ICPC Team Notebook (2018-19)

Contents

1 Combinatorial optimization

1.1 Dense max-flow

```
// Adjacency matrix implementation of Dinic's blocking flow algorithm.
//
// Running time:
//     O(|V|^4)
//
// INPUT:
//     - graph, constructed using AddEdge()
//     - source
//     - sink
//
// OUTPUT:
//     - maximum flow value
//     - To obtain the actual flow, look at positive values only.
typedef vector<int> VI;
typedef vector<VI> VVI;

const int INF = 1000000000;

struct MaxFlow {
  int N;
  VVI cap, flow;
  VI dad, Q;

  MaxFlow(int N) :
    N(N), cap(N, VI(N)), flow(N, VI(N)), dad(N), Q(N) {}

  void AddEdge(int from, int to, int cap) {
    this->cap[from][to] += cap;
  }

  int BlockingFlow(int s, int t) {
    fill(dad.begin(), dad.end(), -1);
    dad[s] = -2;

    int head = 0, tail = 0;
    Q[tail++] = s;
    while (head < tail) {
      int x = Q[head++];
      for (int i = 0; i < N; i++) {
        if (dad[i] == -1 && cap[x][i] - flow[x][i] > 0) {
          dad[i] = x;
          Q[tail++] = i;
        }
      }
    }

    if (dad[t] == -1) return 0;

    int totflow = 0;
    for (int i = 0; i < N; i++) {
      if (dad[i] == -1) continue;
      int amt = cap[i][t] - flow[i][t];
      for (int j = i; amt && j != s; j = dad[j])
        amt = min(amt, cap[dad[j]][j] - flow[dad[j]][j]);
      if (amt == 0) continue;
      flow[i][t] += amt;
      flow[t][i] -= amt;
      for (int j = i; j != s; j = dad[j]) {
        flow[dad[j]][j] += amt;
        flow[j][dad[j]] -= amt;
      }
      totflow += amt;
    }

    return totflow;
  }

  int GetMaxFlow(int source, int sink) {
    int totflow = 0;
    while (int flow = BlockingFlow(source, sink))
      totflow += flow;
    return totflow;
  }
};

int main() {

  MaxFlow mf(5);
  mf.AddEdge(0, 1, 3);
  mf.AddEdge(0, 2, 4);
  mf.AddEdge(0, 3, 5);
  mf.AddEdge(0, 4, 5);
  mf.AddEdge(1, 2, 2);
  mf.AddEdge(2, 3, 4);
  mf.AddEdge(2, 4, 1);
  mf.AddEdge(3, 4, 10);

  // should print out "15"
  cout << mf.GetMaxFlow(0, 4) << endl;
}

// BEGIN CUT
// The following code solves SPOJ problem #203: Potholers (POTHOLE)

#ifdef COMMENT
int main() {
  int t;
  cin >> t;
  for (int i = 0; i < t; i++) {
    int n;
    cin >> n;
    MaxFlow mf(n);
    for (int j = 0; j < n-1; j++) {
      int m;
      cin >> m;
      for (int k = 0; k < m; k++) {
        int p;
        cin >> p;
        p--;
        int cap = (j == 0 || p == n-1) ? 1 : INF;
        mf.AddEdge(j, p, cap);
      }
    }

    cout << mf.GetMaxFlow(0, n-1) << endl;
  }
  return 0;
}
#endif

// END CUT
```

1.2 Sparse max-flow

```
// Adjacency list implementation of Dinic's blocking flow algorithm.
// This is very fast in practice, and only loses to push-relabel flow.
//
// Running time:
//     O(|V|^2 |E|)
//
// INPUT:
//     - graph, constructed using AddEdge()
//     - source and sink
//
// OUTPUT:
//     - maximum flow value
//     - To obtain actual flow values, look at edges with capacity > 0
//       (zero capacity edges are residual edges).
typedef long long LL;

struct Edge {
  int u, v;
  LL cap, flow;
  Edge() {}
  Edge(int u, int v, LL cap): u(u), v(v), cap(cap), flow(0) {}
};

struct Dinic {
  int N;
  vector<Edge> E;
  vector<vector<int>> g;
  vector<int> d, pt;

  Dinic(int N): N(N), E(0), g(N), d(N), pt(N) {}

  void AddEdge(int u, int v, LL cap) {
    if (u != v) {
```

```
    E.emplace_back(u, v, cap);
    g[u].emplace_back(E.size() - 1);
    E.emplace_back(v, u, 0);
    g[v].emplace_back(E.size() - 1);
  }
}

bool BFS(int S, int T) {
  queue<int> q({S});
  fill(d.begin(), d.end(), N + 1);
  d[S] = 0;
  while(!q.empty()) {
    int u = q.front(); q.pop();
    if (u == T) break;
    for (int k: g[u]) {
      Edge &e = E[k];
      if (e.flow < e.cap && d[e.v] > d[e.u] + 1) {
        d[e.v] = d[e.u] + 1;
        q.emplace(e.v);
      }
    }
  }
  return d[T] != N + 1;
}

LL DFS(int u, int T, LL flow = -1) {
  if (u == T || flow == 0) return flow;
  for (int &i = pt[u]; i < g[u].size(); ++i) {
    Edge &e = E[g[u][i]];
    Edge &oe = E[g[u][i]^1];
    if (d[e.v] == d[e.u] + 1) {
      LL amt = e.cap - e.flow;
      if (flow != -1 && amt > flow) amt = flow;
      if (LL pushed = DFS(e.v, T, amt)) {
        e.flow += pushed;
        oe.flow -= pushed;
        return pushed;
      }
    }
  }
  return 0;
}

LL MaxFlow(int S, int T) {
  LL total = 0;
  while (BFS(S, T)) {
    fill(pt.begin(), pt.end(), 0);
    while (LL flow = DFS(S, T))
      total += flow;
  }
  return total;
}
};

// BEGIN CUT
// The following code solves SPOJ problem #4110: Fast Maximum Flow (
    FASTFLOW)

int main()
{
  int N, E;
  scanf("%d%d", &N, &E);
  Dinic dinic(N);
  for(int i = 0; i < E; i++)
  {
    int u, v;
    LL cap;
    scanf("%d%d%lld", &u, &v, &cap);
    dinic.AddEdge(u - 1, v - 1, cap);
    dinic.AddEdge(v - 1, u - 1, cap);
  }
  printf("%lld\n", dinic.MaxFlow(0, N - 1));
  return 0;
}

// END CUT
```

1.3 Min-cost max-flow

```
// Implementation of min cost max flow algorithm using adjacency
// matrix (Edmonds and Karp 1972).  This implementation keeps track of
// forward and reverse edges separately (so you can set cap[i][j] !=
// cap[j][i]).  For a regular max flow, set all edge costs to 0.
//
// Running time, O(|V|^2) cost per augmentation
//     max flow:           O(|V|^3) augmentations
//     min cost max flow:  O(|V|^4 * MAX_EDGE_COST) augmentations
//
// INPUT:
//     - graph, constructed using AddEdge()
//     - source
//     - sink
//
// OUTPUT:
//     - (maximum flow value, minimum cost value)
//     - To obtain the actual flow, look at positive values only.
typedef vector<int> VI;
typedef vector<VI> VVI;
typedef long long L;
typedef vector<L> VL;
typedef vector<VL> VVL;
typedef pair<int, int> PII;
typedef vector<PII> VPII;

const L INF = numeric_limits<L>::max() / 4;

struct MinCostMaxFlow {
  int N;
  VVL cap, flow, cost;
  VI found;
  VL dist, pi, width;
  VPII dad;

  MinCostMaxFlow(int N) :
    N(N), cap(N, VL(N)), flow(N, VL(N)), cost(N, VL(N)),
    found(N), dist(N), pi(N), width(N), dad(N) {}

  void AddEdge(int from, int to, L cap, L cost) {
    this->cap[from][to] = cap;
    this->cost[from][to] = cost;
  }

  void Relax(int s, int k, L cap, L cost, int dir) {
    L val = dist[s] + pi[s] - pi[k] + cost;
    if (cap && val < dist[k]) {
      dist[k] = val;
      dad[k] = make_pair(s, dir);
      width[k] = min(cap, width[s]);
    }
  }

  L Dijkstra(int s, int t) {
    fill(found.begin(), found.end(), false);
    fill(dist.begin(), dist.end(), INF);
    fill(width.begin(), width.end(), 0);
    dist[s] = 0;
    width[s] = INF;

    while (s != -1) {
      int best = -1;
      found[s] = true;
      for (int k = 0; k < N; k++) {
        if (found[k]) continue;
        Relax(s, k, cap[s][k] - flow[s][k], cost[s][k], 1);
        Relax(s, k, flow[k][s], -cost[k][s], -1);
        if (best == -1 || dist[k] < dist[best]) best = k;
      }
      s = best;
    }

    for (int k = 0; k < N; k++)
      pi[k] = min(pi[k] + dist[k], INF);
    return width[t];
  }

  pair<L, L> GetMaxFlow(int s, int t) {
    L totflow = 0, totcost = 0;
    while (L amt = Dijkstra(s, t)) {
      totflow += amt;
      for (int x = t; x != s; x = dad[x].first) {
        if (dad[x].second == 1) {
          flow[dad[x].first][x] += amt;
          totcost += amt * cost[dad[x].first][x];
        } else {
          flow[x][dad[x].first] -= amt;
          totcost -= amt * cost[x][dad[x].first];
        }
      }
    }
    return make_pair(totflow, totcost);
  }
};

// BEGIN CUT
// The following code solves UVA problem #10594: Data Flow

int main() {
  int N, M;

  while (scanf("%d%d", &N, &M) == 2) {
    VVL v(M, VL(3));
    for (int i = 0; i < M; i++)
      scanf("%Ld%Ld%Ld", &v[i][0], &v[i][1], &v[i][2]);
    L D, K;
    scanf("%Ld%Ld", &D, &K);

    MinCostMaxFlow mcmf(N+1);
    for (int i = 0; i < M; i++) {
      mcmf.AddEdge(int(v[i][0]), int(v[i][1]), K, v[i][2]);
      mcmf.AddEdge(int(v[i][1]), int(v[i][0]), K, v[i][2]);
    }
    mcmf.AddEdge(0, 1, D, 0);

    pair<L, L> res = mcmf.GetMaxFlow(0, N);

    if (res.first == D) {
      printf("%Ld\n", res.second);
    } else {
      printf("Impossible.\n");
    }
  }

  return 0;
}

// END CUT
```

1.4 Push-relabel max-flow

```
// Adjacency list implementation of FIFO push relabel maximum flow
// with the gap relabeling heuristic.  This implementation is
// significantly faster than straight Ford-Fulkerson.  It solves
// random problems with 10000 vertices and 1000000 edges in a few
// seconds, though it is possible to construct test cases that
// achieve the worst-case.
//
// Running time:
//     O(|V|^3)
//
// INPUT:
//     - graph, constructed using AddEdge()
//     - source
//     - sink
//
// OUTPUT:
//     - maximum flow value
//     - To obtain the actual flow values, look at all edges with
//       capacity > 0 (zero capacity edges are residual edges).
typedef long long LL;

struct Edge {
  int from, to, cap, flow, index;
  Edge(int from, int to, int cap, int flow, int index) :
    from(from), to(to), cap(cap), flow(flow), index(index) {}
};

struct PushRelabel {
  int N;
  vector<vector<Edge> > G;
  vector<LL> excess;
  vector<int> dist, active, count;
  queue<int> Q;

  PushRelabel(int N) : N(N), G(N), excess(N), dist(N), active(N),
      count(2*N) {}

  void AddEdge(int from, int to, int cap) {
    G[from].push_back(Edge(from, to, cap, 0, G[to].size()));
    if (from == to) G[from].back().index++;
    G[to].push_back(Edge(to, from, 0, 0, G[from].size() - 1));
  }

  void Enqueue(int v) {
    if (!active[v] && excess[v] > 0) { active[v] = true; Q.push(v); }
  }

  void Push(Edge &e) {
    int amt = int(min(excess[e.from], LL(e.cap - e.flow)));
    if (dist[e.from] <= dist[e.to] || amt == 0) return;
```

```
      e.flow += amt;
      G[e.to][e.index].flow -= amt;
      excess[e.to] += amt;
      excess[e.from] -= amt;
      Enqueue(e.to);
    }

    void Gap(int k) {
      for (int v = 0; v < N; v++) {
        if (dist[v] < k) continue;
        count[dist[v]]--;
        dist[v] = max(dist[v], N+1);
        count[dist[v]]++;
        Enqueue(v);
      }
    }

    void Relabel(int v) {
      count[dist[v]]--;
      dist[v] = 2*N;
      for (int i = 0; i < G[v].size(); i++)
        if (G[v][i].cap - G[v][i].flow > 0)
          dist[v] = min(dist[v], dist[G[v][i].to] + 1);
      count[dist[v]]++;
      Enqueue(v);
    }

    void Discharge(int v) {
      for (int i = 0; excess[v] > 0 && i < G[v].size(); i++) Push(G[v][i
          ]);
      if (excess[v] > 0) {
        if (count[dist[v]] == 1)
          Gap(dist[v]);
        else
          Relabel(v);
      }
    }

    LL GetMaxFlow(int s, int t) {
      count[0] = N-1;
      count[N] = 1;
      dist[s] = N;
      active[s] = active[t] = true;
      for (int i = 0; i < G[s].size(); i++) {
        excess[s] += G[s][i].cap;
        Push(G[s][i]);
      }

      while (!Q.empty()) {
        int v = Q.front();
        Q.pop();
        active[v] = false;
        Discharge(v);
      }

      LL totflow = 0;
      for (int i = 0; i < G[s].size(); i++) totflow += G[s][i].flow;
      return totflow;
    }
};

// BEGIN CUT
// The following code solves SPOJ problem #4110: Fast Maximum Flow (
     FASTFLOW)

int main() {
  int n, m;
  scanf("%d%d", &n, &m);

  PushRelabel pr(n);
  for (int i = 0; i < m; i++) {
   int a, b, c;
    scanf("%d%d%d", &a, &b, &c);
    if (a == b) continue;
    pr.AddEdge(a-1, b-1, c);
    pr.AddEdge(b-1, a-1, c);
  }
  printf("%Ld\n", pr.GetMaxFlow(0, n-1));
  return 0;
}

// END CUT
```

1.5 Min-cost matching

```
///////////////////////////////////////////////////////////////////////
// Min cost bipartite matching via shortest augmenting paths
//
// This is an O(n^3) implementation of a shortest augmenting path
// algorithm for finding min cost perfect matchings in dense
// graphs.  In practice, it solves 1000x1000 problems in around 1
// second.
//
//   cost[i][j] = cost for pairing left node i with right node j
//   Lmate[i] = index of right node that left node i pairs with
//   Rmate[j] = index of left node that right node j pairs with
//
// The values in cost[i][j] may be positive or negative.  To perform
// maximization, simply negate the cost[][] matrix.
///////////////////////////////////////////////////////////////////////
typedef vector<double> VD;
typedef vector<VD> VVD;
typedef vector<int> VI;

double MinCostMatching(const VVD &cost, VI &Lmate, VI &Rmate) {
  int n = int(cost.size());

  // construct dual feasible solution
  VD u(n);
  VD v(n);
  for (int i = 0; i < n; i++) {
    u[i] = cost[i][0];
    for (int j = 1; j < n; j++) u[i] = min(u[i], cost[i][j]);
  }
  for (int j = 0; j < n; j++) {
    v[j] = cost[0][j] - u[0];
    for (int i = 1; i < n; i++) v[j] = min(v[j], cost[i][j] - u[i]);
  }

  // construct primal solution satisfying complementary slackness
  Lmate = VI(n, -1);
  Rmate = VI(n, -1);
  int mated = 0;
  for (int i = 0; i < n; i++) {
    for (int j = 0; j < n; j++) {
      if (Rmate[j] != -1) continue;
      if (fabs(cost[i][j] - u[i] - v[j]) < 1e-10) {
        Lmate[i] = j;
        Rmate[j] = i;
        mated++;
        break;
      }
    }
  }

  VD dist(n);
  VI dad(n);
  VI seen(n);

  // repeat until primal solution is feasible
  while (mated < n) {

    // find an unmatched left node
    int s = 0;
    while (Lmate[s] != -1) s++;

    // initialize Dijkstra
    fill(dad.begin(), dad.end(), -1);
    fill(seen.begin(), seen.end(), 0);
    for (int k = 0; k < n; k++)
      dist[k] = cost[s][k] - u[s] - v[k];

    int j = 0;
    while (true) {

      // find closest
      j = -1;
      for (int k = 0; k < n; k++) {
        if (seen[k]) continue;
        if (j == -1 || dist[k] < dist[j]) j = k;
      }
      seen[j] = 1;

      // termination condition
      if (Rmate[j] == -1) break;

      // relax neighbors
      const int i = Rmate[j];
      for (int k = 0; k < n; k++) {
        if (seen[k]) continue;
        const double new_dist = dist[j] + cost[i][k] - u[i] - v[k];
        if (dist[k] > new_dist) {
          dist[k] = new_dist;
          dad[k] = j;
        }
      }
    }

    // update dual variables
    for (int k = 0; k < n; k++) {
      if (k == j || !seen[k]) continue;
      const int i = Rmate[k];
      v[k] += dist[k] - dist[j];
      u[i] -= dist[k] - dist[j];
    }
    u[s] += dist[j];

    // augment along path
    while (dad[j] >= 0) {
      const int d = dad[j];
      Rmate[j] = Rmate[d];
      Lmate[Rmate[j]] = j;
      j = d;
    }
    Rmate[j] = s;
    Lmate[s] = j;

    mated++;
  }

  double value = 0;
  for (int i = 0; i < n; i++)
    value += cost[i][Lmate[i]];

  return value;
}
```

