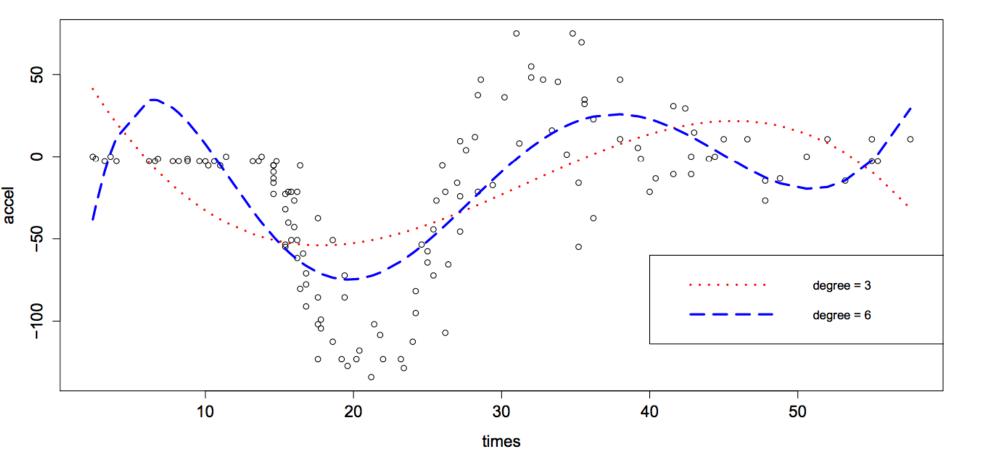
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Q: This represents a nonlinear relationship. Is it still a linear model?

A: Yes, because it's linear in the  $\beta$ 's!

"Although polynomial regression fits a nonlinear model to the data, as a statistical estimation problem it is linear, in the sense that the regression function E(y|x) is linear in the unknown parameters that are estimated from the data. For this reason, polynomial regression is considered to be a special case of multiple linear regression." -- Wikipedia

#### **POLYNOMIAL REGRESSION**



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Q: Does anyone know what it is?

A: This model violates one of the assumptions of linear regression!

#### **POLYNOMIAL REGRESSION**



This model displays multicollinearity, which means the predictor variables are highly correlated with each other.  $y = \alpha + \beta_1 x + \beta_2 x^2 + ... + \beta_n x^n + \epsilon$ 

Multicollinearity causes the linear regression model to break down, because it can't tell the predictor variables apart.

A: Replace the correlated predictors with uncorrelated predictors.

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$$y = \alpha + \beta_1 f_1(x) + \beta_2 f_2(x^2) + ... + \beta_n f_n(x^n) + \varepsilon$$

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#### **OPTIONAL NOTE**

These polynomial functions form an *orthogonal basis* of the function space.

So far, we've seen how polynomial regression allows us to fit complex nonlinear relationships, and even to avoid multicollinearity (by using basis functions).

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Q: Can a regression model be too complex?

# III: REGULARIZATION

## Recall our earlier discussion of overfitting.

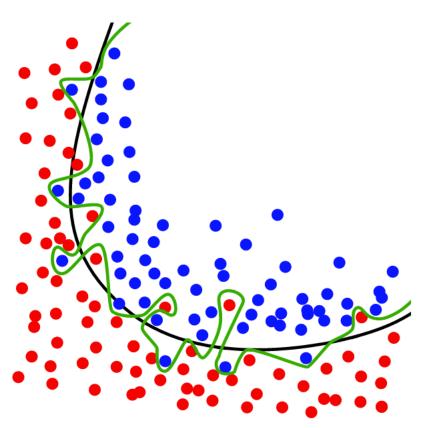
Recall our earlier discussion of overfitting.

When we talked about this in the context of classification, we said that it was a result of matching the training set too closely.

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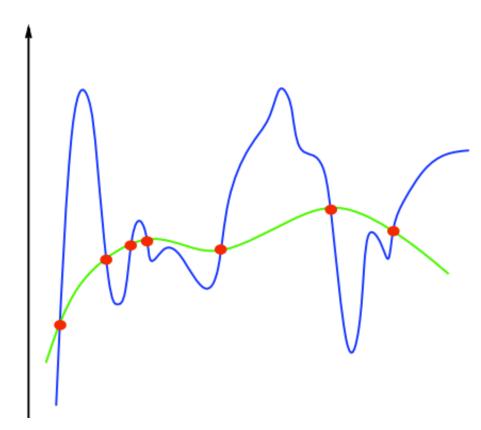
In other words, an overfit model matches the noise in the dataset instead of the signal.



The same thing can happen in regression.

It's possible to design a regression model that matches the noise in the data instead of the signal.

This happens when our model becomes too complex for the data to support.



