Assignment 3

# Part 1: 0-1 Knapsack Problem, Dynamic Solution

To qualify the problem to be solved using Dynamic programming it needs to exhibit both optimal substructure and have overlapping sub problems. Defining a solution as well as a sub problem will allow the above properties to be proven. A candidate solution to the problem can be defined as follows:

Where represents an item with some value, and weight, that is in the solution . is a candidate solution when . is an optimal solution if .

A sub problem of , is a solution to the problem where .

## Optimal Substructure:

Given an optimal solution and its sub problem assume that isn’t optimal. This means that there is some solution to the sub problem where . As isn’t optimal we can assume . But since and are both solutions to the sub problem with weight then and are both solutions to the problem which is supposed to be an optimal solution to. However, since and it follows that and so implies a contradiction to S being optimal. This implies that must be optimal if is optimal. This follows for the sub problems of down to the trivial problem where the sub problem must satisfy .

## Recursive Solution:

Following that the problem exhibits optimal substructure, the solution set can be built up by sub problems to the total problem, making sure that each sub solution is optimal. The trivial sub problem is when the weight is 0. In this case the optimal solution is the empty set. From then on the solution to sub problem with weight can be found by examining each potential item not a part of the current sub solution. For each item it will either be a part of the sub solution or not, with the value of the optimal solution given the remaining items will be the max of the following cases:

1. The value of the sub solution with the same weight as this one but excluding this item.
2. Value of this item plus the value from the sub problem with weight

When the weight of the current item is greater then only case 1 applies.

## Overlapping Sub problems:

Based on the above recursive solution the problem will repeat sub problems when calculating greater sub solutions.

As the 0-1 knapsack problem exhibits both optimal substructure and overlapping sub problems a dynamic program will be suitable. Based on the above recursive solution a dynamic program which stores the solution to sub problems is guaranteed to give an optimal solution.