1.6 Max bipartite matchine

```
// This code performs maximum bipartite matching.
//
// Running time: O(|E| |V|) -- often much faster in practice
//
//   INPUT: w[i][j] = edge between row node i and column node j
//   OUTPUT: mr[i] = assignment for row node i, -1 if unassigned
//           mc[j] = assignment for column node j, -1 if unassigned
//           function returns number of matches made
typedef vector<int> VI;
typedef vector<VI> VVI;

bool FindMatch(int i, const VVI &w, VI &mr, VI &mc, VI &seen) {
  for (int j = 0; j < w[i].size(); j++) {
    if (w[i][j] && !seen[j]) {
      seen[j] = true;
      if (mc[j] < 0 || FindMatch(mc[j], w, mr, mc, seen)) {
        mr[i] = j;
        mc[j] = i;
        return true;
      }
    }
  }
  return false;
}

int BipartiteMatching(const VVI &w, VI &mr, VI &mc) {
  mr = VI(w.size(), -1);
  mc = VI(w[0].size(), -1);

  int ct = 0;
  for (int i = 0; i < w.size(); i++) {
    VI seen(w[0].size());
    if (FindMatch(i, w, mr, mc, seen)) ct++;
  }
  return ct;
}
```

1.7 Global min-cut

```
// Adjacency matrix implementation of Stoer-Wagner min cut algorithm.
//
// Running time:
//     O(|V|^3)
//
// INPUT:
```

```
//     - graph, constructed using AddEdge()
//
// OUTPUT:
//     - (min cut value, nodes in half of min cut)
typedef vector<int> VI;
typedef vector<VI> VVI;

const int INF = 1000000000;

pair<int, VI> GetMinCut(VVI &weights) {
  int N = weights.size();
  VI used(N), cut, best_cut;
  int best_weight = -1;

  for (int phase = N-1; phase >= 0; phase--) {
    VI w = weights[0];
    VI added = used;
    int prev, last = 0;
    for (int i = 0; i < phase; i++) {
      prev = last;
      last = -1;
      for (int j = 1; j < N; j++)
        if (!added[j] && (last == -1 || w[j] > w[last])) last = j;
      if (i == phase-1) {
        for (int j = 0; j < N; j++) weights[prev][j] += weights[last][
              j];
        for (int j = 0; j < N; j++) weights[j][prev] = weights[prev][j
              ];
        used[last] = true;
        cut.push_back(last);
        if (best_weight == -1 || w[last] < best_weight) {
          best_cut = cut;
          best_weight = w[last];
        }
      } else {
        for (int j = 0; j < N; j++)
          w[j] += weights[last][j];
        added[last] = true;
      }
    }
  }
  return make_pair(best_weight, best_cut);
}

// BEGIN CUT
// The following code solves UVA problem #10989: Bomb, Divide and
      Conquer
int main() {
  int N;
  cin >> N;
  for (int i = 0; i < N; i++) {
    int n, m;
    cin >> n >> m;
    VVI weights(n, VI(n));
    for (int j = 0; j < m; j++) {
      int a, b, c;
      cin >> a >> b >> c;
      weights[a-1][b-1] = weights[b-1][a-1] = c;
    }
    pair<int, VI> res = GetMinCut(weights);
    cout << "Case #" << i+1 << ": " << res.first << endl;
  }
}
// END CUT
```

1.8 Graph cut inference

```
// Special-purpose {0,1} combinatorial optimization solver for
// problems of the following by a reduction to graph cuts:
//
//        minimize         sum_i  psi_i(x[i])
//  x[1]...x[n] in {0,1}      + sum_{i < j}  phi_{ij}(x[i], x[j])
//
// where
//      psi_i : {0, 1} --> R
//   phi_{ij} : {0, 1} x {0, 1} --> R
//
// such that
//   phi_{ij}(0,0) + phi_{ij}(1,1) <= phi_{ij}(0,1) + phi_{ij}(1,0)
      (*)
//
// This can also be used to solve maximization problems where the
// direction of the inequality in (*) is reversed.
//
// INPUT: phi -- a matrix such that phi[i][j][u][v] = phi_{ij}(u, v)
//        psi -- a matrix such that psi[i][u] = psi_i(u)
//        x -- a vector where the optimal solution will be stored
//
// OUTPUT: value of the optimal solution
//
// To use this code, create a GraphCutInference object, and call the
// DoInference() method.  To perform maximization instead of
      minimization,
// ensure that #define MAXIMIZATION is enabled.
typedef vector<int> VI;
typedef vector<VI> VVI;
typedef vector<VVI> VVVI;
typedef vector<VVVI> VVVVI;

const int INF = 1000000000;

// comment out following line for minimization
#define MAXIMIZATION

struct GraphCutInference {
  int N;
  VVI cap, flow;
  VI reached;

  int Augment(int s, int t, int a) {
    reached[s] = 1;
    if (s == t) return a;
    for (int k = 0; k < N; k++) {
      if (reached[k]) continue;
      if (int aa = min(a, cap[s][k] - flow[s][k])) {
        if (int b = Augment(k, t, aa)) {
          flow[s][k] += b;
          flow[k][s] -= b;
          return b;
        }
      }
    }
    return 0;
  }

  int GetMaxFlow(int s, int t) {
    N = cap.size();
    flow = VVI(N, VI(N));
    reached = VI(N);

    int totflow = 0;
    while (int amt = Augment(s, t, INF)) {
      totflow += amt;
      fill(reached.begin(), reached.end(), 0);
    }
    return totflow;
  }

  int DoInference(const VVVVI &phi, const VVI &psi, VI &x) {
    int M = phi.size();
    cap = VVI(M+2, VI(M+2));
    VI b(M);
    int c = 0;

    for (int i = 0; i < M; i++) {
      b[i] += psi[i][1] - psi[i][0];
      c += psi[i][0];
      for (int j = 0; j < i; j++)
        b[i] += phi[i][j][1][1] - phi[i][j][0][1];
      for (int j = i+1; j < M; j++) {
        cap[i][j] = phi[i][j][0][1] + phi[i][j][1][0] - phi[i][j
              ][0][0] - phi[i][j][1][1];
        b[i] += phi[i][j][1][0] - phi[i][j][0][0];
        c += phi[i][j][0][0];
      }
    }

#ifdef MAXIMIZATION
    for (int i = 0; i < M; i++) {
      for (int j = i+1; j < M; j++)
        cap[i][j] *= -1;
      b[i] *= -1;
    }
    c *= -1;
#endif

    for (int i = 0; i < M; i++) {
      if (b[i] >= 0) {
        cap[M][i] = b[i];
      } else {
        cap[i][M+1] = -b[i];
        c += b[i];
      }
    }

    int score = GetMaxFlow(M, M+1);
    fill(reached.begin(), reached.end(), 0);
    Augment(M, M+1, INF);
    x = VI(M);
    for (int i = 0; i < M; i++) x[i] = reached[i] ? 0 : 1;
    score += c;
#ifdef MAXIMIZATION
    score *= -1;
#endif

    return score;
  }

};

int main() {

  // solver for "Cat vs. Dog" from NWERC 2008

  int numcases;
  cin >> numcases;
  for (int caseno = 0; caseno < numcases; caseno++) {
    int c, d, v;
    cin >> c >> d >> v;

    VVVVI phi(c+d, VVVI(c+d, VVI(2, VI(2))));
    VVI psi(c+d, VI(2));
    for (int i = 0; i < v; i++) {
      char p, q;
      int u, v;
      cin >> p >> u >> q >> v;
      u--; v--;
      if (p == 'C') {
        phi[u][c+v][0][0]++;
        phi[c+v][u][0][0]++;
      } else {
        phi[v][c+u][1][1]++;
        phi[c+u][v][1][1]++;
      }
    }

    GraphCutInference graph;
    VI x;
    cout << graph.DoInference(phi, psi, x) << endl;
  }

  return 0;
}
```

1.9 General Matching

```
// Unweighted general matching.
// Vertices are numbered from 1 to V.
// G is an adjlist.
// G[x][0] contains the number of neighbours of x.
// The neigbours are then stored in G[x][1] .. G[x][G[x][0]].
// Mate[x] will contain the matching node for x.
// V and E are the number of edges and vertices.
// Slow Version (2x on random graphs) of Gabow's implementation
// of Edmonds' algorithm (O(V^3)).
const int MAXV = 250;
int G[MAXV][MAXV];
int VLabel[MAXV];
int  Queue[MAXV];
int   Mate[MAXV];
int   Save[MAXV];
int   Used[MAXV];
int    Up, Down;
int           V;

void ReMatch(int x, int y)
{
  int m = Mate[x]; Mate[x] = y;
  if (Mate[m] == x)
    {
      if (VLabel[x] <= V)
        {
          Mate[m] = VLabel[x];
          ReMatch(VLabel[x], m);
        }
      else
        {
```

```
        int a = 1 + (VLabel[x] - V - 1) / V;
        int b = 1 + (VLabel[x] - V - 1) % V;
        ReMatch(a, b); ReMatch(b, a);
      }
    }
}

void Traverse(int x)
{
  for (int i = 1; i <= V; i++) Save[i] = Mate[i];
  ReMatch(x, x);
  for (int i = 1; i <= V; i++)
    {
      if (Mate[i] != Save[i]) Used[i]++;
      Mate[i] = Save[i];
    }
}

void ReLabel(int x, int y)
{
  for (int i = 1; i <= V; i++) Used[i] = 0;
  Traverse(x); Traverse(y);
  for (int i = 1; i <= V; i++)
    {
      if (Used[i] == 1 && VLabel[i] < 0)
        {
          VLabel[i] = V + x + (y - 1) * V;
          Queue[Up++] = i;
        }
    }
}

// Call this after constructing G
void Solve()
{
  for (int i = 1; i <= V; i++)
    if (Mate[i] == 0)
      {
        for (int j = 1; j <= V; j++) VLabel[j] = -1;
        VLabel[i] = 0; Down = 1; Up = 1; Queue[Up++] = i;
        while (Down != Up)
          {
            int x = Queue[Down++];
            for (int p = 1; p <= G[x][0]; p++)
              {
                int y = G[x][p];
                if (Mate[y] == 0 && i != y)
                  {
                    Mate[y] = x; ReMatch(x, y);
                    Down = Up; break;
                  }
                if (VLabel[y] >= 0)
                  {
                    ReLabel(x, y);
                    continue;
                  }
                if (VLabel[Mate[y]] < 0)
                  {
                    VLabel[Mate[y]] = x;
                    Queue[Up++] = Mate[y];
                  }
              }
          }
      }
}

// Call this after Solve(). Returns number of edges in matching (half
      the number of matched vertices)
int Size()
{
  int Count = 0;
  for (int i = 1; i <= V; i++)
    if (Mate[i] > i) Count++;
  return Count;
}
```

1.10 Stable Marriage Problem

```
// Gale-Shapley algorithm for the stable marriage problem.
// madj[i][j] is the jth highest ranked woman for man i.
// fpref[i][j] is the rank woman i assigns to man j.
// Returns a pair of vectors (mpart, fpart), where mpart[i] gives the
      partner of man i, and fpart is analogous
pair<vector<int>, vector<int> > stable_marriage(vector<vector<int> >&
      madj, vector<vector<int> >& fpref) {
        int n = madj.size();
        vector<int> mpart(n, -1), fpart(n, -1);
        vector<int> midx(n);
        queue<int> mfree;
        for (int i = 0; i < n; i++) {
                mfree.push(i);
        }
        while (!mfree.empty()) {
                int m = mfree.front(); mfree.pop();
                int f = madj[m][midx[m]++];
                if (fpart[f] == -1) {
                        mpart[m] = f; fpart[f] = m;
                } else if (fpref[f][m] < fpref[f][fpart[f]]) {
                        mpart[fpart[f]] = -1; mfree.push(fpart[f]);
                        mpart[m] = f; fpart[f] = m;
                } else {
                        mfree.push(m);
                }
        }
        return make_pair(mpart, fpart);
}
```

2 Geometry

2.1 Convex hull

```
// Compute the 2D convex hull of a set of points using the monotone
      chain
// algorithm.  Eliminate redundant points from the hull if
      REMOVE_REDUNDANT is
// #defined.
//
// Running time: O(n log n)
//
//   INPUT:   a vector of input points, unordered.
//   OUTPUT:  a vector of points in the convex hull, counterclockwise,
       starting
//            with bottommost/leftmost point

#define REMOVE_REDUNDANT

typedef double T;
const T EPS = 1e-7;
struct PT {
  T x, y;
  PT() {}
  PT(T x, T y) : x(x), y(y) {}
  bool operator<(const PT &rhs) const { return make_pair(y,x) <
      make_pair(rhs.y,rhs.x); }
  bool operator==(const PT &rhs) const { return make_pair(y,x) ==
      make_pair(rhs.y,rhs.x); }
};

T cross(PT p, PT q) { return p.x*q.y-p.y*q.x; }
T area2(PT a, PT b, PT c) { return cross(a,b) + cross(b,c) + cross(c,a
      ); }

#ifdef REMOVE_REDUNDANT
bool between(const PT &a, const PT &b, const PT &c) {
  return (fabs(area2(a,b,c)) < EPS && (a.x-b.x)*(c.x-b.x) <= 0 && (a.y
      -b.y)*(c.y-b.y) <= 0);
}
#endif

void ConvexHull(vector<PT> &pts) {
  sort(pts.begin(), pts.end());
  pts.erase(unique(pts.begin(), pts.end()), pts.end());
  vector<PT> up, dn;
  for (int i = 0; i < pts.size(); i++) {
    while (up.size() > 1 && area2(up[up.size()-2], up.back(), pts[i])
        >= 0) up.pop_back();
    while (dn.size() > 1 && area2(dn[dn.size()-2], dn.back(), pts[i])
        <= 0) dn.pop_back();
    up.push_back(pts[i]);
    dn.push_back(pts[i]);
  }
  pts = dn;
  for (int i = (int) up.size() - 2; i >= 1; i--) pts.push_back(up[i]);

#ifdef REMOVE_REDUNDANT
  if (pts.size() <= 2) return;
  dn.clear();
  dn.push_back(pts[0]);
  dn.push_back(pts[1]);
  for (int i = 2; i < pts.size(); i++) {
    if (between(dn[dn.size()-2], dn[dn.size()-1], pts[i])) dn.pop_back
        ();
    dn.push_back(pts[i]);
  }
  if (dn.size() >= 3 && between(dn.back(), dn[0], dn[1])) {
    dn[0] = dn.back();
    dn.pop_back();
  }
  pts = dn;
#endif
}

// BEGIN CUT
// The following code solves SPOJ problem #26: Build the Fence (BSHEEP
      )

int main() {
  int t;
  scanf("%d", &t);
  for (int caseno = 0; caseno < t; caseno++) {
    int n;
    scanf("%d", &n);
    vector<PT> v(n);
    for (int i = 0; i < n; i++) scanf("%lf%lf", &v[i].x, &v[i].y);
    vector<PT> h(v);
    map<PT,int> index;
    for (int i = n-1; i >= 0; i--) index[v[i]] = i+1;
    ConvexHull(h);

    double len = 0;
    for (int i = 0; i < h.size(); i++) {
      double dx = h[i].x - h[(i+1)%h.size()].x;
      double dy = h[i].y - h[(i+1)%h.size()].y;
      len += sqrt(dx*dx+dy*dy);
    }

    if (caseno > 0) printf("\n");
    printf("%.2f\n", len);
    for (int i = 0; i < h.size(); i++) {
      if (i > 0) printf(" ");
      printf("%d", index[h[i]]);
    }
    printf("\n");
  }
}

// END CUT
```

