Primal estimated sub-gradient solver for SVM

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Advanced Topics in Machine Learning

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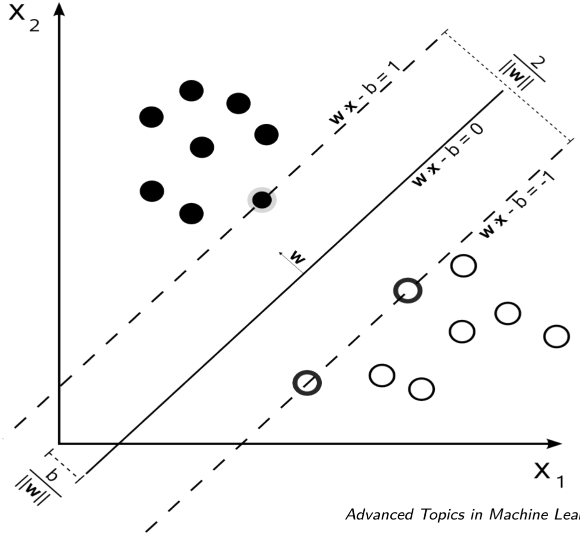
Motivating example



Support Vector Machine

Definition

In machine learning many models to do classification and regression analysis are of the following form. Given a training set $(\boldsymbol{x}_i, y_i)_{i=1}^n$ with $\boldsymbol{x}_i \in \mathbb{R}^d$, $y_i \in \{-1, +1\}$, learning is formulated as the task of minimizing the following objective function:

$$f(\boldsymbol{w}) := \frac{1}{n}\sum_{i=1}^{n} L_i(\boldsymbol{w}) + \frac{\lambda}{2}\|\boldsymbol{w}\|^2$$

Loss Function and Subgradient

Definition

- Loss: $L_i := \ell(\langle \boldsymbol{x}_i, \boldsymbol{w} \rangle, y_i)$
- Subgradient: $l'(\langle \boldsymbol{x}_i, \boldsymbol{w} \rangle, y_i)$

Use the notation $z = \langle \boldsymbol{w}, \boldsymbol{x}_i \rangle$, sample loss functions:

Loss function	Subgradient
$l(z, y_i) = \max\{0, 1 - y_i z\}$	ss
$l(z, y_i) = \log(1 + e^{-y_i z})$	1
$l(z, y_i) = \max\{0, \lvert y_i - z \rvert - \epsilon\}$	1

Stochastic Gradient Descent

Description

Following the basic Pegasos algorithm, $\boldsymbol{w}$ is set to be 0 initially. In each round, we pick a random training example $(\boldsymbol{x}_i, y_i)$ in which i is picked with probability p_i, s.t. $p_i \geq 0$ and $\sum_{i=1}^{n} p_i = 1$.

Basic Pegasos Algorithm

Algorithm

1. Choose $\boldsymbol{w}_1 = 0 \in \mathbb{R}^d$
2. Iterate for $t = 1, 2, \ldots, T$
   1. Choose $A_t \subset S = \{1, 2, \ldots, n\}, |A_t| = b$, uniformly at random
   2. Set stepsize $\eta_t \leftarrow \frac{1}{\lambda t}$
   3. Update $w^{(t+1)} \leftarrow w^{(t)} - \eta$

Theorem For $\overline{\boldsymbol{w}} = \frac{1}{T} \sum_{t=1}^{T} \boldsymbol{w}_t$, we have:

$$\mathbb{E}[f(\overline{\boldsymbol{w}})] \leq f(w^*) + c \log(T) \times \frac{1}{\lambda T}$$

where $c = (\sqrt(\lambda) + 1)^2$.

- Technically
- xx
- Methodologically
- xx

Thank You!

Q&A

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Reference

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