# Chapter 4

## Probability Concepts

# Section 4.1

## Probability Basics

Probability is a measure of uncertainty and is the proportion of times an event happens when an experiment is repeated a large number of times

# Definition 4.1

**Probability for Equally Likely Outcomes ($f/N$ Rule)**

Suppose an experiment has $N$ possible outcomes, all equally likely. An event that can occur in $f$ ways has probability $f/N$ of occurring:

$$\text{Probability of an event} = \frac{f}{N}.$$

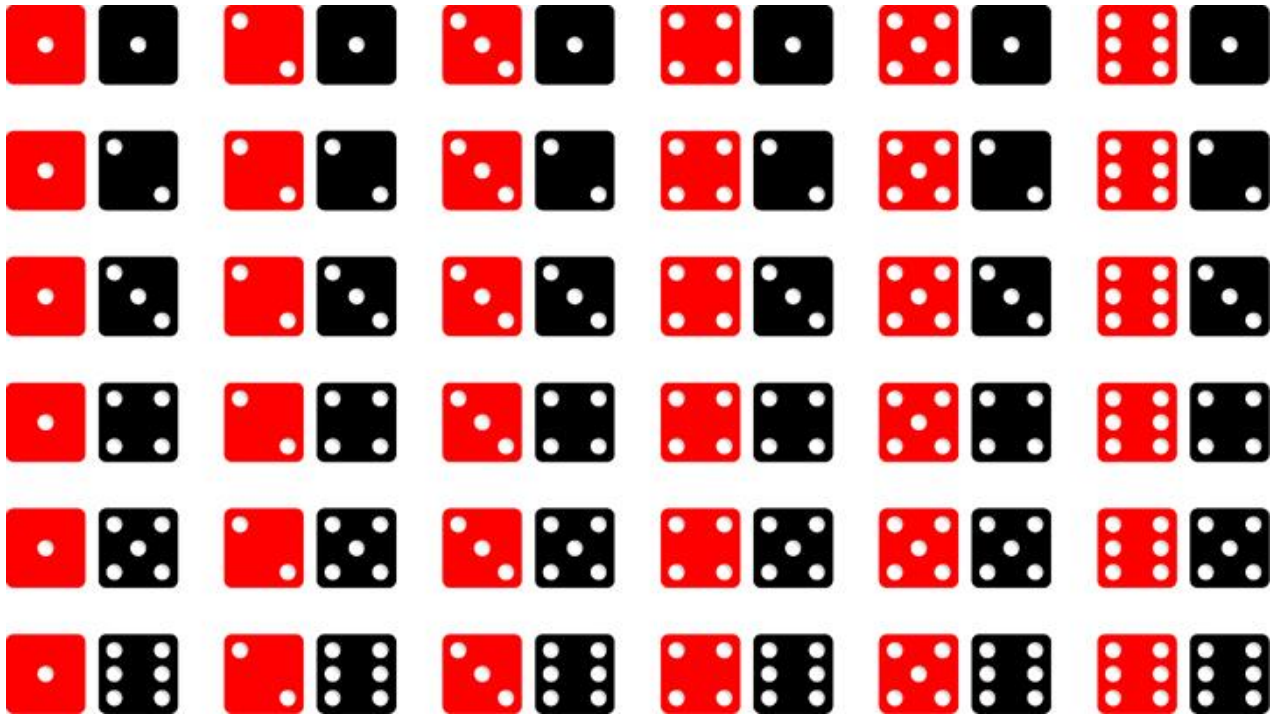
$f$: Number of ways event can occur

$N$: Total number of possible outcomes