2.2 Miscellaneous geometry

```
// C++ routines for computational geometry.
double INF = 1e100;
double EPS = 1e-12;

struct PT {
  double x, y;
  PT() {}
  PT(double x, double y) : x(x), y(y) {}
  PT(const PT &p) : x(p.x), y(p.y)    {}
  PT operator + (const PT &p)  const { return PT(x+p.x, y+p.y); }
  PT operator - (const PT &p)  const { return PT(x-p.x, y-p.y); }
  PT operator * (double c)     const { return PT(x*c,   y*c  ); }
  PT operator / (double c)     const { return PT(x/c,   y/c  ); }
};

double dot(PT p, PT q)     { return p.x*q.x+p.y*q.y; }
double dist2(PT p, PT q)   { return dot(p-q,p-q); }
double cross(PT p, PT q)   { return p.x*q.y-p.y*q.x; }
ostream &operator<<(ostream &os, const PT &p) {
  return os << "(" << p.x << "," << p.y << ")";
}

// rotate a point CCW or CW around the origin
PT RotateCCW90(PT p)   { return PT(-p.y,p.x); }
PT RotateCW90(PT p)    { return PT(p.y,-p.x); }
PT RotateCCW(PT p, double t) {
  return PT(p.x*cos(t)-p.y*sin(t), p.x*sin(t)+p.y*cos(t));
}

// project point c onto line through a and b
// assuming a != b
```

```
PT ProjectPointLine(PT a, PT b, PT c) {
  return a + (b-a)*dot(c-a, b-a)/dot(b-a, b-a);
}

// project point c onto line segment through a and b
PT ProjectPointSegment(PT a, PT b, PT c) {
  double r = dot(b-a,b-a);
  if (fabs(r) < EPS) return a;
  r = dot(c-a, b-a)/r;
  if (r < 0) return a;
  if (r > 1) return b;
  return a + (b-a)*r;
}

// compute distance from c to segment between a and b
double DistancePointSegment(PT a, PT b, PT c) {
  return sqrt(dist2(c, ProjectPointSegment(a, b, c)));
}

// compute distance between point (x,y,z) and plane ax+by+cz=d
double DistancePointPlane(double x, double y, double z,
                          double a, double b, double c, double d)
{
  return fabs(a*x+b*y+c*z-d)/sqrt(a*a+b*b+c*c);
}

// determine if lines from a to b and c to d are parallel or collinear
bool LinesParallel(PT a, PT b, PT c, PT d) {
  return fabs(cross(b-a, c-d)) < EPS;
}

bool LinesCollinear(PT a, PT b, PT c, PT d) {
  return LinesParallel(a, b, c, d)
      && fabs(cross(a-b, a-c)) < EPS
      && fabs(cross(c-d, c-a)) < EPS;
}

// determine if line segment from a to b intersects with
// line segment from c to d
bool SegmentsIntersect(PT a, PT b, PT c, PT d) {
  if (LinesCollinear(a, b, c, d)) {
    if (dist2(a, c) < EPS || dist2(a, d) < EPS ||
      dist2(b, c) < EPS || dist2(b, d) < EPS) return true;
    if (dot(c-a, c-b) > 0 && dot(d-a, d-b) > 0 && dot(c-b, d-b) > 0)
      return false;
    return true;
  }
  if (cross(d-a, b-a) * cross(c-a, b-a) > 0) return false;
  if (cross(a-c, d-c) * cross(b-c, d-c) > 0) return false;
  return true;
}

// compute intersection of line passing through a and b
// with line passing through c and d, assuming that unique
// intersection exists; for segment intersection, check if
// segments intersect first
PT ComputeLineIntersection(PT a, PT b, PT c, PT d) {
  b=b-a; d=c-d; c=c-a;
  assert(dot(b, b) > EPS && dot(d, d) > EPS);
  return a + b*cross(c, d)/cross(b, d);
}

// compute center of circle given three points
PT ComputeCircleCenter(PT a, PT b, PT c) {
  b=(a+b)/2;
  c=(a+c)/2;
  return ComputeLineIntersection(b, b+RotateCW90(a-b), c, c+RotateCW90
       (a-c));
}

// determine if point is in a possibly non-convex polygon (by William
// Randolph Franklin); returns 1 for strictly interior points, 0 for
// strictly exterior points, and 0 or 1 for the remaining points.
// Note that it is possible to convert this into an *exact* test using
// integer arithmetic by taking care of the division appropriately
// (making sure to deal with signs properly) and then by writing exact
// tests for checking point on polygon boundary
bool PointInPolygon(const vector<PT> &p, PT q) {
  bool c = 0;
  for (int i = 0; i < p.size(); i++){
    int j = (i+1)%p.size();
    if ((p[i].y <= q.y && q.y < p[j].y ||
      p[j].y <= q.y && q.y < p[i].y) &&
      q.x < p[i].x + (p[j].x - p[i].x) * (q.y - p[i].y) / (p[j].y - p[
         i].y))
      c = !c;
  }
  return c;
}

// determine if point is on the boundary of a polygon
bool PointOnPolygon(const vector<PT> &p, PT q) {
  for (int i = 0; i < p.size(); i++)
    if (dist2(ProjectPointSegment(p[i], p[(i+1)%p.size()], q), q) <
         EPS)
      return true;
    return false;
}

// compute intersection of line through points a and b with
// circle centered at c with radius r > 0
vector<PT> CircleLineIntersection(PT a, PT b, PT c, double r) {
  vector<PT> ret;
  b = b-a;
  a = a-c;
  double A = dot(b, b);
  double B = dot(a, b);
  double C = dot(a, a) - r*r;
  double D = B*B - A*C;
  if (D < -EPS) return ret;
  ret.push_back(c+a+b*(-B+sqrt(D+EPS))/A);
  if (D > EPS)
    ret.push_back(c+a+b*(-B-sqrt(D))/A);
  return ret;
}

// compute intersection of circle centered at a with radius r
// with circle centered at b with radius R
vector<PT> CircleCircleIntersection(PT a, PT b, double r, double R) {
  vector<PT> ret;
  double d = sqrt(dist2(a, b));
  if (d > r+R || d+min(r, R) < max(r, R)) return ret;
  double x = (d*d-R*R+r*r)/(2*d);
  double y = sqrt(r*r-x*x);
  PT v = (b-a)/d;
  ret.push_back(a+v*x + RotateCCW90(v)*y);
  if (y > 0)
    ret.push_back(a+v*x - RotateCCW90(v)*y);
  return ret;
}

// This code computes the area or centroid of a (possibly nonconvex)
// polygon, assuming that the coordinates are listed in a clockwise or
// counterclockwise fashion.  Note that the centroid is often known as
// the "center of gravity" or "center of mass".
double ComputeSignedArea(const vector<PT> &p) {
  double area = 0;
  for(int i = 0; i < p.size(); i++) {
    int j = (i+1) % p.size();
    area += p[i].x*p[j].y - p[j].x*p[i].y;
  }
  return area / 2.0;
}

double ComputeArea(const vector<PT> &p) {
  return fabs(ComputeSignedArea(p));
}

PT ComputeCentroid(const vector<PT> &p) {
  PT c(0,0);
  double scale = 6.0 * ComputeSignedArea(p);
  for (int i = 0; i < p.size(); i++){
    int j = (i+1) % p.size();
    c = c + (p[i]+p[j])*(p[i].x*p[j].y - p[j].x*p[i].y);
  }
  return c / scale;
}

// tests whether or not a given polygon (in CW or CCW order) is simple
bool IsSimple(const vector<PT> &p) {
  for (int i = 0; i < p.size(); i++) {
    for (int k = i+1; k < p.size(); k++) {
      int j = (i+1) % p.size();
      int l = (k+1) % p.size();
      if (i == l || j == k) continue;
      if (SegmentsIntersect(p[i], p[j], p[k], p[l]))
        return false;
    }
  }
  return true;
}

int main() {

  // expected: (-5,2)
  cerr << RotateCCW90(PT(2,5)) << endl;

  // expected: (5,-2)
  cerr << RotateCW90(PT(2,5)) << endl;

  // expected: (-5,2)
  cerr << RotateCCW(PT(2,5),M_PI/2) << endl;

  // expected: (5,2)
  cerr << ProjectPointLine(PT(-5,-2), PT(10,4), PT(3,7)) << endl;

  // expected: (5,2) (7.5,3) (2.5,1)
  cerr << ProjectPointSegment(PT(-5,-2), PT(10,4), PT(3,7)) << " "
       << ProjectPointSegment(PT(7.5,3), PT(10,4), PT(3,7)) << " "
       << ProjectPointSegment(PT(-5,-2), PT(2.5,1), PT(3,7)) << endl;

  // expected: 6.78903
  cerr << DistancePointPlane(4,-4,3,2,-2,5,-8) << endl;

  // expected: 1 0 1
  cerr << LinesParallel(PT(1,1), PT(3,5), PT(2,1), PT(4,5)) << " "
       << LinesParallel(PT(1,1), PT(3,5), PT(2,0), PT(4,5)) << " "
       << LinesParallel(PT(1,1), PT(3,5), PT(5,9), PT(7,13)) << endl;

  // expected: 0 0 1
  cerr << LinesCollinear(PT(1,1), PT(3,5), PT(2,1), PT(4,5)) << " "
       << LinesCollinear(PT(1,1), PT(3,5), PT(2,0), PT(4,5)) << " "
       << LinesCollinear(PT(1,1), PT(3,5), PT(5,9), PT(7,13)) << endl;

  // expected: 1 1 1 0
  cerr << SegmentsIntersect(PT(0,0), PT(2,4), PT(3,1), PT(-1,3)) << "
       "
       << SegmentsIntersect(PT(0,0), PT(2,4), PT(4,3), PT(0,5)) << " "
       << SegmentsIntersect(PT(0,0), PT(2,4), PT(2,-1), PT(-2,1)) << "
            "
       << SegmentsIntersect(PT(0,0), PT(2,4), PT(5,5), PT(1,7)) <<
            endl;

  // expected: (1,2)
  cerr << ComputeLineIntersection(PT(0,0), PT(2,4), PT(3,1), PT(-1,3))
         << endl;

  // expected: (1,1)
  cerr << ComputeCircleCenter(PT(-3,4), PT(6,1), PT(4,5)) << endl;

  vector<PT> v;
  v.push_back(PT(0,0));
  v.push_back(PT(5,0));
  v.push_back(PT(5,5));
  v.push_back(PT(0,5));

  // expected: 1 1 1 0 0
  cerr << PointInPolygon(v, PT(2,2)) << " "
       << PointInPolygon(v, PT(2,0)) << " "
       << PointInPolygon(v, PT(0,2)) << " "
       << PointInPolygon(v, PT(5,2)) << " "
       << PointInPolygon(v, PT(2,5)) << endl;

  // expected: 0 1 1 1 1
  cerr << PointOnPolygon(v, PT(2,2)) << " "
       << PointOnPolygon(v, PT(2,0)) << " "
       << PointOnPolygon(v, PT(0,2)) << " "
       << PointOnPolygon(v, PT(5,2)) << " "
       << PointOnPolygon(v, PT(2,5)) << endl;

  // expected: (1,6)
  //           (5,4) (4,5)
  //           blank line
  //           (4,5) (5,4)
  //           blank line
  //           (4,5) (5,4)
  vector<PT> u = CircleLineIntersection(PT(0,6), PT(2,6), PT(1,1), 5);
  for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl
       ;
  u = CircleLineIntersection(PT(0,9), PT(9,0), PT(1,1), 5);
  for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl
       ;
  u = CircleCircleIntersection(PT(1,1), PT(10,10), 5, 5);
  for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl
       ;
  u = CircleCircleIntersection(PT(1,1), PT(8,8), 5, 5);
  for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl
       ;
  u = CircleCircleIntersection(PT(1,1), PT(4.5,4.5), 10, sqrt(2.0)
       /2.0);
  for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl
       ;
  u = CircleCircleIntersection(PT(1,1), PT(4.5,4.5), 5, sqrt(2.0)/2.0)
       ;
  for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl
       ;
```

```
    // area should be 5.0
    // centroid should be (1.1666666, 1.166666)
    PT pa[] = { PT(0,0), PT(5,0), PT(1,1), PT(0,5) };
    vector<PT> p(pa, pa+4);
    PT c = ComputeCentroid(p);
    cerr << "Area: " << ComputeArea(p) << endl;
    cerr << "Centroid: " << c << endl;

    return 0;
}
```

2.3 Area of Polygon

```
double area(int x[], int y[], int n) {
    double a = 0;
    for(int i = 0; i < n - 1; i++) {
        a = a + 1.0*x[i]*y[i+1] - 1.0*x[i+1]*y[i];
    }
    a = a + 1.0*x[n-1]*y[0] - 1.0*x[0]*y[n-1];
    return a
}
```

2.4 Java geometry

```
// In this example, we read an input file containing three lines, each
// containing an even number of doubles, separated by commas.  The
      first two
// lines represent the coordinates of two polygons, given in
      counterclockwise
// (or clockwise) order, which we will call "A" and "B".  The last
      line
// contains a list of points, p[1], p[2], ...
//
// Our goal is to determine:
//   (1) whether B - A is a single closed shape (as opposed to
      multiple shapes)
//   (2) the area of B - A
//   (3) whether each p[i] is in the interior of B - A
//
// INPUT:
//   0 0 10 0 0 10
//   0 0 10 10 10 0
//   8 6
//   5 1
//
// OUTPUT:
//   The area is singular.
//   The area is 25.0
//   Point belongs to the area.
//   Point does not belong to the area.

import java.util.*;
import java.awt.geom.*;
import java.io.*;

public class JavaGeometry {

    // make an array of doubles from a string
    static double[] readPoints(String s) {
        String[] arr = s.trim().split("\\s++");
        double[] ret = new double[arr.length];
        for (int i = 0; i < arr.length; i++) ret[i] = Double.
              parseDouble(arr[i]);
        return ret;
    }

    // make an Area object from the coordinates of a polygon
    static Area makeArea(double[] pts) {
        Path2D.Double p = new Path2D.Double();
        p.moveTo(pts[0], pts[1]);
        for (int i = 2; i < pts.length; i += 2) p.lineTo(pts[i], pts[i
              +1]);
        p.closePath();
        return new Area(p);
    }

    // compute area of polygon
    static double computePolygonArea(ArrayList<Point2D.Double> points)
          {
        Point2D.Double[] pts = points.toArray(new Point2D.Double[
              points.size()]);
        double area = 0;
        for (int i = 0; i < pts.length; i++){
            int j = (i+1) % pts.length;
            area += pts[i].x * pts[j].y - pts[j].x * pts[i].y;
        }
        return Math.abs(area)/2;
    }

    // compute the area of an Area object containing several disjoint
          polygons
    static double computeArea(Area area) {
        double totArea = 0;
        PathIterator iter = area.getPathIterator(null);
        ArrayList<Point2D.Double> points = new ArrayList<Point2D.
              Double>();

        while (!iter.isDone()) {
            double[] buffer = new double[6];
            switch (iter.currentSegment(buffer)) {
            case PathIterator.SEG_MOVETO:
            case PathIterator.SEG_LINETO:
                points.add(new Point2D.Double(buffer[0], buffer[1]));
                break;
            case PathIterator.SEG_CLOSE:
                totArea += computePolygonArea(points);
                points.clear();
                break;
            }
            iter.next();
        }
        return totArea;
    }

    // notice that the main() throws an Exception -- necessary to
    // avoid wrapping the Scanner object for file reading in a
    // try { ... } catch block.
    public static void main(String args[]) throws Exception {

        Scanner scanner = new Scanner(new File("input.txt"));
        // also,
        //   Scanner scanner = new Scanner (System.in);

        double[] pointsA = readPoints(scanner.nextLine());
        double[] pointsB = readPoints(scanner.nextLine());
        Area areaA = makeArea(pointsA);
        Area areaB = makeArea(pointsB);
        areaB.subtract(areaA);
        // also,
        //   areaB.exclusiveOr (areaA);
        //   areaB.add (areaA);
        //   areaB.intersect (areaA);

        // (1) determine whether B - A is a single closed shape (as
        //     opposed to multiple shapes)
        boolean isSingle = areaB.isSingular();
        // also,
        //   areaB.isEmpty();

        if (isSingle)
            System.out.println("The area is singular.");
        else
            System.out.println("The area is not singular.");

        // (2) compute the area of B - A
        System.out.println("The area is " + computeArea(areaB) + ".");

        // (3) determine whether each p[i] is in the interior of B - A
        while (scanner.hasNextDouble()) {
            double x = scanner.nextDouble();
            assert(scanner.hasNextDouble());
            double y = scanner.nextDouble();

            if (areaB.contains(x,y)) {
                System.out.println ("Point belongs to the area.");
            } else {
                System.out.println ("Point does not belong to the area
                      .");
            }
        }

        // Finally, some useful things we didn't use in this example:
        //
        //   Ellipse2D.Double ellipse = new Ellipse2D.Double (double x
              , double y,
        //                                                    double w
              , double h);
        //
        //     creates an ellipse inscribed in box with bottom-left
              corner (x,y)
        //     and upper-right corner (x+y,w+h)
        //
        //   Rectangle2D.Double rect = new Rectangle2D.Double (double
              x, double y,
        //                                                     double
              w, double h);
        //
        //     creates a box with bottom-left corner (x,y) and upper-
              right
        //     corner (x+y,w+h)
        //
        // Each of these can be embedded in an Area object (e.g., new
              Area (rect)).

    }
}
```

2.5 3D geometry

```
public class Geom3D {
  // distance from point (x, y, z) to plane aX + bY + cZ + d = 0
  public static double ptPlaneDist(double x, double y, double z,
      double a, double b, double c, double d) {
    return Math.abs(a*x + b*y + c*z + d) / Math.sqrt(a*a + b*b + c*c);
  }

  // distance between parallel planes aX + bY + cZ + d1 = 0 and
  // aX + bY + cZ + d2 = 0
  public static double planePlaneDist(double a, double b, double c,
      double d1, double d2) {
    return Math.abs(d1 - d2) / Math.sqrt(a*a + b*b + c*c);
  }

  // distance from point (px, py, pz) to line (x1, y1, z1)-(x2, y2, z2
      )
  // (or ray, or segment; in the case of the ray, the endpoint is the
  // first point)
  public static final int LINE = 0;
  public static final int SEGMENT = 1;
  public static final int RAY = 2;
  public static double ptLineDistSq(double x1, double y1, double z1,
      double x2, double y2, double z2, double px, double py, double pz
      ,
      int type) {
    double pd2 = (x1-x2)*(x1-x2) + (y1-y2)*(y1-y2) + (z1-z2)*(z1-z2);

    double x, y, z;
    if (pd2 == 0) {
      x = x1;
      y = y1;
      z = z1;
    } else {
      double u = ((px-x1)*(x2-x1) + (py-y1)*(y2-y1) + (pz-z1)*(z2-z1))
           / pd2;
      x = x1 + u * (x2 - x1);
      y = y1 + u * (y2 - y1);
      z = z1 + u * (z2 - z1);
      if (type != LINE && u < 0) {
        x = x1;
        y = y1;
        z = z1;
      }
      if (type == SEGMENT && u > 1.0) {
        x = x2;
        y = y2;
        z = z2;
      }
    }

    return (x-px)*(x-px) + (y-py)*(y-py) + (z-pz)*(z-pz);
  }

  public static double ptLineDist(double x1, double y1, double z1,
      double x2, double y2, double z2, double px, double py, double pz
      ,
      int type) {
    return Math.sqrt(ptLineDistSq(x1, y1, z1, x2, y2, z2, px, py, pz,
        type));
  }
}
```

