FUNCTIONS ANSWER SHEET

Team:

Referee:

F1

Consider the function f_1 from $\{1, \ldots, 10\} \times \{1, \ldots, 10\}$ to the positive integers.

Inputs 1:

Inputs 2:

Outputs:

Description:

F2

Consider the function f_2 from $\{1, \ldots, 10\} \times \{1, \ldots, 10\}$ to the integers.

Inputs 1:

Inputs 2:

Outputs:

Description:

F3

Consider the function f_3 from $\{1, \ldots, 10\}$ to the positive integers.

Inputs:

Outputs:

Description:

F4

Consider the function f_4 from $\{1, \ldots, 10\}$ to the positive integers.

Inputs:

Outputs:

Description:

F5

Consider the function f_5 from $\{1, \ldots, 10\}$ to the positive integers.

Inputs:

Outputs:

Description:

SHUTTLE — A1 AND A3

A1

A set S of positive integers is *exciting* if, for all $n \in S$, any two distinct elements of S are distinct mod n. Find the largest size of an exciting subset of $\{1, 2, \ldots, 2025\}$.

Pass on your answer to A1 as X.

A3

Y is the number you will receive.

A triangle ABC satisfies $AB = 40$, $AC = Y$ and $BC = 24$. Points D and E on sides AB and AC respectively satisfy $AD = DE = EC$. If the length of AD is $\frac{p}{q}$ for positive coprime integers p, q, find $p + q$.

Pass on your answer to A3 as Z.

SHUTTLE — A2 AND A4

A2

X is the number you will receive.

A sequence of positive integers $(a_1, a_2, a_3, \ldots)$ satisfies $a_1 = 2$ and for each $n \geq 1$, $(a_{n+1} - a_n^2)(a_{n+1} - a_n + 1) = 0$. What is the minimal possible N such that $a_N = X$?

Pass on your answer to A2 as Y.

A4

Z is the number you will receive.

In a $2 \times N$ grid, let a_N be the number of ways to fill the grid with the numbers from 1 to $2N$ such that each cell contains a larger number than the cell below it and the cell to its left. If $a_Z/a_{Z-1} = p/q$ for coprime positive integers p, q, what is $p + q$?

Shuttle — B1 and B3

B1

Josiah has a cube. On each face, he writes down one of the numbers $\{1, 2, 4, 8, 16, 32\}$, such that no two faces have the same number. Then on each edge, he writes down the absolute difference between the two numbers on the adjacent faces. Then he calculates the sum s of the numbers on each edge. How many distinct values of s can Josiah obtain in this way?

Pass on your answer to B1 as X.

B3

Y is the number you will receive.

The parabola $y = x^2$ is tangent to the parabola $y = -0.5x^2 + Yx + C$ for some real number C. Let (a, b) be the point of tangency. What is $a + 3b$?

Pass on your answer to B3 as Z.

SHUTTLE — B2 AND B4

B2

X is the number you will receive.

For a positive integer n, let $S(n)$ be the sum of the digits of n. Over all $1 \leq n \leq 10^X$, find the maximum value of $S(n) - S(2n)$.

Pass on your answer to B2 as Y.

B4

Z is the number you will receive.

A triangle T has incircle ω_1 of radius 35, and excircle ω_2 of radius Z (ω_2 is tangent to the extensions of the sides of T, but lies outside of T). Additionally ω_1 and ω_2 are tangent. If the area of T is $\frac{p}{q}$ for coprime positive integers p, q, what is $p + q$?

Shuttle — Answer sheet A

Team:

Referee:

A1   4   3   0

A2   4   3   0

A3   4   3   0

A4   4   3   0

Time: 2   1   0

Final Score: / 18

Shuttle — Answer sheet B

Team:

Referee:

B1   4 3 0

B2   4 3 0

B3   4 3 0

B4   4 3 0

Time: 2 1 0

Final Score: / 18

A 4×4 grid with 16 cells is given. How many ways are there to mark some cells, so that exactly two cells in each row and each column are marked?

