

FUNCTIONS — SOLUTIONS

- Teams may try the functions in any order, and may come back to functions later.
- Where a 2D table is used, the first column represents the x -values and the first row represents the y -values.

F1. $f(x, y) = \gcd(x, y)$.

$x \setminus y$	2	3	4	5	6	7	8	9	10
2	2	1	2	1	2	1	2	1	2
3	1	3	1	1	3	1	1	3	1
4	2	1	4	1	2	1	4	1	2
5	1	1	1	5	1	1	1	1	5
6	2	3	2	1	6	1	2	3	2
7	1	1	1	1	1	7	1	1	1
8	2	1	4	1	2	1	8	1	2
9	1	3	1	1	3	1	1	9	1
10	2	1	2	5	2	1	2	1	10

F2. $f(x, y) = 2x \bmod y$.

$x \setminus y$	1	2	3	4	5	6	7	8	9	10
1	0	0	2	2	2	2	2	2	2	2
2	0	0	1	0	4	4	4	4	4	4
3	0	0	0	2	1	0	6	6	6	6
4	0	0	2	0	3	2	1	0	8	8
5	0	0	1	2	0	4	3	2	1	0
6	0	0	0	0	2	0	5	4	3	2
7	0	0	2	2	4	2	0	6	5	4
8	0	0	1	0	1	4	2	0	7	6
9	0	0	0	2	3	0	4	2	0	8
10	0	0	2	0	0	2	6	4	2	0

F3. $f(x, y) = \text{largest prime } \leq x \text{ multiplied by smallest prime } \geq y$

$x \setminus y$	10	11	12	13	14	15	16	17	18	19	20
10	77	77	91	91	119	119	119	119	133	133	161
11	121	121	143	143	187	187	187	187	209	209	253
12	121	121	143	143	187	187	187	187	209	209	253
13	143	143	169	169	221	221	221	221	247	247	299
14	143	143	169	169	221	221	221	221	247	247	299
15	143	143	169	169	221	221	221	221	247	247	299
16	143	143	169	169	221	221	221	221	247	247	299
17	187	187	221	221	289	289	289	289	323	323	391
18	187	187	221	221	289	289	289	289	323	323	391
19	209	209	247	247	323	323	323	323	361	361	437
20	209	209	247	247	323	323	323	323	361	361	437

F4. $f(n) = \text{the binary representation of } n \text{ considered as a number in base 3}$

n	$f(n)$	n	$f(n)$	n	$f(n)$
1	1	11	31	21	91
2	3	12	36	22	93
3	4	13	37	23	94
4	9	14	39	24	108
5	10	15	40	25	109
6	12	16	81	26	111
7	13	17	82	27	112
8	27	18	84	28	117
9	28	19	85	29	118
10	30	20	90	30	120

F5. $f(x) = \text{smallest positive integer } y \text{ such that } x^2 + y^2 \text{ is a perfect square.}$

n	$f(n)$	n	$f(n)$	n	$f(n)$
3	4	11	60	19	180
4	3	12	5	20	15
5	12	13	84	21	20
6	8	14	48	22	120
7	24	15	8	23	264
8	6	16	12	24	7
9	12	17	144	25	60
10	24	18	24	26	168

SHUTTLE — SOLUTIONS

- If an answer is incorrect, make sure the other pair is aware.
- Circle the corresponding number: 4 if correct on the first attempt, 3 if correct on the second attempt, and 0 otherwise.
- Record the time that the team finished. If the team finished in faster than 8 minutes, circle 2, if faster than 10 minutes circle 1, otherwise circle 0.
- Fill in the total number of points achieved.

A1. 863

A2. 70

A3. 1000

A4. 5

B1. 12

B2. 3

B3. 2023

B4. 69

RELAY — SOLUTIONS

Team:

Referee:

(Circle the corresponding numbers and add them up at the end.)

R1 $5/\sqrt{2}$ (or $5\sqrt{2}/2$)

4 3 0

R2 92

4 3 0

R3 $35/8$

4 3 0

R4 $202/91$

4 3 0

R5 2020

4 3 0

R6 2023×2^{674}

4 3 0

Time

4 2 0

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CROSSNUMBER — SOLUTIONS

- The pairs are supposed to only attempt one square at a time. If many squares are filled in at once, stop marking at the first incorrect square and consult the pair.
- If the answer for a square is correct, tick the circle in the lower right and make sure both pairs' grids are synced.
- If the answer for a square is incorrect, write the correct answer for that square in each team's grid.
- The two pairs are not allowed to communicate on any matters related to the questions themselves, but may encourage the other team to, for example, work on a specific clue.
- Teams are not allowed to communicate what values they think are possible for a square.

1 3	2 9	3 7	2		4 3	
	5 4	8			6 5	7 1
		1		8 4	9	0
9 2		10 2	9	8		2
11 7	12 1	5		6		
13 9	3			14 2	15 4	
	8		16 3	0	0	3

QUIZDLE — SOLUTIONS

- Allow and fix minor spelling mistakes.
- If the answer is incorrect, don't give the correct answer, and the team is not allowed to attempt that word again, even using easier clues.
- Let your team fill in the crossword – your job is just to mark the Answer Sheet.
- The Across and Down clues of a particular difficulty must be given out at the same time.

Across

4. CALCULUS
7. MIRZAKHANI
8. PARKER
13. SINE
14. CROSS
16. BERNOULLI
17. BUTTERFLY
18. QUANTIFIER
19. ESCHER
20. ARROW

Down

1. PYTHAGORAS
2. DAVID
3. FLAT
5. GROUP
6. THREEBODYPROLEM
9. EINSTEIN
10. RAMANUJAN
11. TWENTYONE
12. PAUL
15. GEOMETRY