2.6 Slow Delaunay triangulation

```
// Slow but simple Delaunay triangulation. Does not handle
// degenerate cases (from O'Rourke, Computational Geometry in C)
//
// Running time: O(n^4)
//
// INPUT:    x[] = x-coordinates
//           y[] = y-coordinates
//
// OUTPUT:   triples = a vector containing m triples of indices
//                     corresponding to triangle vertices
typedef double T;

struct triple {
    int i, j, k;
    triple() {}
    triple(int i, int j, int k) : i(i), j(j), k(k) {}
};

vector<triple> delaunayTriangulation(vector<T>& x, vector<T>& y) {
        int n = x.size();
        vector<T> z(n);
        vector<triple> ret;

        for (int i = 0; i < n; i++)
            z[i] = x[i] * x[i] + y[i] * y[i];

        for (int i = 0; i < n-2; i++) {
            for (int j = i+1; j < n; j++) {
                for (int k = i+1; k < n; k++) {
                    if (j == k) continue;
                    double xn = (y[j]-y[i])*(z[k]-z[i]) - (y[k]-y[i])
                        *(z[j]-z[i]);
                    double yn = (x[k]-x[i])*(z[j]-z[i]) - (x[j]-x[i])
                        *(z[k]-z[i]);
                    double zn = (x[j]-x[i])*(y[k]-y[i]) - (x[k]-x[i])
                        *(y[j]-y[i]);
                    bool flag = zn < 0;
                    for (int m = 0; flag && m < n; m++)
                        flag = flag && ((x[m]-x[i])*xn +
                                        (y[m]-y[i])*yn +
                                        (z[m]-z[i])*zn <= 0);
                    if (flag) ret.push_back(triple(i, j, k));
                }
            }
        }
        return ret;
}

int main()
{
    T xs[]={0, 0, 1, 0.9};
    T ys[]={0, 1, 0, 0.9};
    vector<T> x(&xs[0], &xs[4]), y(&ys[0], &ys[4]);
    vector<triple> tri = delaunayTriangulation(x, y);

    //expected: 0 1 3
    //          0 3 2

    int i;
    for(i = 0; i < tri.size(); i++)
        printf("%d %d %d\n", tri[i].i, tri[i].j, tri[i].k);
    return 0;
}
```

2.7 Closest Pair

```
// Source: e-maxx.ru
#define upd_ans(x, y) {}
#define MAXN 100
double mindist = 1e20; // will be the result
void rec(int l, int r, Point a[]) {
    if (r - l <= 3) {
        for (int i=l; i<=r; ++i)
            for (int j=i+1; j<=r; ++j)
                    upd_ans(a[i], a[j]);
        sort(a+l, a+r+1); // compare by y
        return;
    }
    int m = (l + r) >> 1;
    int midx = a[m].x;
    rec(l, m, a), rec(m+1, r, a);
    static Point t[MAXN];
    merge(a+l, a+m+1, a+m+1, a+r+1, t); // compare by y
    copy(t, t+r-l+1, a+l);

    int tsz = 0;
    for (int i=l; i<=r; ++i)
        if (fabs(a[i].x - midx) < mindist) {
            for (int j=tsz-1; j>=0 && a[i].y - t[j].y < mindist; --j)
                upd_ans(a[i], t[j]);
            t[tsz++] = a[i];
        }
}
```

2.8 Rotating Caliphers

```
// Rotating calipers
double convex_diameter(Polygon pt) {
    const int n = pt.size();
    int is = 0, js = 0;
    for (int i = 1; i < n; ++i) {
        if (pt[i].y > pt[is].y) is = i;
        if (pt[i].y < pt[js].y) js = i;
    }
    double maxd = (pt[is]-pt[js]).norm();
    int i, maxi, j, maxj;
    i = maxi = is;
    j = maxj = js;
    do {
        int jj = j+1; if (jj == n) jj = 0;
        if ((pt[i] - pt[jj]).norm() > (pt[i] - pt[j]).norm()) j = (j
            +1) % n;
        else i = (i+1) % n;
        if ((pt[i]-pt[j]).norm() > maxd) {
            maxd = (pt[i]-pt[j]).norm();
            maxi = i; maxj = j;
        }
    } while (i != is || j != js);
    return maxd; /* farthest pair is (maxi, maxj). */
}
```

3 Numerical algorithms

3.1 Number theory (modular, Chinese remainder, linear Diophantine)

```
// This is a collection of useful code for solving problems that
// involve modular linear equations.  Note that all of the
// algorithms described here work on nonnegative integers.
typedef vector<int> VI;
typedef pair<int, int> PII;

// return a % b (positive value)
int mod(int a, int b) {
        return ((a%b) + b) % b;
}

// computes gcd(a,b)
int gcd(int a, int b) {
        while (b) { int t = a%b; a = b; b = t; }
        return a;
}

// computes lcm(a,b)
int lcm(int a, int b) {
        return a / gcd(a, b)*b;
}

// (a^b) mod m via successive squaring
int powermod(int a, int b, int m)
{
        int ret = 1;
        while (b)
        {
                if (b & 1) ret = mod(ret*a, m);
                a = mod(a*a, m);
                b >>= 1;
        }
        return ret;
}

// returns g = gcd(a, b); finds x, y such that d = ax + by
int extended_euclid(int a, int b, int &x, int &y) {
        int xx = y = 0;
        int yy = x = 1;
        while (b) {
                int q = a / b;
                int t = b; b = a%b; a = t;
                t = xx; xx = x - q*xx; x = t;
                t = yy; yy = y - q*yy; y = t;
        }
        return a;
}

// finds all solutions to ax = b (mod n)
VI modular_linear_equation_solver(int a, int b, int n) {
        int x, y;
        VI ret;
        int g = extended_euclid(a, n, x, y);
        if (!(b%g)) {
                x = mod(x*(b / g), n);
                for (int i = 0; i < g; i++)
                        ret.push_back(mod(x + i*(n / g), n));
        }
        return ret;
}

// computes b such that ab = 1 (mod n), returns -1 on failure
int mod_inverse(int a, int n) {
        int x, y;
        int g = extended_euclid(a, n, x, y);
        if (g > 1) return -1;
        return mod(x, n);
}

// Chinese remainder theorem (special case): find z such that
// z % m1 = r1, z % m2 = r2.  Here, z is unique modulo M = lcm(m1, m2)
    .
// Return (z, M).  On failure, M = -1.
PII chinese_remainder_theorem(int m1, int r1, int m2, int r2) {
        int s, t;
        int g = extended_euclid(m1, m2, s, t);
        if (r1%g != r2%g) return make_pair(0, -1);
        return make_pair(mod(s*r2*m1 + t*r1*m2, m1*m2) / g, m1*m2 / g)
            ;
}

// Chinese remainder theorem: find z such that
// z % m[i] = r[i] for all i.  Note that the solution is
// unique modulo M = lcm_i (m[i]).  Return (z, M). On
// failure, M = -1. Note that we do not require the a[i]'s
// to be relatively prime.
PII chinese_remainder_theorem(const VI &m, const VI &r) {
        PII ret = make_pair(r[0], m[0]);
        for (int i = 1; i < m.size(); i++) {
                ret = chinese_remainder_theorem(ret.second, ret.first,
                        m[i], r[i]);
                if (ret.second == -1) break;
        }
        return ret;
}

// computes x and y such that ax + by = c
// returns whether the solution exists
bool linear_diophantine(int a, int b, int c, int &x, int &y) {
        if (!a && !b)
        {
                if (c) return false;
                x = 0; y = 0;
                return true;
        }
        if (!a)
        {
                if (c % b) return false;
                x = 0; y = c / b;
                return true;
        }
        if (!b)
        {
                if (c % a) return false;
                x = c / a; y = 0;
                return true;
        }
```

```
    int g = gcd(a, b);
    if (c % g) return false;
    x = c / g * mod_inverse(a / g, b / g);
    y = (c - a*x) / b;
    return true;
}

int main() {
    // expected: 2
    cout << gcd(14, 30) << endl;

    // expected: 2 -2 1
    int x, y;
    int g = extended_euclid(14, 30, x, y);
    cout << g << " " << x << " " << y << endl;

    // expected: 95 451
    VI sols = modular_linear_equation_solver(14, 30, 100);
    for (int i = 0; i < sols.size(); i++) cout << sols[i] << " ";
    cout << endl;

    // expected: 8
    cout << mod_inverse(8, 9) << endl;

    // expected: 23 105
    //           11 12
    PII ret = chinese_remainder_theorem(VI({ 3, 5, 7 }), VI({ 2,
         3, 2 }));
    cout << ret.first << " " << ret.second << endl;
    ret = chinese_remainder_theorem(VI({ 4, 6 }), VI({ 3, 5 }));
    cout << ret.first << " " << ret.second << endl;

    // expected: 5 -15
    if (!linear_diophantine(7, 2, 5, x, y)) cout << "ERROR" <<
         endl;
    cout << x << " " << y << endl;
    return 0;
}
```

3.2 Systems of linear equations, matrix inverse, determinant

```
// Gauss-Jordan elimination with full pivoting.
//
// Uses:
//   (1) solving systems of linear equations (AX=B)
//   (2) inverting matrices (AX=I)
//   (3) computing determinants of square matrices
//
// Running time: O(n^3)
//
// INPUT:    a[][] = an nxn matrix
//           b[][] = an nxm matrix
//
// OUTPUT:   X      = an nxm matrix (stored in b[][])
//           A^{-1} = an nxn matrix (stored in a[][])
//           returns determinant of a[][]
const double EPS = 1e-10;

typedef vector<int> VI;
typedef double T;
typedef vector<T> VT;
typedef vector<VT> VVT;

T GaussJordan(VVT &a, VVT &b) {
  const int n = a.size();
  const int m = b[0].size();
  VI irow(n), icol(n), ipiv(n);
  T det = 1;

  for (int i = 0; i < n; i++) {
    int pj = -1, pk = -1;
    for (int j = 0; j < n; j++) if (!ipiv[j])
      for (int k = 0; k < n; k++) if (!ipiv[k])
        if (pj == -1 || fabs(a[j][k]) > fabs(a[pj][pk])) { pj = j; pk
             = k; }
    if (fabs(a[pj][pk]) < EPS) { cerr << "Matrix is singular." << endl
         ; exit(0); }
    ipiv[pk]++;
    swap(a[pj], a[pk]);
    swap(b[pj], b[pk]);
    if (pj != pk) det *= -1;
    irow[i] = pj;
    icol[i] = pk;

    T c = 1.0 / a[pk][pk];
    det *= a[pk][pk];
    a[pk][pk] = 1.0;
    for (int p = 0; p < n; p++) a[pk][p] *= c;
    for (int p = 0; p < m; p++) b[pk][p] *= c;
    for (int p = 0; p < n; p++) if (p != pk) {
      c = a[p][pk];
      a[p][pk] = 0;
      for (int q = 0; q < n; q++) a[p][q] -= a[pk][q] * c;
      for (int q = 0; q < m; q++) b[p][q] -= b[pk][q] * c;
    }
  }

  for (int p = n-1; p >= 0; p--) if (irow[p] != icol[p]) {
    for (int k = 0; k < n; k++) swap(a[k][irow[p]], a[k][icol[p]]);
  }

  return det;
}

int main() {
  const int n = 4;
  const int m = 2;
  double A[n][n] = { {1,2,3,4},{1,0,1,0},{5,3,2,4},{6,1,4,6} };
  double B[n][m] = { {1,2},{4,3},{5,6},{8,7} };
  VVT a(n), b(n);
  for (int i = 0; i < n; i++) {
    a[i] = VT(A[i], A[i] + n);
    b[i] = VT(B[i], B[i] + m);
  }

  double det = GaussJordan(a, b);

  // expected: 60
  cout << "Determinant: " << det << endl;

  // expected: -0.233333 0.166667 0.133333 0.0666667
  //           0.166667 0.166667 0.333333 -0.333333
  //           0.233333 0.833333 -0.133333 -0.0666667
  //           0.05 -0.75 -0.1 0.2
  cout << "Inverse: " << endl;
  for (int i = 0; i < n; i++) {
    for (int j = 0; j < n; j++)
      cout << a[i][j] << ' ';
    cout << endl;
  }

  // expected: 1.63333 1.3
  //           -0.166667 0.5
  //           2.36667 1.7
  //           -1.85 -1.35
  cout << "Solution: " << endl;
  for (int i = 0; i < n; i++) {
    for (int j = 0; j < m; j++)
      cout << b[i][j] << ' ';
    cout << endl;
  }
}
```

3.3 Reduced row echelon form, matrix rank

```
// Reduced row echelon form via Gauss-Jordan elimination
// with partial pivoting.  This can be used for computing
// the rank of a matrix.
//
// Running time: O(n^3)
//
// INPUT:    a[][] = an nxm matrix
//
// OUTPUT:   rref[][] = an nxm matrix (stored in a[][])
//           returns rank of a[][]
const double EPSILON = 1e-10;

typedef double T;
typedef vector<T> VT;
typedef vector<VT> VVT;

int rref(VVT &a) {
  int n = a.size();
  int m = a[0].size();
  int r = 0;
  for (int c = 0; c < m && r < n; c++) {
    int j = r;
    for (int i = r + 1; i < n; i++)
      if (fabs(a[i][c]) > fabs(a[j][c])) j = i;
    if (fabs(a[j][c]) < EPSILON) continue;
    swap(a[j], a[r]);

    T s = 1.0 / a[r][c];
    for (int j = 0; j < m; j++) a[r][j] *= s;
    for (int i = 0; i < n; i++) if (i != r) {
      T t = a[i][c];
      for (int j = 0; j < m; j++) a[i][j] -= t * a[r][j];
    }
    r++;
  }
  return r;
}

int main() {
  const int n = 5, m = 4;
  double A[n][m] = {
    {16,  2,  3, 13},
    { 5, 11, 10,  8},
    { 9,  7,  6, 12},
    { 4, 14, 15,  1},
    {13, 21, 21, 13}};
  VVT a(n);
  for (int i = 0; i < n; i++)
    a[i] = VT(A[i], A[i] + m);

  int rank = rref(a);

  // expected: 3
  cout << "Rank: " << rank << endl;

  // expected: 1 0 0 1
  //           0 1 0 3
  //           0 0 1 -3
  //           0 0 0 3.10862e-15
  //           0 0 0 2.22045e-15
  cout << "rref: " << endl;
  for (int i = 0; i < 5; i++) {
    for (int j = 0; j < 4; j++)
      cout << a[i][j] << ' ';
    cout << endl;
  }
}
```

3.4 Fast Fourier transform

```
struct cpx
{
  cpx(){}
  cpx(double aa):a(aa),b(0){}
  cpx(double aa, double bb):a(aa),b(bb){}
  double a;
  double b;
  double modsq(void) const
  {
    return a * a + b * b;
  }
  cpx bar(void) const
  {
    return cpx(a, -b);
  }
};

cpx operator +(cpx a, cpx b)
{
  return cpx(a.a + b.a, a.b + b.b);
}

cpx operator *(cpx a, cpx b)
{
  return cpx(a.a * b.a - a.b * b.b, a.a * b.b + a.b * b.a);
}

cpx operator /(cpx a, cpx b)
{
  cpx r = a * b.bar();
  return cpx(r.a / b.modsq(), r.b / b.modsq());
}

cpx EXP(double theta)
```

```
{
  return cpx(cos(theta),sin(theta));
}

const double two_pi = 4 * acos(0);

// in:     input array
// out:    output array
// step:   {SET TO 1} (used internally)
// size:   length of the input/output {MUST BE A POWER OF 2}
// dir:    either plus or minus one (direction of the FFT)
// RESULT: out[k] = \sum_{j=0}^{size - 1} in[j] * exp(dir * 2pi * i *
       j * k / size)
void FFT(cpx *in, cpx *out, int step, int size, int dir)
{
  if(size < 1) return;
  if(size == 1)
  {
    out[0] = in[0];
    return;
  }
  FFT(in, out, step * 2, size / 2, dir);
  FFT(in + step, out + size / 2, step * 2, size / 2, dir);
  for(int i = 0 ; i < size / 2 ; i++)
  {
    cpx even = out[i];
    cpx odd = out[i + size / 2];
    out[i] = even + EXP(dir * two_pi * i / size) * odd;
    out[i + size / 2] = even + EXP(dir * two_pi * (i + size / 2) /
          size) * odd;
  }
}

// Usage:
// f[0...N-1] and g[0..N-1] are numbers
// Want to compute the convolution h, defined by
// h[n] = sum of f[k]g[n-k] (k = 0, ..., N-1).
// Here, the index is cyclic; f[-1] = f[N-1], f[-2] = f[N-2], etc.
// Let F[0...N-1] be FFT(f), and similarly, define G and H.
// The convolution theorem says H[n] = F[n]G[n] (element-wise product)
     .
// To compute h[] in O(N log N) time, do the following:
//   1. Compute F and G (pass dir = 1 as the argument).
//   2. Get H by element-wise multiplying F and G.
//   3. Get h by taking the inverse FFT (use dir = -1 as the argument)
//      and *dividing by N*. DO NOT FORGET THIS SCALING FACTOR.

int main(void)
{
  printf("If rows come in identical pairs, then everything works.\n");

  cpx a[8] = {0, 1, cpx(1,3), cpx(0,5), 1, 0, 2, 0};
  cpx b[8] = {1, cpx(0,-2), cpx(0,1), 3, -1, -3, 1, -2};
  cpx A[8];
  cpx B[8];
  FFT(a, A, 1, 8, 1);
  FFT(b, B, 1, 8, 1);

  for(int i = 0 ; i < 8 ; i++)
  {
    printf("%7.2lf%7.2lf", A[i].a, A[i].b);
  }
  printf("\n");
  for(int i = 0 ; i < 8 ; i++)
  {
    cpx Ai(0,0);
    for(int j = 0 ; j < 8 ; j++)
    {
      Ai = Ai + a[j] * EXP(j * i * two_pi / 8);
    }
    printf("%7.2lf%7.2lf", Ai.a, Ai.b);
  }
  printf("\n");

  cpx AB[8];
  for(int i = 0 ; i < 8 ; i++)
    AB[i] = A[i] * B[i];
  cpx aconvb[8];
  FFT(AB, aconvb, 1, 8, -1);
  for(int i = 0 ; i < 8 ; i++)
    aconvb[i] = aconvb[i] / 8;
  for(int i = 0 ; i < 8 ; i++)
  {
    printf("%7.2lf%7.2lf", aconvb[i].a, aconvb[i].b);
  }
  printf("\n");
  for(int i = 0 ; i < 8 ; i++)
  {
    cpx aconvbi(0,0);
    for(int j = 0 ; j < 8 ; j++)
    {
      aconvbi = aconvbi + a[j] * b[(8 + i - j) % 8];
    }
    printf("%7.2lf%7.2lf", aconvbi.a, aconvbi.b);
  }
  printf("\n");

  return 0;
}
```