First attempt

Second attempt

Find
$$\int_{1/10}^{1} \left\lceil \frac{1}{x} \right\rceil \left\lfloor \frac{1}{x} \right\rfloor^{4} \, \mathrm{d}x.$$

First attempt

Second attempt

RELAY — R3

How many integers $1 \leq n \leq 2025$ have an odd number of divisors congruent to 1 (mod 4), and an even number of divisors congruent to 3 (mod 4)?

First attempt

Second attempt

RELAY — R4

A square of side-length two contains an equilateral triangle. If the largest possible area of the triangle is $a\sqrt{3} + b$ for integers a, b, find $a - b$.

First attempt

Second attempt

Josiah has yet another cube. On each face, he chooses two diagonally opposite vertices uniformly at random, and draws the line segment between them in a red pen. Let p be the probability that a red triangle is drawn on the cube. Find $64 \times p$.

First attempt

Second attempt
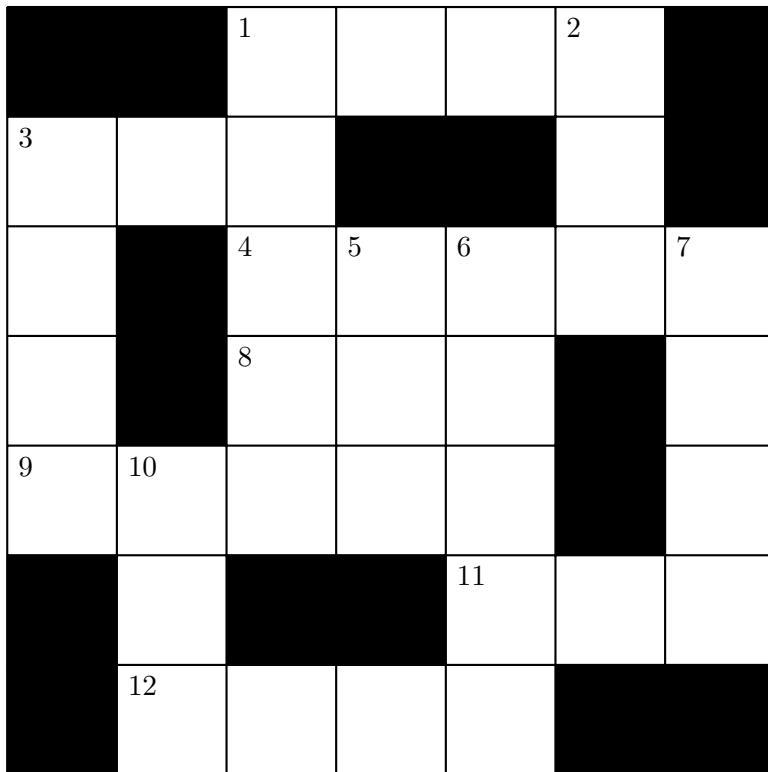
Let $S = \{1, 2, \ldots, 2025\}$. A subset $A \subseteq S$ is chosen uniformly at random. Find

$$\mathbb{E}\left[\left(\sum_{a \in A} a - \sum_{b \in S \setminus A} b\right)^2\right] \pmod{2025}.$$