# Figure 4.1

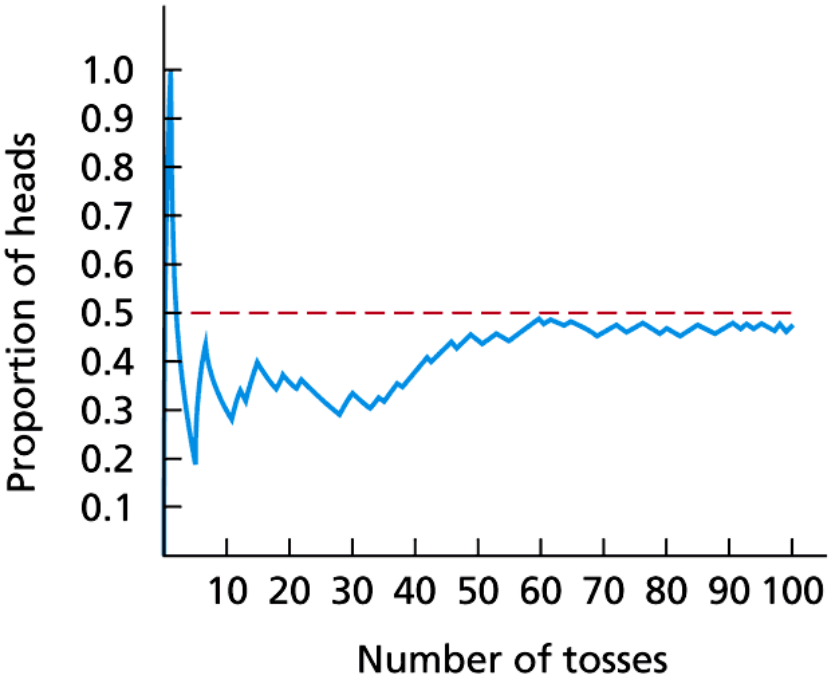
Possible outcomes for rolling a pair of dice

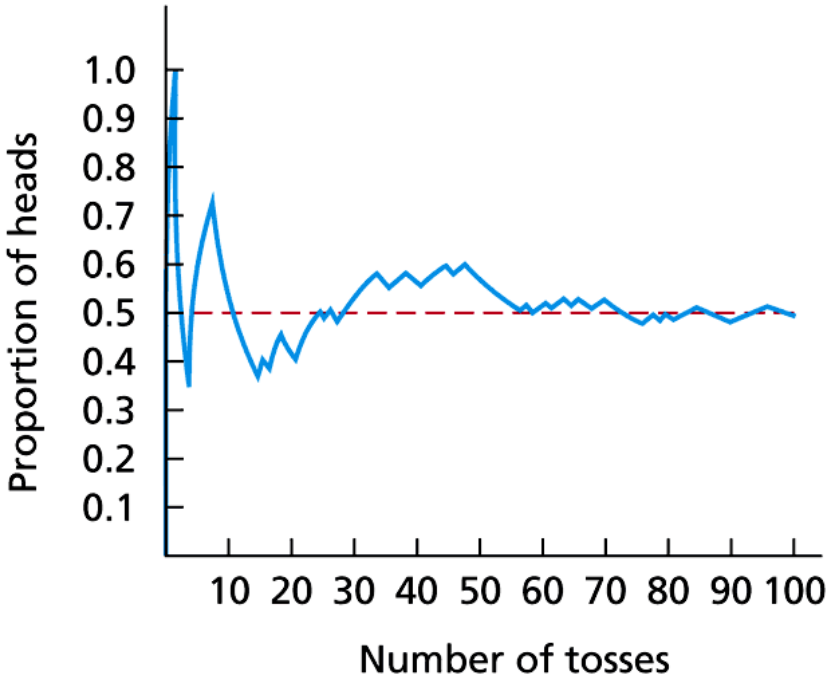


# Figure 4.2

Two computer simulations of tossing a balanced coin 100 times



# Key Fact 4.1

## Basic Properties of Probabilities

**Property 1:** The probability of an event is always between 0 and 1, inclusive.

**Property 2:** The probability of an event that cannot occur is 0. (An event that cannot occur is called an **impossible event.**)

**Property 3:** The probability of an event that must occur is 1. (An event that must occur is called a **certain event.**)