3.5 Simplex algorithm

```
// Two-phase simplex algorithm for solving linear programs of the form
//
//     maximize     c^T x
//     subject to   Ax <= b
//                  x >= 0
//
// INPUT: A -- an m x n matrix
//        b -- an m-dimensional vector
//        c -- an n-dimensional vector
//        x -- a vector where the optimal solution will be stored
//
// OUTPUT: value of the optimal solution (infinity if unbounded
//         above, nan if infeasible)
//
// To use this code, create an LPSolver object with A, b, and c as
// arguments.  Then, call Solve(x).
using namespace std;

typedef long double DOUBLE;
typedef vector<DOUBLE> VD;
typedef vector<VD> VVD;
typedef vector<int> VI;

const DOUBLE EPS = 1e-9;

struct LPSolver {
  int m, n;
  VI B, N;
  VVD D;

  LPSolver(const VVD &A, const VD &b, const VD &c) :
    m(b.size()), n(c.size()), N(n + 1), B(m), D(m + 2, VD(n + 2)) {
    for (int i = 0; i < m; i++) for (int j = 0; j < n; j++) D[i][j] =
          A[i][j];
    for (int i = 0; i < m; i++) { B[i] = n + i; D[i][n] = -1; D[i][n +
          1] = b[i]; }
    for (int j = 0; j < n; j++) { N[j] = j; D[m][j] = -c[j]; }
    N[n] = -1; D[m + 1][n] = 1;
  }

  void Pivot(int r, int s) {
    double inv = 1.0 / D[r][s];
    for (int i = 0; i < m + 2; i++) if (i != r)
      for (int j = 0; j < n + 2; j++) if (j != s)
        D[i][j] -= D[r][j] * D[i][s] * inv;
    for (int j = 0; j < n + 2; j++) if (j != s) D[r][j] *= inv;
    for (int i = 0; i < m + 2; i++) if (i != r) D[i][s] *= -inv;
    D[r][s] = inv;
    swap(B[r], N[s]);
  }

  bool Simplex(int phase) {
    int x = phase == 1 ? m + 1 : m;
    while (true) {
      int s = -1;
      for (int j = 0; j <= n; j++) {
        if (phase == 2 && N[j] == -1) continue;
        if (s == -1 || D[x][j] < D[x][s] || D[x][j] == D[x][s] && N[j]
              < N[s]) s = j;
      }
      if (D[x][s] > -EPS) return true;
      int r = -1;
      for (int i = 0; i < m; i++) {
        if (D[i][s] < EPS) continue;
        if (r == -1 || D[i][n + 1] / D[i][s] < D[r][n + 1] / D[r][s]
              ||
            (D[i][n + 1] / D[i][s]) == (D[r][n + 1] / D[r][s]) && B[i] <
                 B[r]) r = i;
      }
      if (r == -1) return false;
      Pivot(r, s);
    }
  }

  DOUBLE Solve(VD &x) {
    int r = 0;
    for (int i = 1; i < m; i++) if (D[i][n + 1] < D[r][n + 1]) r = i;
    if (D[r][n + 1] < -EPS) {
      Pivot(r, n);
      if (!Simplex(1) || D[m + 1][n + 1] < -EPS) return -
            numeric_limits<DOUBLE>::infinity();
      for (int i = 0; i < m; i++) if (B[i] == -1) {
        int s = -1;
        for (int j = 0; j <= n; j++)
          if (s == -1 || D[i][j] < D[i][s] || D[i][j] == D[i][s] && N[
                j] < N[s]) s = j;
        Pivot(i, s);
      }
    }
    if (!Simplex(2)) return numeric_limits<DOUBLE>::infinity();
    x = VD(n);
    for (int i = 0; i < m; i++) if (B[i] < n) x[B[i]] = D[i][n + 1];
    return D[m][n + 1];
  }
};

int main() {

  const int m = 4;
  const int n = 3;
  DOUBLE _A[m][n] = {
    { 6, -1, 0 },
    { -1, -5, 0 },
    { 1, 5, 1 },
    { -1, -5, -1 }
  };
  DOUBLE _b[m] = { 10, -4, 5, -5 };
  DOUBLE _c[n] = { 1, -1, 0 };

  VVD A(m);
  VD b(_b, _b + m);
  VD c(_c, _c + n);
  for (int i = 0; i < m; i++) A[i] = VD(_A[i], _A[i] + n);

  LPSolver solver(A, b, c);
  VD x;
  DOUBLE value = solver.Solve(x);

  cerr << "VALUE: " << value << endl; // VALUE: 1.29032
  cerr << "SOLUTION:"; // SOLUTION: 1.74194 0.451613 1
  for (size_t i = 0; i < x.size(); i++) cerr << " " << x[i];
  cerr << endl;
  return 0;
}
```

4 Graph algorithms

4.1 Dijkstra and Floyd's algorithm (C++)

```
typedef double T;
typedef vector<T> VT;
typedef vector<VT> VVT;

typedef vector<int> VI;
typedef vector<VI> VVI;

// This function runs Dijkstra's algorithm for single source
// shortest paths.  No negative cycles allowed!
//
// Running time: O(|V|^2)
//
//   INPUT:   start, w[i][j] = cost of edge from i to j
//   OUTPUT:  dist[i] = min weight path from start to i
//            prev[i] = previous node on the best path from the
//                      start node

void Dijkstra (const VVT &w, VT &dist, VI &prev, int start){
  int n = w.size();
  VI found (n);
  prev = VI(n, -1);
```

```
  dist = VT(n, 1000000000);
  dist[start] = 0;

  while (start != -1){
    found[start] = true;
    int best = -1;
    for (int k = 0; k < n; k++) if (!found[k]){
      if (dist[k] > dist[start] + w[start][k]){
        dist[k] = dist[start] + w[start][k];
        prev[k] = start;
      }
      if (best == -1 || dist[k] < dist[best]) best = k;
    }
    start = best;
  }
}

// This function runs the Floyd-Warshall algorithm for all-pairs
// shortest paths.  Also handles negative edge weights.  Returns true
// if a negative weight cycle is found.
//
// Running time: O(|V|^3)
//
//   INPUT:  w[i][j] = weight of edge from i to j
//   OUTPUT: w[i][j] = shortest path from i to j
//           prev[i][j] = node before j on the best path starting at i

bool FloydWarshall (VVT &w, VVI &prev){
  int n = w.size();
  prev = VVI (n, VI(n, -1));

  for (int k = 0; k < n; k++){
    for (int i = 0; i < n; i++){
      for (int j = 0; j < n; j++){
        if (w[i][j] > w[i][k] + w[k][j]){
          w[i][j] = w[i][k] + w[k][j];
          prev[i][j] = k;
        }
      }
    }
  }

  // check for negative weight cycles
  for(int i=0;i<n;i++)
    if (w[i][i] < 0) return false;
  return true;
}
```

4.2 Fast Dijkstra's algorithm

```
// Implementation of Dijkstra's algorithm using adjacency lists
// and priority queue for efficiency.
//
// Running time: O(|E| log |V|)
using namespace std;
const int INF = 2000000000;
typedef pair<int, int> PII;

int main() {

        int N, s, t;
        scanf("%d%d%d", &N, &s, &t);
        vector<vector<PII> > edges(N);
        for (int i = 0; i < N; i++) {
                int M;
                scanf("%d", &M);
                for (int j = 0; j < M; j++) {
                        int vertex, dist;
                        scanf("%d%d", &vertex, &dist);
                        edges[i].push_back(make_pair(dist, vertex));
                                // note order of arguments here
                }
        }

        // use priority queue in which top element has the "smallest"
              priority
        priority_queue<PII, vector<PII>, greater<PII> > Q;
        vector<int> dist(N, INF), dad(N, -1);
        Q.push(make_pair(0, s));
        dist[s] = 0;
        while (!Q.empty()) {
                PII p = Q.top();
                Q.pop();
                int here = p.second;
                if (here == t) break;
                if (dist[here] != p.first) continue;

                for (vector<PII>::iterator it = edges[here].begin();
                       it != edges[here].end(); it++) {
                         if (dist[here] + it->first < dist[it->second])
                                {
                                 dist[it->second] = dist[here] + it->
                                        first;
                                 dad[it->second] = here;
                                 Q.push(make_pair(dist[it->second], it
                                        ->second));
                         }
                }
        }

        printf("%d\n", dist[t]);
        if (dist[t] < INF)
                for (int i = t; i != -1; i = dad[i])
                        printf("%d%c", i, (i == s ? '\n' : ' '));
        return 0;
}

/*
Sample input:
5 0 4
2 1 2 3 1
2 2 4 4 5
3 1 4 3 3 4 1
2 0 1 2 3
2 1 5 2 1

Expected:
5
4 2 3 0
*/
```

4.3 Strongly connected components

```
#include<memory.h>
struct edge{int e, nxt;};
int V, E;
edge e[MAXE], er[MAXE];
int sp[MAXV], spr[MAXV];
int group_cnt, group_num[MAXV];
bool v[MAXV];
int stk[MAXV];
void fill_forward(int x)
{
  int i;
  v[x]=true;
  for(i=sp[x];i;i=e[i].nxt) if(!v[e[i].e]) fill_forward(e[i].e);
  stk[++stk[0]]=x;
}
void fill_backward(int x)
{
  int i;
  v[x]=false;
  group_num[x]=group_cnt;
  for(i=spr[x];i;i=er[i].nxt) if(v[er[i].e]) fill_backward(er[i].e);
}
void add_edge(int v1, int v2) //add edge v1->v2
{
  e [++E].e=v2; e [E].nxt=sp [v1]; sp [v1]=E;
  er[  E].e=v1; er[E].nxt=spr[v2]; spr[v2]=E;
}
void SCC()
{
  int i;
  stk[0]=0;
  memset(v, false, sizeof(v));
  for(i=1;i<=V;i++) if(!v[i]) fill_forward(i);
  group_cnt=0;
  for(i=stk[0];i>=1;i--) if(v[stk[i]]){group_cnt++; fill_backward(stk[
        i]);}
}
```

4.4 Eulerian path

```
struct Edge;
typedef list<Edge>::iterator iter;

struct Edge
{
        int next_vertex;
        iter reverse_edge;

        Edge(int next_vertex)
                :next_vertex(next_vertex)
                { }
};

const int max_vertices = ;
int num_vertices;
list<Edge> adj[max_vertices];           // adjacency list

vector<int> path;

void find_path(int v)
{
        while(adj[v].size() > 0)
        {
                int vn = adj[v].front().next_vertex;
                adj[vn].erase(adj[v].front().reverse_edge);
                adj[v].pop_front();
                find_path(vn);
        }
        path.push_back(v);
}

void add_edge(int a, int b)
{
        adj[a].push_front(Edge(b));
        iter ita = adj[a].begin();
        adj[b].push_front(Edge(a));
        iter itb = adj[b].begin();
        ita->reverse_edge = itb;
        itb->reverse_edge = ita;
}
```

4.5 Kruskal's algorithm

```
/*
Uses Kruskal's Algorithm to calculate the weight of the minimum
      spanning
forest (union of minimum spanning trees of each connected component)
      of
a possibly disjoint graph, given in the form of a matrix of edge
      weights
(-1 if no edge exists). Returns the weight of the minimum spanning
forest (also calculates the actual edges - stored in T). Note: uses a
disjoint-set data structure with amortized (effectively) constant time
       per
union/find. Runs in O(E*log(E)) time.
*/
typedef int T;

struct edge
{
  int u, v;
  T d;
};

struct edgeCmp
{
  int operator()(const edge& a, const edge& b) { return a.d > b.d; }
};

int find(vector <int>& C, int x) { return (C[x] == x) ? x : C[x] =
      find(C, C[x]); }

T Kruskal(vector <vector <T> >& w)
{
  int n = w.size();
  T weight = 0;

  vector <int> C(n), R(n);
  for(int i=0; i<n; i++) { C[i] = i; R[i] = 0; }

  vector <edge> T;
  priority_queue <edge, vector <edge>, edgeCmp> E;

  for(int i=0; i<n; i++)
    for(int j=i+1; j<n; j++)
```

```
      if(w[i][j] >= 0)
      {
        edge e;
        e.u = i; e.v = j; e.d = w[i][j];
        E.push(e);
      }

  while(T.size() < n-1 && !E.empty())
  {
    edge cur = E.top(); E.pop();

    int uc = find(C, cur.u), vc = find(C, cur.v);
    if(uc != vc)
    {
      T.push_back(cur); weight += cur.d;

      if(R[uc] > R[vc]) C[vc] = uc;
      else if(R[vc] > R[uc]) C[uc] = vc;
      else { C[vc] = uc; R[uc]++; }
    }
  }

  return weight;
}

int main()
{
  int wa[6][6] = {
    { 0, -1, 2, -1, 7, -1 },
    { -1, 0, -1, 2, -1, -1 },
    { 2, -1, 0, -1, 8, 6 },
    { -1, 2, -1, 0, -1, -1 },
    { 7, -1, 8, -1, 0, 4 },
    { -1, -1, 6, -1, 4, 0 } };

  vector <vector <int> > w(6, vector <int>(6));

  for(int i=0; i<6; i++)
    for(int j=0; j<6; j++)
      w[i][j] = wa[i][j];

  cout << Kruskal(w) << endl;
  cin >> wa[0][0];
}
```

4.6 Minimum spanning trees

```
// This function runs Prim's algorithm for constructing minimum
// weight spanning trees.
//
// Running time: O(|V|^2)
//
//   INPUT:   w[i][j] = cost of edge from i to j
//
//            NOTE: Make sure that w[i][j] is nonnegative and
//            symmetric.  Missing edges should be given -1
//            weight.
//
//   OUTPUT:  edges = list of pair<int,int> in minimum spanning tree
//            return total weight of tree
typedef double T;
typedef vector<T> VT;
typedef vector<VT> VVT;

typedef vector<int> VI;
typedef vector<VI> VVI;
typedef pair<int,int> PII;
typedef vector<PII> VPII;

T Prim (const VVT &w, VPII &edges){
  int n = w.size();
  VI found (n);
  VI prev (n, -1);
  VT dist (n, 1000000000);
  int here = 0;
  dist[here] = 0;

  while (here != -1){
    found[here] = true;
    int best = -1;
    for (int k = 0; k < n; k++) if (!found[k]){
      if (w[here][k] != -1 && dist[k] > w[here][k]){
        dist[k] = w[here][k];
        prev[k] = here;
      }
      if (best == -1 || dist[k] < dist[best]) best = k;
    }
    here = best;
  }

  T tot_weight = 0;
  for (int i = 0; i < n; i++) if (prev[i] != -1){
    edges.push_back (make_pair (prev[i], i));
    tot_weight += w[prev[i]][i];
  }
  return tot_weight;
}

int main(){
  int ww[5][5] = {
    {0, 400, 400, 300, 600},
    {400, 0, 3, -1, 7},
    {400, 3, 0, 2, 0},
    {300, -1, 2, 0, 5},
    {600, 7, 0, 5, 0}
  };
  VVT w(5, VT(5));
  for (int i = 0; i < 5; i++)
    for (int j = 0; j < 5; j++)
      w[i][j] = ww[i][j];

  // expected: 305
  //           2 1
  //           3 2
  //           0 3
  //           2 4

  VPII edges;
  cout << Prim (w, edges) << endl;
  for (int i = 0; i < edges.size(); i++)
    cout << edges[i].first << " " << edges[i].second << endl;
}
```