First attempt

Second attempt
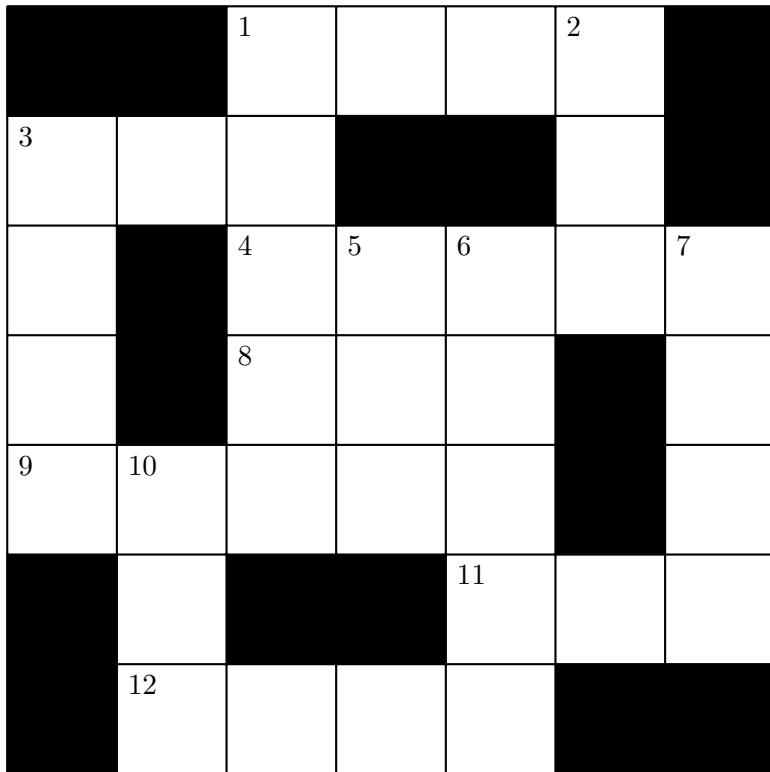
CROSSNUMBER — ACROSS

Across

1. The number of primes below 9905.

3. A palindrome with the following property: the product of the digits is a multiple of, but not equal to, the product of the digits in the second leftmost column.

4. An odd number.

8. A number.

9. This number's digits in total add up to more than the sum of the digits of (3 Across). In addition, one of the digits of this number does not appear anywhere else in the grid.

11. $x^4 + x^3 + x^2 + x$, where $x = \frac{(7 \text{ Down})}{(3 \text{ Across}) - 1}$.

12. When Turbo the Snail wrote this number's binary expansion and erased the right-most digit, he got a ten-digit palindrome.

CROSSNUMBER — DOWN

Down

1. The product of ten distinct integers.

2. The sum of the sixth powers of the digits of (1 Across).

3. $11 \times$ (3 Across).

5. The sum of all the correct digits in each square of this crossnumber (including those not yet filled in).

6. Melonie bought 2^n melons, where n is some positive integer. She then dropped 10 of them in the skibidi toilet, never to be recovered. This number is how many she has left.

7. Exactly one prime factor of this number is one more than a multiple of four. No prime factor of this number is more than 30. Exactly two of this number's factors are square (including 1), and one of these is between 30 and 300.

10. A number.

CROSSNUMBER — ANSWER SHEET

Team:

Referee:

Totals

/4

/4

/6

/5

/6

/4
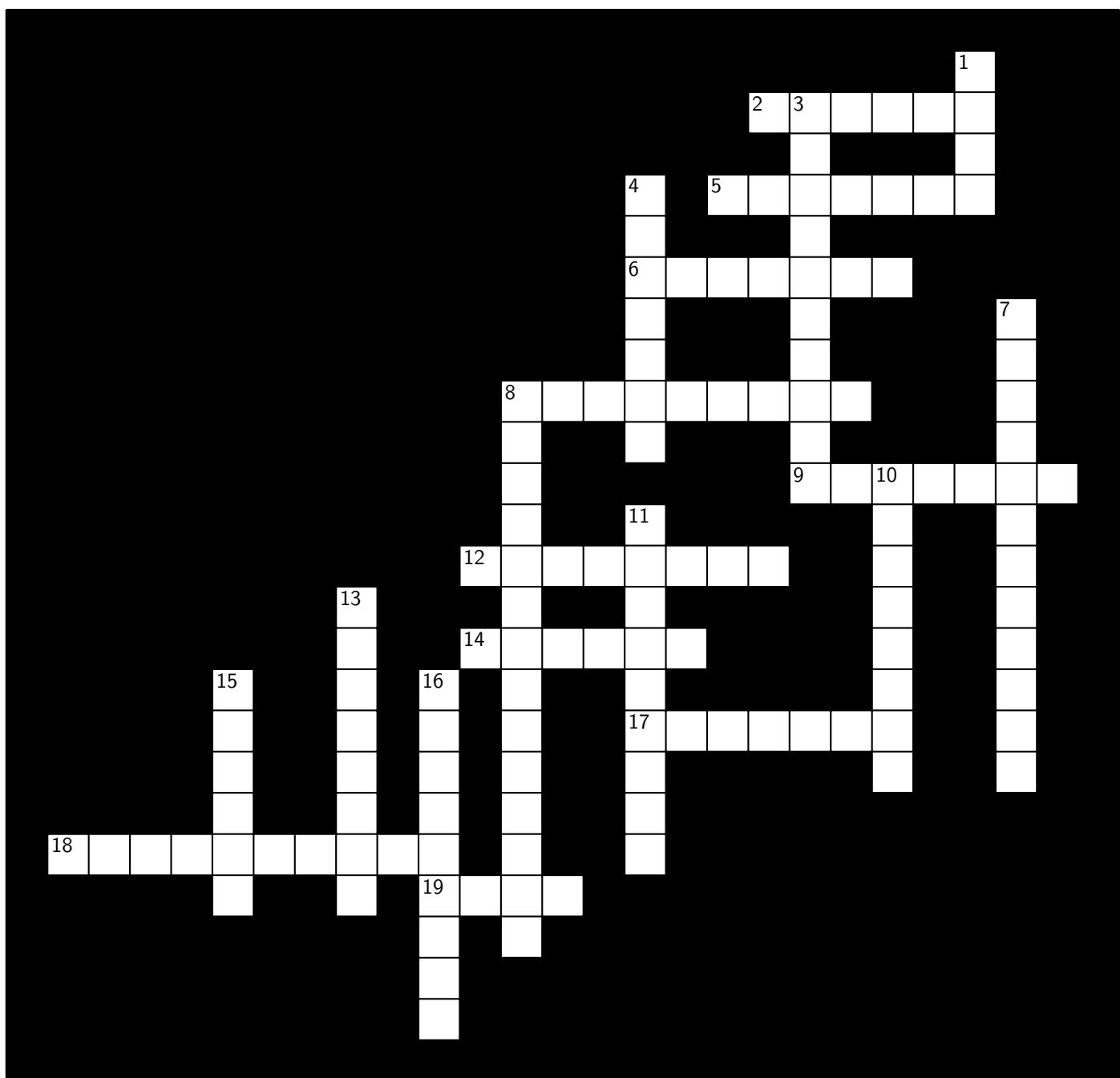
/4

/33

Quizdle — Crossword Sheet

Team:

Referee:

Quizdle — Answer Sheet

Team:

Referee:

Across		H	M	E
2		$1\frac{1}{2}$	1	$\frac{1}{2}$
5		$1\frac{1}{2}$	1	$\frac{1}{2}$
6		$1\frac{1}{2}$	1	$\frac{1}{2}$
8		$1\frac{1}{2}$	1	$\frac{1}{2}$
9		$1\frac{1}{2}$	1	$\frac{1}{2}$
12		$1\frac{1}{2}$	1	$\frac{1}{2}$
14		$1\frac{1}{2}$	1	$\frac{1}{2}$
17		$1\frac{1}{2}$	1	$\frac{1}{2}$
18		$1\frac{1}{2}$	1	$\frac{1}{2}$
19		$1\frac{1}{2}$	1	$\frac{1}{2}$

Down		H	M	E
1		$1\frac{1}{2}$	1	$\frac{1}{2}$
3		$1\frac{1}{2}$	1	$\frac{1}{2}$
4		$1\frac{1}{2}$	1	$\frac{1}{2}$
7		$1\frac{1}{2}$	1	$\frac{1}{2}$
8		$1\frac{1}{2}$	1	$\frac{1}{2}$
10		$1\frac{1}{2}$	1	$\frac{1}{2}$
11		$1\frac{1}{2}$	1	$\frac{1}{2}$
13		$1\frac{1}{2}$	1	$\frac{1}{2}$
15		$1\frac{1}{2}$	1	$\frac{1}{2}$
16		$1\frac{1}{2}$	1	$\frac{1}{2}$

Minor spelling mistakes are allowed and will be fixed by the marker.