Ex 4.22 # of Nobel Prize winners by country

| Country | Winners (f) |
|---|---|
| US | 338 |
| Uk | 119 |
| Germany | 103 |
| France | 59 |
| Sweden | 29 |
| Switzerland | 25 |
| other countries | 166 |
| Total | 839 = N |

a) Prob(selecting a Nobel Laureate from Sweden) $= \frac{29}{839} = \frac{f}{N} = 0.034$

b) P( " from either France or Germany) $= \frac{59+103}{839} = \frac{162}{839}$

$\therefore = 0.193$

c) P( " any other country other than US) $= \frac{839-338}{839} = \frac{501}{839} = 0.597$

# Section 4.2

## Events

# Definition 4.2

**Sample Space and Event**

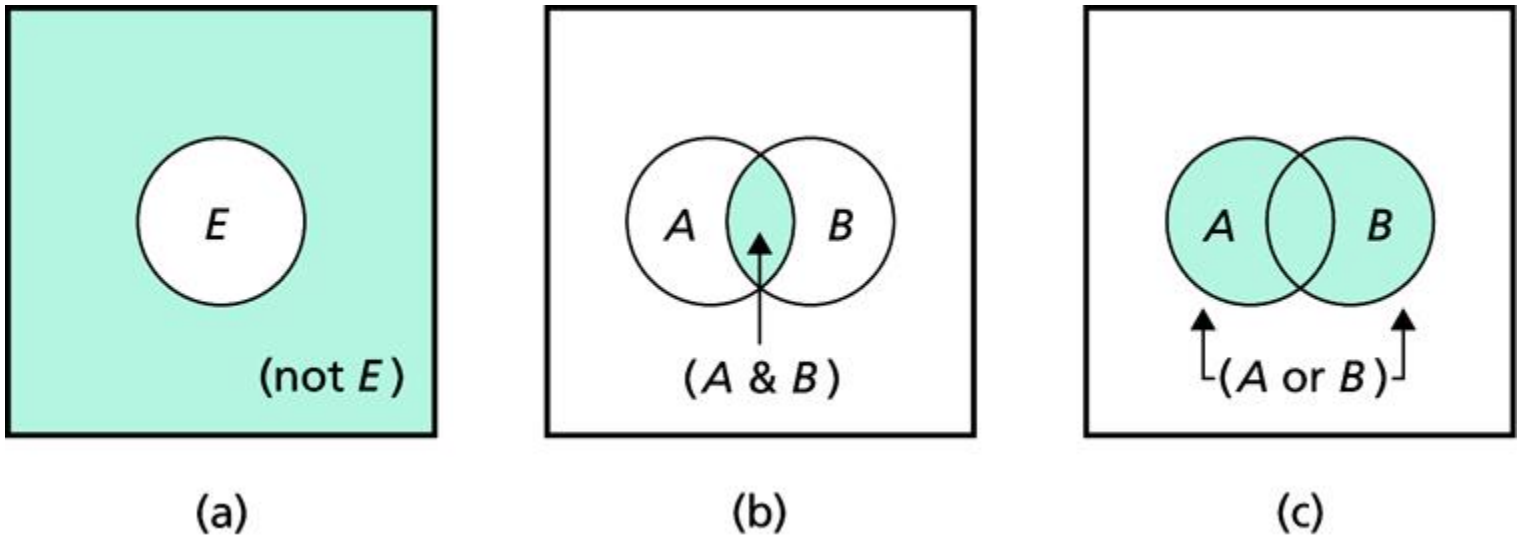
**Sample space:** The collection of all possible outcomes for an experiment.

**Event:** A collection of outcomes for the experiment, that is, any subset of the sample space. An event **occurs** if and only if the outcome of the experiment is a member of the event.

# Figure 4.9

Venn diagrams for (a) event (not E), (b) event (A & B), and (c) event (A or B)



