

Warm up

• Solve for x . $2^x = 32$

Write 32 as 2 to the power of something

$$2^x = 32$$

$$2^x = 2^5 \Rightarrow x = 5$$

Exponentials, a^x , has a unique solution, assuming $a > 0$ & $a \neq 1$

$$2^a = 2^b \Rightarrow a = b$$

• $3^{7x} = 27$

$$3^{7x} = 3^3 \Rightarrow 7x = 3$$

$$\Rightarrow x = \frac{3}{7}$$

• $4^{3x-5} = 16$

$$4^{3x-5} = 4^2 \Rightarrow 3x-5 = 2$$

$$\Rightarrow 3x = 7$$

$$\Rightarrow x = \frac{7}{3}$$

• $3^{x^2+4x} = \frac{1}{27}$

Hint: $3^{-1} = \frac{1}{3}$

$$3^{x^2+4x} = \frac{1}{3^3} = 3^{-3} \Rightarrow x^2+4x = -3$$

$$\Rightarrow x^2+4x+3 = 0$$

$$(x+3)(x+1) = 0$$

$$x = -3$$

$$x = -1$$

Recall logarithms:

$$a^x = y$$

$$\log_a y = x$$

We can go back and forth between the two. Can be helpful when trying to solve some equations.

Ex

① $\log_3 9$

$$\begin{aligned}\log_3 9 = x &\Rightarrow 3^x = 9 \\ 3^x &= 3^2 \\ x &= 2\end{aligned}$$

② $\log_8 2$

$$\begin{aligned}\log_8 2 = x &\Rightarrow 8^x = 2 \\ 8^x &= 8^{1/3} \\ x &= \frac{1}{3}\end{aligned}$$

$$2^3 = 8 \Rightarrow \sqrt[3]{8} = 2 \Rightarrow 8^{1/3} = 2$$

③ $\log_{49} 7$

$$\begin{aligned}\log_{49} 7 = x &\Rightarrow 49^x = 7 \\ 49^x &= 49^{1/2} \\ x &= \frac{1}{2}\end{aligned}$$

Recall: \log & exponentials are "inverse functions" of each other.

In general, inverses "cancel" each other out

$$\log_a b = c$$

$$a^c = b$$

$$\log_a (a^b) = c$$

$$a^c = a^b \Rightarrow c = b$$

$$\Rightarrow \log_a (a^b) = b$$

$$\log_2 2^8 = x \Rightarrow 2^x = 2^8 \Rightarrow x = 8$$

$$\text{Similarly, } a^{\log_a b} = c \Rightarrow \log_a c = \log_a b \Rightarrow c = b$$

$$\Rightarrow a^{\log_a b} = b$$

$$2^{\log_2 8} = x \Rightarrow \log_2 x = \log_2 8 \Rightarrow x = 8$$

Ex

$$\textcircled{1} \log_5 5^4 = x$$

$$\text{Cancels out} \Rightarrow x = 4$$

$$\textcircled{2} \ln \sqrt{e} = x$$

$$e = 2.7182818...$$

$$= \log_e e^{1/2} \Rightarrow x = 1/2$$

$$\textcircled{3} e^{\ln \sqrt{e}} = x$$

$$e^{\log_e \sqrt{e}} \Rightarrow x = \sqrt{e}$$