Round down sum of scores to the nearest integer to get...

Final Score: ⬜ / 30

QUIZDLE — ACROSS (HARD)

2. Earth was represented by this in ancient Chinese cosmology. Hindu yantras often feature these, which are adorned with gates. In music, Gregorian chants were notated using neumes, with noteheads resembling these in a notation named after them.

5. Pink noise is sometimes known as this kind of noise. In computer graphics, Perlin noise is commonly used with landscape generators like Terragen to create this kind of landscape, while in mathematics, the chaos game creates these.

6. This person, despite studying mathematics at Cambridge, won the 1950 Nobel Prize in Literature "in recognition of [their] varied and significant writings in which [they] champion humanitarian ideals and freedom of thought". A vocal advocate of pacifism and social reform, they opposed war and nuclear weapons and were imprisoned during World War I for their pacifist activism.

8. Write this number by its digits only, for example, "FOURTWO" for 42. This is the number widely considered to be the most likely candidate for Plato's number. An ISO standard with this number defines paper sizes, including A4.

9. In computer science, examples of technologies which use the peer-to-peer type of this structure include BitTorrent and Bitcoin. In economics and technology, an "effect" known by this name describes how a product or service becomes more valuable as more people use it, as in the case of telephones and software.

12. This mathematician is the namesake of an infinite set of orthogonal polynomials formed by performing the Gram–Schmidt process on $1, x, x^2, x^3, \ldots$. He is also known for the still unsolved conjecture that there is a prime between any two consecutive positive square numbers.

14. This theory in algebraic geometry was proposed by Grothendieck to unify the vast array of cohomology theories that behaved similarly. It is sometimes used in its original French spelling, whereupon it becomes another word in English meaning "pattern". Examples of such "patterns" in other fields include granny squares in crochet, meanders in art, and exchange sacrifices in chess.

17. A type of democracy described by this adjective emphasises participation and deliberation, and its most notable strand of thought is the agonistic strand. It is sometimes also used to describe ancient Greek democracy under Pericles. In algebra, an ideal with the property that if some power of an element belongs to it, then the element itself does too, is named for this term.

18. In technical drawing, types of this include multiview, axonometric, oblique, and perspective. In cartography, Tissot's indicatrix is used to measure the distortion of one of these.

19. The adjective derived from this mathematician's name is not capitalised. Among other things, they are the namesake of a category, a transform and a test for the convergence of an infinite series.

Quizdle — Down (Hard)

1. A philosopher with this last name wrote *Gravity and Grace* and explored themes of mysticism, political activism, and social justice before her early death in 1943. A mathematician with this last name was the first to represent a null set as $\varnothing$. He is depicted in the picture to the right:

3. The defining features of these mathematical objects were etched onto a stone of Broom Bridge, Dublin on 16th October, 1843.

4. This word unites all of the following. Karl Popper stated that a society must be intolerant toward intolerance to protect itself. Hans Moravec stated that tasks easy for humans, like object recognition, are hard for machines, and vice versa. Cyrus Levinthal demonstrated that although a protein has an astronomical number of possible conformations, it rapidly folds into its functional form.

7. In mathematics, the Dirac Delta function is an example of this, the general theory of which extends extends differentiation to non-smooth functions in the theory developed by Laurent Schwartz. On the internet, a system of servers that improves content delivery speed and reliability is sometimes called a Content _____ Network. Zipf's law concerns a Zipfian or zeta one of these which is used to model this aspect of a frequency table of data drawn from a corpus.

