Project: Laplace in a rectangular region

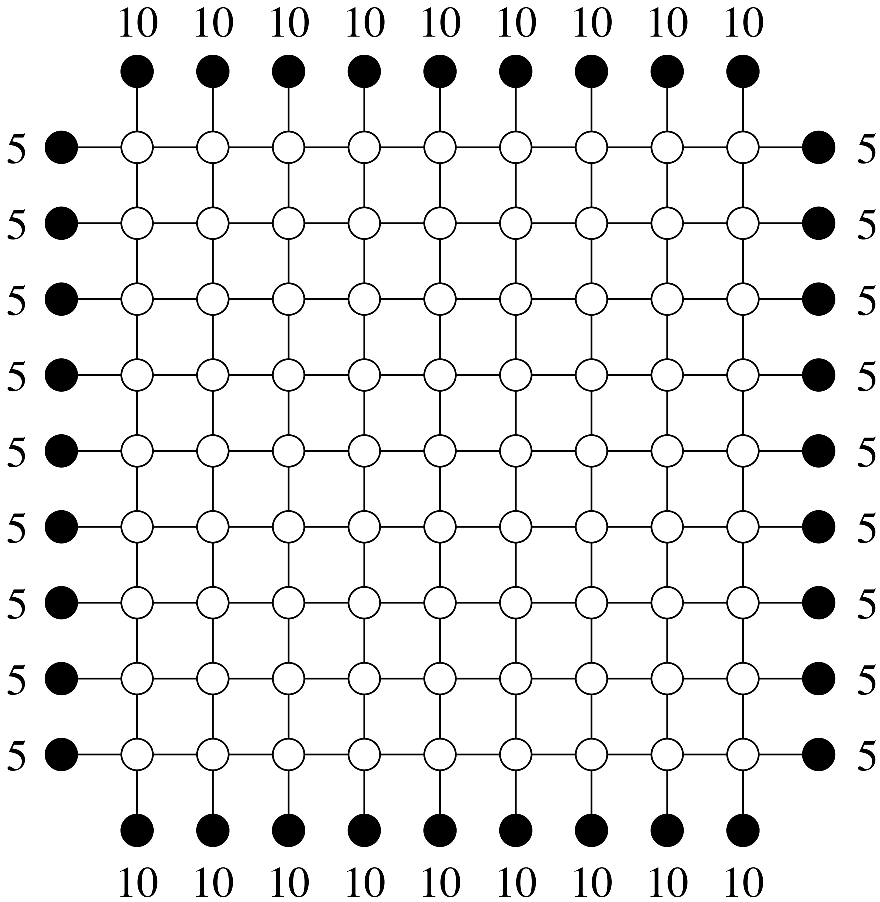
Problem 10.10. Numerical solution of the potential within a rectangular region

a. Modify `LaplaceApp` to determine the potential $V(x, y)$ in a square region with linear dimension $L = 10$. The boundary of the square is at a potential $V = 10$. Choose the grid size $\Delta x = \Delta y = 1$. Before you run the program, guess the exact form of $V(x, y)$ and set the initial values of the interior potential 10% lower than the exact answer. How many iterations are necessary to achieve 1% accuracy? Decrease the grid size by a factor of two, and determine the number of iterations that are now necessary to achieve 1% accuracy.

b. Consider the same geometry as in part (a), but set the initial potential at the interior sites equal to zero except for the center site whose potential is set equal to four. Does the potential distribution evolve to the same values as in part (a)? What is the effect of a poor initial guess? Are the final results independent of your initial guess?

c. Modify `LaplaceApp` so that the value of the potential at the four sides is 5, 10, 5, and 10, respectively (see Figure 10.1). Sketch the equipotential surfaces. What happens if the potential is 10 on three sides and 0 on the fourth? Start with a reasonable guess for the initial values of the potential at the interior sites and iterate until 1% accuracy is obtained.

10x10 grid setup



Project: Gauss-Seidel relaxation

Problem 10.11. Gauss-Seidel relaxation

a. Modify the program that you used in Problem 10.10 so that the potential at each site is updated sequentially. That is, after the average potential of the nearest neighbor sites of site i is computed, update the potential at i immediately. In this way the new potential of the next site is computed using the most recently computed values of its nearest neighbor potentials. Are your results better, worse, or about the same as for the simple relaxation method?

b. Imagine coloring the alternate sites of a grid red and black, so that the grid resembles a checkerboard. Modify the program so that all the red sites are updated first, and then all the black sites are updated. This ordering is repeated for each iteration. Do your results converge any more quickly than in part (a)?

Project: Random-walk solution of Laplace's equation

Problem 10.17.

a. Consider the square region shown in Figure 10.1 and compare the results of the random walk method with the results of the relaxation method (see Problem 10.10c). Try $n = 100$ and $n = 1000$ walkers, and choose a point near the center of the square.

b. Repeat part (a) for other points within the square. Do you need more or less walkers when the potential near the surface is desired? How quickly do your answers converge as a function of n?

Problem 10.18. Green's function solution of Laplace's equation

(a) Compute the Green's function $G(x, y, x_b, y_b)$ for the same geometry considered in Problem 10.17. Use at least 200 walkers at each interior site to estimate G. Because of the symmetry of the geometry, you can determine some of the values of G from other values without doing an additional calculation. Store your results for G in a file.

(b) Use your results for G found in part (a) to determine the potential at each interior site when the boundary potential is the same as in part (a), except for five boundary sites which are held at $V = 20$. Find the locations of the five boundary sites that maximize the potential at the interior site located at $(3, 5)$. Repeat the calculation to maximize the potential at $(5, 3)$. Use trial and error guided by your physical intuition. □