5 Data structures

5.1 Suffix array

```
// Suffix array construction in O(L log^2 L) time.  Routine for
// computing the length of the longest common prefix of any two
// suffixes in O(log L) time.
//
// INPUT:   string s
//
// OUTPUT:  array suffix[] such that suffix[i] = index (from 0 to L-1)
//          of substring s[i...L-1] in the list of sorted suffixes.
//          That is, if we take the inverse of the permutation suffix
//     [],
//          we get the actual suffix array.
struct SuffixArray {
  const int L;
  string s;
  vector<vector<int> > P;
  vector<pair<pair<int,int>,int> > M;

  SuffixArray(const string &s) : L(s.length()), s(s), P(1, vector<int
      >(L, 0)), M(L) {
    for (int i = 0; i < L; i++) P[0][i] = int(s[i]);
    for (int skip = 1, level = 1; skip < L; skip *= 2, level++) {
      P.push_back(vector<int>(L, 0));
      for (int i = 0; i < L; i++)
        M[i] = make_pair(make_pair(P[level-1][i], i + skip < L ? P[
            level-1][i + skip] : -1000), i);
      sort(M.begin(), M.end());
      for (int i = 0; i < L; i++)
        P[level][M[i].second] = (i > 0 && M[i].first == M[i-1].first)
            ? P[level][M[i-1].second] : i;
    }
  }

  vector<int> GetSuffixArray() { return P.back(); }

  // returns the length of the longest common prefix of s[i...L-1] and
      s[j...L-1]
  int LongestCommonPrefix(int i, int j) {
    int len = 0;
    if (i == j) return L - i;
    for (int k = P.size() - 1; k >= 0 && i < L && j < L; k--) {
      if (P[k][i] == P[k][j]) {
        i += 1 << k;
        j += 1 << k;
        len += 1 << k;
      }
    }
    return len;
  }
};

// BEGIN CUT
// The following code solves UVA problem 11512: GATTACA.
#define TESTING
#ifdef TESTING
int main() {
  int T;
  cin >> T;
  for (int caseno = 0; caseno < T; caseno++) {
    string s;
    cin >> s;
    SuffixArray array(s);
    vector<int> v = array.GetSuffixArray();
    int bestlen = -1, bestpos = -1, bestcount = 0;
    for (int i = 0; i < s.length(); i++) {
      int len = 0, count = 0;
      for (int j = i+1; j < s.length(); j++) {
        int l = array.LongestCommonPrefix(i, j);
        if (l >= len) {
          if (l > len) count = 2; else count++;
          len = l;
        }
      }
      if (len > bestlen || len == bestlen && s.substr(bestpos, bestlen
          ) > s.substr(i, len)) {
        bestlen = len;
        bestcount = count;
        bestpos = i;
      }
    }
    if (bestlen == 0) {
      cout << "No repetitions found!" << endl;
    } else {
      cout << s.substr(bestpos, bestlen) << " " << bestcount << endl;
    }
  }
}

#else
// END CUT
int main() {

  // bobocel is the 0'th suffix
  //  obocel is the 5'th suffix
  //   bocel is the 1'st suffix
  //    ocel is the 6'th suffix
  //     cel is the 2'nd suffix
  //      el is the 3'rd suffix
  //       l is the 4'th suffix
  SuffixArray suffix("bobocel");
  vector<int> v = suffix.GetSuffixArray();

  // Expected output: 0 5 1 6 2 3 4
  //                  2
  for (int i = 0; i < v.size(); i++) cout << v[i] << " ";
  cout << endl;
  cout << suffix.LongestCommonPrefix(0, 2) << endl;
}
// BEGIN CUT
#endif
// END CUT
```

5.2 Binary Indexed Tree

```
// BIT with range updates, inspired by Petr Mitrichev
struct BIT {
    int n;
    vector<int> slope;
    vector<int> intercept;
    // BIT can be thought of as having entries f[1], ..., f[n]
    // which are 0-initialized
    BIT(int n): n(n), slope(n+1), intercept(n+1) {}
    // returns f[1] + ... + f[idx-1]
    // precondition idx <= n+1
```

```
    int query(int idx) {
        int m = 0, b = 0;
        for (int i = idx-1; i > 0; i -= i&-i) {
            m += slope[i];
            b += intercept[i];
        }
        return m*idx + b;
    }
    // adds amt to f[i] for i in [idx1, idx2)
    // precondition 1 <= idx1 <= idx2 <= n+1 (you can't update element
          0)
    void update(int idx1, int idx2, int amt) {
        for (int i = idx1; i <= n; i += i&-i) {
            slope[i] += amt;
            intercept[i] -= idx1*amt;
        }
        for (int i = idx2; i <= n; i += i&-i) {
            slope[i] -= amt;
            intercept[i] += idx2*amt;
        }
    }
};

// BIT with range updates, inspired by Petr Mitrichev
class FenwickTree {
private: vi ft1, ft2;
    int query(vi &ft, int b) {
        int sum = 0; for (; b; b -= LSOne(b)) sum += ft[b];
        return sum; }
    void adjust(vi &ft, int k, int v) {
        for (; k < (int)ft.size(); k += LSOne(k)) ft[k] += v; }
public:
    FenwickTree() {}
    FenwickTree(int n) { ft1.assign(n + 1, 0); ft2.assign(n+1, 0);}
    int query(int a) { return a * query(ft1, a) - query(ft2, a); }
    int query(int a, int b) { return query(b) - (a == 1 ? 0 : query(a
         -1)); }
    void adjust(int a, int b, int value){
        adjust(ft1, a, value);
        adjust(ft1, b+1, -value);
        adjust(ft2, a, value * (a-1));
        adjust(ft2, b+1, -1 * value * b);
    }
    int get(int n) {
        return query(n) - query(n-1);}
};
```

5.3 2D Binary Indexed Tree

```
// WARNING NOT FIELD TESTED YET
class FenwickTree {
private:
  vi ft;

public:
  FenwickTree() {}
  // initialization: n + 1 zeroes, ignore index 0
  FenwickTree(int n) { ft.assign(n + 1, 0); }

  int rsq(int b) {                                  // returns RSQ
        (1, b)
    int sum = 0; for (; b; b -= LSOne(b)) sum += ft[b];
    return sum; }

  int rsq(int a, int b) {                           // returns RSQ(
        a, b)
    return rsq(b) - (a == 1 ? 0 : rsq(a - 1)); }

  // adjusts value of the k-th element by v (v can be +ve/inc or -ve/
        dec)
  void adjust(int k, int v) {                    // note: n = ft.size
        () - 1
    for (; k < (int)ft.size(); k += LSOne(k)) ft[k] += v; }
};

class FenwickTree2D {
private:
  vector<FenwickTree> ft2d;

public:
  FenwickTree2D() {}
  FenwickTree2D(int n) { ft2d.assign(n+1, FenwickTree(n)); }

  int rsq(int r, int c) {
    int sum = 0;
    for(; r; r -= LSOne(r)) sum += ft2d[r].rsq(c);
    return sum;
  }

  // top left, bottom right
  int rsq(int r1, int c1, int r2, int c2) {
    return rsq(r2, c2) - rsq(r2, c1-1) - rsq(r1-1, c2) + rsq(r1-1, c2
          -1);
  }

  void adjust(int r, int c, int v) {
    for (; r < (int)ft2d.size(); r += LSOne(r)) ft2d[r].adjust(c, v);
  }
};


int main() {
  FenwickTree2D ft2d(4);
  ft2d.adjust(1, 1, 1);
  ft2d.adjust(2, 2, 1);
  ft2d.adjust(3, 3, 1);
  ft2d.adjust(4, 4, 1);
  printf("%d\n", ft2d.rsq(1,1));      // 1
  printf("%d\n", ft2d.rsq(2,2));      // 2
  printf("%d\n", ft2d.rsq(3,3));      // 3
  printf("%d\n", ft2d.rsq(2,2,3,3));  // 2
  return 0;
}
```

5.4 Union-find SegmentTreeLazy

```
struct UnionFind {
    vector<int> C;
    UnionFind(int n) : C(n) { for (int i = 0; i < n; i++) C[i] = i; }
    int find(int x) { return (C[x] == x) ? x : C[x] = find(C[x]); }
    void merge(int x, int y) { C[find(x)] = find(y); }
};
int main()
{
    int n = 5;
    UnionFind uf(n);
    uf.merge(0, 2);
    uf.merge(1, 0);
    uf.merge(3, 4);
    for (int i = 0; i < n; i++) cout << i << " " << uf.find(i) << endl
          ;
    return 0;
}
```

5.5 KD-tree

```
// -----------------------------------------------------------------
// A straightforward, but probably sub-optimal KD-tree implmentation
// that's probably good enough for most things (current it's a
// 2D-tree)
//
//  - constructs from n points in O(n lg^2 n) time
//  - handles nearest-neighbor query in O(lg n) if points are well
//    distributed
//  - worst case for nearest-neighbor may be linear in pathological
//    case
// -----------------------------------------------------------------

#include <iostream>
#include <vector>
#include <limits>
#include <cstdlib>

using namespace std;

// number type for coordinates, and its maximum value
typedef long long ntype;
const ntype sentry = numeric_limits<ntype>::max();

// point structure for 2D-tree, can be extended to 3D
struct point {
    ntype x, y;
    point(ntype xx = 0, ntype yy = 0) : x(xx), y(yy) {}
};

bool operator==(const point &a, const point &b)
{
    return a.x == b.x && a.y == b.y;
}

// sorts points on x-coordinate
bool on_x(const point &a, const point &b)
{
    return a.x < b.x;
}

// sorts points on y-coordinate
bool on_y(const point &a, const point &b)
{
    return a.y < b.y;
}

// squared distance between points
ntype pdist2(const point &a, const point &b)
{
    ntype dx = a.x-b.x, dy = a.y-b.y;
    return dx*dx + dy*dy;
}

// bounding box for a set of points
struct bbox
{
    ntype x0, x1, y0, y1;

    bbox() : x0(sentry), x1(-sentry), y0(sentry), y1(-sentry) {}

    // computes bounding box from a bunch of points
    void compute(const vector<point> &v) {
        for (int i = 0; i < v.size(); ++i) {
            x0 = min(x0, v[i].x);   x1 = max(x1, v[i].x);
            y0 = min(y0, v[i].y);   y1 = max(y1, v[i].y);
        }
    }

    // squared distance between a point and this bbox, 0 if inside
    ntype distance(const point &p) {
        if (p.x < x0) {
            if (p.y < y0)       return pdist2(point(x0, y0), p);
            else if (p.y > y1)  return pdist2(point(x0, y1), p);
            else                return pdist2(point(x0, p.y), p);
        }
        else if (p.x > x1) {
            if (p.y < y0)       return pdist2(point(x1, y0), p);
            else if (p.y > y1)  return pdist2(point(x1, y1), p);
            else                return pdist2(point(x1, p.y), p);
        }
        else {
            if (p.y < y0)       return pdist2(point(p.x, y0), p);
            else if (p.y > y1)  return pdist2(point(p.x, y1), p);
            else                return 0;
        }
    }
};

// stores a single node of the kd-tree, either internal or leaf
struct kdnode
{
    bool leaf;      // true if this is a leaf node (has one point)
    point pt;       // the single point of this is a leaf
    bbox bound;     // bounding box for set of points in children

    kdnode *first, *second; // two children of this kd-node

    kdnode() : leaf(false), first(0), second(0) {}
    ~kdnode() { if (first) delete first; if (second) delete second; }

    // intersect a point with this node (returns squared distance)
    ntype intersect(const point &p) {
        return bound.distance(p);
    }

    // recursively builds a kd-tree from a given cloud of points
    void construct(vector<point> &vp)
    {
        // compute bounding box for points at this node
        bound.compute(vp);

        // if we're down to one point, then we're a leaf node
        if (vp.size() == 1) {
            leaf = true;
            pt = vp[0];
        }
        else {
```

```
            // split on x if the bbox is wider than high (not best
                heuristic...)
            if (bound.x1-bound.x0 >= bound.y1-bound.y0)
                sort(vp.begin(), vp.end(), on_x);
            // otherwise split on y-coordinate
            else
                sort(vp.begin(), vp.end(), on_y);

            // divide by taking half the array for each child
            // (not best performance if many duplicates in the middle)
            int half = vp.size()/2;
            vector<point> vl(vp.begin(), vp.begin()+half);
            vector<point> vr(vp.begin()+half, vp.end());
            first = new kdnode();   first->construct(vl);
            second = new kdnode();  second->construct(vr);
        }
    }
};

// simple kd-tree class to hold the tree and handle queries
struct kdtree
{
    kdnode *root;

    // constructs a kd-tree from a points (copied here, as it sorts
        them)
    kdtree(const vector<point> &vp) {
        vector<point> v(vp.begin(), vp.end());
        root = new kdnode();
        root->construct(v);
    }
    ~kdtree() { delete root; }

    // recursive search method returns squared distance to nearest
        point
    ntype search(kdnode *node, const point &p)
    {
        if (node->leaf) {
            // commented special case tells a point not to find itself
//            if (p == node->pt) return sentry;
//            else
                return pdist2(p, node->pt);
        }

        ntype bfirst = node->first->intersect(p);
        ntype bsecond = node->second->intersect(p);

        // choose the side with the closest bounding box to search
            first
        // (note that the other side is also searched if needed)
        if (bfirst < bsecond) {
            ntype best = search(node->first, p);
            if (bsecond < best)
                best = min(best, search(node->second, p));
            return best;
        }
        else {
            ntype best = search(node->second, p);
            if (bfirst < best)
                best = min(best, search(node->first, p));
            return best;
        }
    }

    // squared distance to the nearest
    ntype nearest(const point &p) {
        return search(root, p);
    }
};

//
    --------------------------------------------------------------------------

// some basic test code here

int main()
{
    // generate some random points for a kd-tree
    vector<point> vp;
    for (int i = 0; i < 100000; ++i) {
        vp.push_back(point(rand()%100000, rand()%100000));
    }
    kdtree tree(vp);

    // query some points
    for (int i = 0; i < 10; ++i) {
        point q(rand()%100000, rand()%100000);
        cout << "Closest squared distance to (" << q.x << ", " << q.y
              << ")"
             << " is " << tree.nearest(q) << endl;
    }

    return 0;
}

//
    --------------------------------------------------------------------------
```

5.6 Splay tree

```
const int N_MAX = 130010;
const int oo = 0x3f3f3f3f;
struct Node
{
  Node *ch[2], *pre;
  int val, size;
  bool isTurned;
} nodePool[N_MAX], *null, *root;

Node *allocNode(int val)
{
  static int freePos = 0;
  Node *x = &nodePool[freePos ++];
  x->val = val, x->isTurned = false;
  x->ch[0] = x->ch[1] = x->pre = null;
  x->size = 1;
  return x;
}

inline void update(Node *x)
{
  x->size = x->ch[0]->size + x->ch[1]->size + 1;
}

inline void makeTurned(Node *x)
{
  if(x == null)
    return;
  swap(x->ch[0], x->ch[1]);
  x->isTurned ^= 1;
}

inline void pushDown(Node *x)
{
  if(x->isTurned)
  {
    makeTurned(x->ch[0]);
    makeTurned(x->ch[1]);
    x->isTurned ^= 1;
  }
}

inline void rotate(Node *x, int c)
{
  Node *y = x->pre;
  x->pre = y->pre;
  if(y->pre != null)
    y->pre->ch[y == y->pre->ch[1]] = x;
  y->ch[!c] = x->ch[c];
  if(x->ch[c] != null)
    x->ch[c]->pre = y;
  x->ch[c] = y, y->pre = x;
  update(y);
  if(y == root)
    root = x;
}

void splay(Node *x, Node *p)
{
  while(x->pre != p)
  {
    if(x->pre->pre == p)
      rotate(x, x == x->pre->ch[0]);
    else
    {
      Node *y = x->pre, *z = y->pre;
      if(y == z->ch[0])
      {
        if(x == y->ch[0])
          rotate(y, 1), rotate(x, 1);
        else
          rotate(x, 0), rotate(x, 1);
      }
      else
      {
        if(x == y->ch[1])
          rotate(y, 0), rotate(x, 0);
        else
          rotate(x, 1), rotate(x, 0);
      }
    }
  }
  update(x);
}

void select(int k, Node *fa)
{
  Node *now = root;
  while(1)
  {
    pushDown(now);
    int tmp = now->ch[0]->size + 1;
    if(tmp == k)
      break;
    else if(tmp < k)
      now = now->ch[1], k -= tmp;
    else
      now = now->ch[0];
  }
  splay(now, fa);
}

Node *makeTree(Node *p, int l, int r)
{
  if(l > r)
    return null;
  int mid = (l + r) / 2;
  Node *x = allocNode(mid);
  x->pre = p;
  x->ch[0] = makeTree(x, l, mid - 1);
  x->ch[1] = makeTree(x, mid + 1, r);
  update(x);
  return x;
}

int main()
{
  int n, m;
  null = allocNode(0);
  null->size = 0;
  root = allocNode(0);
  root->ch[1] = allocNode(oo);
  root->ch[1]->pre = root;
  update(root);

  scanf("%d%d", &n, &m);
  root->ch[1]->ch[0] = makeTree(root->ch[1], 1, n);
  splay(root->ch[1]->ch[0], null);

  while(m --)
  {
    int a, b;
    scanf("%d%d", &a, &b);
    a ++, b ++;
    select(a - 1, null);
    select(b + 1, root);
    makeTurned(root->ch[1]->ch[0]);
  }

  for(int i = 1; i <= n; i ++)
  {
    select(i + 1, null);
    printf("%d ", root->val);
  }
}
```