8. Write this number by its digits only, for example, "FOURTWO" for 42. This number appears in the former name of the developer Halo Studios, known for developing Halo from Halo 4 onwards after Bungie became independent from Microsoft. It is also the number of women who signed a petition written by Simone de Beauvoir in 1971 demanding abortion rights. This number is the 10th Friedman number in base 10 – numbers that can be expressed non-trivially in terms of their digits using the basic arithmetic operations.

10. The Christian Eye of Providence, Hindu Sri Yantra, and Norse Valknut all prominently feature this. New York's Flatiron Building resembles this when viewed from above. A US Cold War foreign policy involving China and the Soviet Union was named for this. The Knaster–Kuratowski fan (also known as Cantor's leaky tent) resembles this.

11. Anthropologists use this term to describe cultural traits that appear in all human societies, such as property, divination, and music. In mathematics, this kind of algebra studies general structures rather than specific examples of mathematical objects. Noam Chomsky proposed a theory known as this kind of grammar to describe innate principles underlying all human languages.

13. In complex analysis, a function related to a given function by the Cauchy-Riemann equations is called one of these conjugates. A point D is this kind of conjugate of C with respect to A and B if $AC/CB = -AD/DB$. On stringed instruments, a special technique involving two points of contact on a single string produces artificial ones of these.

15. A professor at Cardiff University with this surname introduced the subdivision method which gave the best large values estimate for Dirichlet Polynomials until the work of Maynard–Guth in 2024. Hodgkin and a biologist with this surname first described the conductance-based model of action potentials in neurons, for which they won the Nobel Prize in 1963.

16. A mathematical object named after this mathematician is referenced by the time signatures, prosody, and lyrics in the song *Lateralus* by Tool. The mathematical object was first described in India in the work of the mathematician Pingala.

Quizdle — Across (Medium)

2. A diagram used in genetics to predict offspring traits is named for this and was developed by Reginald Punnett. Examples of this in the real world include ones named Red in Moscow, Tahrir in Cairo, and Syntagma in Athens. This is also the name of the protagonist in the book *Flatland*.

5. The Weierstrass function, an example of a real-valued function that is continuous everywhere but differentiable nowhere, was one of the first examples of this kind of mathematical object. Other examples include Cantor dust, Apollonian gasket, and well-known examples of continuous bijections $f : [0,1] \to [0,1] \times [0,1]$. The coastline paradox concerns this feature of coastlines.

6. This person was one of the founders of analytic philosophy along Gottlob Frege, G. E. Moore, and their student Ludwig Wittgenstein. In a thought experiment now known as their "teapot", they compare belief in God to an undetectable teapot orbiting the Sun.

8. Write this number by its digits only, for example, "FOURTWO" for 42. This number is the first non-trivial counterexample to Euler's sum of powers conjecture. It is also the number of colours in the web-safe colour palette used in the mid-1990s. In the Hepburn romanisation of Japanese, it can be written as "nihyaku jūroku."

9. In mathematics, a theory known by this name is often the result of applying graph theory to the real world – for example, the max-flow min-cut theorem is a result on the flow type of these. A 2010 film directed by David Fincher dramatized the rise of Facebook and includes this term in its title.