# Definition 4.3

**Relationships Among Events**

**(not *E*):** The event "*E* does not occur"

**(*A* & *B*):** The event "both *A* and *B* occur"

**(*A* or *B*):** The event "either *A* or *B* or both occur"

# Definition 4.4

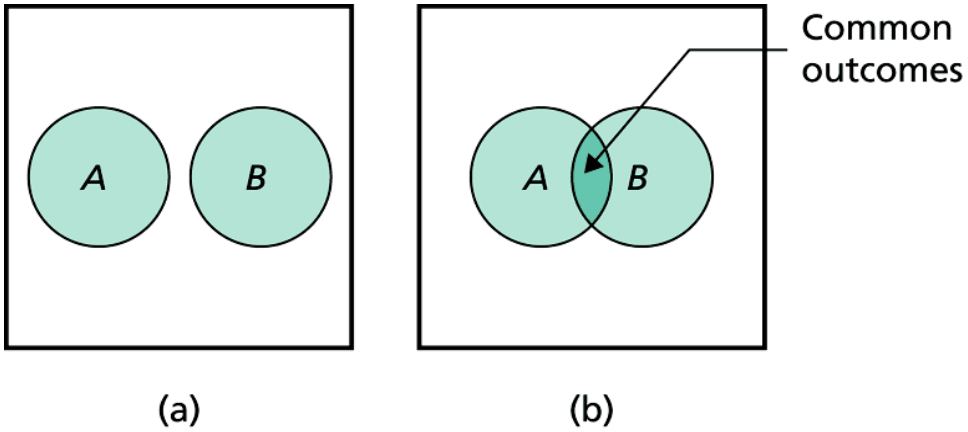
**Mutually Exclusive Events**

Two or more events are **mutually exclusive events** if no two of them have outcomes in common.

# Figure 4.14

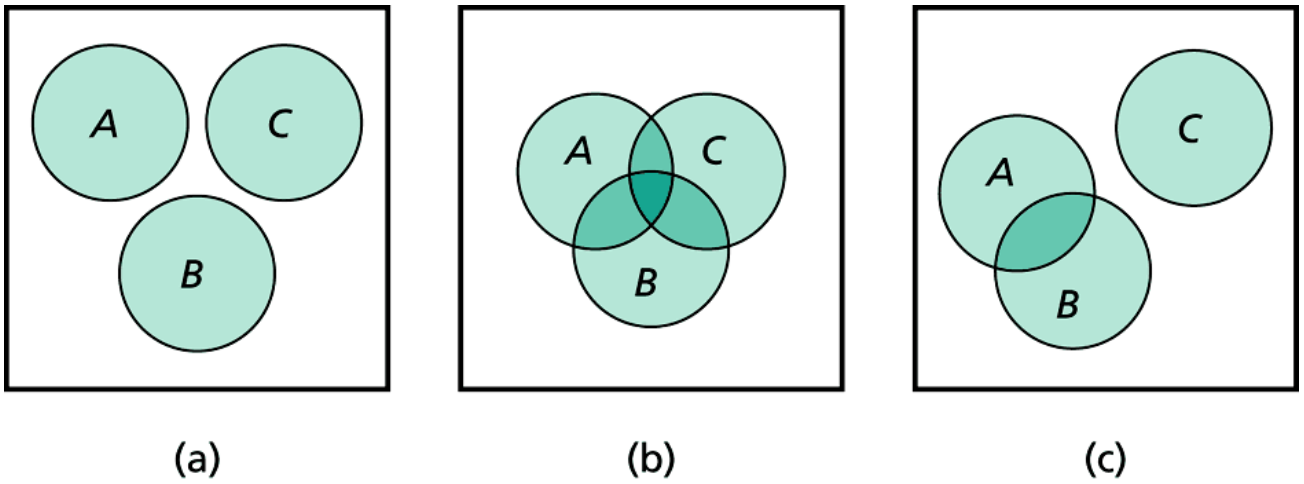
(a) Two mutually exclusive events;
(b) two non–mutually exclusive events



# Figure 4.15

(a) Three mutually exclusive events;
(b) three non–mutually exclusive events;
(c) three non–mutually exclusive events



Ex 4.53. A die is rolled

$S = \{1, 2, 3, 4, 5, 6\}$

A = event die comes up even = $\{2, 4, 6\}$

B = " " " " 4 or more = $\{4, 5, 6\}$

C = " " " " at most 2 = $\{1, 2\}$

D = " " " " 3 = $\{3\}$

Ex 4.57 a) not A = $\{1, 3, 5\}$ → event that the die comes up odd

b) A and B = $\{4, 6\}$ → set of outcomes that are common to both A and B

c) B or C = $\{1, 2, 4, 5, 6\}$ → Set of outcomes that are either in A or in B or in both

Ex 4.67 a) Events A and B are <u>not</u> mutually exclusive.

b) Events B and C are mutually exclusive.

c) A and C are not mutually exclusive, but C and D are.

d) 3 mutually exclusive events B, C, D. Not four.