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Ex 1:  $\Sigma |\beta_i|$ 

Ex 2:  $\sum \beta_i^2$ 

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Ex 1:  $\Sigma |\beta_i|$  this is called the L1-norm

Ex 2:  $\sum \beta_i^2$  this is called the L2-norm

L1 regularization:  $y = \sum \beta_i x_i + \varepsilon$  st.  $\sum |\beta_i| < s$ 

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 st.  $\sum \beta_i^2 < s$ 

Regularization refers to the method of preventing overfitting by explicitly controlling model complexity.

#### These regularization problems can also be expressed as:

L1 regularization:  $min(||y - x\beta||^2 + \lambda ||\beta||)$ 

L2 regularization:  $min(||y - x\beta||^2 + \lambda ||\beta||^2)$ 

#### These regularization problems can also be expressed as:

L1 regularization (Lasso):  $min(||y - x\beta||^2 + \lambda ||x||)$ 

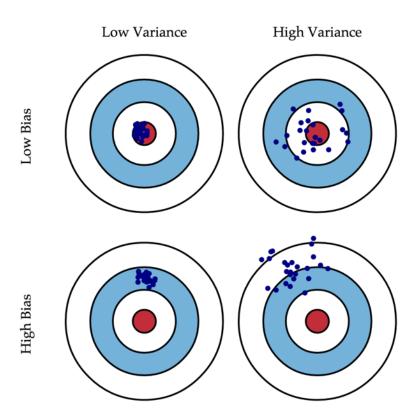
L2 regularization (Ridge):  $min(||y - x\beta||^2 + \lambda ||x||^2)$ 

This (Lagrangian) formulation reflects the fact that there is a cost associated with regularization.

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Variance refers to predictions that are generally inaccurate.

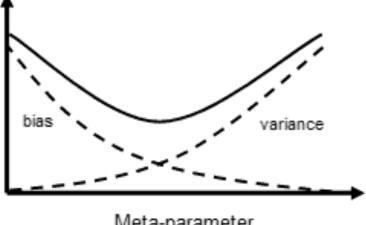


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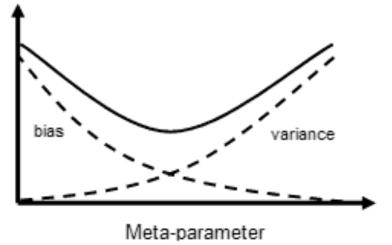
Variance refers to predictions that are generally inaccurate.

It turns out (after some math) that the generalization error in our model can be decomposed into a bias component and variance component.

This is another example of the bias-variance tradeoff.



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#### NOTE

The "meta-parameter" here is the lambda we saw above.

A more typical term is "hyperparameter".

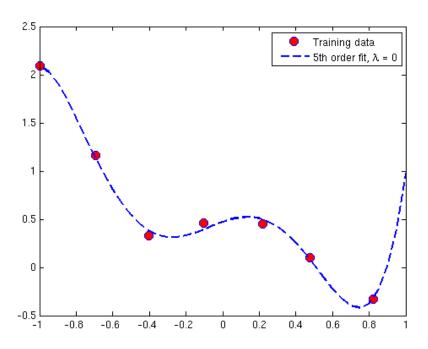
# This tradeoff is regulated by a hyperparameter $\lambda$ , which we've already seen:

L1 regularization: 
$$y = \sum \beta_i x_i + \epsilon$$
 st.  $\sum |\beta_i| < \lambda$ 

L2 regularization: 
$$y = \sum \beta_i x_i + \varepsilon$$
 st.  $\sum \beta_i^2 < \lambda$ 

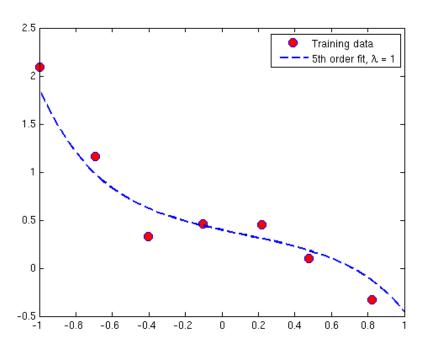
So regularization represents a method to trade away some variance for a little bias in our model, thus achieving a better overall fit.

## L2 regularization: $\min(\|\mathbf{y} - \mathbf{x}\boldsymbol{\beta}\|^2 + \lambda \|\mathbf{x}\|^2)$



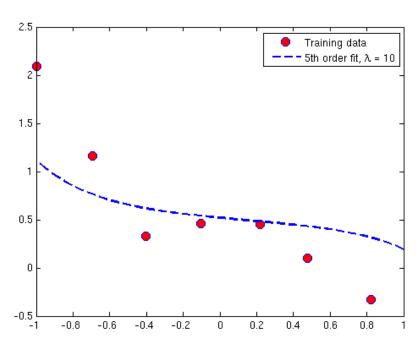
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- Linear regression
- Multiple regression
- Polynomial regression
- The concept of minimizing some error or "cost" function
- Regularization

# LAB: POLYNOMIAL REGRESSION & REGULARIZATION