5.7 Lazy segment tree

```
public class SegmentTreeRangeUpdate {
        public long[] leaf;
        public long[] update;
        public int origSize;
        public SegmentTreeRangeUpdate(int[] list)       {
                origSize = list.length;
                leaf = new long[4*list.length];
                update = new long[4*list.length];
```

```
        build(1,0,list.length-1,list);
    }
    public void build(int curr, int begin, int end, int[] list)
            {
        if(begin == end)
            leaf[curr] = list[begin];
        else    {
            int mid = (begin+end)/2;
            build(2 * curr, begin, mid, list);
            build(2 * curr + 1, mid+1, end, list);
            leaf[curr] = leaf[2*curr] + leaf[2*curr+1];
        }
    }
    public void update(int begin, int end, int val) {
        update(1,0,origSize-1,begin,end,val);
    }
    public void update(int curr,  int tBegin, int tEnd, int begin,
        int end, int val)      {
        if(tBegin >= begin && tEnd <= end)
            update[curr] += val;
        else    {
            leaf[curr] += (Math.min(end,tEnd)-Math.max(
                begin,tBegin)+1) * val;
            int mid = (tBegin+tEnd)/2;
            if(mid >= begin && tBegin <= end)
                update(2*curr, tBegin, mid, begin, end
                    , val);
            if(tEnd >= begin && mid+1 <= end)
                update(2*curr+1, mid+1, tEnd, begin,
                    end, val);
        }
    }
    public long query(int begin, int end)   {
        return query(1,0,origSize-1,begin,end);
    }
    public long query(int curr, int tBegin, int tEnd, int begin,
        int end)   {
        if(tBegin >= begin && tEnd <= end)      {
            if(update[curr] != 0)   {
                leaf[curr] += (tEnd-tBegin+1) * update
                    [curr];
                if(2*curr < update.length){
                    update[2*curr] += update[curr
                        ];
                    update[2*curr+1] += update[
                        curr];
                }
                update[curr] = 0;
            }
            return leaf[curr];
        }
        else    {
            leaf[curr] += (tEnd-tBegin+1) * update[curr];
            if(2*curr < update.length){
                update[2*curr] += update[curr];
                update[2*curr+1] += update[curr];
            }
            update[curr] = 0;
            int mid = (tBegin+tEnd)/2;
            long ret = 0;
            if(mid >= begin && tBegin <= end)
                ret += query(2*curr, tBegin, mid,
                    begin, end);
            if(tEnd >= begin && mid+1 <= end)
                ret += query(2*curr+1, mid+1, tEnd,
                    begin, end);
            return ret;
        }
    }
}
```

5.8 Lowest common ancestor

```
const int max_nodes, log_max_nodes;
int num_nodes, log_num_nodes, root;

vector<int> children[max_nodes];        // children[i] contains the
    children of node i
int A[max_nodes][log_max_nodes+1];      // A[i][j] is the 2^j-th
    ancestor of node i, or -1 if that ancestor does not exist
int L[max_nodes];                       // L[i] is the distance
    between node i and the root

// floor of the binary logarithm of n
int lb(unsigned int n)
{
    if(n==0)
        return -1;
    int p = 0;
    if (n >= 1<<16) { n >>= 16; p += 16; }
    if (n >= 1<< 8) { n >>=  8; p +=  8; }
    if (n >= 1<< 4) { n >>=  4; p +=  4; }
    if (n >= 1<< 2) { n >>=  2; p +=  2; }
    if (n >= 1<< 1) {           p +=  1; }
    return p;
}

void DFS(int i, int l)
{
    L[i] = l;
    for(int j = 0; j < children[i].size(); j++)
        DFS(children[i][j], l+1);
}

int LCA(int p, int q)
{
    // ensure node p is at least as deep as node q
    if(L[p] < L[q])
        swap(p, q);

    // "binary search" for the ancestor of node p situated on the same
        level as q
    for(int i = log_num_nodes; i >= 0; i--)
        if(L[p] - (1<<i) >= L[q])
            p = A[p][i];

    if(p == q)
        return p;

    // "binary search" for the LCA
    for(int i = log_num_nodes; i >= 0; i--)
        if(A[p][i] != -1 && A[p][i] != A[q][i])
        {
            p = A[p][i];
            q = A[q][i];
        }

    return A[p][0];
}

int main(int argc,char* argv[])
{
    // read num_nodes, the total number of nodes
    log_num_nodes=lb(num_nodes);

    for(int i = 0; i < num_nodes; i++)
    {
        int p;
        // read p, the parent of node i or -1 if node i is the root

        A[i][0] = p;
        if(p != -1)
            children[p].push_back(i);
        else
            root = i;
    }

    // precompute A using dynamic programming
    for(int j = 1; j <= log_num_nodes; j++)
        for(int i = 0; i < num_nodes; i++)
            if(A[i][j-1] != -1)
                A[i][j] = A[A[i][j-1]][j-1];
            else
                A[i][j] = -1;

    // precompute L
    DFS(root, 0);


    return 0;
}
```

5.9 Sparse Table

```
#include <algorithm>
#include <cmath>
#include <cstdio>
using namespace std;
#define MAX_N 1000                          // adjust this value as
    needed
#define LOG_TWO_N 10              // 2^10 > 1000, adjust this value as
    needed

class RMQ {                                        // Range Minimum
    Query
private:
  int _A[MAX_N], SpT[MAX_N][LOG_TWO_N];
public:
  RMQ(int n, int A[]) {    // constructor as well as pre-processing
      routine
    for (int i = 0; i < n; i++) {
      _A[i] = A[i];
      SpT[i][0] = i; // RMQ of sub array starting at index i + length
          2^0=1
    }
    // the two nested loops below have overall time complexity = O(n
        log n)
    for (int j = 1; (1<<j) <= n; j++) // for each j s.t. 2^j <= n, O(
        log n)
      for (int i = 0; i + (1<<j) - 1 < n; i++)    // for each valid i,
          O(n)
        if (_A[SpT[i][j-1]] < _A[SpT[i+(1<<(j-1))][j-1]])
            // RMQ
          SpT[i][j] = SpT[i][j-1];    // start at index i of length
              2^(j-1)
        else                  // start at index i+2^(j-1) of length
            2^(j-1)
          SpT[i][j] = SpT[i+(1<<(j-1))][j-1];
  }

  int query(int i, int j) {
    int k = (int)floor(log((double)j-i+1) / log(2.0));    // 2^k <= (j
        -i+1)
    if (_A[SpT[i][k]] <= _A[SpT[j-(1<<k)+1][k]]) return SpT[i][k];
    else                                         return SpT[j-(1<<k)
        +1][k];
} };

int main() {
  // same example as in chapter 2: segment tree
  int n = 7, A[] = {18, 17, 13, 19, 15, 11, 20};
  RMQ rmq(n, A);
  for (int i = 0; i < n; i++)
    for (int j = i; j < n; j++)
      printf("RMQ(%d, %d) = %d\n", i, j, rmq.query(i, j));

  return 0;
}
```

6 Miscellaneous

6.1 Nifty Tricks

```
// remove duplicated from vector
#define UNIQUE(x) x.erase(unique(x.begin(), x.end()), x.end())
// filters.erase(unique(filters.begin(), filters.end()), filters.end()
    );

// convert string to int
int myint = stoi("123");

// memset
int res[MAX_V][MAX_V];
memset(res, 0, sizeof res);
fill (myvector.begin(),myvector.begin()+4,5);
int myint1 = stoi(str1); // convert string to int

// Convert int to binary string
cout << bitset<32>(val).to_string() << endl;

// Generate all permutations
sort(nodes.begin(), nodes.end());
do {
  int sum = 0;
  for(int i = 1; i < nodes.size(); i++)
    sum += __builtin_popcount(nodes[i] & nodes[i-1]);
  best = min(best, sum);
 } while(next_permutation(nodes.begin(), nodes.end()));

// Generate all set of n elements
```

```
unsigned next_set_n(unsigned x) {
  unsigned smallest, ripple, new_smallest, ones;
  if(x==0) return 0;
  smallest = (x & -x);
  ripple = x + smallest;
  new_smallest = (ripple & -ripple);
  ones = ((new_smallest/smallest) >> 1 ) - 1;
  return ripple | ones;
}
```

6.2 C++ input/output

```
#include <iostream>
#include <iomanip>

using namespace std;

#define db(x) cerr << #x << "=" << x << endl
#define db2(x, y) cerr << #x << "=" << x << "," << #y << "=" << y <<
      endl
#define db3(x, y, z) cerr << #x << "=" << x << "," << #y << "=" << y
      << "," << #z << "=" << z << endl

#define LSOne(S) (S & (-S))

// remove duplicated from vector
#define UNIQUE(x) x.erase(unique(x.begin(), x.end()), x.end())

// Generate all set of n elements
unsigned next_set_n(unsigned x) {
    unsigned smallest, ripple, new_smallest, ones;
    if(x==0) return 0;
    smallest = (x & -x);
    ripple = x + smallest;
    new_smallest = (ripple & -ripple);
    ones = ((new_smallest/smallest) >> 1 ) - 1;
    return ripple | ones;
}


int main()
{
    // Ouput a specific number of digits past the decimal point,
    // in this case 5
    cout.setf(ios::fixed); cout << setprecision(5);
    cout << 100.0/7.0 << endl;
    cout.unsetf(ios::fixed);

    // Output the decimal point and trailing zeros
    cout.setf(ios::showpoint);
    cout << 100.0 << endl;
    cout.unsetf(ios::showpoint);

    // Output a '+' before positive values
    cout.setf(ios::showpos);
    cout << 100 << " " << -100 << endl;
    cout.unsetf(ios::showpos);

    // Output numerical values in hexadecimal
    cout << hex << 100 << " " << 1000 << " " << 10000 << dec << endl;

    // Convert int to binary string
    cout << bitset<32>(val).to_string() << endl;

}
```

6.3 Longest increasing subsequence

```
// Given a list of numbers of length n, this routine extracts a
// longest increasing subsequence.
//
// Running time: O(n log n)
//
//   INPUT: a vector of integers
//   OUTPUT: a vector containing the longest increasing subsequence
typedef vector<int> VI;
typedef pair<int,int> PII;
typedef vector<PII> VPII;

#define STRICTLY_INCREASNG

VI LongestIncreasingSubsequence(VI v) {
  VPII best;
  VI dad(v.size(), -1);

  for (int i = 0; i < v.size(); i++) {
#ifdef STRICTLY_INCREASNG
    PII item = make_pair(v[i], 0);
    VPII::iterator it = lower_bound(best.begin(), best.end(), item);
    item.second = i;
#else
    PII item = make_pair(v[i], i);
    VPII::iterator it = upper_bound(best.begin(), best.end(), item);
#endif
    if (it == best.end()) {
      dad[i] = (best.size() == 0 ? -1 : best.back().second);
      best.push_back(item);
    } else {
      dad[i] = it == best.begin() ? -1 : prev(it)->second;
      *it = item;
    }
  }

  VI ret;
  for (int i = best.back().second; i >= 0; i = dad[i])
    ret.push_back(v[i]);
  reverse(ret.begin(), ret.end());
  return ret;
}
```

6.4 Median Max/Min Heap

```
#include <bits/stdc++.h>
using namespace std;

int main() {
        priority_queue<int> maxPQ;
        priority_queue<int, vector<int>, greater<int> > minPQ;
        string s;
        while(cin >> s) {
                if (s == "#") {
                        int m = minPQ.top(); minPQ.pop();
                        if (minPQ.size() != maxPQ.size()) {
                                minPQ.push(maxPQ.top());
                                maxPQ.pop();
                        }
                        cout << m << endl;
                } else {
                        int c = stoi(s);
                        if(!minPQ.empty() && c > minPQ.top()) {
                                minPQ.push(c);
                                if(minPQ.size() > maxPQ.size() + 1) {
                                        int d = minPQ.top(); minPQ.pop
                                            ();
                                        maxPQ.push(d);
                                }
                        } else {
                                maxPQ.push(c);
                                if (maxPQ.size() > minPQ.size()) {
                                        minPQ.push(maxPQ.top());
                                        maxPQ.pop();
                                }
                        }
                }
        }
}
```

6.5 Dates

```
// Routines for performing computations on dates.  In these routines,
// months are expressed as integers from 1 to 12, days are expressed
// as integers from 1 to 31, and years are expressed as 4-digit
// integers.
string dayOfWeek[] = {"Mon", "Tue", "Wed", "Thu", "Fri", "Sat", "Sun"
    };

// converts Gregorian date to integer (Julian day number)
int dateToInt (int m, int d, int y){
  return
    1461 * (y + 4800 + (m - 14) / 12) / 4 +
    367 * (m - 2 - (m - 14) / 12 * 12) / 12 -
    3 * ((y + 4900 + (m - 14) / 12) / 100) / 4 +
    d - 32075;
}

// converts integer (Julian day number) to Gregorian date: month/day/
    year
void intToDate (int jd, int &m, int &d, int &y){
  int x, n, i, j;

  x = jd + 68569;
  n = 4 * x / 146097;
  x -= (146097 * n + 3) / 4;
  i = (4000 * (x + 1)) / 1461001;
  x -= 1461 * i / 4 - 31;
  j = 80 * x / 2447;
  d = x - 2447 * j / 80;
  x = j / 11;
  m = j + 2 - 12 * x;
  y = 100 * (n - 49) + i + x;
}

// converts integer (Julian day number) to day of week
string intToDay (int jd){
  return dayOfWeek[jd % 7];
}

int main (int argc, char **argv){
  int jd = dateToInt (3, 24, 2004);
  int m, d, y;
  intToDate (jd, m, d, y);
  string day = intToDay (jd);

  // expected output:
  //    2453089
  //    3/24/2004
  //    Wed
  cout << jd << endl
    << m << "/" << d << "/" << y << endl
    << day << endl;
}
```

6.6 Regular expressions

```
// Code which demonstrates the use of Java's regular expression
    libraries.
// This is a solution for
//
//   Loglan: a logical language
//   http://acm.uva.es/p/v1/134.html
//
// In this problem, we are given a regular language, whose rules can
    be
// inferred directly from the code.  For each sentence in the input,
    we must
// determine whether the sentence matches the regular expression or
    not.  The
// code consists of (1) building the regular expression (which is
    fairly
// complex) and (2) using the regex to match sentences.

import java.util.*;
import java.util.regex.*;

public class LogLan {

    public static String BuildRegex (){
        String space = " +";

        String A = "([aeiou])";
        String C = "([a-z&&[^aeiou]])";
        String MOD = "(g" + A + ")";
        String BA = "(b" + A + ")";
        String DA = "(d" + A + ")";
        String LA = "(l" + A + ")";
        String NAM = "([a-z]*" + C + ")";
        String PREDA = "(" + C + C + A + C + A + "|" + C + A + C + C +
             A + ")";

        String predstring = "(" + PREDA + "(" + space + PREDA + ")*)";
        String predname = "(" + LA + space + predstring + "|" + NAM +
            ")";
        String preds = "(" + predstring + "(" + space + A + space +
            predstring + ")*)";
```

```
        String predclaim = "(" + predname + space + BA + space + preds
               + "|" + DA + space +
            preds + ")";
        String verbpred = "(" + MOD + space + predstring + ")";
        String statement = "(" + predname + space + verbpred + space +
               predname + "|" +
            predname + space + verbpred + ")";
        String sentence = "(" + statement + "|" + predclaim + ")";

        return "^" + sentence + "$";
    }

    public static void main (String args[]){

        String regex = BuildRegex();
        Pattern pattern = Pattern.compile (regex);

        Scanner s = new Scanner(System.in);
        while (true) {

            // In this problem, each sentence consists of multiple
                  lines, where the last
            // line is terminated by a period.  The code below reads
                  lines until
            // encountering a line whose final character is a '.'.
                  Note the use of
            //
            //    s.length() to get length of string
            //    s.charAt() to extract characters from a Java string
            //    s.trim() to remove whitespace from the beginning and
                   end of Java string
            //
            // Other useful String manipulation methods include
            //
            //    s.compareTo(t) < 0 if s < t, lexicographically
            //    s.indexOf("apple") returns index of first occurrence
                   of "apple" in s
            //    s.lastIndexOf("apple") returns index of last
                  occurrence of "apple" in s
            //    s.replace(c,d) replaces occurrences of character c
                  with d
            //    s.startsWith("apple) returns (s.indexOf("apple") ==
                  0)
            //    s.toLowerCase() / s.toUpperCase() returns a new
                  lower/uppercased string
            //
            //    Integer.parseInt(s) converts s to an integer (32-bit
                  )
            //    Long.parseLong(s) converts s to a long (64-bit)
            //    Double.parseDouble(s) converts s to a double

            String sentence = "";
            while (true){
                sentence = (sentence + " " + s.nextLine()).trim();
                if (sentence.equals("#")) return;
                if (sentence.charAt(sentence.length()-1) == '.') break
                      ;
            }

            // now, we remove the period, and match the regular
                  expression

            String removed_period = sentence.substring(0, sentence.
                  length()-1).trim();
            if (pattern.matcher (removed_period).find()){
                System.out.println ("Good");
            } else {
                System.out.println ("Bad!");
            }
        }
    }
}
```

6.7 Prime numbers

```
typedef unsigned long long ll;
typedef vector<ll> vll;
typedef vector<int> vi;

ll _sieve_size;
bitset<10000010> bs;
vll primes;

void sieve(ll upper) {
    _sieve_size = upper + 1;
    bs.set(); // set all to one
    bs[0] = bs[1] = 0;
    for(ll i = 2; i < _sieve_size; i++) if (bs[i]) {
            for(ll j = i*i; j < _sieve_size; j+= i) {
                bs[j] = 0;
            }
            primes.push_back((int) i);
        }
}

bool isPrime(ll n) {
    if (n <= _sieve_size) return bs[n];
    for(int i = 0; i < (int) primes.size(); i++) {
        if (n % primes[i] == 0) return false;
        if (primes[i] * primes[i] > n) return true;
    }
    return true;
}

bool isPrime_slow(ll n) {
    if(n < 2) return false;
    if(n == 2 || n == 3) return true;
    if(n % 2 == 0 || n % 3 == 0) return false;
    int limit = sqrt(n);
    for(int i = 5; i <= limit; i += 6) {
        if(n % i == 0 || n % (i+2) == 0)
            return false;
    }
    return true;
}


vi primeFactors(ll N) {
    vi factors;
    ll PF_index = 0; ll PF = primes[PF_index];
    while(PF*PF <= N) {
        while(N%PF == 0) {
            N /= PF; factors.push_back(PF);
        }
        PF = primes[++PF_index];
    }
    if(N != 1) factors.push_back(N);
    return factors;
}


// Primes less than 1000:
//      2     3     5     7    11    13    17    19    23    29    31
         37
//     41    43    47    53    59    61    67    71    73    79    83
         89
//     97   101   103   107   109   113   127   131   137   139   149
        151
//    157   163   167   173   179   181   191   193   197   199   211
        223
//    227   229   233   239   241   251   257   263   269   271   277
        281
//    283   293   307   311   313   317   331   337   347   349   353
        359
//    367   373   379   383   389   397   401   409   419   421   431
        433
//    439   443   449   457   461   463   467   479   487   491   499
        503
//    509   521   523   541   547   557   563   569   571   577   587
        593
//    599   601   607   613   617   619   631   641   643   647   653
        659
//    661   673   677   683   691   701   709   719   727   733   739
        743
//    751   757   761   769   773   787   797   809   811   821   823
        827
//    829   839   853   857   859   863   877   881   883   887   907
        911
//    919   929   937   941   947   953   967   971   977   983   991
        997

// Other primes: largest prime smaller than X is Y
//    10 is 7.
//    100 is 97.
//    1000 is 997.
//    10000 is 9973.
//    100000 is 99991.
//    1000000 is 999983.
//    10000000 is 9999991.
//    100000000 is 99999989.
//    1000000000 is 999999937.
//    10000000000 is 9999999967.
//    100000000000 is 99999999977.
//    1000000000000 is 999999999989.
//    10000000000000 is 9999999999971.
//    100000000000000 is 99999999999973.
//    1000000000000000 is 999999999999989.
//    10000000000000000 is 9999999999999937.
//    100000000000000000 is 99999999999999997.
//    1000000000000000000 is 999999999999999989.
```