## Ex 4.63 World Series

| Games required | Frequency |
|---|---|
| 4 | 21 |
| 5 | 24 |
| 6 | 24 |
| 7 | 36 |

A: World Series decided in 4 games
B: " " " < 6 games
C: " " " in 7 games

a) not A : Event that World Series was decided in 5 or more games → 84 (24+24+36) World Series make up the event 'not A'

b) A and B: World Series was decided in 4 games → 21 World Series make up the event 'A and B'

c) A or C: World Series was decided in either 4 or 7 games → 57 (21+36) World Series make up the event 'A or C'

d) A and C: World Series was decided in both 4 and 7 games which is impossible → No world Series make up this event

# Section 4.3

## Some Rules of Probability

# Definition 4.5

**Probability Notation**

If *E* is an event, then ***P*(*E*)** represents the probability that event *E* occurs. It is read “the probability of *E*.”

# Formula 4.1

**The Special Addition Rule**

If event $A$ and event $B$ are mutually exclusive, then

$$P(A \text{ or } B) = P(A) + P(B).$$

More generally, if events $A$, $B$, $C$, ... are mutually exclusive, then

$$P(A \text{ or } B \text{ or } C \text{ or } \cdots) = P(A) + P(B) + P(C) + \cdots .$$

# Formula 4.2

**The Complementation Rule**

For any event $E$,

$$P(E) = 1 - P(\text{not } E).$$

Ex 4.89

| Cigs per day | Percentage | Event |
|---|---|---|
| None | 73.3 | A |
| less than 1 | 9.8 | B |
| 1-5 | 7.8 | C |
| 6-14 | 5.3 | D |
| 15-25 | 2.8 | E |
| 26-34 | 0.7 | F |
| 35 or more | 0.3 | G |
| | 100.0 | |

a) $P(\text{Selected student smoked}) = P(\text{not } A) = P(B)+P(C)+P(D)+P(E)+P(F)+P(G)$

$= 100 - 73.3 = \underline{26.7\%} = \underline{0.267}$ Complement Rule

b) $P(\text{student smoked at least one cig per day} \Rightarrow 1 \text{ or more}) = 100 - 73.3 - 9.8 = \underline{16.9\%} = \underline{0.169}$

$= P(C)+P(D)+P(E)+P(F)+P(G)$ Complement rule

c) $P(\text{student smoked between 6 and 34 cigs/day inclusive}) = 5.3 + 2.8 + 0.7 = \underline{8.8\%} = \underline{0.088}$

$= P(D)+P(E)+P(F)$

Events A through G are all mutually exclusive

# Formula 4.3

**The General Addition Rule**

If $A$ and $B$ are any two events, then

$$P(A \text{ or } B) = P(A) + P(B) - P(A \,\&\, B).$$

Refer Ex 4.53 A die is rolled

$S = \{1, 2, 3, 4, 5, 6\}$

$A = \{2, 4, 6\}$

$B = \{4, 5, 6\}$

$C = \{1, 2\}$

$D = \{3\}$

a) $P(B \text{ or } C) = P(B) + P(C)$ Special Addition Rule

$= \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$

'B' and 'C' are mutually exclusive events, they cannot happen at the same time

b) $P(A \text{ or } C) = P(A) + P(C) - P(A \text{ and } C)$ General Addition Rule

$= \frac{3}{6} + \frac{2}{6} - \frac{1}{6} = \frac{4}{6}$

'A' and 'C' are not mutually exclusive events

c) $P(\text{not } D) = P(\text{getting any number other than } 3)$

$= P(1) + P(2) + P(4) + P(5) + P(6) = \frac{5}{6}$

$= 1 - P(D)$

$= 1 - \frac{1}{6} = \frac{5}{6}$ Complement Rule