12. The only surviving picture of this mathematician is the following caricature:
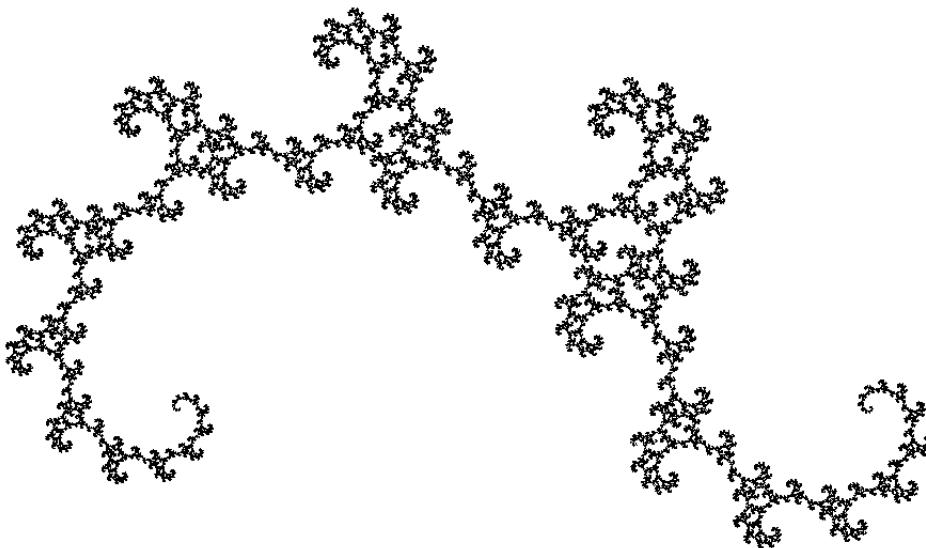
14. This word is a British slang term referring to an event or activity which you do with your friends. In criminal investigation, this word, along with means and opportunity, are the three indicators of suspicion sometimes used to convince juries of guilt.

17. A feminist movement advocating for the dismantling of patriarchy and structural gender inequality is named by this term. In Chinese dictionaries, characters are ordered by the "indexing component" of the character, which is also known by this word.

18. In psychology, this term describes the defence mechanism in which a person attributes their own thoughts or feelings to others. In engineering, the orthographic type of this represents three-dimensional objects in two dimensions

19. His most well-known result, named for him and Ruffini, proved that general quintic equations cannot be solved by radicals. A 17th century Dutch explorer who was the namesake of the Tasman sea has this mathematician's name as their first name.

Quizzle — Down (Medium)

1. In mathematics, a set of namesake conjectures in algebraic geometry led to an influential multi-decade program, ultimately ending with their proof by Pierre Deligne. They strengthened a theorem now named after Mordell and themselves to show that the group of rational points on an abelian variety such as an elliptic curve forms a finitely generated abelian group. This mathematician also co-founded the Bourbaki group.

3. These objects are useful in describing rotations and so are extensively applied in computer graphics. They are also the name for a group of order 8 notable for being a non-abelian group all of whose subgroups are normal.

4. This word unites all the following. In statistics, an effect discovered by Edward Simpson shows that trends evident within several groups can reverse when those groups are combined. A famous probability problem named for Saint Petersburg involves a gambling game which has an infinite expected payoff. In traffic network theory, Dietrich Braess discovered that adding an extra route can worsen overall traffic flow.

7. In economics, this term wealth or income one of these describe how these resources are allocated among individuals or groups. Some examples of this in mathematics include ones called Boltzmann, Cauchy, and beta.

8. Write this number by its digits only, for example, "FOURTWO" for 42. It is the maximal number of 1's an invertible 19×19 matrix whose entries are in $\{0, 1\}$. It also cannot be expressed as the sum of three squares. It can be expressed in dozenal as $4\mathcal{Z} + 1\mathcal{E}9$.

10. The Atlantic slave trade is an example of a trade route named for this. In psychology, Robert Sternberg developed a theory of love describes different types of relationships using a model based on this. There is an online encyclopedia documenting over sixty thousand centres of these.

11. In science and mathematics, the domain of discourse is known as this kind of set, and is sometimes denoted $\mathbb{D}$. This word appears in the name of a company, which, along with Sony Music Entertainment and Warner Music Group, form one of the "big three" music corporations.

13. The Euler–Mascheroni constant is the limiting difference between this series and the natural logarithm. This term also describes the integer-multiple overtones of a fundamental frequency in music.