6.8 Miller-Rabin Primality Test (C)

```
// Randomized Primality Test (Miller-Rabin):
//   Error rate: 2^(-TRIAL)
//   Almost constant time. srand is needed

#include <stdlib.h>
#define EPS 1e-7

typedef long long LL;

LL ModularMultiplication(LL a, LL b, LL m)
{
        LL ret=0, c=a;
        while(b)
        {
                if(b&1) ret=(ret+c)%m;
                b>>=1; c=(c+c)%m;
        }
        return ret;
}
LL ModularExponentiation(LL a, LL n, LL m)
{
        LL ret=1, c=a;
        while(n)
        {
                if(n&1) ret=ModularMultiplication(ret, c, m);
                n>>=1; c=ModularMultiplication(c, c, m);
        }
        return ret;
}
bool Witness(LL a, LL n)
{
        LL u=n-1;
  int t=0;
        while(!(u&1)){u>>=1; t++;}
        LL x0=ModularExponentiation(a, u, n), x1;
        for(int i=1;i<=t;i++)
        {
                x1=ModularMultiplication(x0, x0, n);
                if(x1==1 && x0!=1 && x0!=n-1) return true;
                x0=x1;
        }
        if(x0!=1) return true;
        return false;
}
LL Random(LL n)
{
  LL ret=rand(); ret*=32768;
        ret+=rand(); ret*=32768;
        ret+=rand(); ret*=32768;
        ret+=rand();
  return ret%n;
}
bool IsPrimeFast(LL n, int TRIAL)
{
  while(TRIAL--)
  {
    LL a=Random(n-2)+1;
    if(Witness(a, n)) return false;
  }
  return true;
}
```

6.9 Knuth-Morris-Pratt

```
/*
Finds all occurrences of the pattern string p within the
text string t. Running time is O(n + m), where n and m
are the lengths of p and t, respecitvely.
*/
typedef vector<int> VI;
```

```
void buildPi(string& p, VI& pi)
{
  pi = VI(p.length());
  int k = -2;
  for(int i = 0; i < p.length(); i++) {
    while(k >= -1 && p[k+1] != p[i])
      k = (k == -1) ? -2 : pi[k];
    pi[i] = ++k;
  }
}

int KMP(string& t, string& p)
{
  VI pi;
  buildPi(p, pi);
  int k = -1;
  for(int i = 0; i < t.length(); i++) {
    while(k >= -1 && p[k+1] != t[i])
      k = (k == -1) ? -2 : pi[k];
    k++;
    if(k == p.length() - 1) {
      // p matches t[i-m+1, ..., i]
      cout << "matched at index " << i-k << ": ";
      cout << t.substr(i-k, p.length()) << endl;
      k = (k == -1) ? -2 : pi[k];
    }
  }
  return 0;
}

int main()
{
  string a = "AABAACAADAABAABA", b = "AABA";
  KMP(a, b); // expected matches at: 0, 9, 12
  return 0;
}
```

6.10 Latitude/longitude

```
/*
Converts from rectangular coordinates to latitude/longitude and vice
versa. Uses degrees (not radians).
*/
struct ll
{
  double r, lat, lon;
};

struct rect
{
  double x, y, z;
};

ll convert(rect& P)
{
  ll Q;
  Q.r = sqrt(P.x*P.x+P.y*P.y+P.z*P.z);
  Q.lat = 180/M_PI*asin(P.z/Q.r);
  Q.lon = 180/M_PI*acos(P.x/sqrt(P.x*P.x+P.y*P.y));

  return Q;
}

rect convert(ll& Q)
{
  rect P;
  P.x = Q.r*cos(Q.lon*M_PI/180)*cos(Q.lat*M_PI/180);
  P.y = Q.r*sin(Q.lon*M_PI/180)*cos(Q.lat*M_PI/180);
  P.z = Q.r*sin(Q.lat*M_PI/180);

  return P;
}

int main()
{
  rect A;
  ll B;

  A.x = -1.0; A.y = 2.0; A.z = -3.0;

  B = convert(A);
  cout << B.r << " " << B.lat << " " << B.lon << endl;

  A = convert(B);
  cout << A.x << " " << A.y << " " << A.z << endl;
}
```

6.11 Topological sort (C++)

```
// This function uses performs a non-recursive topological sort.
//
// Running time: O(|V|^2).  If you use adjacency lists (vector<map<int
//     > >),
//              the running time is reduced to O(|E|).
//
//   INPUT:   w[i][j] = 1 if i should come before j, 0 otherwise
//   OUTPUT:  a permutation of 0,...,n-1 (stored in a vector)
//            which represents an ordering of the nodes which
//            is consistent with w
//
// If no ordering is possible, false is returned.
typedef double T;
typedef vector<T> VT;
typedef vector<VT> VVT;

typedef vector<int> VI;
typedef vector<VI> VVI;

bool TopologicalSort (const VVI &w, VI &order){
  int n = w.size();
  VI parents (n);
  queue<int> q;
  order.clear();

  for (int i = 0; i < n; i++){
    for (int j = 0; j < n; j++)
      if (w[j][i]) parents[i]++;
      if (parents[i] == 0) q.push (i);
  }

  while (q.size() > 0){
    int i = q.front();
    q.pop();
    order.push_back (i);
    for (int j = 0; j < n; j++) if (w[i][j]){
      parents[j]--;
      if (parents[j] == 0) q.push (j);
    }
  }

  return (order.size() == n);
}
```

6.12 Random STL stuff

```
// Example for using stringstreams and next_permutation
int main(void){
  vector<int> v;

  v.push_back(1);
  v.push_back(2);
  v.push_back(3);
  v.push_back(4);

  // Expected output: 1 2 3 4
  //                  1 2 4 3
  //                  ...
  //                  4 3 2 1
  do {
    ostringstream oss;
    oss << v[0] << " " << v[1] << " " << v[2] << " " << v[3];

    // for input from a string s,
    //   istringstream iss(s);
    //   iss >> variable;

    cout << oss.str() << endl;
  } while (next_permutation (v.begin(), v.end()));

  v.clear();

  v.push_back(1);
  v.push_back(2);
  v.push_back(1);
  v.push_back(3);

  // To use unique, first sort numbers.  Then call
  // unique to place all the unique elements at the beginning
  // of the vector, and then use erase to remove the duplicate
  // elements.

  sort(v.begin(), v.end());
  v.erase(unique(v.begin(), v.end()), v.end());

  // Expected output: 1 2 3
  for (size_t i = 0; i < v.size(); i++)
    cout << v[i] << " ";
  cout << endl;
}
```

6.13 Constraint satisfaction problems

```
// Constraint satisfaction problems
#define DONE   -1
#define FAILED -2

typedef vector<int> VI;
typedef vector<VI> VVI;
typedef vector<VVI> VVVI;

typedef set<int> SI;

// Lists of assigned/unassigned variables.
VI assigned_vars;
SI unassigned_vars;

// For each variable, a list of reductions (each of which a list of
//     eliminated
// variables)
VVVI reductions;

// For each variable, a list of the variables whose domains it reduced
//      in
// forward-checking.
VVI forward_mods;

// need to implement ----------------------------
int Value(int var);

void SetValue(int var, int value);
void ClearValue(int var);

int DomainSize(int var);
void ResetDomain(int var);
void AddValue(int var, int value);
void RemoveValue(int var, int value);

int NextVar() {
  if ( unassigned_vars.empty() ) return DONE;

  // could also do most constrained...
  int var = *unassigned_vars.begin();
  return var;
}

int Initialize() {
  // setup here
  return NextVar();
}
// ------------------------- end -- need to implement


void UpdateCurrentDomain(int var) {
  ResetDomain(var);
  for (int i = 0; i < reductions[var].size(); i++) {
    vector<int>& red = reductions[var][i];
    for (int j = 0; j < red.size(); j++) {
      RemoveValue(var, red[j]);
    }
  }
}


void UndoReductions(int var) {
  for (int i = 0; i < forward_mods[var].size(); i++) {
    int other_var = forward_mods[var][i];
    VI& red = reductions[other_var].back();
    for (int j = 0; j < red.size(); j++) {
```

```
      AddValue(other_var, red[j]);
    }
    reductions[other_var].pop_back();
  }
  forward_mods[var].clear();
}


bool ForwardCheck(int var, int other_var) {
  vector<int> red;

  foreach value in current_domain(other_var) {
    SetValue(other_var, value);
    if ( !Consistent(var, other_var) ) {
      red.push_back(value);
      RemoveValue(other_var, value);
    }
    ClearValue(other_var);
  }
  if ( !red.empty() ) {
    reductions[other_var].push_back(red);
    forward_mods[var].push_back(other_var);
  }

  return DomainSize(other_var) != 0;
}


pair<int, bool> Unlabel(int var) {
  assigned_vars.pop_back();
  unassigned_vars.insert(var);

  UndoReductions(var);
  UpdateCurrentDomain(var);

  if ( assigned_vars.empty() ) return make_pair(FAILED, true);

  int prev_var = assigned_vars.back();
  RemoveValue(prev_var, Value(prev_var));
  ClearValue(prev_var);
  if ( DomainSize(prev_var) == 0 ) {
    return make_pair(prev_var, false);
  } else {
    return make_pair(prev_var, true);
  }
}


pair<int, bool> Label(int var) {
  unassigned_vars.erase(var);
  assigned_vars.push_back(var);

  bool consistent;
  foreach value in current_domain(var) {
    SetValue(var, value);
    consistent = true;
    for (int j=0; j<unassigned_vars.size(); j++) {
      int other_var = unassigned_vars[j];
      if ( !ForwardCheck(var, other_var) ) {
        RemoveValue(var, value);
        consistent = false;
        UndoReductions(var);
        ClearValue(var);
        break;
      }
    }
    if ( consistent ) return (NextVar(), true);
  }
  return make_pair(var, false);
}


void BacktrackSearch(int num_var) {
  // (next variable to mess with, whether current state is consistent)
  pair<int, bool> var_consistent = make_pair(Initialize(), true);
  while ( true ) {
    if ( var_consistent.second ) var_consistent = Label(var_consistent
          .first);
    else var_consistent = Unlabel(var_consistent.first);

    if ( var_consistent.first == DONE ) return; // solution found
    if ( var_consistent.first == FAILED ) return; // no solution
  }
}
```

6.14 Fast exponentiation

```
/*
Uses powers of two to exponentiate numbers and matrices. Calculates
n^k in O(log(k)) time when n is a number. If A is an n x n matrix,
calculates A^k in O(n^3*log(k)) time.
*/
typedef double T;
typedef vector<T> VT;
typedef vector<VT> VVT;

T power(T x, int k) {
  T ret = 1;

  while(k) {
    if(k & 1) ret *= x;
    k >>= 1; x *= x;
  }
  return ret;
}

VVT multiply(VVT& A, VVT& B) {
  int n = A.size(), m = A[0].size(), k = B[0].size();
  VVT C(n, VT(k, 0));

  for(int i = 0; i < n; i++)
    for(int j = 0; j < k; j++)
      for(int l = 0; l < m; l++)
        C[i][j] += A[i][l] * B[l][j];

  return C;
}

VVT power(VVT& A, int k) {
  int n = A.size();
  VVT ret(n, VT(n)), B = A;
  for(int i = 0; i < n; i++) ret[i][i]=1;

  while(k) {
    if(k & 1) ret = multiply(ret, B);
    k >>= 1; B = multiply(B, B);
  }
  return ret;
}

int main()
{
  /* Expected Output:
     2.37^48 = 9.72569e+17

     376 264 285 220 265
     550 376 529 285 484
     484 265 376 264 285
     285 220 265 156 264
     529 285 484 265 376 */
  double n = 2.37;
  int k = 48;

  cout << n << "^" << k << " = " << power(n, k) << endl;

  double At[5][5] = {
    { 0, 0, 1, 0, 0 },
    { 1, 0, 0, 1, 0 },
    { 0, 0, 0, 0, 1 },
    { 1, 0, 0, 0, 0 },
    { 0, 1, 0, 0, 0 } };

  vector <vector <double> > A(5, vector <double>(5));
  for(int i = 0; i < 5; i++)
    for(int j = 0; j < 5; j++)
      A[i][j] = At[i][j];

  vector <vector <double> > Ap = power(A, k);

  cout << endl;
  for(int i = 0; i < 5; i++) {
    for(int j = 0; j < 5; j++)
      cout << Ap[i][j] << " ";
    cout << endl;
  }
}
```

6.15 Longest common subsequence

```
/*
Calculates the length of the longest common subsequence of two vectors
    .
Backtracks to find a single subsequence or all subsequences. Runs in
O(m*n) time except for finding all longest common subsequences, which
may be slow depending on how many there are.
*/
typedef int T;
typedef vector<T> VT;
typedef vector<VT> VVT;

typedef vector<int> VI;
typedef vector<VI> VVI;

void backtrack(VVI& dp, VT& res, VT& A, VT& B, int i, int j)
{
  if(!i || !j) return;
  if(A[i-1] == B[j-1]) { res.push_back(A[i-1]); backtrack(dp, res, A,
       B, i-1, j-1); }
  else
  {
    if(dp[i][j-1] >= dp[i-1][j]) backtrack(dp, res, A, B, i, j-1);
    else backtrack(dp, res, A, B, i-1, j);
  }
}

void backtrackall(VVI& dp, set<VT>& res, VT& A, VT& B, int i, int j)
{
  if(!i || !j) { res.insert(VI()); return; }
  if(A[i-1] == B[j-1])
  {
    set<VT> tempres;
    backtrackall(dp, tempres, A, B, i-1, j-1);
    for(set<VT>::iterator it=tempres.begin(); it!=tempres.end(); it++)
    {
      VT temp = *it;
      temp.push_back(A[i-1]);
      res.insert(temp);
    }
  }
  else
  {
    if(dp[i][j-1] >= dp[i-1][j]) backtrackall(dp, res, A, B, i, j-1);
    if(dp[i][j-1] <= dp[i-1][j]) backtrackall(dp, res, A, B, i-1, j);
  }
}

VT LCS(VT& A, VT& B)
{
  VVI dp;
  int n = A.size(), m = B.size();
  dp.resize(n+1);
  for(int i=0; i<=n; i++) dp[i].resize(m+1, 0);

  for(int i=1; i<=n; i++)
    for(int j=1; j<=m; j++)
    {
      if(A[i-1] == B[j-1]) dp[i][j] = dp[i-1][j-1]+1;
      else dp[i][j] = max(dp[i-1][j], dp[i][j-1]);
    }

  VT res;
  backtrack(dp, res, A, B, n, m);
  reverse(res.begin(), res.end());
  return res;
}

set<VT> LCSall(VT& A, VT& B)
{
  VVI dp;
  int n = A.size(), m = B.size();
  dp.resize(n+1);
  for(int i=0; i<=n; i++) dp[i].resize(m+1, 0);
  for(int i=1; i<=n; i++)
    for(int j=1; j<=m; j++)
    {
      if(A[i-1] == B[j-1]) dp[i][j] = dp[i-1][j-1]+1;
      else dp[i][j] = max(dp[i-1][j], dp[i][j-1]);
    }
  set<VT> res;
  backtrackall(dp, res, A, B, n, m);
  return res;
}
```

```cpp
int main()
{
  int a[] = { 0, 5, 5, 2, 1, 4, 2, 3 }, b[] = { 5, 2, 4, 3, 2, 1, 2,
        1, 3 };
  VI A = VI(a, a+8), B = VI(b, b+9);
  VI C = LCS(A, B);

  for(int i=0; i<C.size(); i++) cout << C[i] << " ";
  cout << endl << endl;

  set <VI> D = LCSall(A, B);
  for(set<VI>::iterator it = D.begin(); it != D.end(); it++)
  {
    for(int i=0; i<(*it).size(); i++) cout << (*it)[i] << " ";
    cout << endl;
  }
}
```

6.16 Binary Search

```cpp
// n is size of array, c is value looking for
// sematically equiv to std::lower_bound and std::upper_bound
int lower_bound(int A[], int n, int c) {
        int l = 0, r = n;
        while(l < r) {
                int m = (r-l)/2+l;
                if(A[m] < c) l = m+1; else r=m;
        }
        return l;
}

int upper_bound(int A[], int n, int c) {
        int l = 0, r = n;
        while(l < r) {
                int m = (r-l)/2+l;
                if(A[m] <= c) l = m+1; else r=m;
        }
        return l;
}
```