15. This surname is shared by a biologist known as "Darwin's Bulldog" for his advocacy of Darwin's theory of evolution, and by his grandson, who wrote the dystopian novel Brave New World.

16. This mathematician was also known as Leonardo of Pisa. They wrote the book *Liber Abaci* which introduced base 10 and the Hindu-Arabic numeral system to Europe.

Quizdle — Across (Easy)

2. Many physical laws, such as those for gravity and light intensity, follow an inverse form of this kind of law. A statistical method used in regression analysis to minimize error is named for the least of these. In Italian cuisine, ravioli commonly resemble these.

5. One way to define these objects is as spaces whose Hausdorff dimension exceeds their topological dimension. An example of this mathematical object is shown below:

6. Within mathematics, he is most well-known as a logician. He co-authored *Principia Mathematica* with Alfred North Whitehead, attempting to derive all of mathematics from logical principles. Together with 4 Down (which you may not fill in), this names an idea concerning the set of all sets that do not contain themselves, which challenged naive set theory.

8. Write this number by its digits only, for example, "FOURTWO" for 42. This is the smallest cube number which is the sum of three other cube numbers.

9. A system of interconnected artificial neurons used in machine learning is called one of these. As a social activity, this term refers to the building of professional or personal connections, which may occur at "events" described by this word.

12. It is easy to be confused between this mathematician and two of his contemporaries: Laplace and Lagrange. He can rightly be called a legendary figure.

14. The definition of this word in English is a reason for doing something. In mathematics, a variation of this word is used to talk about the inspiration behind outside-the-box thinking.

17. In geometry, the line passing through the two intersections of two circles is known as this kind of axis. In chemistry, a molecule or atom with an unpaired electron is a "free" one of these.

18. In mathematics, this term describes linear transformations that satisfy idempotency. In cartography, types of this such as Mercator or cylindrical are used.

19. This man, whose name can be a pun on the fact that certain algebraic structures are able to commute, also gives their name to a prize considered the mathematical equivalent of the Nobel Prize. Their name is also shared with a biblical figure, the second son of Adam and Eve, who was slain by his brother Cain.

Quizdle — Down (Easy)

1. This mathematician's name closely resembles that of Andrew Wiles, and the philosopher in the hard clue is his better-known sister.

3. Sir William Rowan Hamilton discovered these objects while trying to find a three-dimensional analogue for the complex numbers. Multiplication with these mathematical objects are closely related to the three-dimensional cross-product.

4. This word unites all the following. Gabriel's Horn is a mathematical solid that encloses a finite volume while possessing an infinite surface area. A well-known result shows that a solid ball can be split into a finite number of pieces and reassembled into two identical copies of the original ball. Together with 6 Across (which you may not fill in), this names an idea concerning the set of all sets that do not contain themselves, which challenged naive set theory.

7. In probability, a function describing the likelihood of different outcomes is called one of these. In film, a company responsible for releasing and marketing a movie is said to handle this process.

8. Write this number by its digits only, for example, "FOURTWO" for 42. This is a cube number.

10. Examples of this in geography include the Bermuda and Golden ones. A lithograph by M.C. Escher features a never-ending waterfall constructed from "Penrose" versions of these. Foods like onigiri and hamantashen often resemble this. In mathematics, this gives its name to a certain subset of matrices and an inequality.

11. In technology, the "U" in USB stands for this word. Some economic proposals advocate a guaranteed basic income for all citizens which can be described by this term. A major Hollywood film studio and theme park chain is named for this term.

13. The reciprocal of the arithmetic mean of reciprocals is this kind of mean. In physics, a mass-spring system undergoing periodic motion exhibits the simple form of this type of oscillation. The series known by this name begins $1 + \frac{1}{2} + \frac{1}{3} + \cdots$.

15. This is the name of the building you're in right now!

16. This mathematician gives their name to the sequence of numbers which describes a common puzzle in mathematics involving a population of rabbits. The ratio of successive terms in the sequence approach the golden ratio.